Where it is shown that the Oscillation Symmetry is also verified in the physical properties of the Periodic table of the Atomic Elements

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Abstract

The oscillation symmetry is applied with success to some physical atomic properties of many Periodic Elements. It allows to tentatively predict possible values for several unknown properties.

A regularity is observed between oscillating periods. These values, according to different studied bodies, take discrete values as if they were quantified.

1. Introduction

The validity of the oscillation symmetry applied to masses of fundamental particles and nuclei is justified since they result from Schrödinger equation containing two opposite forces: kinetic and potential [1]. As in classical physics the existence of opposite interactions involve oscillations.

Although there is no theory linking between themselves the masses, or widths of fundamantal particles, or of nuclei, or of astrophysical bodies, it was shown that they too are submitted to such symmetry [1]. The common property between particles [2], nuclei [3], and astrophysical bodies [4] is the existence of acting opposite forces. In the astrophysical field they are gravitational forces and centrifugal forces related to their kinetic energies. Up to now, there is no known theoretical justification for such well observed and widely applied oscillation symmetry.

The data of a given system are first classified in increasing order (increasing Atomic number here). The possible oscillations are studied

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using the following relation:

$$m_{(n+1)} - m_n = f[(m_{(n+1)} + m_n)/2] \tag{1}$$

where n indicates the increasing data order. The differences between two successive data are plotted versus their corresponding mean values. Such study is named "property data" where here property may be either mass, density, Melting point, Boiling point, or else.

A simple normalised cosine function is used for the fits of the data. M is the variable $(\mathrm{m}_{(n+1)} + m_n)/2$ and Δ M is the function for "mass data" studies.

When all studied data are positive, their oscillations are fitted using the following formula:

$$\Delta M = \alpha(1 + cos(M/M_1))exp(\beta.M) \tag{2}$$

where $\mathrm{M/M_1}$ is defined within 2π. The oscillation period $\mathrm{P} = 2\ \pi M_1$.

α, β, and M_1 are the three fitted parameters. There values are given in Table A1.

The common property of these bodies is that they are compound of smaller bodies. Several physical properties of many Atomic Elements are studied by Groups of the Periodic Table of the Elements. Indeed their properties are defined by the number of electrons of their external shell. This number is obtained by Group number minus 10. An exception concern Helium since it has only two electrons and belongs to Group 18.

The Oscillation Symmetry method cannot be applied to Elements from Group 3 up to Group 12, since they include only four elements preventing to define a clear oscillation. The Celsius values of the Melting points and the Boiling points vary sometimes from negative to positive ^{0}C temperatures, justifying the use of K units. In several groups, when the Melting points and Boiling points for the concerned elements are either negative, either positive the oscillations are fitted using the following formula:

$$\Delta M = (\alpha_0 + \alpha_1 cos(M/M_1))exp(\beta.M) \tag{3}$$

The concerned four parameters are reported in Table A2.

Density stp means (Standard Temperature and Pressure) density. In some cases, the values found in tables are not the same. The values used are all read using the references [5], where 'Element' is the name of the Atomic Element which physical properties are concerned.

2. Group 18 (Noble Gases)

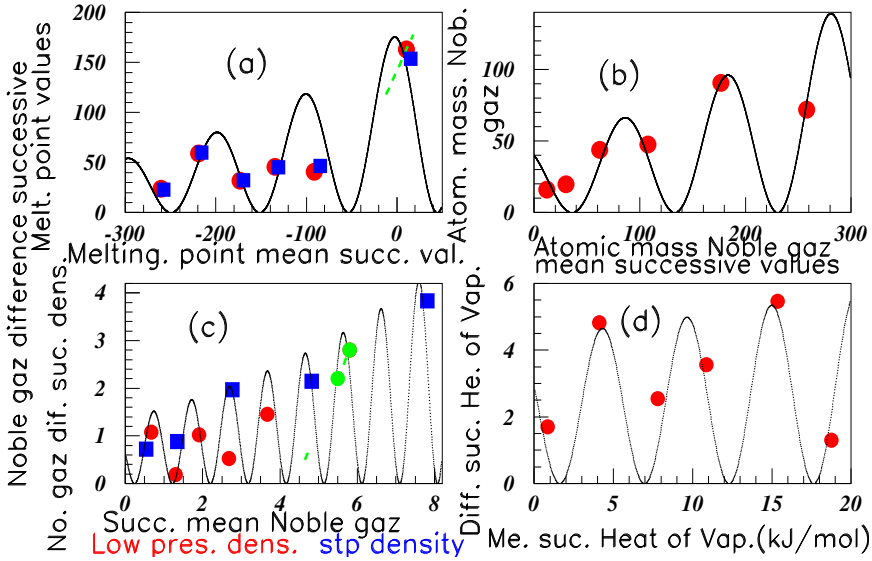

Figure 1: Color on line. Several Noble gas physical properties allowing to tentatively predict unknown values. The unit in insert (a) is ^{0}C. The unit in insert (c) is g/L. (See text).

Several properties of Noble gases (or Inert gases) are known [6]. In addition to helium, neon, argon, krypton, xenon, and radon, a new element: oganesson [7] (Og) was recently found with a small number of known properties: charge number Z=118, atomic number A=294, and a few others. Some possible properties are discussed using fig. 1.

The Melting points of Noble gases (full red circles in fig.1(a)) are nearly the same as the Noble gaz Boiling points (full blue squares in fig.1(a). Its value for (Og) is predicted to be situated between 320 and 380 K. In order to tentatively explicit this value, the "melting point" data are plotted in fig.1(a) with successive introduction of these both limits. These limits allows to get the green dashed line. The other "Melting point data" are fitted by the curve obtained with period

P=98 ^{0}C (98 K). The intersection between dashed line and curve is obtained for X=15.15 and Y=153.7 obtained when Melt.(Og)=92 ^{0}C (365 K).

Fig.1(b) shows the "Atomic number data" of Noble Gases. The corresponding possible value A=294 for (Og) is very well fitted (last point) by the curve with the Period P=97.4 defined by the other values.

Fig.1(c) shows "stp density data" (blue full squares) in g/L of the Noble gases and "low pression density data" (red full circles) fitted with the period $P_{dens.}$=0.98 g/L. Two predictions for the (Og) liquid low pression density exist in tables. The first one predicts this density to be located between 4.9 and 5.1 g/cm^3. This range will involve the last data in "low pression density data" (fig.1(c)) shown by green point at X=4.7 and Y=0.6 to be totally outside the curve obtained with other data. The second range predicts the (Og) density to be located in the range 6.6-7.4. These limits lead to the dashed green line in the range X=5.5-5.9 and Y=2.2-3 in fig.1(c). Therefore two possible solutions for the (Og) density are either d(Og) 6.6 or 7.1 g/cm^3, corresponding to the intersection of the curve with the green line and leading to the two green marks in fig. 1(c).

