MODELING OF THE SPATIAL STRUCTURE OF THE ATOM.
PART 2. ATOMIC NUCLEUS

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Summary
Based on the model of nuclear shells, a computer model of the spatial structure of the atomic nucleus was constructed. The application of a new approach, by analogy with the modeling of the electron shell of an atom, made it possible to visualize the structure of the atomic nucleus. The resulting spatial arrangement of nucleons in the nucleus simulates the presence of an ultrafine structure of levels. The adequacy of the constructed model is confirmed by correlations of the dependences of the radius and asymmetry of the shape of the nucleus on the charge number. Based on the results obtained, the equations for estimating the limit number of isotopes in atoms for the N-Z diagram are determined.

Key words: atomic nucleus, model of the nucleus structure, nucleon, quantum number, magic number, radius, asymmetry, isotope.

1. INTRODUCTION

At the present stage of the development of science, the study of features in the structure of atomic nuclei is of particular interest. Here, the atomic nucleus is understood as a system of interconnected nucleons, in which various changes in their relative position and the nature of movement are possible. [[1]. The complexity of the behavior of nucleons in various processes has led to the fact that a consistent theory of the atomic nucleus has not yet been developed. Therefore, in order to simplify the theoretical description of atomic nuclei, it is customary to use models based on the representation of the nucleus as an object with well-known characteristic properties.

Nominally, nuclear models are divided into two classes [2] – microscopic and collective. The behavior of individual nucleons of the nucleus is considered in microscopic models. Moreover, it is believed that the movement of a single nucleon in the nucleus does not depend on the movement of the rest. The collective models already take into account the consistent behavior of large groups of nucleons.

The most common microscopic model of the nucleus is the shell model [3], in which the energy of the nucleons takes certain discrete values, that is, the nucleons are at certain energy levels. The shell model of the nucleus describes the structure of nuclei in a mostly stable state. According to this model, the nucleons in the nucleus are arranged in shells, and only a certain number of nucleons can be on each shell. There are two systems of nucleon energy levels: one for neutrons, the other for protons, and each system is filled with nucleons independently of each other. With an increase in the number of protons or neutrons in the nucleus, certain numbers of nucleons appear, at which the binding energy of the next nucleon is much less than that of the last one inhabited. Atomic nuclei with fully filled nucleon shells have augmented stability. Atomic nuclei containing magic numbers are particularly stable 2, 8, 20, 28, 50, 82 for protons and neutrons, as well as 126 for neutrons [4].

In the nuclear shell model, single-particle conditions are characterized by a set of quantum numbers: n, l, j, m [5]. Here n is the main quantum number, which numbers one-partial orbits with the same lj in ascending order of their energy; l is the orbital moment of the amount of motion of the nucleon; j is the total moment of the amount of motion of the nucleon and m is the magnetic quantum number, defined as the projection of the moment of the amount of motion on the base axis. At the same time, to estimate the position of nucleons in the nucleus, it is desirable to use such quantum numbers as orbital, magnetic, as well as an additional quantum number introduced, called the basic quantum number [6]. The model of nuclear shells is the basis of modern understanding of the structure of the atomic nucleus, in its essence being a semi-empirical scheme that allows us to understand some patterns in the structure of nuclei. However, it is not able to quantify the basic properties of the core consistently. So, for example, it is not able to find out the order of filling shells theoretically, and, consequently, the magic numbers for nuclear shells. Within the framework of this model, visualization of the ground state of the nucleus, which can be represented in terms of the number of nucleons inhabiting individual subshells, could be of great importance. The obtained information of this type could allow for a targeted verification of the predictive predictions of various theoretical models.

The main tools for describing the characteristics of the nucleus, in addition to theoretical and experimental research methods, can also be graphical methods of visual representation of the structure of the atomic nucleus [7, 8]. A number of authors are actively working in this direction [9]. At the same time, along with the well-known classical models of the nucleus, a whole family of so-called lattice models appeared. Thus, Garai [8] proposed to consider the formation of tetrahedrons consisting of identical spheres arranged in an FCC packing, which can be extended by adding layers of equilateral triangles arranged in a two-dimensional packing. If layers are added to the outer sides of the tetrahedron, then in this case a core shell can be formed. Similar approaches were considered by a number of other authors [7, 9, 10]. In addition, one can note the works of G. Anagnostatos [11], in which neutrons and protons are considered as hard spherical balls. In this semi classical model, the nucleons on each shell are in dynamic equilibrium, and their average positions correspond to the vertices of the polyhedron. The shell dimensions are determined from the requirement of dense packing of nucleons; the wave functions of nucleons are calculated in the potential of a harmonic oscillator, so this model essentially describes atomic nuclei as crystals. The model quite well reproduces the experimental data on the binding energies of nuclei, the root-mean-square radii of the charge and nucleon distributions.

Analyzing the above ways of surveying atomic nuclei, as well as the known models of the nucleus, it was decided to choose as the goal of the work the construction of a computer model for visualizing the spatial formation of the structure of the atomic nucleus, when it is described within the framework of the shell model in accordance with a set of quantum numbers characterizing nuclear shells.

2. GENERAL FORMULATION OF THE PROBLEM

During the visualization process, we will take the statement about the possibility of considering the atomic nucleus as a collection of nucleons located on one common axis and jointly rotating around this axis as a single whole [12] as a basis. In this case, we will assume that the problem of describing the spatial structure of the atomic nucleus is to determine the location of the nucleons in accordance with their placement near the base axis of rotation of the nucleus. The nucleons themselves can be considered as structureless material objects whose position in space can be set by three quantum numbers: basic, orbital and magnetic (side) [6].

