LIGHT CONES QUANTUM GRAVITY

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Abstract. Gravity can be evaluated by simple rotations of light cones, here I will explore idea of Minkowski space-time rotations. Those rotations lead to natural interpretation of spin that can be thought as components of rotation of parts of light cone. Graviton field then is a field of rotation of light cones and so rotation of Minkowski space-time both in space and time or in space only for spin. This approach does not brake at Planck energy and gives results very close to General Relativity till event horizon of a black hole. I assume anti-matter field that moves backwards in time, that makes model complete in mathematical sense.

Contents

1. Core idea

I will explore simple idea of rotating a light cones [1] to create gravity- it makes process of gravity interaction simple and can be easy extended. But with this idea come few problems. First if I only rotate light cone I will miss Lorentz transformations and it makes gravity limited to only rotation of light cones. But as will be shown later this idea is consistent with General Relativity to a point of singularity- i dismiss all other gravity effects other than just simple rotation of light cone. And from it i get an simple view of gravity that if I use Planck units and tune rotation angles i will get consistent view that lead to picture of singularity where theory does not brake. From this simple idea of rotation I can recover quantum properties of matter- like we don't know in what direction object does move and spin. Spin is in this idea just one part of light cone rotation in space, where i sum a present rotation as sum of half of all possible light cones part that connect with that present. Wave function can be constructed out of rotation of light cone- it can point in any direction when it has some velocity so it moves in many possible paths. Where each paths changes by space direction of rotation in this case. Graviton is a second order tensor field that comes out of rotation of position tensor by rotation operator with probability of each possible rotation angle. All units used here are Planck units set to one, so Planck energy is one, Planck mass, time, length and so on is equal to one. There are not Lorentz transformations but rotation of light cone can explain all effects of Special Relativity without need of those transformations. Form fact that i rotate a light cone there is always speed of light constant preserved even for object that move with speed of light. Cone lines always show what path light follows from point of view of that object- by light i mean al mass-less particles. From it i can build a casual structure from point of view of any object by rotating it's light cone so it matches it's energy and how fast in moves. It means that each reference frame can have any rotation angle of light cone and from that rotation angle comes how it sees casual structure of events. I do not rotate all other light cones to match that one from point of view im watching events- it means time axis will rotate but from point of view any observer time axis is always how it moves, so from it's point of view it remains stationary. There is no global transformation in this hypothesis like Lorentz one that makes all says how all object see events- transformation is always local rotation of light cones. Where object move no longer around time normal axis but around light cone central line that was equal to normal time coordinate.

2. Rotated Minkowski space-time

First and most important future of this idea is to rotate a light cone, but i need to first explain what graviton is. Light cones can be split into future and past light cone- but from fact that graviton is spin two i need two more light cones. Each light cone part gives spin one half, so i need four parts or two full light cones. To do so i will use new light cone that is perpendicular to normal light cone. First i write equation for distance of first two light cones that are normal one not perpendicular to normal time axis:

$$ds^2_+\left(\mathbf{x}\right) = \sum_i \frac{1}{N_i^2} R^{\mu\nu}_{\alpha\beta}\left(\frac{1}{2}\alpha^+_{01,i}, .., \frac{1}{2}\alpha^+_{\gamma\delta,i}, \mathbf{x}\right) dx^\alpha dx^\beta \eta_{\mu\nu}\Big|_0^{x^0} \tag{2.1}$$

$$ds^2_-\left(\mathbf{x}\right) = \sum_j \frac{1}{N_j^2} R^{\mu\nu}_{\alpha\beta}\left(\frac{1}{2}\alpha^-_{01,j}, .., \frac{1}{2}\alpha^-_{\gamma\delta,j}, \mathbf{x}\right) dx^\alpha dx^\beta \eta_{\mu\nu}\Big|_{-x^0}^{0} \tag{2.2}$$

