Detection of the continuous gravitational wave of HM Cancri

Herbert Weidner, Am Stutz 3, D-63864 Glattbach, Germany [1]

HM Cancri is expected to be be one of the brightest sources of gravitational waves in our galaxy. Despite its known frequency, the radiation could not be detected so far. A novel technique can compensate for phase modulation and detect this CGW in the records of superconducting gravimeters beyond doubt. This novel observational window will allow a deeper understanding of the enigmatic stellar system.

1 Introduction

In the binary star system HM Cancri (RX J0806.3+1527), two white dwarfs orbit each other at a very close distance, generating a CGW of frequency $f_{CGW} = 6.22$ mHz, which is within the measurement range of superconducting gravimeters.

The reception frequency here on earth changes in annual rhythm, because the high orbital velocity around the sun causes a periodic Doppler shift. This manifests itself as a phase modulation of the signal with $f_{year} = 31.69$ nHz and is a reliable signature of any CGW. This modulation produces sidebands, which is why the signal appears in the spectrum no longer as an isolated line, but as a bundle of closely spaced lines (Figure 1). The modulation index η determines how many lines with which individual amplitudes can be expected. A typical spectrum hardly differs from noise and is difficult to identify. The method described below solves this problem without knowledge of the spectrum.

CGW can be successfully detected if the S/N is sufficiently high. The proportion $S/N \propto h_0\sqrt{T/B}$ provides hints on how to improve the S/N. h_0 means the amplitude of the CGW signal, T is the observation period, and B is the bandwidth of the receiver. Superconducting gravimeters have been recording minute changes in gravity for years and are therefore an excellent sources of long-term data. A reduction in bandwidth can only be achieved if phase modulation can be eliminated.

2 Frequency or phase modulation

In communications engineering, phase-modulated signals are used to transmit data. The method is so closely related to frequency modulation that the two are often confused. The advantage is that the respective sets of formulas can be converted into each other.

Although HM Cancri does not transmit data, we still receive a phase modulated signal because the earth moves in the CGW wave field. This phase modulation provides reliable evidence that the received signal is not generated in our solar system and also allows us to determine the direction to the source.

[1]eMail: herbertweidner@gmx.de

The phase modulation formulas require that the duration of the record spans several years. Then, the modulation frequency is equal to the reciprocal of the orbital period around the Sun and the spectrum does not consist of a single line, but it resembles a "picket fence" of many lines with mutual spacing $f_{year} = 31.69$ nHz (Fig 1). The following formula defines the modulation index η, which determines the amplitudes of the spectral lines:

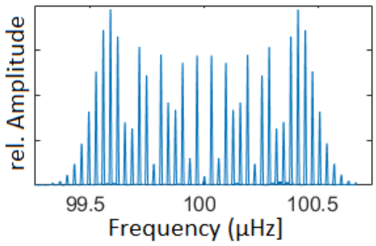

Figure 1): *The spectrum of a phase modulated oscillation of sufficiently long duration with $\eta = 10$ fills a wide band. The modulation frequency determines the spacing of the lines.*

$$\eta = \frac{\Delta f}{f_{MOD}} = \frac{\Delta f}{f_{year}} \tag{1}$$

The maximum frequency deviation Δf results from the relativistic Doppler effect due to the orbital velocity of the Earth. Knowing this value, it is possible to calculate the maximum frequency deviation Δf of the GW source HM Cancri, which is located almost in the plane of the ecliptic. On April 24 and October 27, the orbital velocities of the Earth are about the same and have the value 29626.5 ± 109 m/s [8]. This allows the Doppler shift to be calculated.

$$\Delta f = f_{CGW} \cdot \left(\sqrt{\frac{c + v_{Earth}}{c - v_{Earth}}} - 1 \right) \approx 614 nHz \tag{2}$$

The modulation index η reaches the surprisingly high value 20 and the corresponding spectrum claims the Carson bandwidth of 1.4 µHz. It does not seem very promising to search in the noise for a set of about 50 spectral lines with unknown amplitude distribution. In addition, for certain values of η the amplitude of the carrier frequency disappears (zeros of the Bessel function).

