ON RIEMANN HYPOTHESIS

LUCIAN M. IONESCU

Abstract. A line of study of the Riemann Hypothesis is proposed, based on a comparison with Weil zeros and a categorification of the duality between Riemann zeros and prime numbers.

Contents

1. Introduction

The Riemann Hypothesis (RH) in finite characteristic is proved part of the Weil Conjectures", now Theorems; see bibliography.

A natural idea is to understand the relation between Riemann zeros and Weil zeros, as well as to look for the objects behind them. Specifically, R-zeros are in duality with prime powers in the sense of distributions [7, 5], which are just sizes (integrals, hence "periods") of finite Abelian groups (Category Abf). It is expected that R-zeros, which are better understood as poles of $1/\zeta = DT(\mu)$ via the convolution identity involving the inverse of the Moebus function $\mu \star 1 = \delta$ and Dirichlet Transform (DT), may also have a similar interpretation as "counting something", i.e. periods.

To give meaning to less obvious numbers and relations, a powerful tool is Categorification (see Khovanov's work related to homology). By categorification one should look for a duality between the category Abf and a "natural category" which realize R-zeros as periods. If Tannaka-Krein duality associates to a Tannakian category a Hopf algebra, here we would look at group rings of finite Abelian groups as Hopf algebras. Their representation category is thus TK-dual to the original finite Abelian group.

In what follows we only try to expand the above and point to some related work.

2. Primes Numbers and Riemann Zeros

2.1. The Duality.

This duality is well documented in [7]; see also [5] for the Fourier transform interpretation of the duality, extended to the context of distributions. An important point is that all the prime powers are involved as dual to R-zeros.

The bicharacter that needs to be studied is $z^s : (C^*, \cdot) \times (C, +) \to C^*$.

2.2. Categorifying Prime Numbers.

Prime powers are sizes of primary abelian groups Z/p^k.

When considering the symmetries of $(Z/p, +)$ symmetries we get the finite fields F_p (see refs.) and derive the POSet structure of the prime numbers.

The higher powers of primes are categorified as truncations of p-adic groups. The relation to finite fields F_{p^k} is very rich: Deformation Theory (p-adic numbers) meets algebraic extensions theory (Galois Theory).

2.3. Lessons from Weil Conjectures.

Weil zeros are zeros of generating functions in the finite characteristic, as part of Weil Conjectures (see Modern Introduction to Number Theory). The associated RH is more amenable to interpretation, meaning that we expect the real part to be $1/2$ since we expect $\sqrt{p}$ to enter into the picture.

It is natural to look for a meaning for the R-zeros, in char zero. The two cases are of course related within the theory of adeles. One deforms Z to the "right" (reals[1]) or "left" (p-adic numbers[2]). How are these two cases related?

3. Now what!?

The relation between field extensions of F_p in characteristic p ($Aut(Z/p, +)$) and deformations of the same object as a tangent space in the sense of Deformation Theory, yielding the p-adic numbers form a 2-dim categorical diagram; the relation to Witt vectors and the corresponding convolution algebra is essential to be understood.

Other pieces of the puzzle are: the group ring of Z/p, cyclotomic units and Gauss periods; quadratic reciprocity exhibits $\sqrt{p}$ as involved fro possibly common reasons.

4. Conclusions

We apologise for the scattered ideas presented, but they were encountered while studying the subject by the author.

The Polya-Hilbert suggestion regarding the interpretation of R-zeros is too vague / generic to be of direct use; a more structured program was presented above, based on the duality between the prime numbers powers and R-zeros, to be categorified to a TK-duality. The comparison with Weil zeros and their role in Weil Conjectures is

[1] Decimal representation with carryover 2-cocycle going against the topological 10-adic grading.

[2] 2-cocycle carryover compatible with the topological grading.

essential; perhaps one should look there for a categorification that is general enough to be transferred from local-to-global cases. Since the overall impression is that this is a "quadratic case" (quadratic reciprocity and RH) one may hope the transfer is possible, the abstract setting controlling both cases. The adeles containing these both cases play the role of a multiplicative duality at the number level (the product 1 of components of an adele behaves like the augmentation morphism).

A cohomological interpretation is clear for Weil zeros of zeta function of algebraic varieties; what is the zero char homological interpretation (think of Galois extensions and groups as algebraic fundamental group).

The bibliography contains many more stepping stones that helped the author reach the above considerations, and are included as a "menu" for the reader's choice. Additional references can be found in author's "old" drafts on RZ and Weil Conjectures available at https://vixra.org/author/lucian_m_ionescu

Hopefully this draft will be expanded if time will allow it ...

$char/duality$	0 ($Global$)	p ($Local$)	Both: Adeles
$Categorification:$	Ab_f vs. H-Mod (TK)	l-adic cohomology (Poincare)	?
$Numbers$	$Primes\ vs.\ R-zeros$	$@p\ Weil\ zeros$	?

Note that "finite characteristic" is misleading: primary finite fields are just infinitesimal tangent spaces for p-adic numbers as deformations. Hence *"everything" happens in characteristic zero*: p-adic numbers as algebraic deformations Z_p have real numbers "Z_0" as a projective completion in some sense, captured by the adeles.

References

[1] K. Ireland, M. Rosen, A Classical Introduction to Modern Number Theory, GTM Series #84, Springer New York, 2010.