Let protons and neutrons inside nuclei exist not only in the form of independent particles, but also bound proton-neutron pairs – deuterons (more complex nuclei in the framework of this model will not be considered) [13]. A deuteron is formed by the fusion of a proton and a neutron into a separate particle, and its shape may have some elongation exceeding the scope of a strong interaction. This fact affects the size of the deuteron, whose radius is the value: R_d = 2.13 *fm*. It should be noted that the radii of the proton and neutron are approximately the same and equal to $r \approx 0.84$ *fm* [9]. In addition, the deuteron has a small mass defect, which allows us to assert that it is not a mechanically coupled *p-n* system, but can be considered as an independent particle with its own characteristics.

In further consideration, we confine ourselves to the condition that all nucleons in the first approximation have a shape close to spherical. However, at the same time, we note that the difference in size of deuterons from protons and neutrons will have a significant impact on the properties of the atomic nucleus, which will lead to some shift in the position of a number of nucleons.

At the next step of our consideration, we will determine a possible correspondence between quantum numbers and spatial coordinates. Thus, the axis of rotation associated with the base quantum number *x* is the basis for the placement of *s*-particles [12]. Another quantum number *l*, related to the orbital momentum, indicates the possibility of separating the nuclear structure into separate layers, such as *p*-, *d*-, *f*-layer, etc. In this case, it is necessary to consider the nucleus as a system of particles, the position of which is determined by using a cylindrical coordinate system [6]. In this case, the magnetic quantum number m will be responsible for the number of particles installed in each of the considered layers, for certain values of the base and orbital quantum numbers.

Summarizing the above surveyed, we can conclude that for us the most convenient coordinate system, as well as for the electron shell of an atom, is a cylindrical system having a dedicated base axis, which is associated with the base quantum number *x*. The other two components will be the quantum numbers *l* and *m*, which are also analogs of coordinates [6].

3. SIMULATION

Let us assume that the nucleus of an atom in a normal condition consists of neutrons *N* and protons *Z*, as well as their possible combination in the form of deuterons. For the purpose of simplification, we will consider only spherical nucleons. We will assume that the nucleons are placed on the corresponding shells and subshells. Each nucleon is characterized by the following basic parameters: charge (proton), radius of finite size and its position inside the nucleus, determined on the basis of quantum numbers. For convenience of representation, taking into account the selected value of the orbital quantum number, each of the protons can be associated with a specific color identifier (Table 1).

Table 1.

Color identification of protons, according to the values of the orbital quantum number

Orbital quantum number	*s*	*p*	*d*	*f*	*g*	*h*
Colour	Yellow	Green	Red	Blue	Azure	Violet

We will look for a solution to the problem of forming a 3D-dimensional structure of the nucleus in a way that uses a graphical representation of the shell model developed to represent the electron shell of an atom [6]. However, the presence of two parallel shells – proton and neutron – makes it difficult to build a model defining the nucleus from common unified positions. The way out of this situation is possible when considering the scheme of the nucleus in the form of a set of structural cells with subsequent settlement of their nucleons. To this end, we will divide the spatial region near the base axis of the nucleus into separate cells, in each of which it will be possible to populate either only a proton or a neutron, or together in the form of a deuteron.

In the first approximation, we will assume that the deuteron is spherically symmetric, although, as noted above, in reality it also has an ellipsoidal (cylindrical) shape. This statement is due to the fact that the deuteron spends a certain part of its time in a form close to a sphere, but the "recession" of its constituent nucleons makes it possible to consider its shape as cylindrical.

Let's consider the general model of the atomic nucleus. It should be noted that according to this model, the nucleus contains the so-called base nucleus zone consisting of 1*s*-protons, as well as the periphery zone represented by 1*p-protons*. During the formation of a new shell, bonds arise at the level of individual protons of various subshells.

If we consider the structure of the 1*p*-shell consisting of deuterons (Fig. 1), it can be noted that only two of the three deuterons are in direct contact with the protons of the 1*s*-shell. This fact leads to the presence of a set of 1*p*-protons having two different energy levels. Of course, such a graphical description of the core structure is approximate, but it will be convenient enough for its visual representation.

For fixation of the static nature of the structure, we will assume that there are no processes of reconfiguration of protons near the nucleus in this model, with the exception of the occupation of subshells by nuclear kainosymmetrics. There is no provision for taking into account the mechanical inertia of particles determined by their mass, as well as taking into account their own rotational motion of particles. We will also assume that all kinetic energy is concentrated only in the rotational motion of the entire atom as a whole.

Fig. 1 Diagram of the spatial arrangement of deuterons in the structure of the $1p$-shell (the graphic image is shown perpendicular to the direction of the base axis)

Summarizing all the explanations considered, it was decided, when conducting computer modeling of the spatial structure of the atomic nucleus, to accept the following assumptions:

1. We can consider only nuclei that are in a normal state.
2. Nucleons have a spherical shape and a finite radius.
3. The nucleus as a whole is in a state of rotation around some selected base axis.
4. The modified Pauli principle determines the spatial arrangement of nucleons relative to each other and is based on three quantum numbers: *x*, *l*, and *m*.
5. The magnetic quantum number m makes it possible to determine the number of nucleons placed on a common plane perpendicular to the base axis at given values of the base and orbital quantum numbers.
6. The principle of minimum energy determines the order of settlement of nucleons having different values of potential energy.
7. The settlement of nucleons in the atomic nucleus occurs in accordance with previously established principles and rules.
8. The mechanism of paired interaction of protons and neutrons is assumed, taking into account the effects of proton-neutron pairing [13].

4. RESULTS AND DISCUSSION

4.1 The location of protons in the nuclear shell model

Let us consider the possible configurations of closed and semi-closed subshells of the spatial structures of atomic nuclei that arise during the occupation of the proton shell.

Atomic number *Z* = 2. Two protons cannot form a nucleus in the absence of at least one neutron (Fig. 2).

To form a bond between protons, it is necessary to place at least one additional neutron in one of the spatial cells. It is this neutron, which forms a neutron together with a proton, and will be considered as the basis for the beginning of the formation of the nucleus of an atom.