3 The modified superhet principle MSH

In radio engineering, a high receive frequency f_E is reduced to a lower value f_{IF} by mixing it with a locally generated frequency f_{OSZ}, because it can be investigated more advantageously. For the frequencies, $f_{IF} = |f_E - f_{OSZ}|$ is valid.

Usually, the value f_{OSZ} is constant in order not to change the modulation content of the signal. In the search for CGW, the opposite is true: one must remove the known but unwanted phase modulation in order to reduce the bandwidth. Therefore, one modulates the frequency f_{OSZ} with the goal of obtaining a *constant* diffence frequency f_{IF}. When the modulation of the received signal and the oscillator coincide, the "picket fence-like" spectrum turns into a single high-amplitude spectral line. Illustratively speaking: The many spectral lines that are adjacent to each other in Figure 1 are rearranged so that they add up to a large total length. Then the following statements are valid:

- With a broadband FM signal, the total energy is distributed over many spectral lines, each with low amplitudes. For an unmodulated signal, the total energy is concentrated on a single line of high amplitude.

- A constant frequency appears in the spectrum as an isolated line that can be easily and unambiguously identified in the noise.
- If it is possible to compensate the phase modulation completely, one may reduce the receive bandwidth strongly to improve the S/N further. In the present case, the bandwidth may be reduced by a factor of about 400.

Every CGW has two signatures: a slow frequency drift and phase modulation at 31.69 nHz. The additional phase modulation as a consequence of the intrinsic rotation of the Earth can hardly be detected because of the low velocity at the equator. Therefore, the approach (4) for the locally generated oscillation is limited to only two properties, which we discuss below.

4 The data basis of the investigations

Can gravitational waves be measured with gravimeters? Probably yes. Gravitational data have been recorded for decades and the IGETS Potsdam [1] stores correspondingly long data series. So far, the gravimeters have been used for earthquake research, are mounted directly on the ground, and therefore respond to distant, minute ground motions. Stronger earthquakes overload the sensors and cause data gaps of several minutes. These irregular tremors of the ground affect the quality of all data sets. Prior to release, the signals from the sensors pass through low-pass filters of cutoff frequency 16 mHz, which is why the data series from the gravimeters include the range in which the CGW from HM Cancri is expected.

The gravimeters are not identical and the data sets differ because the raw data are apparently processed by different methods: For a few, the noise floor in the frequency range of interest around 6 mHz is much too large, others contain data gaps that are too wide, and occasionally experiments were performed on the gravimeter during the recording period. Some data errors can be mitigated by high-pass filters.

The IGETS stores gravity data in a variety of formats. When searching for CGW in the frequency range above 1 mHz, one can ignore the influence of the variable air mass above the gravimeter and focus on the gapless data series in the ninth column. After concatenation to a single file, a high-pass filter removes the interfering tides. Last, we reduce the frequency from 6.22 mHz to 6 µHz and the bandwidth to 7 µHz in two steps. This does not change the modulation content of the signal (phase modulation and drift), but has the advantage that we can extend the sampling interval by a factor of 500. This shortens the computing time of the following iterations because the file lengths are reduced by the same factor.

5 First search experience

In the "raw state" no data set of the 17 gravimeters shows prominent spectral lines in the frequency range around 6.22 mHz which could be interpreted as CGW. But even if some lines would stand out in the noise, they could not be assigned to a clear cause due to the lack of a signature.

