General relativity under the condition of extremely weak gravitational field

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Abstract

It is generally believed that under the condition of relatively strong gravitational field, the effect of general relativity will be more significant. Under the condition that the gravitational field is relatively weak, we can directly use Newton's theory of gravity. However, if we think that dark matter is a kind of fluid, then general relativity can also be used to explain the more special space-time structure formed by the dark matter fluid. According to the characteristics of the fluid, when the fluid flows, two different flow phenomena, laminar flow and turbulent flow, can be formed. In this study, the Zou metric is analyzed, and it is believed that the space-time formed by dark matter turbulence can be well described by the Zou metric. On this basis, this study calculates the pulsation frequency and amplitude of dark matter turbulence in space-time, and predicts the possible impact of dark matter turbulence on visible matter. For the space-time formed by the laminar flow of dark matter, this paper attempts to establish a new two-dimensional metric. This two-dimensional metric has a structure similar to that of a uniform gravitational field. But the two-dimensional metric in this study are obtained entirely from the properties of the space-time structure. In this metric, therefore, the resulting acceleration is due to the structure of the space-time itself. When the speed of visible matter is slower than that of the dark matter fluid, the visible matter will be pulled by the dark matter fluid and continue to accelerate. Conversely, if the velocity of visible matter is lower than that of dark matter fluid, the velocity of visible matter will continue to slow down due to the dissipation of energy. In addition, this dark matter laminar space-time metric also has a very interesting phenomenon, that is, if the velocity of the visible matter is equal to the velocity of the dark matter fluid, the metric will degenerate into the flat metric of Minkowski space-time.

1 Introduction

It is generally believed that general relativity should be able to show its observable effects under a strong gravitational field. Under the condition of weak gravitational field, general relativity will be approximated to Newton's theory of gravity. However, some studies have shown that the nonlinear effects of general relativity exist even under the conditions of extremely weak gravitational fields [1,2]. For example, by introducing a new nonlinear metric [1], or introducing the self-interaction of the gravitational field, etc. [2].

This study believes that general relativity under the condition of this extremely weak gravitational field may be related to dark matter. If we adopt the dark matter fluid model, due to the different flow states of the dark matter fluid: laminar flow or turbulent flow, etc., it will lead to nonlinear changes in the space-time where the visible matter is located. This nonlinear change can be handled using general relativity. At present, Zou solves the relationship between the galaxy velocity curve and the galaxy matter density distribution by establishing a nonlinear metric in the weak gravitational field.

For dark matter fluid, its flow mode can be divided into two cases: laminar flow and turbulent flow. For spacetime in a laminar dark matter fluid state, a flat spacetime metric can be used to describe it. However, since the matter is in the dark matter fluid, the visible matter still has a gravitational interaction with the dark matter fluid. This gravitational interaction causes energy dissipation in the dark matter fluid, which in turn transfers some of that energy to visible matter. This can cause acceleration and deceleration of visible matter. Therefore, in this laminar dark matter fluid space-time, a special energy dissipation term needs to be introduced.

For the turbulent state of dark matter fluids. Its shape will be more complicated. The relatively simple random turbulence appears as a very random turbulent flow state of the fluid. This random turbulence may cause random curvature of spacetime.

Another relatively ordered dark matter turbulent state is the vortex state of dark matter fluid. This eddy state is also a type of turbulent flow. But in form, it shows a certain orderly rotating flow state. By establishing a nonlinear metric [1], Zou can be used to describe the space-time formed by this dark matter fluid in a vortex state.

We can also get an inspiration from the Zou metric, that is, when we calculate the influence of dark matter on space-time, we can not only simply do the approximate work of Newtonian gravity, but also directly consider the influence of dark matter on the space-time metric. For example, the existence of dark matter is regarded as an interference term of the space-time metric for computational analysis [3,4]. It can also be regarded as an important component of time and space.

2 Zou Metrics

Here is a more typical example of nonlinear general relativity, which was proposed by Zou in 2022. In his dissertation, he proposed a rotation metric that could explain the relationship between galaxy velocity curves and galaxy mass density distributions.