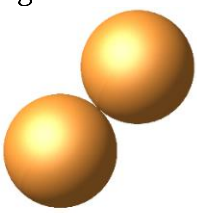


Fig. 2 Diagram of the arrangement of protons in the nucleus model with Z=2 in the $1s^2$ nuclear configuration in a plane passing through the axis of rotation of the nucleus at an angle of 45^0

In other words, a deuteron can be taken as the power center of the nucleus, to which other nucleons will be added.

Atomic number Z = 8. Figure 3 shows the structure of the nucleus with a charge number Z = 8.

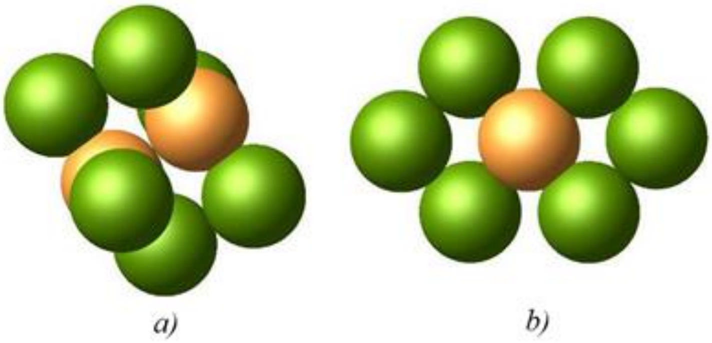


Fig. 3 Diagram of the arrangement of protons in the nucleus model with Z=8 in two projections: *a*) in plane passing through the axis of rotation of the nucleus at an angle of 45^0; *b*) in plane perpendicular to the axis of rotation and passing through the center of the nucleus. – *s*-protons, – *p*-protons

When surveying this closed shell, one can clearly see the division of the 1*p* shell into two parts, having 4 protons in contact with the lower 1*s*-shell and 2 protons located at a slightly greater distance from the 1*s*-protons (Fig. 3b). For example, in the 1*p*-subshell, the first 2 protons occupy cells with a positive base quantum number, the next 2 with a negative value of this number. The remaining 2 protons, respectively, also have the same potential energy values when occupied, for different signs of the base quantum number.

Atomic number Z = 20. To obtain a closed shell with atomic number Z = 20, formed by colonizing with protons a 1*d*-shell with a capacity of up to 10 protons (Fig. 4).

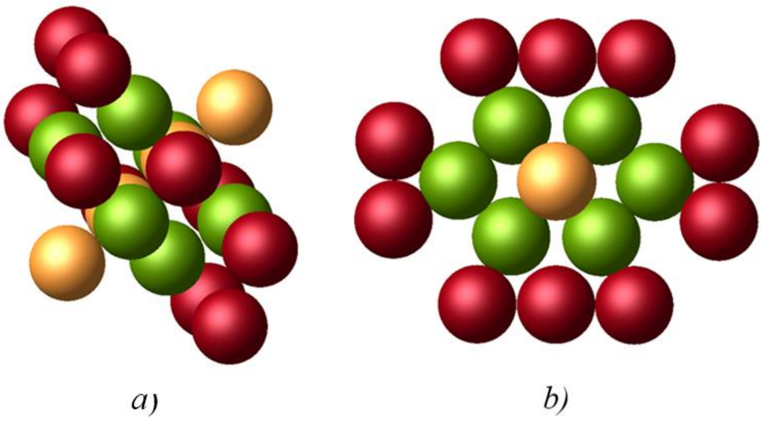
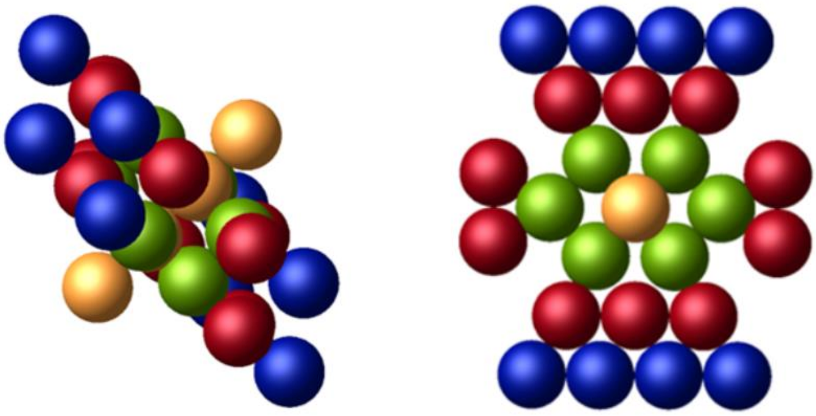


Fig. 4 The arrangement of protons in the model of the nucleus with Z = 20 in two projections: *a)* in plane passing through the axis of rotation of the nucleus at an angle of 45^0; *b)* in plane perpendicular to the axis of rotation and passing through the center of the nucleus. – *s*-protons, – *p*-protons, – *d*-protons

Here it is also possible to obtain the separation of the 1*d*-shell into two parts having, respectively, 6 and 4 protons (Fig. 4b).

Atomic number Z = 28. Adding 8 more *f*-subshell protons to the nucleus, we get the following magic number Z = 28 (Fig. 5).

Let us turn our attention to the shape of the nucleus (Fig. 5*b*). Unlike nuclei with Z = 8 and Z = 20, which have a slight elongation in the horizontal position, there is also some asymmetry of the nucleus, but in the vertical direction.

