Unity Formula that connect the Fine Structure constant and the Proton to Electron Mass Ratio

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Abstract

In this paper will be presented the unity Formulas that connect the Fine Structure constant α and the Proton to Electron Mass Ratio μ. The equations are simple,elegant and symmetrical in a great physical meaning. At the beginning we will make a review of the last works. We will suggest the exact formula for the Fine Structure constant α with the Golden Angle and the Fifth Power of the Golden Mean and also we propose a simple and accurate expression for the Fine Structure constant α in terms of the Archimedes constant π. After we propose two exact mathematical expressions for the Proton to Electron Mass Ratio. It will be presented the mathematical Formula that connects the Proton to Electron Mass Ratio μ,the Fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro number N_A,the Gravitational coupling constant α_G for the electron and the gravitational coupling constant of proton $\alpha_{G(p)}$. Also we will find a new formula for the Avogadro number N_A and a new formula for Gravitational Constant G.

It will be explained that the product $\mu \cdot \alpha^{-1}$ is one of the roots of the following trigonometric equation:

$$2 \cdot 10^2 \cdot \cos(\mu \cdot \alpha^{-1}) + 13^2 = 0 \tag{1}$$

It will also be shown that another way to show this equation is the following exponential form of the equation:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + 13^2 = 0 \tag{2}$$

This exponential form can also be written with the form:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) = 13^2 \cdot e^{i\pi} \tag{3}$$

So the beautiful formula that connect the Fine Structure constant,the Proton to Electron Mass Ratio and the Fifth Power of the Golden Mean is:

$$5^2 \cdot (5 \cdot \varphi^{-2} + \varphi^{-5})^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + (5 \cdot \varphi^2 - \varphi^{-5})^2 = 0 \tag{4}$$

Also the formula that connect the Fine Structure constant,the Proton to Electron Mass Ratio and Mathematical constants π,φ,e,i is:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) = (5 \cdot \varphi^2 - \varphi^{-5})^2 \cdot e^{i\pi} \tag{5}$$

Also the unity Formula can also be written in the form:

$$10 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha})^{1/2} = 13 \cdot i \tag{6}$$

Finally we will show that the radio $\omega = (c/\upsilon_s)^2$ of the speed of light and the maximum speed of sound depends only from the Fine Structure constant α and not from the Proton to Electron Mass Ratio μ:

$$10^2 \cdot (e^{i\omega\alpha/2} + e^{-i\omega\alpha/2}) + 13^2 = 0 \tag{7}$$

All these equations are simple,elegant and symmetrical in a great physical meaning.

Keywords

Fine-Structure Constant, Proton to Electron Mass Ratio , Dimensionless physical constants , Maximum speed of sound , Speed of light , Trigonometric Functions, Exponential Function

1. Introduction

One of the most important numbers in physics is the Fine Structure constant ɑ which defines the strength of the electro-magnetic field. It is a dimensionless number independent of how we define our units of mass,length,time or electric charge. A change in these units of measurement leaves the dimensionless constant unchanged. The number can be seen as the chance that an electron emits or absorbs a photon. It's a pure number that shapes the universe to an astonishing degree. Paul Dirac considered the origin of the number «the most fundamental unsolved problem of physics». The constant is everywhere because it characterizes the strength of the electromagnetic force affecting charged particles such as electrons and protons.

Many eminent physicists and philosophers of science have pondered why ɑ itself has the value that it does, because the value shows up in so many important scenarios and aspects of physics. Nobody has come up with any ideas that are even remotely convincing. A similar situation occurs with the proton-electron mass ratio μ,not because of its ubiquity,but rather how chemistry can be based on two key electrically charged particles of opposite electric charge that are opposite but of seemingly identical magnitude while their masses have a ratio that is quite large yet finite. These two questions have a huge bearing on why physics and chemistry behave the way they do. The product of the two quantities appears,at least at first glance,not to be so important.

In Physics,the ratio of the mass of a proton to an electron is simply the remainder of the mass of the proton divided by that of the electron,from the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious ratio of mass between a proton and an electron. The values of me and mp,and the equilibrium between them,govern nuclear reactions such as the decay of protons and the nuclear synthesis of stars,leading to the formation of basic biochemical elements,including carbon. The space where stars and planets form and support life and molecular structures can appear. The mass ratio of protons to electrons,two constant particles that make up about 95% of the visible Universe,may be related to the total computational value of the Universe. Thus,as pure numbers they are supposed to be associated with prime numbers,entropy,binary and complexity.