For this reason, a different method was chosen to look for signs of CGW: Neglecting the frequency drift of the CGW, the equation (4) contains only three selectable parameters: The frequency of the CGW and the two parameters η and φ, from which the direction to the source can be calculated.

$$y = sin(2\pi t f_{CGW} + \eta sin(2\pi t f_{Year} + \varphi)) \tag{3}$$

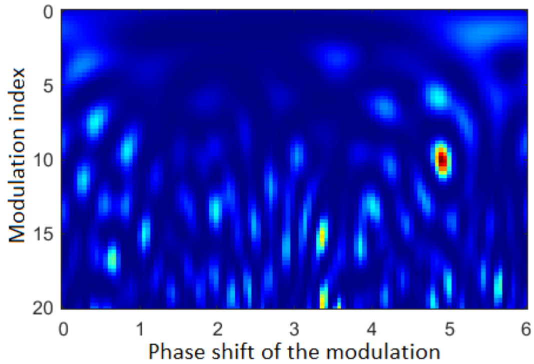

Figure 2): *A surprising intensity peak of the spectral line f_{IF}=5 ÂµHz after compensation of the sidebands as a function of η and φ*

It is tedious to scan the noise for signs of CGW with the constraints $0 < \eta < 20$ and $0 < \varphi < 2\pi$ and 6220 µHz $< f_{CGW} <$ 6221 µHz. Therefore, a full automated scan was programmed with step size $\Delta f = 1$ nHz, yielding a single prominent feature: for parameter values $\eta \approx 10$, $\varphi \approx 4.9$, and $f_{CGW} \approx 6220.64$ µHz, the spectrum shows a prominent maximum with S/N = 4 (Fig 2) in the period 2009 to 2018. No other parameter combination leads to a comparable good result, which can also be found in the data sets of other gravimeters. The value of η does not match the expected value for HM Cancri, but encouraged further investigation, proceeding in the following steps:

- The data recording of a low-noise superconducting gravimeter starts at 0 o'clock on a January 1 and ends ten years later. During this period, the spectrum of the 6220.64 µHz environment shows no abnormalities. This is also true for the data of other gravimeters.

- After reducing the frequency to 6 µHz by mixing with a *frequency-stable* oscillator, no abnormalities are detected in the spectrum of the intermediate frequency. So far the procedure corresponds to a superhet receiver of usual design.

- If the frequency-stable oscillator is replaced by a *phase-modulated* oscillator of the same frequency (modified superhet), the spectrum of the intermediate frequency changes fundamentally when the phase modulation meets the following criteria: The modulation frequency is 31.69 nHz, the modulation index has the value 10±0.1 and the phase shift has the value 4.9±0.1. Only then one observes a clear amplitude maximum at $f_{IF} = 1 \pm 0.001$ µHz (compare Fig 3).

The best explanation of this maximum is: the recordings of the gravimeter contain a phase-modulated signal of frequency 6220.64 µHz, whose spectrum looks similar to Fig 1. The broadband signal cannot be identified because no spectral line of the weak signal reaches the noise level of the gravimeters. As soon as the phase modulation of the MSH's auxiliary oscillator matches the corresponding values of the signal, everything changes: the previously wide spectrum (embedded in the noise of the intermediate frequency) contracts into a single line with high amplitude. This corresponds to a constant intermediate frequency whose intensity significantly exceeds the noise level. All parameters of the measured phase modulation indicate that the signal source is neither on Earth nor in the solar system. The values of the parameters rule out HM Cancri as the signal source. Although all parameters indicate an intense CGW, a more detailed investigation was deferred to focus on HM Cancri.

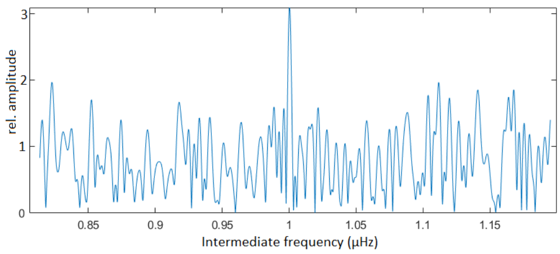

Figure 3): *Typical spectrum of the lowest intermediate frequency of the MSH. Originally, the total energy of the CGW is distributed over about 50 spectral lines; after compensation of the phase modulation, the total energy is concentrated on a single spectral line.*