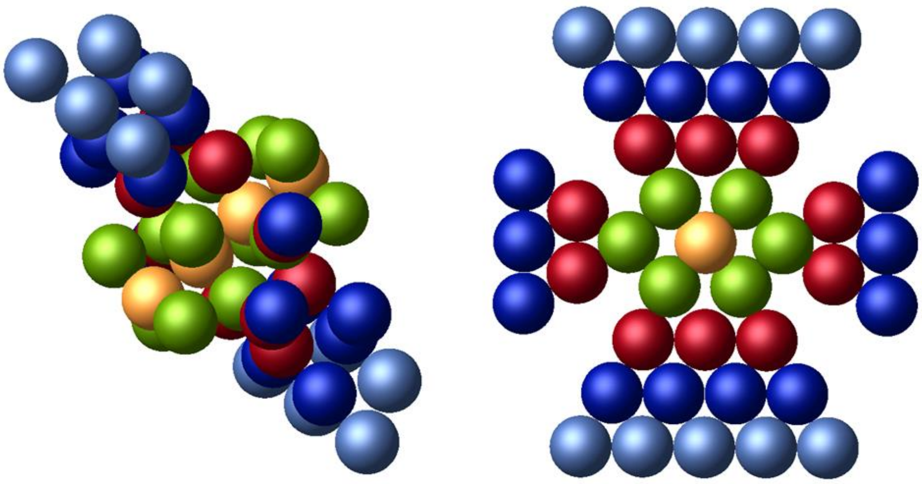


a) *b)*

Fig. 5 The arrangement of protons in the model of the nucleus with Z = 28:
a) in plane passing through the axis of rotation of the nucleus at an angle of 45^0;
b) in plane perpendicular to the axis of rotation and passing through the center of the nucleus.
– *s*-protons, – *p*-protons, – *d*-protons, – *f*-protons

Both in the previously considered nuclei, and in this case, the difference between the shape of the nucleus and the symmetric one is minimal.

Atomic number *Z* = 50. Let's move on to another semi-closed 1*g*-shell, which is part of the nucleus having atomic number Z = 50 (Fig. 6).



a) *b)*

Fig. 6 The arrangement of protons in the model of the nucleus with Z = 50:
a) in plane passing through the axis of rotation of the nucleus at an angle of 45^0;
b) in a plane perpendicular to the axis of rotation and passing through the center of the nucleus.
– *s*-protons, – *p*-protons, – *d*-protons, – *f*-protons, – *g*-protons

Here, the occupation of spatial cells is carried out in accordance with the scheme of the arrangement of the energy levels of protons [14]. First, 2*p*-subshell protons and the remaining 1*f*-subshell protons are placed, and then part of the cells and 1*g*-subshells are occupied.

Atomic number *Z* = 82. The nucleus with a charge number Z = 82 (Fig. 7) at first glance does not have a completely symmetrical shape. However, the horizontal “extension” of protons compensates for this fact, bringing the nucleus with the considered atomic number to an almost symmetrical shape.

Due to the uncertainty in the order of proton population, we will take as a basis the known experimental data on the determination of the electric quadrupole moment [14]. When structural cells are occupied with protons from Z= 54 to Z= 70, they are simultaneously occupied with neutrons, which distort the order of filling the proton shell.

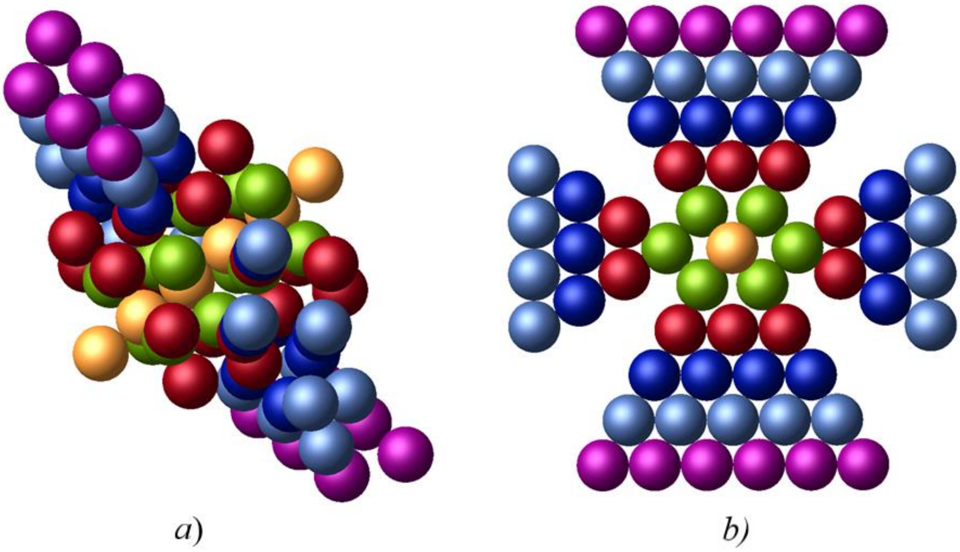


Fig. 7 The arrangement of protons in the model of the nucleus with Z = 82:
a) in plane passing through the axis of rotation of the nucleus at an angle of 45^0;
b) in a plane perpendicular to the axis of rotation and passing through the center of the nucleus.
– *s*-protons, – *p*-protons, – *d*-protons, – *f*-protons, – *g*-protons, – *h*-protons

This leads to the initial occupation of all protons in cells having a positive base quantum number up to and including 1*h*-subshells. It should be noted here that in the process of proton occupation at this interval, neutron occupation of cells with both positive and negative base quantum numbers occurs in parallel. This process leads to the appearance of nucleus asymmetry, which is reflected in the magnitude of the electric quadrupole moment. With the further filling of the structural cells, the proton occupation of cells with a negative base quantum number already occurs.

Atomic number *Z* = 112. At the end of our survey of the spatial structure of the atomic nucleus, we will focus on the nucleus with the charge number Z = 112 (Fig. 8).