Fig.1(d) shows the Noble gases "Heat of Vaporisation data". The suggested value of Heat of Vaporisation for (Og) 19.4 kJ/mol leads to the last point in fig 1(d), in agreement with the curve obtained using the previous data.

3. Group 1 (Alkali Metals)

The Alkali metal elements belonging to the first column in the Periodic table of Elements are H, Li, Na, K, Rb, Cs, and Fr. They are studied in all inserts of fig. 2. The H properties are very different from the other 6 and are omitted from this study. Fig.2(a) shows the (-)"Boiling point data" therefore the difference of successive Boiling points multiplied by -1, versus the corresponding mean Boiling points in ^{0}C values. The sign of the values have been inverted in order to get usual figs. They are fitted with the period: P=279 ^{0}C.

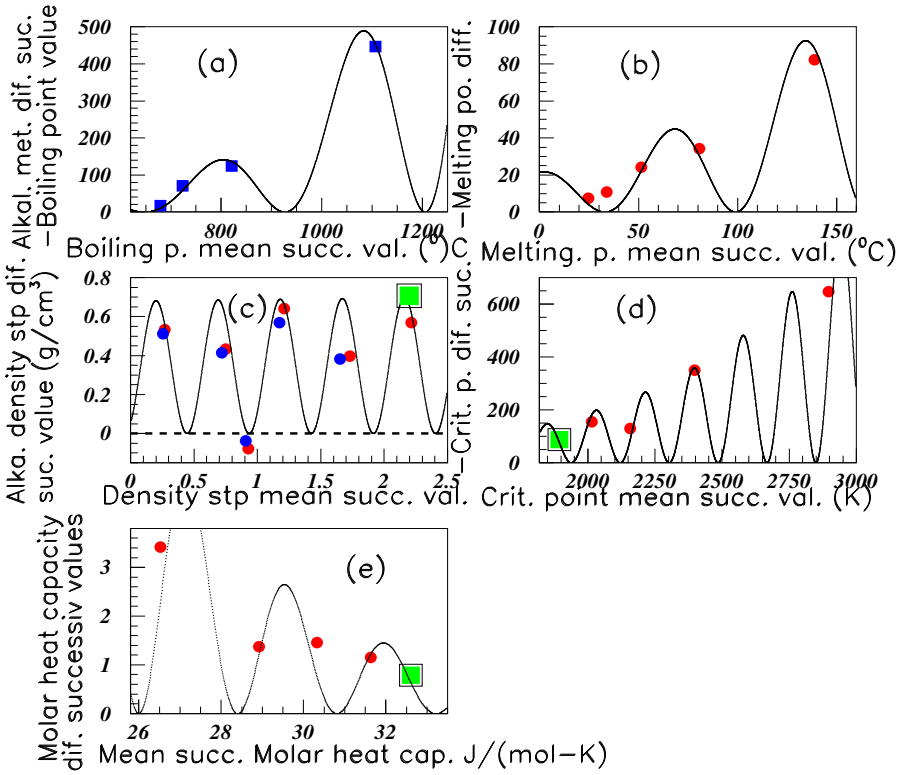

Figure 2: Color on line. (See text). The five inserts show respectively data and fits for: "Boiling point data" in insert (a), "Melting point data" in insert (b), the "stp density data" in insert (c), the "Critical point data" in insert (d), and the "Molar heat capacity" in insert (e).

Fig.2(b) shows the (-)"Melting point data" fitted with the period: P=66 ^{0}C.

Fig.2(c) shows the Alkali standard temperature and pression densities: "density stp data" (red circles) and "density liquid m.p. data" (blue) in g/cm^3. The corresponding oscillation period, determined using the known values up to (Cs), is P=0.49 g/cm^3. The Fr (stp) density value is predicted to be 2.48 g/cm^3. The intersection of the "density data" curve with the possible value close to 2.48 g/cm^3 allows to predict more precisely the density liquid value for Fr to be approximately 2.55 g/cm^3 (full green square mark).

Fig.2(d) shows the (-)"critical point data" fitted with the period P=182.2 K. The predicted possible value for the Fr atom is then: $\approx$2.48 g/cm^3 leading to the green mark in fig. 2(d) at X=1894 and Y=88.

Fig.2(e) shows the Molar heat capacities fitted by the period: P=2.41 J/(mol-K) allowing to predict the possible value for the Fr atom: ≈33 J/(mol-K). The corresponding green mark is at X=32.6 and Y=0.79.

The Heat of fusion data and the Heat of vaporition data are not analysed inside our oscillation method, since they are nearly aligned, involving indetermined oscillations.

4. Group 17 (halogens)

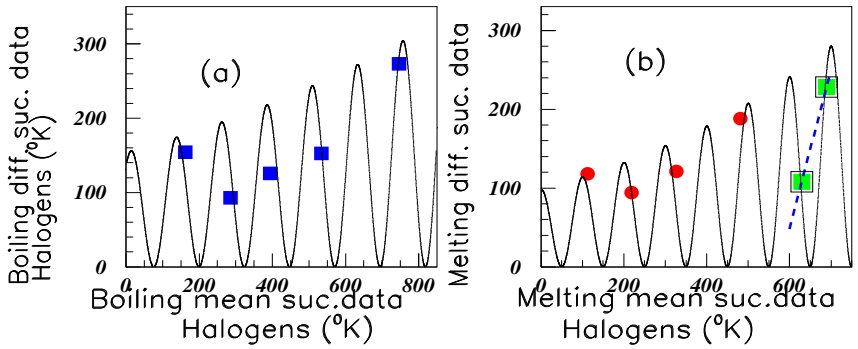

Figure 3: Color on line. (See text). Inserts (a) and (b) show the data and fits for Boiling points and Melting points of Group 17 (Halogens) Elements.

The Elements belonging to group 17 (Halogens) are: Fluorium (F), Chlorine (Cl), Bromine (Br), Iodine (I), Astatine (At), and Tennessine (Ts). Except the Melting points and Boiling points, the other properties are not all known, preventing their study. The (Ts) properties are only predicted or extrapolated.

Fig.3(a) shows the "Boiling point data". They are fitted with P=123.8 K (123.8 ^{0}C).

Fig.3(b) shows the "Melting point data" fitted with P=99.9 K (99.9 ^{0}C). The value of the (Ts) Melting point is predicted to be located inside the range 350-550 ^{0}C. This large uncertainty is reduced using the oscillation symmetry. The dashed blue line corresponds to this range. Its intersection with the curve describing the "melting point data" of the other elements allows to predict the (Ts) Melting point to be close to 410 or 530 ^{0}C.