[2] O. Shanker, "Entropy of Riemann zeta zero sequence", AMO - Advanced Modeling and Optimization, Volume 15, Number 2, 2013, https://camo.ici.ro/journal/vol15/v15b18.pdf

[3] K. Ford and A. Zaharescu, On the distribution of imaginary parts of zeros of the Riemann zeta function, J. reine angew. Math. **579** (2005), 145-158. www.researchgate.net, 2005.

[4] K. Ford, K. Soundararajan, A. Zaharescu, On the distribution of imaginary parts of zeros of the Riemann zeta function, II, 0805.2745, 2009.

[5] L. M. Ionescu, "On prime numbers and Riemann zeros", 2017.

[6] L. M. Ionescu, "A statistics study of Riemann zeros", 2014, to appear.

[7] B. Mazur, W. Stein, *Primes: What is Riemann's Hypothesis?*, Cambridge University Press, 2016; http://modular.math.washington.edu/rh/rh.pdf; new title: "Prime Numbers and the Riemann Hypothesis"

[8] P. Garrett, "Riemann's explicit/exact formula", 2015, http://www-users.math.umn.edu/g̃arrett/m/mfms/notes_2015-16/03_Riemann_and_zeta.pdf

[9] V. Munoz and R. P. Marco, Unified treatment of explicit and trace formulas via Poisson-Newton formula, https://arxiv.org/abs/1309.1449

[10] A. Connes, "An essay on the Riemann Hypothesis", http://www.alainconnes.org/docs/rhfinal.pdf

[11] I. Volovich, Number Theory as the Ultimate Physics Theory.

[12] L.M. Ionescu, "Remarks on physics as number theory", *Proceedings of the 19th National Philosophy Alliance* Vol. 9, pp. 232-244, http://www.gsjournal.net/old/files/4606_Ionescu2.pdf.

[13] Wikipedia, Riemann-Roch duality.

[14] Wikipedia, Jacobian Variety.

[15] H. Rademacher, *Fourier analysis in number theory*, Collected Works, pp.434-458.

[16] M. Kontsevich, D. Zagier, Periods, IHES 2001, http://www.maths.ed.ac.uk/ãar/papers/kontzagi.pdf

[17] L. M. Ionescu, A natural partial order on the prime numbers, Notes on Number Theory and Discrete Mathematics, Volume 21, 2015, Number 1, Pages 1?9; arxiv.org/abs/1407.6659, 2014.

[18] V. R. Pratt, Every prime has a succinct certificate, SIAM J. Comput. Vol.4, No.3, Sept. 1975, 214-220.

[19] L. M. Ionescu, On Prime Numbers and Riemann zeros, https://vixra.org/abs/2204.0105

[20] Wen Wang, Notes on character sums, http://wstein.org/edu/2010/414/projects/wen_wang.pdf

[21] L. M. Ionescu, Topics in Number Theory MAT 410, Fall 2015, Fall 2016, Google presentation.

[22] B. Ossermann, Weil conjectures, https://www.math.ucdavis.edu/ osserman/math/pcm.pdf

[23] L. M. Ionescu, "A natural partial order on the prime numbers", *Notes on Number Theory and Discrete Mathematics*, Volume 21, 2015, Number 1, Pages 1?9; arxiv.org/abs/1407.6659, 2014.

[24] S. S. Kudla, "Tate's thesis", *An Introduction to the Langlands Program*, pp 109-131, Editors: Joseph Bernstein, Stephen Gelbart, Springer, 2004.

[25] K. Conrad, "The character group of Q", http://www.math.uconn.edu/k̃conrad/blurbs/gradnumthy/characterQ.pdf

[26] I. M. Gel'fand M. I. Graev, I. I. Pyateskii-Shapiro, *Representation theory and automorphic forms*, Academic Press, 1990.

[27] F. Gouvea, *p-adic Numbers: An Introduction*, Springer-Verlag, 1993.

[28] L. M. Ionescu, Recent presentations, http://my.ilstu.edu/ lmiones/presentations_drafts.htm

[29] R. Meyer, "A spectral interpretation of the zeros of the Riemann zeta function", math/0412277

[30] J. F. Burnol, "Spectral analysis of the local conductor operator", math/9809119.pdf, 1998.

[31] John Baez, "Quasicrystals and the Riemann Hypothesis", golem.ph.utexas.edu/category/2013/06/quasicrystals_and_the_riemann.html

[32] Freeman Dyson, "Frogs and Birds", AMS 56 (2009), 212-223.

[33] A. M. Odlyzko, "Primes, quantum chaos and computers", *Proc. Symp.*, May 1989, Washington DC, pp.35-46.

[34] Wikipedia: "Riemann Zeta Function".

[35] Y. Tian, Weil conjecture I.

[36] M. Mustaza, Lecture 1: The Hasse-Weil Zeta Functions.

[37] Colin Hayman, The Weil conjectures, Master Thesis 2008.

[38] Eyal Z. Goren, Gauss and Jacobi sums, Weil conjectures, http://www.math.mcgill.ca/goren/SeminarOnCohomology/mycohomologytalk.pdf

[39] F. Thaine, On Gaussian periods that are rational integers.

[40] Wikipedia: 1) Gaussian period; 2) Gauss sum; 3) Jacobian variety.

[41] Paramand Singh, Gauss and regular polygons: Gaussian periods.

[42] W. Duke, S.R. Garcia, B. Lutz, The graphic nature of Gaussian periods.

Department of Mathematics, Illinois State University, IL 61790-4520
Email address: lmiones@ilstu.edu