The proton-to-electron mass ratio μ is a ratio of like-dimensioned physical quantities,it is a dimensionless quantity,a function of the dimensionless physical constants,and has numerical value independent of the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious mass ratio between the proton and electron. The numerical challenge of the mass ratio of proton to electron in the field of elementary particle physics began with the discovery of the electron by JJ Thomson in 1.897,and with the identification of the point nature of the proton by E. Rutherford in 1.911. These two particles have electric charges that are identical in size but opposite charges.

2. The Fibonacci numbers 5 and 13

Among the numbers in the Fibonacci range,the numbers 5 and 13 seem to be the most important. Whereas number 5 is involved in the definition of the golden mean,number 13 is found as a helix repetition number for instance in tubulin protein,thought to be the location from where our thinking and consciousness originates.

Five is the third smallest prime number and a Fermat prime. Therefore,a regular polygon with 5 sides is constructible with a compass and an unmarked straightedge. Five is the third Sophie Germain prime,the first safe prime,the third Catalan number,and the third Mersenne prime exponent. Five is the first Wilson prime and the third factorial prime,also an alternating factorial. Five is the first good prime. It is also the only number that is part of more than one pair of twin primes. Five is also a super-prime,and a congruent number. Five is also the only prime that is the sum of two consecutive primes,namely 2 and 3,with these indeed being the only possible set of two consecutive primes. The number 5 is the fifth Fibonacci number,being 2 plus 3. It is the only Fibonacci number that is equal to its position. Whereas 5 is unique in the Fibonacci sequence,in the Perrin sequence 5 is both the fifth and sixth Perrin numbers. The expressions between number 5 and the golden ratio φ is:

$$5=(2\cdot\varphi-1)^2=\varphi^6-8\cdot\varphi=\varphi^4-3\cdot\varphi^{-1}=3\cdot\varphi+\varphi^{-4}=8\cdot\varphi^{-1}+\varphi^{-6}=(\varphi^2+1)\cdot(\varphi^{-2}+1)=\varphi^2+\varphi^1+2\cdot\varphi^{-2} \quad (8)$$

The number 13 is the sixth prime number,a star number one of only three known Wilson primes,Fibonacci number,the third centered square number,a lucky number,the smallest number whose fourth power can be written as a sum of two consecutive square numbers (119^2+120^2),is the sum and the difference of 2 consecutive squares:

$13=(2^2+3^2)=(7^2-6^2)$

Since:

$5^2+12^2=13^2$

(5,12,13) forms a Pythagorean triple.

The expressions between number 13 and the golden ratio φ is:

$$13=5\cdot\varphi^2-\varphi^{-5}=(2+\varphi)^2-\varphi^{-5}=2+\varphi^5-\varphi^{-5}=\varphi^8-21\cdot\varphi=\varphi^6-8\cdot\varphi^{-1}=\varphi^{-6}+8\cdot\varphi=\varphi^{-8}+21\cdot\varphi^{-1} \quad (9)$$

3. Measurement of the Fine Structure constant

The 2.018 CODATA recommended value of α is:

$$\alpha=q_e^2/4\cdot\pi\cdot\varepsilon_0\cdot\hbar\cdot c=0.0072973525693(11) \quad (10)$$

With standard Uncertainty $0,0000000011\times10^{-3}$ and Relative Standard Uncertainty $1,5\times10^{10}$. For reasons of convenience,historically the value of the reciprocal of the fine-structure constant is often specified. The 2.018 CODATA recommended value is given by:

$$\alpha^{-1}=137,035999084(21) \quad (11)$$

With standard Uncertainty $0,000000021\times10^{-3}$ and Relative Standard Uncertainty $1,5\times10^{10}$. There is general agreement for the value of α,as measured by these different methods. The preferred methods in 2.019 are measurements of electron anomalous magnetic moments and of photon recoil in atom interferometry. The most precise value of α obtained experimentally (as of 2.012) is based on a measurement of g using a one-electron so-called "quantum cyclotron" apparatus,together with a calculation via the theory of QED that involved 12.672 tenth-order Feynman diagrams:

$$\alpha^{-1}=137,035999174(35) \quad (12)$$

This measurement of α has a relative standard uncertainty of $2,5\times10^{-10}$. This value and uncertainty are about the same as the latest experimental results. Further refinement of this work were published by the end of 2.020,giving the value:

$$\alpha^{-1}=137,035999206(11) \quad (13)$$

With a relative accuracy of 81 parts per trillion. We propose in [8] the exact formula for the Fine Structure constant α with the Golden Angle and the Fifth Power of the Golden Mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5} \quad (14)$$

with numerical value:

$\alpha^{-1}=137,03599916476564$..........