5. Group 16 (chalcogens)

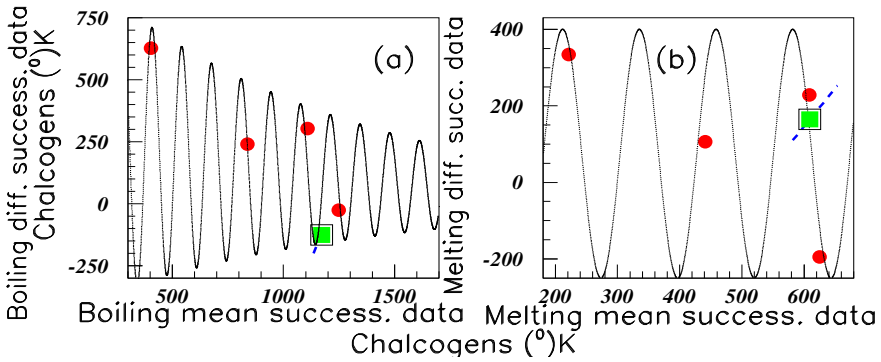

Figure 4: Color on line. (See text). "Boiling data" in insert(a) and "Melting data" in insert (b) in Kelvin degrees for Chalcogen elements.

The group 16 (chalcogen) Elements are: Oxygen (O), Sulfur (S), Selenium (Sl), Tellurium (Tl), Polonium (Po), and Livermorium (Lv). The (Lv) properties are only predicted or extrapolated.

Fig.4(a) shows the "Boiling point data" of the Elements of the chalcogen group, fitted on the first data with the period P=133.8 K (133.8 ^{0}C). The intersection of this fit with the small dashed blue line corresponding to range (762-862 ^{0}C), attributed to the possible extrapolated Livermorium Boiling point value, allows to predict the corresponding more precise possible value to be close to 835 ^{0}C (see the full green square).

Fig.4(b) shows the "Melting point data" of the group 16 Elements, fitted on the first known five data with P=123.3 K (123.3 ^{0}C). The intersection of this fit with the dashed blue line corresponding to range (364-507 ^{0}C) attributed to the Livermorium Melting point value allows to predict a corresponding more precise possible value to be close to 419 ^{0}C.

6. Group 13 (Boron) physical properties

The group 13 Elements are: Boron (B), Aluminium (Al), Gallium (Ga), Indium (In), Thallium (Tl), and Nihonium (Nh). The (Nh) physical properties are only predicted, except the Molar Heat capacity

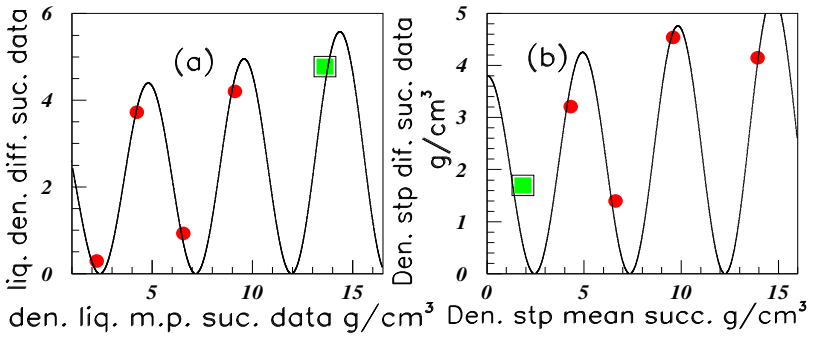

Figure 5: Color on line. (See text). "Density data" of the group 13 (Boron) Elements. Insert (a) shows the liquid m. p. data, insert (b) shows the stp data.

(mhc) and the liquid density (m.p.) which are not given. The known values of the five (mhc) elements prevent to deduce the possible (mhc) value for (Nh). The (B) density (stp) also is not given.

Fig.5(a) shows the data and fit of the "liquid density data" of the other five elements belonging to this group, fitted with P=4.78 g/cm^3. The extrapolation of density versus charge indicates that the (Nh) liquid density should be close to 15. The green square corresponds to the (Nh) density equals to 16.

Fig.5(b) shows the data for the five known "density stp data" fitted with P=4.9 g/cm^3. The extrapolated value for (B) density stp should be close to 1. The green square correspond to this value of (B) density (stp).

7. Group 15 (pnictogens)

The used Elements of Group 15 are: Nitrogen (N), white Phosphorus (P), Arsenic (As), Antimony (Sb), Bismuth (Bi), and Moscovium (Mc). They are studied in fig. 6. The red Phosphorus is not considered.

Fig.6(a) shows the pnictogen "density stp data" (Group 15 Elements) in g/cm^3, fitted with P=4.0 g/cm^3.

Fig.6(b) shows the "Melting point data", not known for (As) and (Mc). The period obtained with these data is 302.2 K. The unknown data value being inside the known values, following our equation (1) two values are obtained. They are not successive because the equation

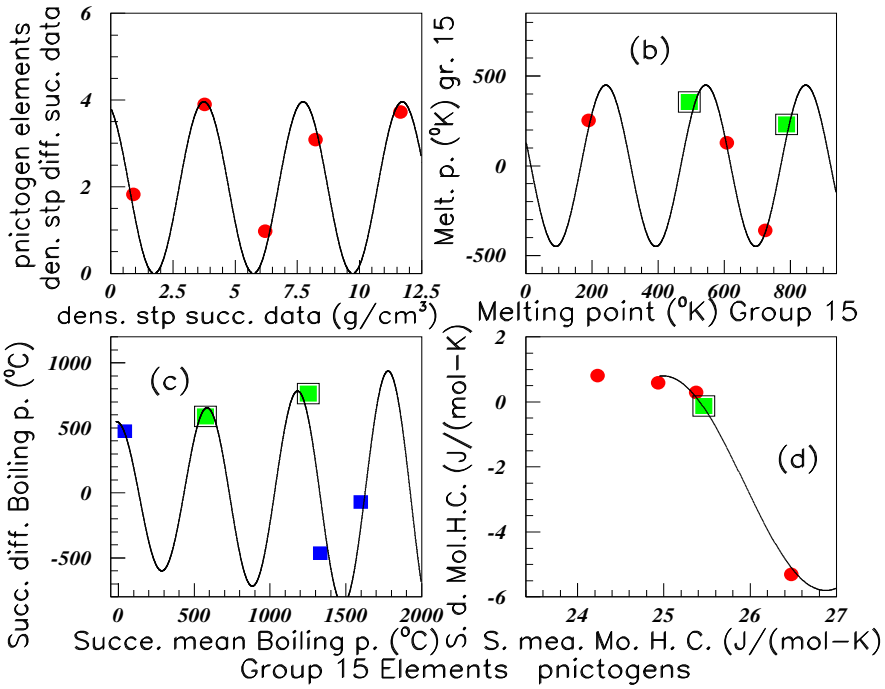

Figure 6: Color on line. (See text). Inserts (a), (b), (c), and (d) show respectively for the pnictogen elements: the "density stp data", the "Melting point data", the "Boiling Point data", and the "Molar Heat Capacity data".