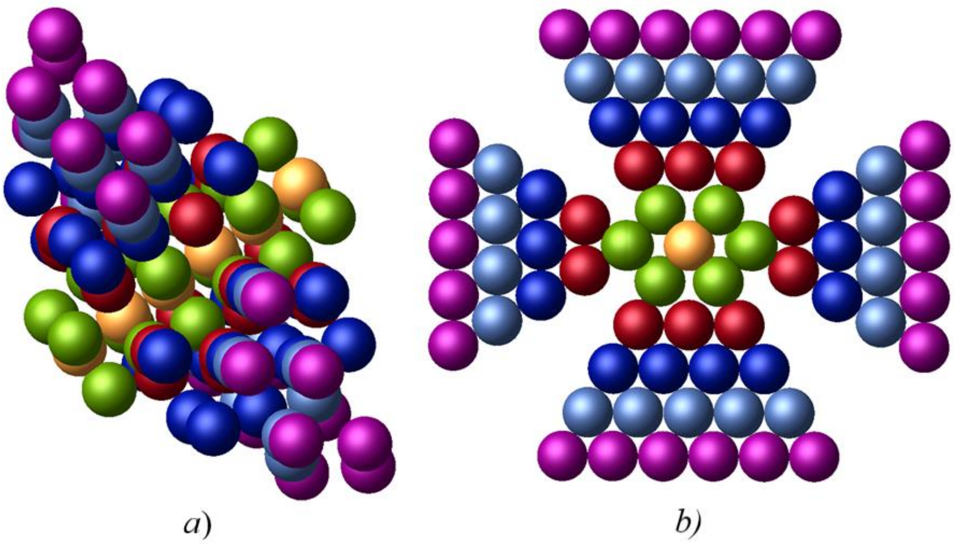


Figure 8 Diagram of the arrangement of protons in the nucleus model with Z= 112:
a) in plane passing through the axis of rotation of the nucleus at an angle of 45^0;
b) in plane perpendicular to the axis of rotation and passing through the center of the nucleus.
– *s*-protons, – *p*-protons, – *d*-protons, – *f*-protons, – *g*-protons, – *h*-protons

When cells are occupied with protons with charge numbers greater than Z = 82, similar processes are repeated, in which there is also a unilateral filling of structural cells with protons having a positive base quantum number. Therefore, initially only 5 protons will be populated in the 1h-subshell, then 7 protons in the 2f-subshell cells and at the end 3 protons in the 3p-subshell cells. And only then will protons fill the cells of the same subshells with negative base quantum numbers.

4.2 Sizing of the atomic nucleus

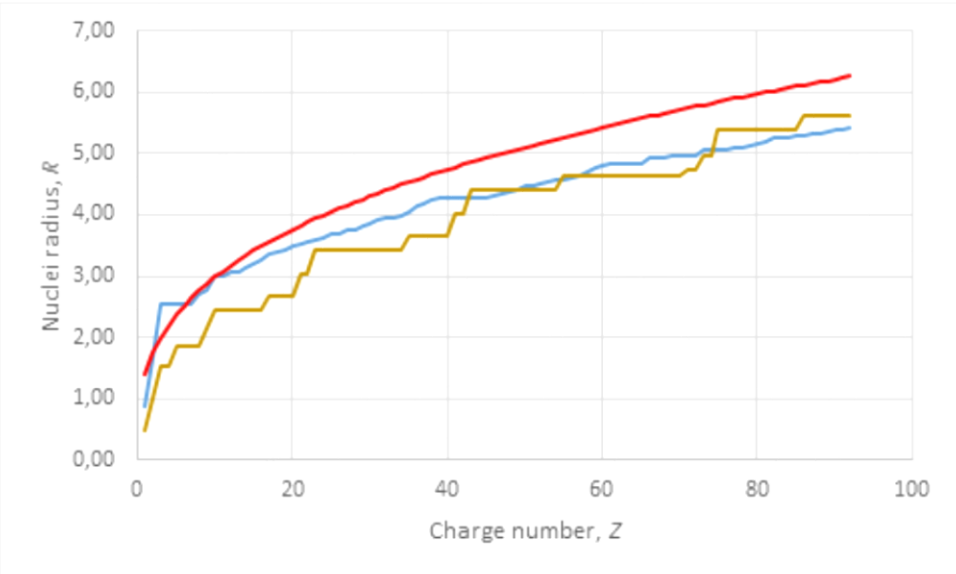
The radius of the nucleus is usually called the distance from its center to the point at which the density decreases by half compared to the density in the center [13]. Currently, data on the size of atomic nuclei obtained by various experimental methods are known [14-16]. The standard deviation of the radius values obtained experimentally lies in the range from 0.106 to 0.023 *fm*. The most accurate results on measuring the size of nuclei were obtained by scattering fast electrons on nuclei and from mesoatom spectra. There, measurements give the so-called root-mean-square electric radius of the nucleus.

Based on the proportionality of the binding energy of the nucleus to the number of nucleons N, it was concluded that the radius of the nucleus R is approximately described by the empirical formula

$$R = r_0 N^{1/3}\ , \qquad (1)$$

where $r_0 = (1.0 \div 1.1)$ *fm*. Another definition of the radius of the core is often used, approximating it with a sphere of uniform density (without a blurred surface layer). In this case: $r_0 = (1.2 \div 1.3)$ *fm*. It follows from formula (1) that the volume of the nucleus is proportional to the number of nucleons located in the nucleus.

To determine the dependence of the radius of the nucleus on the atomic number, within the framework of the model under consideration, it is necessary to estimate the distances of each of the nucleons located in separate subshells from the center of the nucleus. At the same time, it is necessary to take into account that two nucleons can be populated simultaneously in a separate cell – both a proton and a neutron. The experimental data were taken from the work on the estimation of



the radius of the nucleus performed by I. Angeli [15].

Fig. 9 Dependence of the radius of the nucleus on the atomic number:
— experiment [Angeli]; — model (present article); — empirical influence by the formula (1) при r_0 =1.1

As it can be seen from the dependence of the radius of the nucleus on the atomic number presented in Figure 9, a comparison of the sizes of nuclei obtained by various methods allows us to reveal a certain correlation of their trends. The model curve on the graph has a number of "steps" indicating the filling of cells of a certain layer. In a somewhat weakened form, such "steps" also take place with the results of experimental investigation of the radius of the nucleus.