The form of the Zou metric is this

$$g_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & -\beta(r) & 0 \\ 0 & 1 & 0 & 0 \\ -\beta(r) & 0 & r^2 - \dfrac{q}{b}\beta(r) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

If a particle moves in the Zou metric, its velocity

$$v = \frac{rd\phi}{dt} = \frac{r\beta' \pm \sqrt{(r\beta')^2 + (r+\gamma')c^2r^2\alpha'}}{(r+\gamma')}$$

It can be seen from Zou's analysis that $\alpha(r) = 0$; therefore $\alpha' = 0$;

If $r \gg \gamma'$, if the velocity v is not 0, then:

$$v \approx 2\beta'$$

Considering

$$\beta' = b\left(\frac{1}{1+e^{-\frac{r}{s}}} - \frac{1}{1+e^{\frac{w}{s}-\frac{r}{s}}}\right)$$

Therefore, if $r \gg s; w \gg s; r \gg w$, then

$$\beta' \approx b\left(1 - e^{-\frac{r}{s}} - 1 + e^{\frac{w}{s}-\frac{r}{s}}\right) = be^{-\frac{r}{s}}\left(-1 + e^{\frac{w}{s}}\right) \approx be^{\frac{w}{s}-\frac{r}{s}}$$

and

$$\beta(r) = -bsln\left(\frac{e^{-\frac{r}{s}}+1}{e^{\frac{w}{s}-\frac{r}{s}}+1}\right) = bsln\left(\frac{e^{\frac{r}{s}}+1}{e^{\frac{w}{s}}+e^{\frac{r}{s}}}\right)$$

$$\beta(r) \approx bsln\left(\frac{1}{e^{\frac{w}{s}-\frac{r}{s}}+1}\right) = -bsln\left(e^{\frac{w}{s}-\frac{r}{s}}+1\right)$$

$$\beta(r) \approx -bse^{\frac{w}{s}-\frac{r}{s}} - \frac{b^2s^2}{2}e^{2\left(\frac{w}{s}-\frac{r}{s}\right)} \approx -bse^{\frac{w}{s}-\frac{r}{s}}$$

So

$$v \approx 2be^{\frac{w}{s}-\frac{r}{s}}$$

This is to reflect the variation of the velocity in the tangential ϕ direction with the radius. It can be seen that this is a velocity that decreases exponentially.

The inspiration given to us by the Zou metric is that in such a nonlinear general relativity metric, even if gravity is very weak, particles will still produce corresponding tangential motions in this space-time. This is incomprehensible in the linear approximation of general relativity.

If we regard the region of $r < w$ as a vortex region of dark matter fluid, the above-mentioned change of velocity can be regarded as a region away from the vortex. In this region away from the vortex, the tangential velocity begins to decrease. But it still exists.

On the other hand, if we consider the characteristics of turbulence. There may also be turbulent flow of dark matter fluids far from the eddies. At this time, it will also cause a certain degree of curvature of space-time.

3 Randomly curved space-time

If a dark matter fluid composed of space-time is a turbulent flow with strong randomness. Considering the randomness of turbulence, we can set the constant b as a random function *b(t)*. Since it is a random function, it means that the function has a changing amplitude *A*, a changing frequency *f*, and so on.

It is difficult to predict the frequency of this random function. However, the experimentally measured turbulent frequencies are generally between 10^2 and $10^5 Hz$. The amplitude is less than 10% of the average speed [5]

According to the turbulent dissipation rate equation proposed by Jones, W. P and Launder, B. K. in 1972, the eigenfrequency formula of turbulent flow can be obtained [6]

$$f_0 = \frac{1}{c_\mu}\frac{\varepsilon}{k}$$

where k is the kinetic energy of the turbulent flow and ε is the turbulent dissipation of the turbulent energy. And c_μ is a constant.

Turbulent dissipation is proportional to the viscosity coefficient. Kinetic energy is proportional to the square of velocity.