(1) and the Melting point values of the other elements. The two green squares are obtained with the tentatively determination of the Arsenic Melting point value M=400 ^{0}C (673.5 K), less certain than the previously determinations.

Fig.6(c) shows the "Boiling point data" since not known also for (As). The period obtained with these data is 596.9 ^{0}C. The unknown data being inside the known values, following our equation (1) two green squares are again obtained with introduction of the (As) value. They are obtained, in agreement with the fit obtained with known values, when the possible predicted Arsenic Boiling point egals 870 ^{0}C.

Fig.6(d) shows the "Molar Heat Capacity data" of the five first Elements of the Group 15 (pnictogens). These very small number of data cannot be described by an oscillation. The corresponding value is not known for (Mc). The green square corresponds to the possible predicted value of MHC for (Mc) approximativement equal to 25.4 J/(mol-K).

8. Actinide and Lanthanide serie Elements

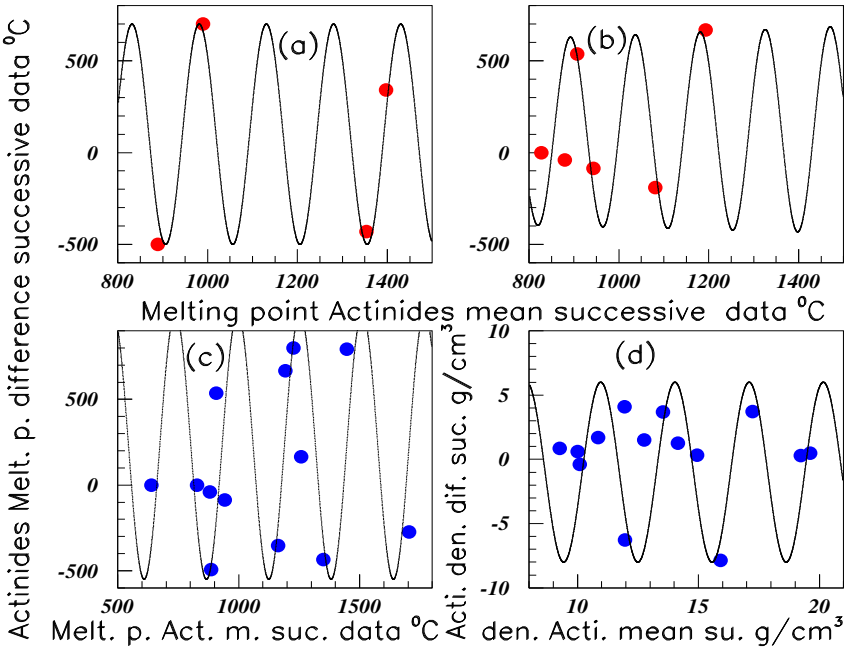

Figure 7: Color on line. (See text). Actinide "Melting point data" in insert(a), (b), and (c) in centigrad degrees. "Actinide density data" in insert (d) in g/cm^3.

Fig.7 shows the studies of Actinide and Lanthanide Boiling point and Melting point properties. All Elements of the Actinide serie contain the same number of electrons per the first shells: 2, 8, 18, and 32 for K, L, M, and N shells. They all (except Lawrencium) contain 2 electrons in their last shell (Q). The number of electrons vary for the other two shells (O) and (P). Among the fifteen Elements belonging to the Actinide series, I first separate them into two sub series, depending on electron sub-shell configuration.

One sub-serie include the 5 elements containing 9 electrons in the P shell. The number of electrons in the O shell is indicated between parenthesis. They are Actinium (Ac) with 18 electrons in the O shell (18), Protactinium (Pa) (20), Uranium (U) (21), Neptunium (Np) (22), and Curium (Cm) (25). Fig.7(a) shows the "Melting point data" in full red marks fitted with the period P=149.5 ^{0}C.

The second sub-serie include the elements containing 8 electrons in the P shell. They are Plutonium (Pu) (24), Americium (Am) (25),

Berkelium (Bk) (27), Californium (Cf) (28), Einsteinium (Es) (29), Fermium (Fm) (30), Mendelevium (Md) (31), and Nobelium (No) (32). Fig.7(b) studies these "Melting point data" in full red circles fitted with the period P=144.5 ^{0}C.

Since the difference of "Melting point data" periods between both sub-series is small, the following study will avoid such separation. Fig.7(c) shows nearly all "Melting point data", except Protactinium (Pa) and Lawrencium (Lc) which do not fill any of the two previous sub-shells. These data are fitted with period P=245 ^{0}C.

Fig.7(d) shows "Actinide density data" of the two sub-series fitted with P=3.07 g/cm^3. Here the agreement between data and fit is not as good as before.

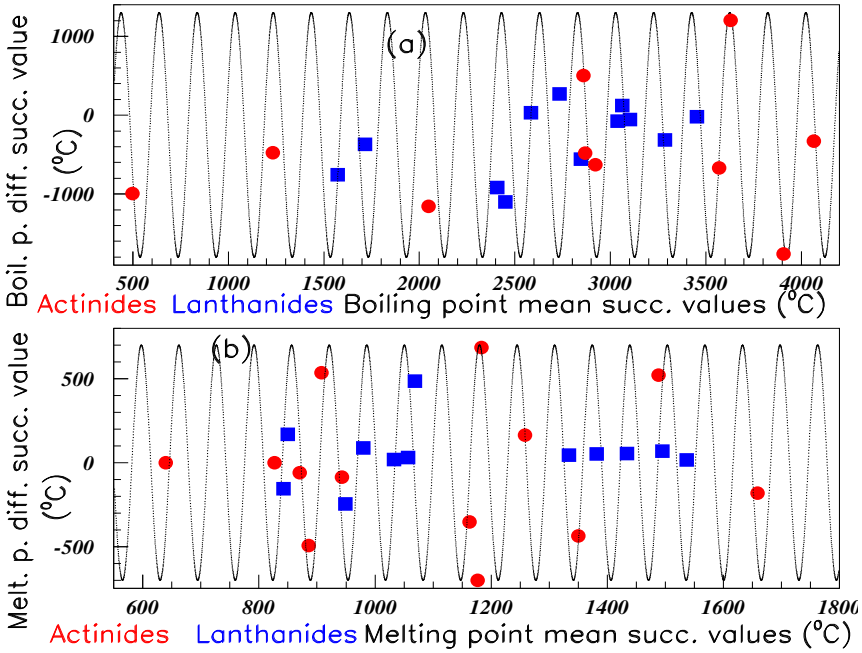

Figure 8: Color on line. (See text). "Boiling and Melting points of Actinide and Lanthanide" elements.