I sum over all possible states, where each state has a probability squared that is equal to one over number N squared. Now i ned to move to those perpendicular light cone that can be again split into two, i will denote that perpendicular coordinates by $\overline{\mathbf{x}}$, so i can write this part of equation as:

$$ds^2_+\left(\overline{\mathbf{x}}\right) = \sum_k \frac{1}{N_k^2} R^{\mu\nu}_{\alpha\beta}\left(\frac{1}{2}\alpha^+_{01,k}, .., \frac{1}{2}\alpha^+_{\gamma\delta,k}, \overline{\mathbf{x}}\right) d\overline{x}^\alpha d\overline{x}^\beta \eta_{\mu\nu}\Big|_0^{\overline{x}^0} \tag{2.3}$$

$$ds^2_-\left(\overline{\mathbf{x}}\right) = \sum_l \frac{1}{N_l^2} R^{\mu\nu}_{\alpha\beta}\left(\frac{1}{2}\alpha^-_{01,l}, .., \frac{1}{2}\alpha^-_{\gamma\delta,l}, \overline{\mathbf{x}}\right) d\overline{x}^\alpha d\overline{x}^\beta \eta_{\mu\nu}\Big|_{-\overline{x}^0}^{0} \tag{2.4}$$

Where again i assign probability for each possible rotation angle, so for each possible state of graviton field. Angles notation i used is all possible axis of rotation so for example notation 01 means rotation around time and first space axis. I take all this space-time intervals in limits from minus time coordinate to zero time coordinate, same with perpendicular coordinates. R is rotation tensor that acts on coordinates [2] , η is flat Minkowski space-time metric tensor [3]. Now from this when i have metric defined i can move to graviton itself. I assume one time coordinate and N space dimensions. Probabilities need to sum to one:

$$\sum_i \sum_j \sum_k \sum_l \frac{1}{N_i^2} + \frac{1}{N_j^2} + \frac{1}{N_k^2} + \frac{1}{N_l^2} = 1 \tag{2.5}$$

3. Graviton

In quantum physics an object is explain by function of possible states of system. Let's say I have a position tensor $x^{\alpha\beta}\left(\mathbf{x}\right)$ that gives each point of field a position. It has to be a second order tensor in order to be consistent with rest of mathematical idea. That tensor can be rotated and this rotation is a wave field of gravitons [4]. I can split rotation into parts like before I will start with normal coordinates:

$$\sum_{i}\frac{1}{N_i^2}\,R^{\mu\nu}_{\alpha\beta}\left(\frac{1}{2}\alpha^{+}_{01,i},..,\frac{1}{2}\alpha^{+}_{\gamma\delta,i},\mathbf{x}\right)x^{\alpha\beta}\left(\mathbf{x}\right)=\psi^{\mu\nu}_{+,i}\left(\mathbf{x}\right)\Big|_{0}^{x^0} \tag{3.1}$$

$$\sum_{j}\frac{1}{N_j^2}\,R^{\mu\nu}_{\alpha\beta}\left(\frac{1}{2}\alpha^{-}_{01,j},..,\frac{1}{2}\alpha^{-}_{\gamma\delta,j},\mathbf{x}\right)x^{\alpha\beta}\left(\mathbf{x}\right)=\psi^{\mu\nu}_{-,j}\left(\mathbf{x}\right)\Big|_{-x^0}^{0} \tag{3.2}$$

Now i can write same relation but for perpendicular coordinates:

$$\sum_{k}\frac{1}{N_k^2}\,R^{\mu\nu}_{\alpha\beta}\left(\frac{1}{2}\alpha^{+}_{01,k},..,\frac{1}{2}\alpha^{+}_{\gamma\delta,k},\overline{\mathbf{x}}\right)x^{\alpha\beta}\left(\overline{\mathbf{x}}\right)=\psi^{\mu\nu}_{+,k}\left(\overline{\mathbf{x}}\right)\Big|_{0}^{\overline{x}^0} \tag{3.3}$$

$$\sum_{l}\frac{1}{N_l^2}\,R^{\mu\nu}_{\alpha\beta}\left(\frac{1}{2}\alpha^{-}_{01,l},..,\frac{1}{2}\alpha^{-}_{\gamma\delta,l},\overline{\mathbf{x}}\right)x^{\alpha\beta}\left(\overline{\mathbf{x}}\right)=\psi^{\mu\nu}_{-,l}\left(\overline{\mathbf{x}}\right)\Big|_{-\overline{x}^0}^{0} \tag{3.4}$$