4.3 Asymmetry of the atomic nucleus

The difference between the distribution of electric charge and the spherically symmetric one in the atomic nucleus is characterized by the electric quadrupole moment of the nucleus Q. If we assume that the nucleus is an ellipsoid of rotation with a charge density constant in volume, then the quadrupole moment can be determined by the formula [14]

$$Q = \frac{2}{5} Ze(b^2 - a^2)\,, \tag{2}$$

where Z is the atomic number of the nucleus; b and a are the semi-axes of the ellipsoid along and perpendicular to the axis of symmetry. It is known that nuclei with closed shells and subshells have zero quadrupole moment, and the greatest values of quadrupole moments are for nuclei where the subshells are only partially filled.

To estimate the magnitude of the nucleus deformity, along with the electric quadrupole moment, we introduce an auxiliary concept – the asymmetry of the nucleus A, which will be determined by summing the ratio of the basic quantum numbers x of protons, each of which makes a certain contribution to the overall value of the asymmetry, to the modules of the same numbers

$$A = \frac{1}{4} \sum_{i=1}^{Z} \frac{x_i}{mod(x_i)}\,, \quad \text{при } x \neq 0\,. \tag{3}$$

In order to compare the data on the electric quadrupole moment [17] with the results of the calculation of the nucleus asymmetry, a coefficient numerically equal to 0.25 was additionally introduced into formula (3).

When constructing the dependence of the asymmetry of the nucleus on the charge number, we do not have a clear unambiguous representation of the order of the nucleon occupation of the atomic nucleus. For this reason, it was decided to fill the proton shell by taking into account the behavior of the dependence of the electric quadrupole moment on the number of installed protons [13]. With this approach, it was found that after $Z = 54$, occupation already takes place unilaterally, without sequential installation of protons in accordance with their energy levels. The probable reason for this "behavior" of protons may be an increased number of neutrons, the presence of which affects the order of occupation of proton shell particles.

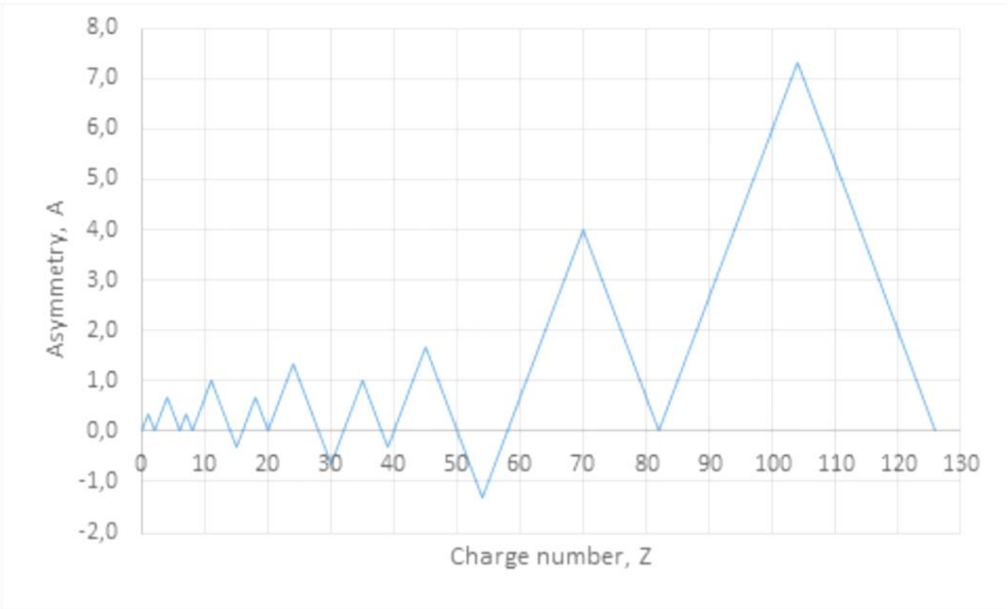


Fig. 10 Dependence of the asymmetry of the nucleus on the atomic number

The analysis of the obtained dependence of the asymmetry of the nucleus A on the charge number Z shows that in this graph (Fig. 10) it can be distinguished as standard magic numbers 2, 8, 20, 28, 50, 82, 126, and additional – 6, 14, 16, 32, 40 and 58. In addition, there are a number of extreme values for the asymmetry of the nucleus, of which 45, 54, 70 and 104 are the most clearly expressed. The calculated data obtained confirm their close correspondence to the known results of experimental studies of both spherical and deformed nuclei [19, 20].

4.4 Ultimate mass numbers of nuclides

Another example confirming the correctness of the chosen approach to the representation of the spatial structure of the atomic nucleus is the possibility of estimating the limiting mass numbers of nuclides on the N-Z diagram. To analyze the limiting mass numbers, we formulate rules that allow us to determine the mass numbers of nuclei. To this end, we will divide all neutrons into four groups. In the first group we place N_d, neutrons, which combine with protons to form deuterons. The second group includes N_s neutrons necessary for binding only *s*-protons. The third group of N_b neutrons allows for the creation of bonds between individual protons (with the exception of *s*-protons) installed in a certain nuclear substructure. The last, fourth group of neutrons N_f can occupy the cells of the proton subshell, which remained free in the selected closed subshell. This neutron separation allows us to propose the following formula for estimating the minimum mass number of A_{min} nuclides represented in the N-Z diagram

$$A_{min} = Z + N_d + N_s + (2B + 1)N_b + N_f \, , \qquad (4)$$

where B is a coefficient numerically equal to: $B = 0$ – for nuclear kainosymmetrics; $B = 1$ – when estimating the number of binding neutrons of other subshells.