Fig.8(a) shows in full red circles the Actinide "Boiling points data" and in full blue squares the Lanthanide "Boiling point data" in (^{0}C). The period of the fit is P=199.2 ^{0}C.

Fig.8(b) shows in full red circles the Actinide "Melting points data" and in full blue squares the Lanthanide "Melting point data" in (^{0}C).

The period of the fit is P=64.7 ^{0}C. The Melting Point values of Actinides and Lanthanides is smaller than the Boiling points by a factor between 2 and 3. This factor is reproduced in the comparison between periods.

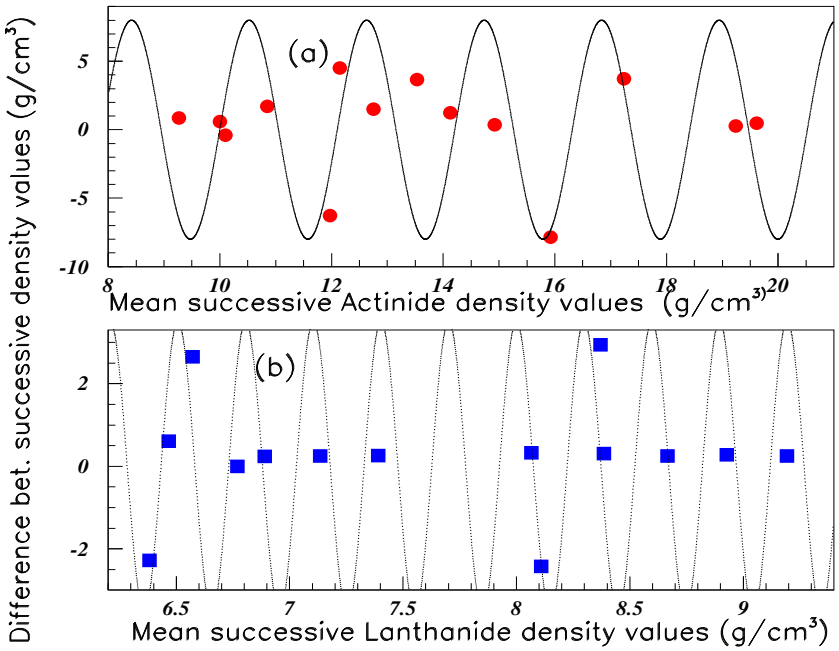

Figure 9: Color on line. (See text). "Density data" of Actinide (insert (a)) and Lanthanide (insert (b)) elements.

Fig.9(a) shows the "Actinide density (near r.t.) data" fitted with the period P=2.1 g/cm^3. The Nobelium and Larencium densities are not measured, however there predictions: den.(No)=9.9 g/cm^3, den.(La)=14.4 g/cm^3 are well fitted.

Fig.9(b) shows the "Lanthanide density (near r.t.) data" fitted with the period P=0.298 g/cm^3.

9. Discussion

The shown figs. concerning Atomic Body "data" confirm that the oscillation properties are widely observed in nature. The oscillation periods are given in following tables.

These "data" studied above are fitted with simple functions which parameters are reported in Table A1 and Table A2. The values of

Table A1: Parameters of the fits done using equation (2). See text.

Fig.	α	β	P
1(a)	89	0.004	98.0 ^{0}C
1(b)	24	0.0038	97.4 ^{0}C
1(c)	0.68	0.15	0.98 g/L
1(d)	2.2	0.013	5.34 kJ/(mol)
2(a)	2.05	0.00445	279 ^{0}C
2(b)	10.7	0.011	66 ^{0}C
2(c)	0.34	0.011	0.49 g/cm^3
2(d)	3.7	0.00162	182.2 K
2(e)	2150	-0.25	2.41 J/(mol-K)
3(a)	77	0.0009	123.8 K
3(b)	77	0.0009	99.9 K
5(a)	1.95	0.025	4.78 g/cm^3
5(b)	1.9	0.023	4.9 g/cm^3
6(a)	1.98	0	4.0 g/cm^3

Table A2: Parameters of the fits done using equation (3). See text.

Fig.	α_0	α_1	β	P
4(a)	-720	286	-0.00085	133.8 K
4(b)	-325	75	0	123.3 K
6(b)	450	0	0	302.2 K
6(c)	550	0	0.0003	596.9 ^{0}C
7(a)	600	100	0	149.5 ^{0}C
7(b)	450	100	0.00015	144.5 ^{0}C
7(c)	650	100	0	257.6 ^{0}C
7(d)	7	-1	0	3.07 g/cm^3
8(a)	1550	-250	0	199.2 ^{0}C
8(b)	700	0	0	64.7 ^{0}C
9(a)	8	0	0	2.1 g/cm^3
9(b)	3	0	0	0.298 g/cm^3

Table A3: Tentative predictions of some unknown physical Atomic Element properties. (See text).

Family	Element	tentative predictions
Noble Gases	Oganesson	Melting point 92 ^{0} C
		density 6.6 or 7.1 g/cm^3
Alkalin Metals	Francium	density 2.55 g/cm^3
		critical point 2.48 g/cm^3
		Molar heat capacity 33 J/(mol-K)
Halogens	Tennessine	Melting point 410 ^{0}C or 530 ^{0}C
Chalcogens	Livermorium	Boiling point 835 ^{0}C
		Melting point 419 ^{0}C
Boron	Nihonium	density 16 g/cm^3
Pnictogens	Arsenic	Melting point 400 ^{0}C
		Boiling point 870 ^{0}C
	Moscovium	Molar heat capacity 25.4 J/(mol-K)

the "data" period of oscillations are the same, both in K unit and in ^{0}C units. The α and β parameters are connected and depend on the starting "data values" used for the fit. Since this is not the case for the periods, it is useful to study an eventual connection between all found value for the periods.

This oscillation symmetry allows to tentatively predict some physical unknown values. These predictions are all the more solid, when the tables indicate the corresponding range of validity. These tentatively predicted values are recalled in Table A3.

The extracted periods are discuted below. Fig. 10(a) shows all "Melting point" and "Boiling point" data periods plotted versus their successive extraction number. We observe that these periods take discrete values, regularly separated by a constant separation ΔP=29.5 ^{0}C.

Fig. 10(b) shows the density periods in g/cm^3 regularly separated by ΔP=0.495 g/cm^3.