To create a wave field i need to combine all those parts of field into one just by summing them, so i will get finally a wave field of graviton field:

$$\Psi^{\mu\nu}\left(\mathbf{x},\overline{\mathbf{x}}\right)=\psi^{\mu\nu}_{+,i}\left(\mathbf{x}\right)+\psi^{\mu\nu}_{-,j}\left(\mathbf{x}\right)+\psi^{\mu\nu}_{+,k}\left(\overline{\mathbf{x}}\right)+\psi^{\mu\nu}_{-,l}\left(\overline{\mathbf{x}}\right) \tag{3.5}$$

Position tensor says how objects move in field- each point of field points to some other point, so position changes from one point of field to another. Simplest case is when object have constant time-time direction only, they are in rest, then rotation created by gravitons makes them move. So graviton field is a field that rotates a position tensor so does it change trajectories of all particles moving in field. Graviton wave field itself depends on two sets of coordinates, first are normal ones , second are perpendicular coordinates to first ones. Each possible state of field has probability build into it and those probabilities need to sum to one:

$$\sum_{i}\sum_{j}\sum_{k}\sum_{l}\frac{1}{N_i^2}+\frac{1}{N_j^2}+\frac{1}{N_k^2}+\frac{1}{N_l^2}=1 \tag{3.6}$$

Those are same probabilities used in previous section. When i do measurement gravity field changes from all possible states to one state with probability gives by those numbers.

4. Antimatter and symmetry matrix

All normal matter moves forward in time, but there is a opposite matter that moves backwards in time: anti-matter [5]. Antimatter in this model literary moves backwards in time. So each particle will have an anti-particle even those ones in Standard model are each others own antiparticles [6]. Equations of motion will be same as for matter but coordinates will switch from normal axis to opposite axis, I can denote antimatter axis $\mathbf{x}^* = -\mathbf{x}$ and i will use this notation later on. So antimatter does not only move backwards in time but backwards in space, but to make sense of antimatter i need some kind of elementary particle model. All this idea is based on light cones parts and their rotation. From it comes that maximum spin [7] of particle can be four, if i have three dimension of space and one of time. I can write all possible states of light cone parts, i can have positive direction of light cone and negative, so there are four combinations of them that i can write as a matrix, a light cone symmetry matrix that is crucial to understanding elementary particles:

$$S_{nm} = \begin{pmatrix} S_0 & S_{-0} \\ S_1 & S_{-1} \\ S_2 & S_{-2} \\ S_3 & S_{-3} \end{pmatrix} \tag{4.1}$$

Where S numbers can be multiplication of one half positive and negative, each part represents future and past light cone , where I denote future as positive and past as negative. For maximum of spin four there can be four light cones each has it's time axis perpendicular to another. From this matrix state i can create all possible particles, but first there is need to explain one property of it, spin number is just absolute value of sum of matrix elements:

$$\sigma = \left| \sum_{n,m} S_{nm} \right| \tag{4.2}$$

Where for antimatter particles i will have opposite symmetry number, it means all states are reversed:

$$\sum_{n,m} S_{nm} = -\sum_{n,m} S^*_{nm} \tag{4.3}$$

So antimatter does not only move backwards in time it has opposite symmetry matrix elements. Both things are in truth same, if matter has positive light cones antimatter has negative so it moves in opposite directions.

5. Field equation for gravitons

There are fundamental relations in field that had to be obeyed. I can write whole space-time interval as combination of all four space-time parts of interval, and i add equality with second part of equation that connects energy in direction $\mu\nu$ and each rotation as four order mixed energy tensor T:

$$ds^2\left(\mathbf{x},\overline{\mathbf{x}}\right)=\sum_{i,j,k,l}\left(\frac{1}{N_i^2}+\frac{1}{N_j^2}+\frac{1}{N_k^2}+\frac{1}{N_l^2}\right)$$
$$\times R^{\mu\nu}_{\alpha\beta}\left(\frac{1}{2}\left(\alpha_{01,i}+\alpha_{01,j}+\alpha_{01,k}+\alpha_{01,l}\right),..,\mathbf{x},\overline{\mathbf{x}}\right)dx^\alpha dx^\beta\eta_{\mu\nu}\Big|_{-x^0,-\overline{x}^0}^{x^0,\overline{x}^0}$$
$$=\sum_{i,j,k,l}\left(\frac{1}{N_i^2}+\frac{1}{N_j^2}+\frac{1}{N_k^2}+\frac{1}{N_l^2}\right)\left(\delta^\mu_\alpha\delta^\nu_\beta dx^\alpha dx^\beta\eta_{\mu\nu}-T^{\mu\nu}_{\alpha\beta}dx^\alpha dx^\beta\eta_{\mu\nu}\right) \tag{5.1}$$