The maximum mass number of the A_{max} nuclide is determined by the sum of protons and neutrons necessary for the formation of deuterons, as well as the number of neutrons in the remaining free cells of the next closed proton subshell of the atomic nucleus

$$A_{max} = Z + N_d + N_s + N_b + 2B(N_b + 2) + N_f \, , \qquad (5)$$

As an example, we will use the proposed formulas (4) and (5) to determine the limiting boundaries of the region of possible existence of isotopes of closed subshells, including the nucleus with a charge number $Z = 1$.

Atomic number *Z*=1. In our case, when the atomic nucleus consists of one stable proton, the presence of neutrons that make up the nucleus is not a prerequisite. Therefore, the minimum mass number will be numerically equal to: $A_{min}(Z{=}1) = 1$.

Let's determine what is the ultimate maximum number of neutrons can be located near a proton. According to our approach, the number of cells free for neutron population near the proton is determined in accordance with the model of nuclear shells. Considering that we can have only one deuteron ($N_d = 1$), formed, then $N_f = 5$ free cells remain for neutron occupation (Fig. 3b). Consequently, for a nuclide consisting of a single proton, the possible maximum mass number will be equal to 7. However, there is some probability of occupying the remaining two cells displaced by a certain distance, also belonging to the 1*p*-subshell, theoretically allowing the appearance of isotopes with mass numbers 8 and 9. Based on this assumption, it can be argued that the maximum value of the mass number of this nuclide can be the value: $A_{max}(Z{=}1) = 9$.

Atomic number *Z*=2. When considering a 1*s*-shell containing two protons, only one binding neutron is needed to obtain the minimum mass number. Therefore, the minimum mass number will be numerically equal to: $A_{min}(Z{=}2) = 3$.

In the case of estimating the maximum value of the mass number for nuclide isotopes, with the atomic number $Z = 2$, it is necessary to form two deuterons ($N_d = 2$) and fill the entire 1p subshell with free neutrons up to the next magic number, adding: $N_f = 6$, and the resulting number can already be considered as the limit value of neutrons placed in this the nucleus. In this case, the maximum mass number will be the value: $A_{Max}(Z{=}2) = 10$.

Atomic number *Z*=8. If we take as a basis, to determine the minimum value, the number of neutrons sufficient to " interlinked" protons in individual subshells, then in our case this number can have a minimum value equal to: N_b = 4. This value is determined by installing a 1*p*-subshell between 3 protons with positive values of the base quantum number of two binding neutrons and two more neutrons will be placed between the protons of this subshell with negative values of the base quantum number (Fig. 3a). To bind the protons of the inner 1*s*-shell, the same neutrons that are installed for the 1*p*-subshell will be used, that is, the number of *s*-neutrons: N_s = 0. In this case, the minimum mass number will be numerically equal to: $A_{min}(Z=8)$ = 12.

The maximum value of the mass number of a nuclide is determined by forming, on the basis of this nucleus, a closed 1*p*-subshell already populated with deuterons with N_d = 8. We also take into account the population of neutrons and 2*s*-subshells, giving an additional two neutrons (N_s = 2), and then add ten neutrons to the free cells of the 1*d*-subshell (N_f = 10), until the next magic number is obtained: N = 20, in the end we get the value of the maximum mass number equal to: $A_{max}(Z=8)$ = 28.

Atomic number *Z*=20. According to formula (4), to estimate the minimality of the neutrons number in this subshell, we will get an additional 8 binding neutrons for the 1*d*-subshell (4 for each value of the base quantum number) and 2 neutrons for the 2*s*-subshell (Fig. 4). Consequently, the total number of neutrons, taking into account the previously populated 4 neutrons, It will be equal to 14 neutrons, which also corresponds to the known minimum value of the mass number for a nuclide containing 20 protons: $A_{min}(Z=20)$ = 34.

The maximum value of the mass number of this nuclide will be obtained similarly to the analysis for the previous charge number. In this case, we will have 20 neutrons and 8 free neutrons in 1*f*-subshell. However, the resulting value of 28 neutrons is underestimated, so it must be assumed that part of the 2*p*-subshell may also be populated. In accordance with formula (5), in addition to the settlement of 4 neutrons, there are also bonds with 1*p*-subshell, giving 8 more neutrons. As a result, for the maximum value of the mass number of the considered nuclide, we obtain the value: $A_{max}(Z=20)$ = 60.

Atomic number *Z*=28. Adding 8 more 1*f*-subshell protons to our nucleus, we get the following magic atomic number Z = 28 (Fig. 5). Here, according to the proposed equation (4), it is necessary to determine the minimum number of nucleons only by using neutron binding of these protons into groups, in accordance with the sign of the base quantum number, that is, 4 protons each. Therefore, we need to add only 6 neutrons, which will allow us to estimate the total number of neutrons, taking into account the previously populated 14 neutrons, equal to the value N = 20, which for this mass number coincides with the known experimental value: $A_{min}(Z=28)$ = 48.

The maximum value of the mass number at the atomic number Z = 28 is determined as follows. Having 2 more free places in the 2*p*-subshell (N_f = 2), and also taking into account the need for their connection with the protons of the 1*p*-subshell (N_b = 4), it is possible to occupy 6 neutrons. Additionally, 6 more neutrons can be populated in 1*f*-subshell. In this case, the total number of neutrons in the nucleus becomes equal to: N = 52. Of course, it is possible to completely populate the 1*g*-subshell, which would include 10 more neutrons in the nucleus, but in reality this position remains vacant here for now: $A_{max}(Z=28)$ = 80.

Atomic number *Z*=50. Let's move on to another semi-closed 1*g*-subshell, which is part of the nucleus having atomic number Z = 50 (Fig. 6). For this nucleus, the minimum value of neutrons can be specified as N = 48 (we will always consider only even values as the number of neutrons). Since we already have 20 neutrons occupied, with this charge number, their number should increase by 28 neutrons. The increase is due to the occupation of 4 binding neutrons inside the 2*p*-subshell, as well as the creation of 12 bonds with 1*p*-protons, since each proton of the 2*p*-subshell contacts two protons belonging to the 1*p*-subshell at once. Therefore, we can add 16 neutrons at once. In addition, we get a bunch of protons of part 1*f*-subshell 4 neutrons and 8 neutrons bind protons of 1*g*-subshell, which together gives us the desired 28 neutrons: $A_{min}(Z=50)$ = 98.