The observation of this possible oscillating period quantification, leads us to look at possible similar observations in periods extracted previously in different papers wich studied oscillations in different bodies. This possible effect of quantification is first investigated for the periods describing the masses and widths of the fundamental parti-

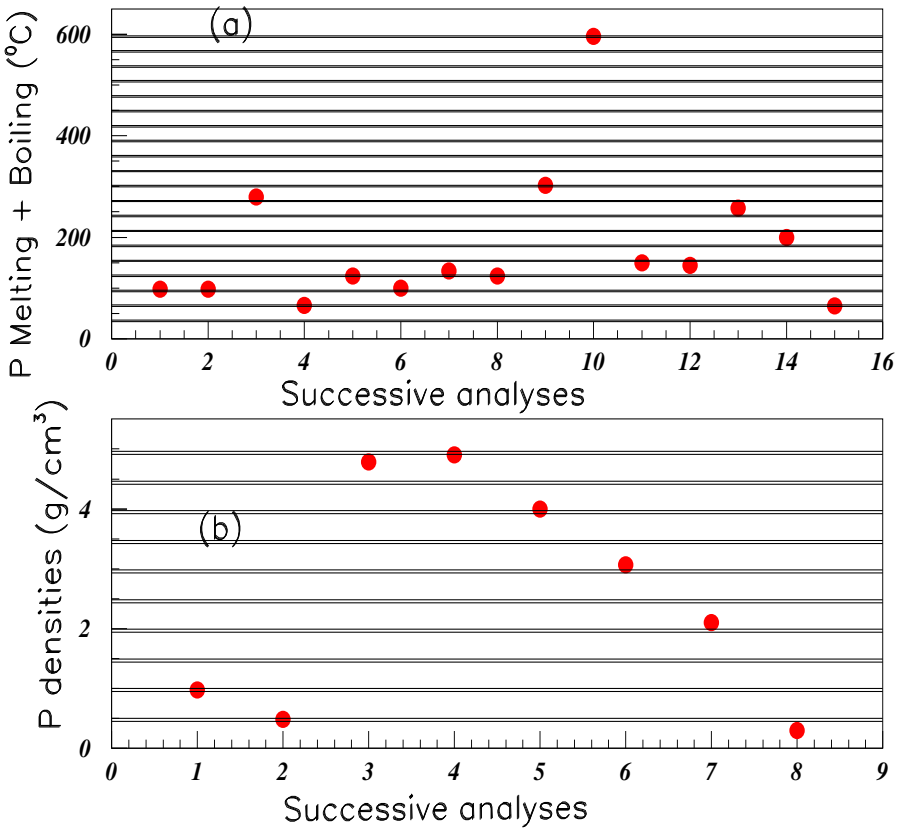

Figure 10: Color on line. (See text). Successive analyses of periods corresponding to Melting points, Boiling points, and densities of Atomic Elements. essai311

cles. These periods have been obtained using values read in references given in previous papers and not repeated here.

9.1. Meson period variations

Fig.11 shows the periods of "meson data" reported in [2].

Fig.11(a) shows these values read in Table A1 of [2], in the order of the table, namely the f_0 "mass period", then f_2 "mass period", a.s.a. We observe that nearly all periods are located in horizontal lines separated by ΔP=64 MeV.

Insert (b) shows the width periods, read in the same order inside Table A1 of [2]. They are located in horizontal lines which values are separated by ΔP=20 MeV.

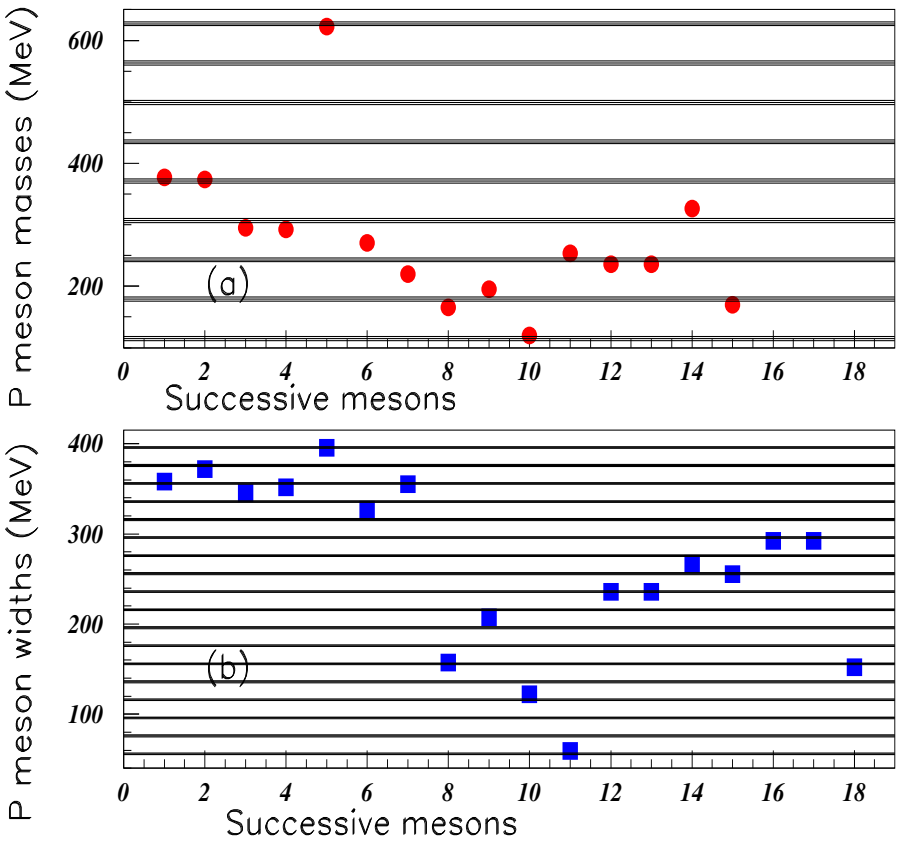

Figure 11: Color on line. (See text). Variation of the successive meson "mass data" periods and widths.

9.2. Baryon period variations

Fig.12 shows the baryons periods read in ref. [2], Insert (a) shows the baryon "mass data" periods read in Table A 2 in [2], and separated by ΔP=23 MeV.

Insert (b) shows the baryon width periods separated by ΔP=20 MeV.

Several remarkable properties have already been observed between several periods in figs. 19 and 20 of ref.[4]. The periods in these figs. concerned the "mass data" of the mesons f_0, f_2, Ξ, and Ξ_C, and also the N(1/2), N(3/2), Δ(1/2), and Δ(3/2) baryons, and also the ^{14}N excited state level masses. Another connection between "mass data periods" of Λ_B, ΣB, and Ξ_B is illustrated in [2].