6 Comparison of previous data

In 1999 the periodic changes of the X-ray emission of the stellar system HM Cancri were discovered [2] in the records of the satellite ROSAT. A first estimation gave the period duration 321.25 s ± 0.25 s. The system is hardly visible in the optical range, so measurements of the period are difficult. Further investigations followed:

- Initial evaluations of the observations in the X-ray region [3] yielded period lengths of 321.5393 ± 0.0004 s and 321.5465 ± 0.0004 s, respectively. This corresponds to GW frequencies of 6.220079474 mHz and 6.219940195 mHz. These 2001 results may have been obtained using short record lengths and should therefore be viewed with caution.

- From optical observations [4] the period 321.5304 s follows. In conjunction with ROSAT data the slightly different value 321.53033 s is calculated. The frequency of the radiated CGW is 6.220251647 mHz or 6.22025307 mHz.

- In the following year, the probably most exact value 3.11013824 mHz ± 0.17 nHz was determined from the Chandra data [5]. At that time the CGW had the frequency 6.22027648 mHz.

- Preliminary tests in the present work revealed a strikingly strong signal of frequency 6.22064 mHz in the records of superconducting gravimeters from 2009 to 2018, which could be a CGW. A comparison with known data rules out that it originates from HM Cancri.

From X-ray data, Strohmayer [5] obtained the value $\frac{df}{dt} = (3.63 \pm 0.06) \times 10^{-16} \frac{Hz}{s} = 11.5 \frac{nHz}{year}$ for the drift of the orbital frequency. For the CGW one has to double this value because $f_{CGW} = 2 \cdot f_{orbit}$.

7 Methodology of measurement of a CGW

The following investigations focus on the frequency range 6220 µHz to 6220.6 µHz. In this narrow search range, there are apparently several CGW that can be distinguished based on their phases and modulation indices. To identify the signals of HM Cancri, one needs accurate initial values. Every year on July 22, Earth - Sun - HMC lie approximately on a straight line. On this day and with sufficient S/N, one could measure the undistorted frequency of the CGW (The small ecliptic latitude 11^0 of HMC shall be neglected for the moment). Three months later, on October 27, the Earth is heading toward HMC at about 29 km/s. Six months later, on April 25, it moves away from HMC [8] at about the same speed. These dates of maximum blueshift or redshift of HMC can be checked with the following approach:

$$y = sin(2\pi t(f_{CGW} + t \cdot k_D) + \eta \cdot sin(2\pi t f_{year} + \varphi)) \tag{4}$$

The formula (4) contains all necessary parameters to describe the receivable CGW. The initial values [5] for HM Cancri are:
$f_{CGW} \approx$ 6,22 mHz (Frequency of CGW; depends on the year)
$k_D \approx$ 23 nHz per year
$\eta \approx$ 20 (Modulation index, follows from the ecliptic latitude of the source)
f_{Jahr} = 31,688 nHz (constant orbital frequency of the earth)
$\varphi \approx$ 1,37 (Gravimeter data recording start on January 1 of each year; astronomers use the vernal equinox of March 21 as a phase reference.)

Data basis were the records of 17 gravimeters, which are characterized by a low noise level in the vicinity of 6 mHz and were in operation almost without gaps in the period 1997 to 2020: BF1+2 (Germany), ST (France), BH (Germany), CO (Austria), DJ (Benin), MC (Italy), MO1+2 (Germany), PE (Czech Republic), SU1+2 (South Africa), OS (Sweden), WE (Germany), YS (Spain), CB (Australia), MB (Belgium).

Records from these sources are used to form 32 series, each starting on January 1 of a year between 1997 and 2012 and spanning ten years. As an aside, the expectation that the addition of data series from different gravimeters increases the amplitude of the CGW has also been confirmed. Each data series is treated in the same way:

1. The center frequency of a narrow range by 6220 µHz is reduced in two steps to the intermediate frequency 6 µHz. This corresponds to a superhet of usual type.