The maximum limit of the number of isotopes for the nucleus in question requires the addition of at least another 38 neutrons. These neutrons will go partially to occupy the 1*g*-subshell (8 neutrons), 2*d*-subshells with a bundle of 1*d*-protons (30 neutrons), 2 neutrons will be located in the cells of the 3*s*-subshell (the cells of the 1*h*-subshell are not populated with neutrons). The total number of neutrons obtained in this case is N = 92, which makes it possible to estimate the value of the maximum mass number: $A_{max}(Z=50) = 142$.

Atomic number *Z*=82. Nuclei with atomic number Z =82 (Fig. 7) have an isotope with a minimum number of neutrons equal to N_{min}=96, which requires the addition of 48 more neutrons to perform proton binding. 6 neutrons must be added for 1*g*-subshell protons, 28 neutrons will be required for 2*d*-subshell, 2 neutrons will bind 3*s*-subshell protons and 10 neutrons for 1*h*-subshell: $A_{min}(Z=82) = 178$.

The maximum number of neutrons in this nucleus is N_{max}=138, therefore, to already existing 82 neutrons included in the deuterons, it is necessary to add another 56 neutrons. We should immediately note that we need to limit ourselves to free cells up to Z = 112, and not up to a value of 126 protons, otherwise it is not possible to obtain normal values close to those set in the N-Z diagram. This number of neutrons can be occupied as follows: 10 neutrons will be placed on the 1*h*-subshell, 14 free cells will be populated on the 2*f*-subshell, as well as 28 neutrons will organize the bonds of 2*f*-protons with the 1*f*-subshell and 12 neutrons will be populated in a similar way on the part consisting of 4 protons of the 3*p*-subshell. The obtained additional value of 64 neutrons indicates a slightly increased limit value of the number of neutrons in the isotopes of this nuclide: $A_{max}(Z=82) = 228$.

Atomic number *Z*=112. At the end of our survey of the spatial structure of the atomic nucleus, we will focus on the nucleus with a charge number Z = 112 (Fig. 8). In the case of determining an isotope with a minimum number of neutrons, we need to occupy additionally, relative to the previous nucleus, about 70 neutrons: 8 binding neutrons will be placed on the 1*h*-subshell, 40 neutrons – on the 2*f*-subshell and 16 neutrons on a 3*p*-subshell. The total number of occupied neutrons will be 64 neutrons, which is 6 neutrons less than is known at this point in time: $A_{min}(Z=112) = 270$.

Now let's move on to determining the isotope with the maximum number of neutrons. Here, as the boundary number of neutrons for our case, we indicate a nucleus having 136 neutrons. The reason for this restriction is the same as in the previous review. The existing limit on the number of isotopes for the nucleus in question requires the addition of at least 62 neutrons. Let's populate neutrons for a nuclear isotope with a charge number Z = 112. So 14 neutrons will be placed on the 1*i*-subshell and 30 neutrons will be placed on the 2*g*-subshell. In addition, the part consisting of 6 protons of the 3*d*-subshell will be populated with 18 neutrons. The value of 62 neutrons almost completely coincides with the experimentally obtained result, which allows us to assert the correctness of the chosen approach to determine the maximum value of the mass number of the isotope of the selected nuclide: $A_{max}(Z=112) = 286$.

All the obtained limit values of mass numbers for nuclides of closed and semi-closed subshells in the N-Z diagram were included in the general table upon completion of their survey (Table 2).

Table 2.

Limit values of mass numbers for nuclides in the N-Z diagram

Nuclide atomic number	Minimum mass number		Maximum mass number	
	Experiment [18]	Computing	Experiment [18]	Computing
1	1	1	7	7 (9)
2	3	3	10	10
8	12	12	28	28
20	34	34	58	60
28	48	48	80	80
50	99	98	139	142
82	178	178	220	228
112	276	270	285	286

A comparative analysis of the experimental and computed data presented in Table 2 shows a fairly good coincidence of most of the limit values indicating the boundaries of the region of existence of atomic nuclei. In a number of data where there is no such coincidence, additional studies may be required to clarify them.

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5. CONCLUSION

The paper develops a new approach for describing the spatial arrangement of nuclones in nuclei, based on the model of nuclear shells. Unlike most of the approaches known from the scientific literature, the method presented by us is applicable to the graphical representation of the atomic nucleus. The computer model of the spatial structure of the atomic nucleus developed on the basis of this method can be considered as a graphical representation of the well-known model of nuclear shells, which is the basis of the modern understanding of the structure of the atomic nucleus. It is shown that the settlement of structural cells of subshells with nucleons of various types occurs in parallel. The inclusion of the atomic nucleus, along with the nucleons, and the deuteron in the consideration of the model made it possible to clearly explain the cause of the appearance of the fine structure of the nucleus. The analysis of the obtained data revealed the features of filling and mutual arrangement of neutron and proton subshells of these nuclei, their relationship with other observed properties is shown: the radius of the nucleus and its asymmetry.

In this paper, the features of the limit values of isotopes in the N-Z diagram are also considered. We have shown that nuclear shells consisting of nucleons, including neutrons and protons, allow us to model the limit values of nuclides as a function of the mass number.

The proposed computer model, in the future, may be suitable for predicting additional magic numbers and studying nuclides far from the region of stable nuclei, as well as for predicting the properties of various processes in poorly studied areas. The approach developed in the work significantly expands the possibilities of studying the behavior of nuclei, for example, the currently actively studied super heavy atoms, and also increases the visibility of their representation. The obtained information of this type will allow for an effective verification of various experimental data and theoretical models.

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