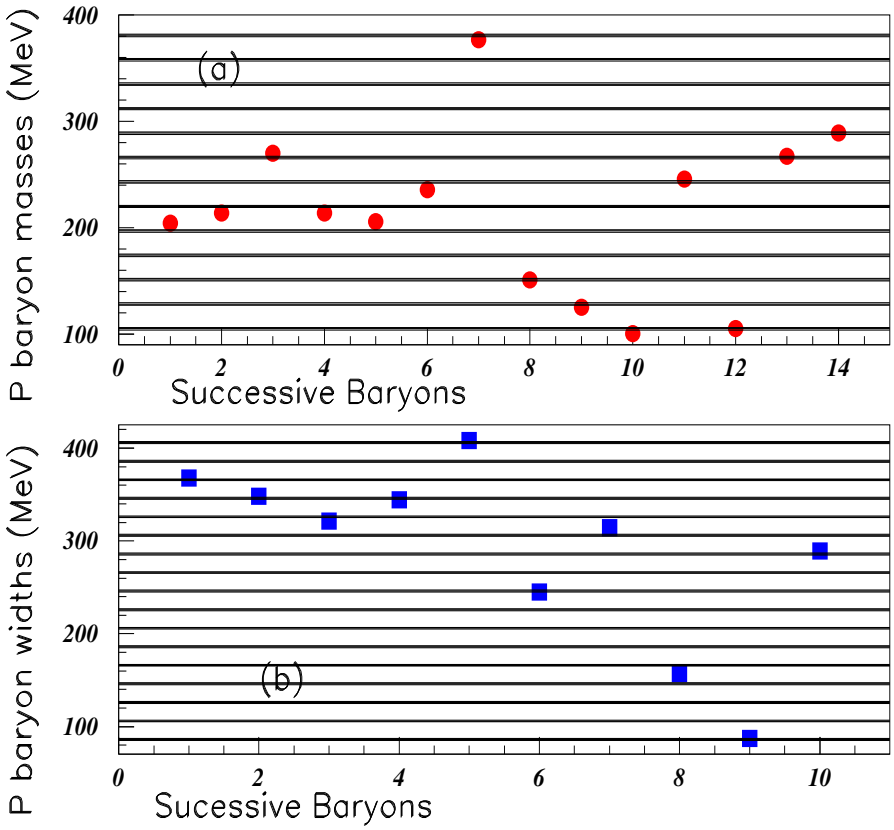

Figure 12: Color on line. (See text). Variation of baryon "mass data" and width periods.

9.3. Nuclear Nuclei period variations

The nuclei "mass data" oscillating periods, read in several Tables in [3] are shown in fig. 13. They are plotted versus the corresponding Atomic numbers. The different marks and colours correspond to different tables. The periods are separated by ΔP=0.162 MeV.

Fig. 13(b) shows the nuclei widths periods read in Table VII and VIII of [3]. The horizontal lines are separated by ΔP=0.162 MeV.

9.4. Electromagnetic Nuclei period variations

Fig.14(a) shows the variation of the Electromagnetic nuclei mass periods plotted versus their nuclei atomic number. Full red circles correspond to electric interactions, full blue squares correspond to magnetic interactions. The quantization is ΔP=0.225 MeV.

Fig.14(b) shows the variation of the Electromagnetic nuclei width periods. Full red circles correspond to electric interactions, full blue

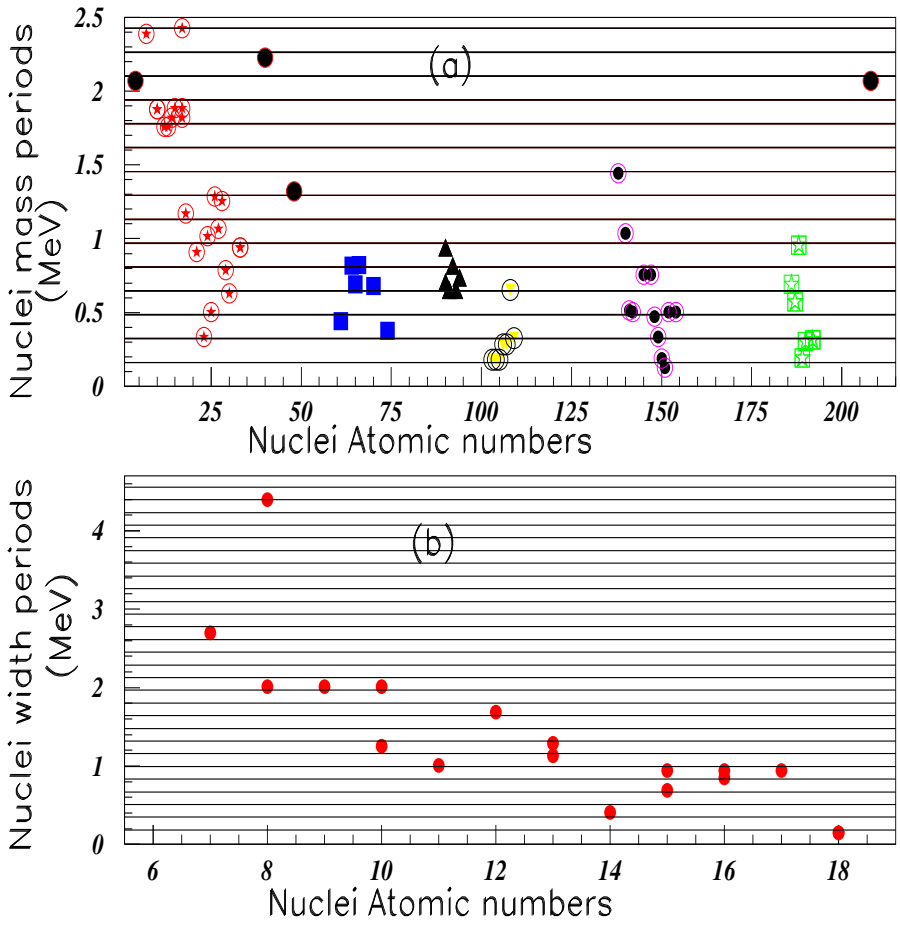

Figure 13: Color on line. (See text).Baryon periods variation.

squares correspond to magnetic interactions. The quantization is also ΔP=0.225 MeV.

9.5. Periods variation in Astrophysics

Fig.15(a) shows the periods of different exoplanet properties versus their masses (in Jupiter mass). Data from [4] are in red, data from [8] are in green. The quantification egals 0.0205 Jupiter mass.

Fig.15(b) shows the periods of different exoplanet properties plotted versus their radii (in Jupiter radius unit) in full blue marks [4] and full green marks [8]. The quantification egals 0.021 Jupiter radius.