To get angles of rotation i need to solve this equation. This is equation for graviton but can be written for any spin particle, i can write wave tensor field of that graviton as:

$$\Psi^{\mu\nu}\left(\mathbf{x},\overline{\mathbf{x}}\right)=\sum_{i,j,k,l}\left(\frac{1}{N_i^2}+\frac{1}{N_j^2}+\frac{1}{N_k^2}+\frac{1}{N_l^2}\right)$$
$$\times R^{\mu\nu}_{\alpha\beta}\left(\frac{1}{2}\left(\alpha_{01,i}+\alpha_{01,j}+\alpha_{01,k}+\alpha_{01,l}\right),..,\mathbf{x},\overline{\mathbf{x}}\right)x^{\alpha\beta}\Big|_{-x^0,-\overline{x}^0}^{x^0,\overline{x}^0}$$
$$=\sum_{i,j,k,l}\left(\frac{1}{N_i^2}+\frac{1}{N_j^2}+\frac{1}{N_k^2}+\frac{1}{N_l^2}\right)\left(\delta^\mu_\alpha\delta^\nu_\beta x^{\alpha\beta}-T^{\mu\nu}_{\alpha\beta}x^{\alpha\beta}\right) \tag{5.2}$$

Field equation says how distance in space-time (metric tensor) rotated is connected to energy of that rotation. Solving it gives angles for rotation, so gives all possible unknowns in equation. I assume that for each summation on right hand-side of equation i will get same energy tensor value so what changes is only direction of that rotation. For any spin particle it will change only by factor of how many summations where for each one half spin i get one summation. Final spin of particle is sum of partials spins.

6. Massive and massless particles and energy tensor

There are only two types of particles in that model that are movement sort by, one are massless another ones are massive, one don't interact with Higgs field another one do, before i come to terms what Higgs field is in that model first i write relations for both particles and energy tensor. Position tensor for massive particles have only one component that is non zero that is time-time component:

$$\eta_{\alpha\beta}x^{\alpha\beta} = x^{00} \tag{6.1}$$

For massless particles i can write that there have to be time component minus space components equals zero:

$$\eta_{\alpha\beta}x^{\alpha\beta} = x^{00} - x^{11} - x^{22} - x^{33} = 0 \tag{6.2}$$

In both cases i used mostly minus metric signature [8] . Now from it i can move to energy tensor. In quantum mechanics there are co called virtual particles that carry forces when real particles interact [9], here they are present in form of energy tensor components. It has two terms with virtual particles, first term is how real matter particles interact with virtual ones and second is how virtual particles interact with them self. Virtual particles are always rotated $\frac{\pi}{2}$ radians of time axis of any particle in positive or negative direction. I will denote real particles part as R first virtual ones as V and second ones V' so i can write energy tensor equation as:

$$T^{\mu\nu}_{\alpha\beta} = \sum_{R}\sum_{V}\sum_{V'}\partial_{\alpha\beta}C^2_R S^{\mu\nu}_{(R)} + \tilde{\partial}_{\alpha\beta}\tilde{C}^2_{RV}\tilde{S}^{\mu\nu}_{(RV)} + \tilde{\partial}_{\alpha\beta}\tilde{C}^2_{VV'}\tilde{S}^{\mu\nu}_{(VV')} \tag{6.3}$$

Where i used constants $C_R, \tilde{C}_{RV}, \tilde{C}_{VV'}$ that are interaction strength constant that say how much compared to Planck energy interaction is, for real particles, real with virtual and virtual with virtual. I take how each term changes with respect to all directions and where i denote axis rotated by $\frac{\pi}{2}$ radians virtual particles axis and change by tilda notation. So virtual particles always go from one point to another with no time from point of view of observer emitting that particle. There is tensor that is main component of that equation that is symmetry tensor, that gives each point of space-time a symmetry value, where it can be thought as symmetry value in direction $\mu\nu$. This simple definition of energy tensor in term of light cones that comes from symmetry matrix, so it can be thought as symmetry matrix sum value tensor. Now when i have energy defined in terms of light cones value i can move to particles itself.