From my estimates in my last paper [7], the viscosity coefficient of dark matter fluids may be 4200 times that of water. The speed is calculated according to the speed of 200km/s of the outer galaxies of the Milky Way, which is 200,000 times the flow velocity of ordinary water at 1m/s. So we can estimate that the frequency of dark matter turbulence is about

$$f_{dark} = \frac{4200}{200000^2} f_{water} = 2.1 \times 10^{-7} f_{water}$$

According to the general conditions, the turbulent frequency of water is generally 100 Hz, and the pulsation frequency of dark matter turbulence can be roughly obtained as:

$$f_{dark} = 2.1 \times 10^{-5} Hz$$

That is to say, the dark matter fluid pulsates once every 47619 seconds. This is equivalent to an average of 0.55 days, or a pulse of 13.23 hours. If it enters this space-time, it means that the entire space-time may have random changes that pulsate every 13 hours or so, resulting in the problem that visible matter may need to be adjusted in direction.

4 The scale of dark matter turbulence

If large-scale galaxies such as the Milky Way are involved, the scale of the resulting turbulence can be very large. And if it is local dark matter turbulence like the solar system, the scale of the turbulence will be much smaller.

For example, if we fly out of the solar system according to the distance from Kuiper to Voyager 2 in 2018 (the detected solar radiation ions have dropped significantly), the distance of the entire area is about:

$$\Delta r = 1.8 \times 10^{10} - 48 \times 1.5 \times 10^{8} \approx 1.08 \times 10^{10} (km)$$

If dark matter turbulence is also equivalent to such a range of visible matter turbulence. It means that there is also a dark matter turbulent region in the range of about $1.08 \times 10^{10} (km)$ outside the solar system. If it runs at a speed of 20km per second, it takes about $5.4 \times 10^{8} s$ to cross the past. It will take about 17 years. However, considering that the further away from the solar system, the lower the intensity of turbulence, the phenomenon of random bending of spacetime will gradually weaken.

5 Dark matter laminar space-time

If dark matter is in a laminar state, then this time can be represented by a flat space-time metric. But if there is an interaction between the visible matter and the dark matter fluid, it will cause the energy dissipation of the dark matter fluid, resulting in a change in the velocity of the visible matter.

So we can also slightly change the flat space-time metric. Assuming that the dark matter fluid has only one direction of the x-axis, if the range of the dark matter fluid is large enough, and the laminar flow has only one flow direction, we can use a two-dimensional metric to express:

$$g_{\mu\nu} = \begin{pmatrix} \alpha(x) & 0 \\ 0 & -\beta(x) \end{pmatrix}$$

The formula of Christoffel symbols

$$\Gamma^{\mu}_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(\frac{\partial g_{\alpha\nu}}{\partial x^{\beta}} + \frac{\partial g_{\nu\beta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{\nu}} \right)$$

We can get

$$\Gamma^{0}_{\alpha\beta} = \frac{1}{2} g^{00} \left(\frac{\partial g_{\alpha 0}}{\partial x^{\beta}} + \frac{\partial g_{0\beta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{0}} \right)$$

The components that are not 0 are

$$\Gamma_{01}^{0} = \frac{1}{2} g^{00} \left(\frac{\partial g_{00}}{\partial x} \right) = \frac{\alpha'(x)}{2\alpha(x)}$$

$$\Gamma_{10}^{0} = \Gamma_{01}^{0} = \frac{\alpha'(x)}{2\alpha(x)}$$

While

$$\Gamma_{\alpha\beta}^{1} = \frac{1}{2} g^{11} \left(\frac{\partial g_{\alpha 1}}{\partial x^{\beta}} + \frac{\partial g_{1\beta}}{\partial x^{\alpha}} - \frac{\partial g_{\alpha\beta}}{\partial x^{1}} \right)$$