Also we propose in [11] a simple and accurate expression for the Fine Structure constant α in terms of the Archimedes constant π:

$$\alpha^{-1}=2\cdot 3\cdot 11\cdot 41\cdot 43^{-1}\cdot \ln 2\cdot \pi \tag{15}$$

with numerical value:

$\alpha^{-1}=137{,}03599907817552$..........

$\alpha=0{,}007297352569594$..........

4. Measurement of the Proton to Electron Mass Ratio

The 2.018 CODATA recommended value of the Proton to Electron Mass Ratio μ is:

$$\mu=mp/me=1.836{,}15267343 \tag{16}$$

With standard Uncertainty 0,00000011 and Relative Standard Uncertainty $6{,}0\times 10^{-11}$. The value of μ is known at about 0,1 parts per billion. The value of μ is a solution of the equation:

$3\cdot\mu^4-5.508\cdot\mu^3-841\cdot\mu^2+10\cdot\mu-2.111=0$

The 2.018 CODATA recommended value of μ^{-1} is:

$\mu^{-1}=me/mp=0{,}000544617021487$

With standard Uncertainty 0,000000000000033 and Relative Standard Uncertainty $6{,}0\times 10^{-11}$. We propose in [9] the exact equivalent mathematical expressions for the Proton to Electron Mass Ratio:

$$\mu=11^{47/32}\cdot 5^{5/2}\cdot 9.349^{5/76}\cdot\varphi^{-21/16} \tag{17}$$

$$\mu^{32}=\varphi^{-42}\cdot(\varphi^5-\varphi^{-5})^{47}\cdot(2\cdot\varphi-1)^{160}\cdot(\varphi^{19}-\varphi^{-19})^{40/19} \tag{18}$$

$$\mu^{32}=\varphi^{-42}\cdot F_5^{\,160}\cdot L_5^{\,47}\cdot L_{19}^{\,40/19} \tag{19}$$

with exact numerical value:

$\mu=1.836{,}15267343$……….

Also we propose in [9] the exact simply mathematical expression for the Proton to Electron Mass Ratio:

$$\mu^3=7^{-1}\cdot(5\cdot 13)^3\cdot[\ln(2\cdot 5)]^{11} \tag{20}$$

with numerical value:

$\mu=1836{,}152673929077$……...

We observe that in expression appears the Fibonacci numbers 5 and 13.

8. The Fine Structure constant and the Proton to Electron Mass Ratio connects other dimensionless physical constants

In physics,a dimensionless physical constant is a physical constant that is dimensionless,a pure number having no units attached and having a numerical value that is independent of whatever system of units may be used. The term fundamental physical constant is used to refer to some universal dimensionless constants. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values.

In the 1.920s and 1.930s,Arthur Eddington embarked upon extensive mathematical investigation into the relations between the fundamental quantities in basic physical theories,later used as part of his effort to construct an overarching theory unifying quantum mechanics and cosmological physics. The mathematician Simon Plouffe has made an extensive search of computer databases of mathematical formulas,seeking formulas for the mass ratios of the fundamental particles. An empirical relation between the masses of the electron,muon and tau has been discovered by physicist Yoshio Koide,but this formula remains unexplained.

We propose in [9] the mathematical Formulas that connects the Proton to Electron Mass Ratio μ,the fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro number N_A,the Gravitational coupling constant α_G for the electron and the gravitational coupling constant of proton $\alpha_{G(p)}$:

$$\alpha=\mu\cdot N_1\cdot \alpha_G \tag{21}$$

$$\alpha\cdot\mu=N_1\cdot \alpha_{G(p)} \tag{22}$$

$$\alpha^2=N_1{}^2\cdot\alpha_G\cdot\alpha_{G(p)} \tag{23}$$

$$\mu\cdot N_1=4\cdot e^2\cdot\alpha^3\cdot N_A{}^2 \tag{24}$$

$$2\cdot e\cdot\alpha\cdot\mu\cdot N_A\cdot\alpha_G{}^{1/2}=1 \tag{25}$$

$$4\cdot e^2\cdot\alpha\cdot\mu\cdot\alpha_G{}^2\cdot N_A{}^2\cdot N_1=1 \tag{26}$$

$$\mu^3=4\cdot e^2\cdot\alpha\cdot\alpha_{G(p)}{}^2\cdot N_A{}^2\cdot N_1 \tag{27}$$

$$\mu=2\cdot e\cdot\alpha_G{}^{1/2}\cdot\alpha_{G(p)}\cdot N_A\cdot N_1 \tag{28}$$

$$\mu=4\cdot e^2\cdot\alpha\cdot\alpha_G\cdot\alpha_{G(p)}\cdot N_A{}^2\cdot N_1 \tag{29}$$