7. Elementary boson fields

Idea is that all elementary particles can be created out of symmetry matrix states. Most base field is Higgs field, I will write all bosons fields symmetry matrix states. First i write Higgs field as, where it can be at three possible states with signs so six in total:

$$H^0 = \begin{bmatrix} 0 & 0 \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ \mp\frac{1}{2} & \pm\frac{1}{2} \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ \mp\frac{1}{2} & \pm\frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ 0 & 0 \\ \mp\frac{1}{2} & \pm\frac{1}{2} \end{bmatrix} \tag{7.1}$$

Now i will move to simplest boson field- photon that is just in one state:

$$\gamma = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{7.2}$$

Now graviton field- that has more states, three:

$$G = \begin{bmatrix} 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{7.3}$$

Now i move to strong field and twenty-four states of gluons:

$$g = \begin{bmatrix} 0 & 0 \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{7.4}$$

$$\begin{bmatrix} 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \\ \pm\frac{1}{2} & \mp\frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ \pm\frac{1}{2} & \mp\frac{1}{2} \end{bmatrix} \tag{7.5}$$

$$\begin{bmatrix} 0 & 0 \\ \pm\frac{1}{2} & \frac{1}{2} \\ \mp\frac{1}{2} & \frac{1}{2} \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \pm\frac{1}{2} & \frac{1}{2} \\ \mp\frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ \pm\frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ \mp\frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{7.6}$$

$$\begin{bmatrix} 0 & 0 \\ \frac{1}{2} & \pm\frac{1}{2} \\ \frac{1}{2} & \mp\frac{1}{2} \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{2} & \pm\frac{1}{2} \\ \frac{1}{2} & \mp\frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ \frac{1}{2} & \pm\frac{1}{2} \\ 0 & 0 \\ \frac{1}{2} & \mp\frac{1}{2} \end{bmatrix} \tag{7.7}$$

And as last i get W and Z bosons so weak force:

$$W = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ \mp\frac{1}{2} & \pm\frac{1}{2} \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ \mp\frac{1}{2} & \pm\frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ 0 & 0 \\ \mp\frac{1}{2} & \pm\frac{1}{2} \end{bmatrix} \tag{7.8}$$

$$Z = \begin{bmatrix} 0 & 0 \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ \mp\frac{1}{2} & \pm\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ \mp\frac{1}{2} & \pm\frac{1}{2} \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ \mp\frac{1}{2} & \pm\frac{1}{2} \end{bmatrix} \tag{7.9}$$

Now i have all elementary boson fields [10], their interaction depends on energy tensor. A fermion particle will be combination of those fields. So i can write fermion field as superposition of bosons field states. For example i can write a simple neutrino state as:

$$S_{nm} = \begin{bmatrix} 0 & 0 \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ \mp\frac{1}{2} & \pm\frac{1}{2} \\ 0 & 0 \end{bmatrix} + \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \end{bmatrix} - i \begin{bmatrix} 0 & 0 \\ \pm\frac{1}{2} & \mp\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ \mp\frac{1}{2} & \pm\frac{1}{2} \end{bmatrix} \right) \tag{7.10}$$

It means that it interact with Higgs field, gravity field and weak field. If its summed it needs to get spin one half as result. In general i can write a field as:

$$S_{nm} = \sum_k c_k S_{nmk} \tag{7.11}$$

$$S_{nmij} = \sum_k \sum_r c_k c_r S_{nmk} S_{ijr} \tag{7.12}$$

Second part of field it can be re-written in tensor form as:

$$S^n_m = c_k S^{nk}_m \tag{7.13}$$

$$S^{ni}_{mj} = c_k c_r S^{nk}_m S^{ir}_j \tag{7.14}$$

And from it finally i can get tensor field equation by summing down indexes:

$$S^{\mu\nu} = \sum_m \sum_j c_k c_r \delta^\mu_n \delta^\nu_i S^{nk}_m S^i_{jr} \tag{7.15}$$

Constants c need to fulfil equation that their sum needs to be equal to one:

$$\sum_k \sum_r c_k c_r = 1 \tag{7.16}$$

References

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