2. A phase modulated auxiliary oscillator reduces the intermediate frequency to 1 µHz. The frequency drift of this oscillator is freely selectable, but should be about

24 µHz per year. This stage corresponds to a modified superhet.

3. One iterates the phase φ, the modulation index η and the drift of the auxiliary oscillator until the amplitude of a spectral line near 1 µHz reaches a maximum. This causes the amplitudes of all other spectral lines of the phase modulated signal to decrease. Their energy flows into the central spectral line. A small frequency deviation ($\Delta f < 10$ nHz) is adjusted. A larger deviation indicates that the signal may not come from HM Cancri and requires a new iteration with changed parameters.

4. As soon as a reproducible result with high S/N is obtained, the parameters of the phase-modulated auxiliary oscillator match the characteristics of the CGW. These are tabulated with the initial date of the record.

8 Results

The formula (4) was applied to 32 records, yielding the following mean values: $\varphi = 1.319 \pm 0.030$, $\eta = 19.510 \pm 0.163$, and $k_{drift} = 24.356 \pm 0.279$ nHz per year. The error bars were calculated using the jackknife method.

The frequency drift agrees well with the slope calculated from the starting points alone (Fig 4) and the value given by Strohmayer [5].

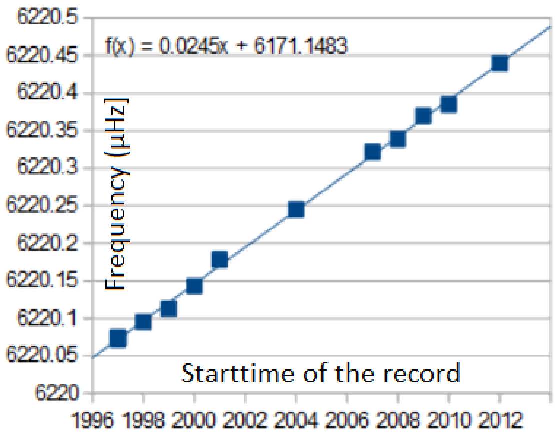

Figure 4): *The start frequencies of all 32 records as a function of the start date. The results differ so little that the differences are barely noticeable. The last data points extend till 2020. The frequency of the CGW increases by 24.5 nHz per year.*

From formula (1), the frequency deviation is calculated

$$\Delta f = \eta \cdot f_{year} = 618.224 \pm 5.165 \ nHz \tag{5}$$

This value corresponds approximately to the relativistic maximum value according to formula (2) and means that the source of CGW is very close to the plane of the ecliptic. The actual ecliptic latitude of HM Cancri is 11^0.

For an MSH, the parameters of the phase-modulated auxiliary oscillator are identical to the characteristics of the CGW, which remains hidden in the noise. The instantaneous frequency of the MSH can be determined at any instant from the spacing of adjacent zero crossings. The figure 5 shows the frequency changes of the auxiliary oscillator corresponding to the CGW. The value φ determines the timing of the extrema:

On every 105th day of a year the frequency of the oscillator is minimum, and on the 289th day of each year it is maximum (measured with gravimeters).

With Chandra one obtains for HM Cancri [8] the values: On every 110th day in the year the frequency of the CGW is minimum, on the 295th day of each year it is maximum. The agreement could not be better.