The non-zero components are:

$$\Gamma_{00}^{1} = -\frac{1}{2} g^{11} \left(\frac{\partial g_{00}}{\partial x} \right) = \frac{\alpha'(x)}{2\beta(x)}$$

$$\Gamma_{11}^{1} = \frac{1}{2} g^{11} \left(2 \frac{\partial g_{11}}{\partial x} - \frac{\partial g_{11}}{\partial x} \right) = \frac{\beta'(x)}{2\beta(x)}$$

if

$$\alpha(x) = e^{\nu}$$

$$\beta(x) = e^{\mu}$$

then

$$\Gamma_{10}^{0} = \Gamma_{01}^{0} = \frac{\nu'}{2}$$

$$\Gamma_{00}^{1} = \frac{\nu'}{2} e^{\nu-\mu}$$

$$\Gamma_{11}^{1} = \frac{\mu'}{2}$$

In this way, we can solve the corresponding Ricci curvature tensor

$$R_{\mu\nu} = \frac{\partial \Gamma_{\mu\lambda}^{\lambda}}{\partial x^{\nu}} - \frac{\partial \Gamma_{\mu\nu}^{\lambda}}{\partial x^{\lambda}} + \Gamma_{\gamma\nu}^{\lambda} \Gamma_{\mu\lambda}^{\gamma} - \Gamma_{\gamma\lambda}^{\lambda} \Gamma_{\mu\nu}^{\gamma}$$

Notice $R_{\alpha\beta} = 0$, where $\alpha \neq \beta$

The non-zero components include

$$R_{00}=\frac{\partial\Gamma_{0\lambda}^{\lambda}}{\partial x^0}-\frac{\partial\Gamma_{00}^{\lambda}}{\partial x^\lambda}+\Gamma_{\gamma 0}^{\lambda}\Gamma_{0\lambda}^{\gamma}-\Gamma_{\gamma\lambda}^{\lambda}\Gamma_{00}^{\gamma}=-\frac{\partial\Gamma_{00}^{1}}{\partial x}+\Gamma_{10}^{0}\Gamma_{00}^{1}+\Gamma_{00}^{1}\Gamma_{01}^{0}-\Gamma_{10}^{0}\Gamma_{00}^{1}-\Gamma_{11}^{1}\Gamma_{00}^{1}$$
$$=-\frac{\partial\Gamma_{00}^{1}}{\partial x}+\Gamma_{10}^{0}\Gamma_{00}^{1}-\Gamma_{11}^{1}\Gamma_{00}^{1}$$
$$=-\frac{e^{\nu-\mu}}{2}\nu''-\frac{e^{\nu-\mu}}{2}(\nu'-\mu')\nu'+\left(\frac{\nu'}{2}-\frac{\mu'}{2}\right)\frac{e^{\nu-\mu}}{2}\nu'$$
$$=-e^{\nu-\mu}\left(\frac{\nu''}{2\nu'}+\frac{\nu'}{4}-\frac{\mu'}{4}\right)\nu'$$

Therefore

$$R_{00}=e^{\nu-\mu}\left[-\frac{\nu''}{2}+\frac{\nu'}{4}(\mu'-\nu')\right]$$

$$R_{11}=\frac{\partial\Gamma_{1\lambda}^{\lambda}}{\partial x^1}-\frac{\partial\Gamma_{11}^{\lambda}}{\partial x^\lambda}+\Gamma_{\gamma 1}^{\lambda}\Gamma_{1\lambda}^{\gamma}-\Gamma_{\gamma\lambda}^{\lambda}\Gamma_{11}^{\gamma}=\frac{\partial\Gamma_{10}^{0}}{\partial x}+\frac{\partial\Gamma_{11}^{1}}{\partial x}-\frac{\partial\Gamma_{11}^{1}}{\partial x}+\Gamma_{01}^{0}\Gamma_{10}^{0}+\Gamma_{11}^{1}\Gamma_{11}^{1}-\Gamma_{10}^{0}\Gamma_{11}^{1}-\Gamma_{11}^{1}\Gamma_{11}^{1}$$
$$=\frac{\partial\Gamma_{10}^{0}}{\partial x}+\Gamma_{01}^{0}\Gamma_{10}^{0}-\Gamma_{10}^{0}\Gamma_{11}^{1}=\frac{\nu''}{2}+\frac{\nu'^2}{4}-\frac{\mu'\nu'}{4}$$