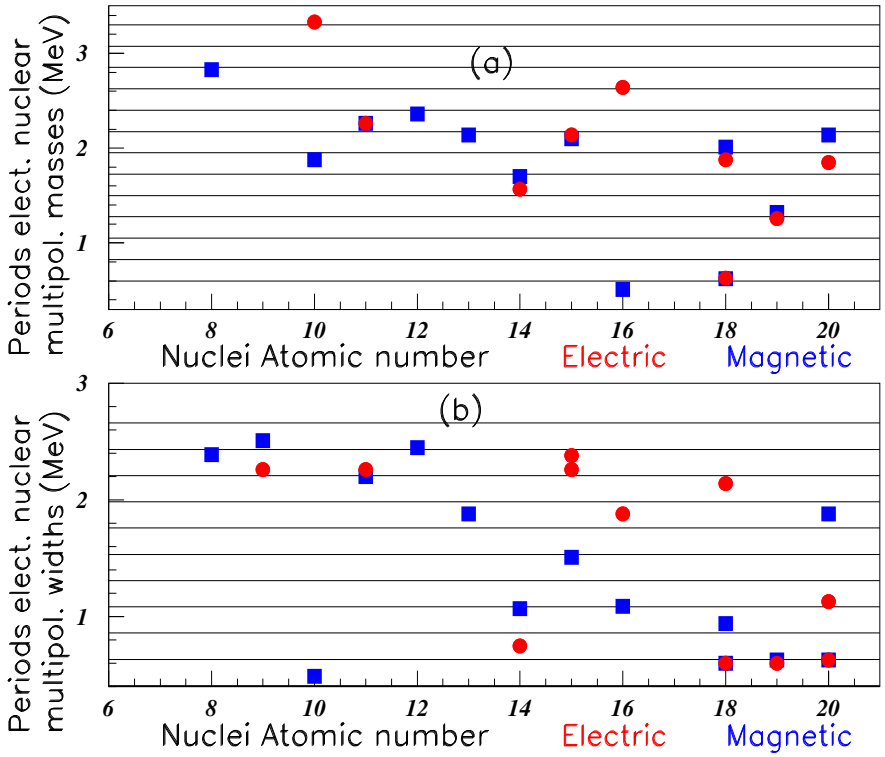

Figure 14: Color on line. (See text). Electromagnetic nuclei mass periods . Full red circles correspond to electric interactions, full blue squares correspond to magnetic interactions.

10. Conclusion

The "data" of the studied properties of many atomic elements of the Periodic Table exhibit oscillations, named Oscillation Symmetry. This is the main result shown in this paper, which generalize similar results concerning particles, nuclei, and astrophysics, studied in previous papers. This strenghtens the ascertainment that such property is widely observed in nature.

A new observation concerns the oscillation periods. These obey, with a rather good precision, to a 'like-quantification' taking values equally seperated when studied in given bodies. For mesons, the quantification periods differ between masses and widths. For other bodies, they are either the same, or not very different as shown in table A4.

The results presented in this paper deserve theoretical studies which are clearly outside the scope of the present work.

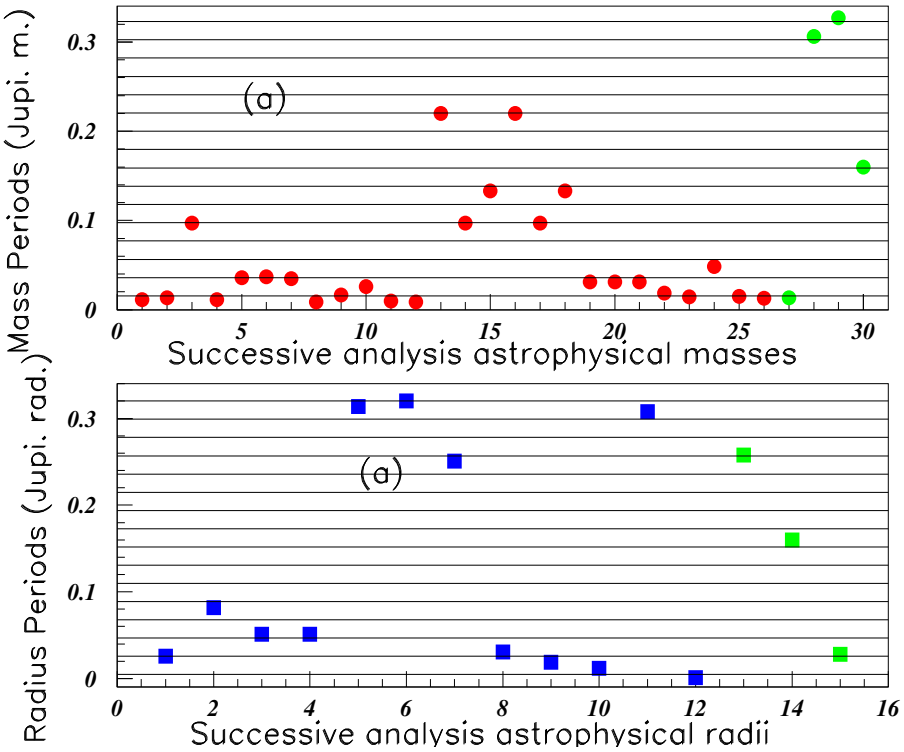

Figure 15: Color on line. (See text). The variation of several mass (radius) astrophysical periods is shown in insert (a) ((b)).

References

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[2] B. Tatischeff, "Hidden properties of meson and baryon masses and widths", https:// vixra.org/abs/2201.0004

[3] B. Tatischeff, "Observation of oscillation symmetry in nuclei excited state masses and widths", vixra 2111.0010v

[4] B. Tatischeff, "Oscillation symmetry applied to some astrophysical masses, and allowing to highlight remarkable relations between masses", Physics and Astronomy International Journal PAIJ-04-00206-2.pdf. DOI:10.1506/paij.2020.04.00206.

Table A4: Quantification parameters

Body	property	parameter value
mesons	masses	64 MeV
	widths	20 MeV
baryons	masses	23 MeV
	widths	20 MeV
nuclei	nuclear masses	0.162 MeV
	nuclear widths	0.162 MeV
	electromagnetic masses	0.225 MeV
	electromagnetic widths	0.225 MeV
astrophysics	masses	0.0205 Jup. mass
	radii	0.021 Jup. width

[5] https://en.wikipedia.org/wiki/Atomic_Element

[6] https://en.wikipedia.org/wiki/Noble_gas

[7] Yu. Ts. Oganessian *et al.*, "Synthesis of the isotopes of elements 118 and 116 in the ^{249}Cf and ^{245}Cm + ^{48}Ca fusion reactions" Phys. Rev. C **74**, 044602. DOI:https://doi.org/10.1103/PhysRevC.74.044602.

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