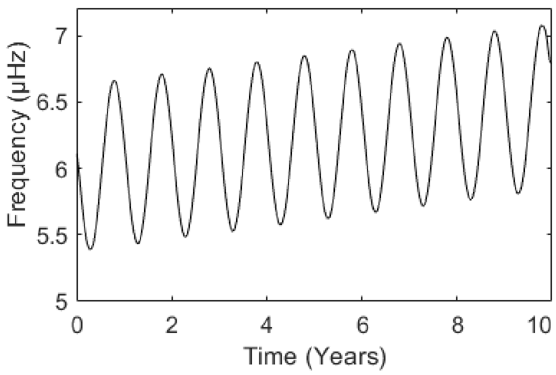

Figure 5): *The frequency of the auxiliary oscillator as a function of time. The frequency changes and the time points of the extrema apply unchanged also to the CGW. The average value of the CGW frequency is 6220 Âμ Hz and was reduced to 6 Âμ Hz before processing. (Principle of the superhet).*

9 Summary

In this work, it was demonstrated that:

- *Each* superconducting gravimeter is sufficiently sensitive and stable over time to measure the CGW of HM Cancri, although (currently) no instrument is acoustically isolated from the ground and therefore picks up numerous disturbances. This type of mounting needs improvement.
- This CGW is phase modulated with f_{year} and therefore has a broad spectrum of many individual lines which disappear in the noise. The CGW can only be detected if a) a multi-year record is chosen and b) the phase modulation is compensated.
- The modified superhet method MSH can compensate the phase modulation and amplify the central spectral line sufficiently to make the CGW measurable.
- The frequency and frequency drift of this measurement are in very good agreement with X-ray astronomy measurements [5] for the HM Cancri stellar system.
- MSH also provides the coordinates of the CGW source. Considering the frequent disturbances of the gravimeters by earthquakes, the agreement with the astronomical coordinates is very satisfactory.

10 Technical details of the data reduction

Superconducting gravimeters measure every second. The records are dominated by tides with frequencies around 11 µHz [6], whose amplitudes are at least a factor of 10^6 higher than the amplitude of the searched CGW. Prior to publication by IGETS, data gaps are filled by synthetic tidal waves, then an initial decimation by a factor of 60 is performed to reduce the data size. Attached data such as barometric pressure do not contain useful information to detect CGW with frequencies above about 1 mHz. Only at lower frequencies, the influence of the air mass must be taken into account.

The intermediate frequency must be higher than the Carson bandwidth of the CGW signal of about 1.4 µHz to process the FM signal without distortion (compare Fig 1). The value of the Carson bandwidth follows from the highest modulation index η given by the relativistic Doppler shift.

Using the commonly used superhet principle, the receive and oscillator frequencies differ by the value of the intermediate frequency : $f_{IF} = f_E \pm f_{OSZ}$. This ambiguity leads to the problem of the mirror frequency, which is foreign to the so-called IQ-procedure [7] and which was therefore used in all investigations. A more detailed account of the procedure is beyond the scope of this article.

The formula (4) is valid for a circular orbit of the earth. Due to the poor S/N of the gravitational data, the implementation of a more precise model did not lead to satisfactory results.

References

[1] IGETS Datenbank, GFZ Potsdam, 2021

[2] Israel G. L. et al., The discovery of 321 s pulsations in the ROSAT HRI light curves of J080622.8+152732 = RX J0806.3+1527, Astron. Astrophys. 349, L1–L4 (1999)

[3] Burwitz V., Reinsch K., 2001, in White N., Malaguti G., Palumbo G., eds, AIP Conf. Proc. Vol. 599. Am. Inst. Phys., New York, p. 522, arxiv.org/pdf/astro-ph/0410453.pdf

[4] Israel et al., Taking the pulse of the shortest orbital period binary system RX J0806.3+1527, Mem. S.A.It. Suppl. Vol. 5, 148, 2004

[5] Strohmayer Tod, Precision X-ray Timing of RX J0806.3+1527 with CHANDRA: Evidence for Gravitational Radiation from an Ultracompact Binary, 2005, arxiv.org/pdf/astro-ph/0504150.pdf

[6] Hartmann and Wenzel, Tidal potential catalogue, 1995

[7] D.K. Weaver, A third method of generation and detection of single-sideband signals, Proc. of IRE, pp. 903-5, 1956

[8] Calculate radial velocities, www.gb.nrao.edu/GBT/setups/radvelcalc.html