So

$$R_{11}=\frac{\nu''}{2}-\frac{\nu'}{4}(\mu'-\nu') \qquad (4-14)$$

By the Einstein's equation

$$R_{\mu\nu}=0$$

We can get

$$\frac{\nu''}{2}-\frac{\nu'}{4}(\mu'-\nu')=0$$

According to the Schwarzschild metric, we can know that when *r* is very, very large, there is a reciprocal relationship between *α(x)* and *β(x)*, that is to say

$$\beta(x)=\frac{1}{\alpha(x)}$$

And when *r* is very large, it is equivalent to a very weak gravitational force.

Because

$$\nu=ln\alpha(x)$$

Therefore

$$\mu = ln\beta(x) = -ln\alpha(x)$$

or

$$\mu = -\nu$$

then

$$\alpha(x) = ax + b$$

$$\beta(x) = \frac{1}{\alpha(x)} = \frac{1}{ax + b}$$

So the metric can be expressed as

$$g_{\mu\nu} = \begin{pmatrix} ax + b & 0 \\ 0 & -\frac{1}{ax + b} \end{pmatrix}$$

This way we can find the distance between two points:

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} = (ax + b)dt^2c^2 - \frac{1}{ax + b}dx^2$$

where a reflects an acceleration factor. With this factor, once a particle enters this space-time, it means that it will continue to accelerate or decelerate. This is similar to the space-time metric in a uniform gravitational field [8~11].

Also if we consider that in the complete absence of this acceleration factor, the entire metric should be approximated by the Minkowski metric. which is

$$s^2 = dt^2c^2 - dx^2$$

So we can get $b = 1$

Then the whole metric will become

$$g_{\mu\nu} = \begin{pmatrix} ax + 1 & 0 \\ 0 & -\frac{1}{ax + 1} \end{pmatrix}$$

$$ds^2 = (ax + 1)dt^2c^2 - \frac{1}{ax + 1}dx^2$$

This is basically consistent with the formula (47) of the literature [8]. Where a reflects the acceleration of a uniform gravitational field.

Of course, in the object of this study, the existence of the acceleration factor a is caused by the gravitational interaction between the visible matter and the dark matter flow. When the velocity of the visible matter is less than the velocity of the dark matter flow, an accelerating viscous force is generated, which causes the visible matter to continue to accelerate. And if the velocity of visible matter is greater than the velocity of the dark matter fluid, a viscous force that hinders the movement will be generated, thus causing the visible matter to continue to slow down.

When the velocity of the visible matter is the same as the velocity of the dark matter fluid, the viscous force of this acceleration or deceleration disappears. At this time, the acceleration factor $a = 0$ in the metric, and the metric will become the Minkowski metric of flat space-time.

6 Conclusions

It is generally believed that under the condition of weak gravitational field, although general relativity can still be used, it is slightly more complicated to deal with problems. Therefore, when conducting research and analysis, Newton's theory of gravity is more widely used.

However, this study shows that the theory of general relativity still needs to be used even under conditions where the gravitational field is very weak. For example, under the conditions of calculating galaxy velocity curves, gravitational lensing effects, etc., if Newton's theory of gravity is used, there will be difficult self-consistent errors. The current solution is to introduce dark matter to generate excess gravity. But if we directly use the general theory of relativity and establish an appropriate metric that adapts to this scale, we can solve the related problems without considering the extra gravitational force generated by dark matter.

There is a contradiction here. Since general relativity is indeed under the condition of weak gravitational field, it can be approximated as Newton's theory of gravity, why under certain conditions, such as at the scale of galaxies, some results cannot be obtained using Newton's theory of gravity?

The fundamental reason for this lies in what kind of dark matter the visible matter of the universe is built on. If dark matter is also regarded as static, dark matter can generate corresponding gravitational force like other visible matter, thereby affecting the gravitational interaction of visible matter. This still seems to be able to solve the related problems in the framework of Newtonian gravity.

And if we regard dark matter as a kind of fluid, the state of this fluid is different, which can lead to changes in the space-time structure of visible matter, then we can create a suitable metric, and we can use the theory of general relativity to solve the current Newtonian gravity theoretically intractable problems. From the calculation results of some literature [1], the results are quite satisfactory.

Using such a new space-time metric, this study analyzes the frequency and magnitude of turbulent pulsations in dark matter fluids. From the current application situation in various fields, the research in this direction is obviously still a blank. In the absence of any relevant theory, I think such research is still very meaningful. It is believed that this can provide a useful reference for the conditions under which human spacecraft have been able to enter the galaxy space.

This study further explores the space-time properties at locations far from the visible matter of the galaxy. Since these regions far from the visible matter of the galaxy are basically filled with dark matter fluid. These dark matter fluids also exhibit the characteristics of laminar flow under undisturbed conditions. Therefore, this study believes that this can be analyzed using a metric similar to a uniform gravitational field. In general relativity, the main study is the spherical symmetric gravitational field. However, there are relatively few studies on uniform gravitational fields. However, there are still some papers on this subject. This study refers to the work of some authors to construct a space-time metric suitable for laminar flow of dark matter. It can be seen from this space-time metric that there is an acceleration factor. This factor is related to the coefficient of viscosity (that is, the gravitational constant) between the dark matter fluid and visible matter. If the visible matter is moving slower than the flow rate of the dark matter fluid, the visible matter will be accelerated. Visible matter, which moves faster than the dark matter fluid, may be slowed down. When the velocity of visible matter is the same as that of dark matter fluid, the laminar space-time metric of this dark matter fluid will degenerate into a flat Minkowski space-time metric.

From the results of this study, we can now do a reflection on the theory of general relativity. Perhaps general relativity should not be seen as a theory of gravity, but rather as a theory of space-time. This is in line with the special theory of relativity.

References

[1] Zou, S. J. A Galactic Spacetime Model to Resolve the Problem Between Mass Density and Rotation Curve. https://vixra.org/abs/2205.0123

[2] Alexandre Deur. Relativistic corrections to the rotation curves of disk galaxies. https://arxiv.org/abs/2004.05905

[3] Stuckey, W.M.; McDevitt, T.; Sten, A.K.; Silberstein, M. Dark Matter as a Metric Perturbation. Preprints 2016, 2016110080 (doi: 10.20944/preprints201611.0080.v1).

[4] Capozziello, S., De Laurentis, M., Francaviglia, M. et al. From Dark Energy & Dark Matter to Dark Metric. Found Phys 39, 1161–1176 (2009). https://doi.org/10.1007/s10701-009-9332-7

[5] 马晴晴．湍流概论．https://blog.sciencenet.cn/blog-2370254-864156.html (Chinese online, can be translated into English by the online translation tools.)

[6] 徐江荣, 姚强, 曹欣玉, & 岑可法. (1998). 湍流脉动的特征频率与频率谱分析．中国工程热物理学会多相流学术会议．中国工程热物理学会(Chinese).

[7] Cheng, Z. A Fluid Model of Dark Matter. https://vixra.org/abs/2205.0018

[8] Alberici, M. (2005). Measures and metrics in uniform gravitational fields. arXiv preprint gr-qc/0503092.

[9] Muñoz, G., & Jones, P. (2010). The equivalence principle, uniformly accelerated reference frames, and the uniform gravitational field. American Journal of Physics, 78(4), 377-383.

[10] Desloge, E. A. (1989). Spatial geometry in a uniformly accelerating reference frame. American Journal of Physics, 57(7), 598-602.

[11] W. Rindler, Relativity: Special, General, and Cosmological, 2nd. ed. (Oxford University Press, 2001).