And the exact Mathematical formulas that connect dimensionless physical constants:

$$2\cdot e\cdot(6\cdot 7\cdot\varphi/5^2\cdot e)^{13/2}\cdot N_A\cdot\alpha\cdot\alpha_G{}^{1/2}=1 \tag{30}$$

$$\mu\cdot N_1=4\cdot e^2\cdot(6\cdot 7\cdot\varphi/5^2\cdot e)^{13}\cdot\alpha^3\cdot N_A{}^2 \tag{31}$$

$$2\cdot e\cdot(6\cdot 7\cdot\varphi/5^2\cdot e)^{13/2}\cdot\alpha\cdot\mu\cdot N_A\cdot\alpha_G{}^{1/2}=1 \tag{32}$$

$$4\cdot e^2\cdot(6\cdot 7\cdot\varphi/5^2\cdot e)^{13}\cdot\alpha\cdot\mu\cdot\alpha_G{}^2\cdot N_A{}^2\cdot N_1=1 \tag{33}$$

$$\mu^3=4\cdot e^2\cdot(6\cdot 7\cdot\varphi/5^2\cdot e)^{13}\cdot\alpha\cdot\alpha_{G(p)}{}^2\cdot N_A{}^2\cdot N_1 \tag{34}$$

$$\mu=2\cdot e\cdot(6\cdot 7\cdot\varphi/5^2\cdot e)^{13/2}\cdot\alpha_G{}^{1/2}\cdot\alpha_{G(p)}\cdot N_A\cdot N_1 \tag{35}$$

A new formula for the Planck length $\ell_{p\ell}$:

$$\ell_{p\ell}=\alpha\cdot\alpha_0\cdot\alpha_G{}^{1/2} \tag{36}$$

New formulas for the Avogadro number N_A:

$$NA=[2\cdot e\cdot(6\cdot 7\cdot \varphi/5^2\cdot e)^{13/2}\cdot \alpha\cdot \alpha G^{1/2})]^{-1}\simeq(2\cdot e\cdot \alpha\cdot \alpha G^{1/2})^{-1} \tag{37}$$

New formulas for Gravitational Constant G:

$$G=[4\cdot e^2\cdot(6\cdot 7\cdot \varphi/5^2\cdot e)^{13}\cdot \alpha^2\cdot NA^2]^{-1}\cdot(\hbar\cdot c/me^2)\simeq(4\cdot e^2\cdot \alpha^2\cdot NA^2)^{-1}\cdot(\hbar\cdot c/me^2) \tag{38}$$

Also the exact mathematical formula that connects 6 dimensionless physical constants is:

$$\mu=4\cdot e^2\cdot(6\cdot 7\cdot \varphi/5^2\cdot e)^{13}\cdot \alpha\cdot \alpha G\cdot \alpha G(p)\cdot NA^2\cdot N1 \tag{39}$$

So:

$$\alpha\cdot \mu^{-1}\cdot \alpha G\cdot \alpha G(p)\cdot NA^2\cdot N1=2^{-15}\cdot 3^{-13}\cdot 5^{26}\cdot 7^{-13}\cdot \varphi^{-13}\cdot e^{11} \tag{40}$$

We observe that in expressions appears the Fibonacci numbers 5 and 13.

5. Roots of the trigonometric equation

We can solve trigonometric equation (1) using the following steps:

$2^3\cdot 5^2\cdot \cos(\mu\cdot \alpha^{-1})+13^2=0$

$2\cdot 10^2\cdot \cos(\mu\cdot \alpha^{-1})+13^2=0$

$2\cos(\mu\cdot \alpha^{-1})+(13\cdot 10^{-1})^2=0$

$\cos(\mu\cdot \alpha^{-1})=-2^{-1}\cdot(13\cdot 10^{-1})^2$

In mathematics,the inverse trigonometric functions or cyclometric functions are the inverse functions of the trigonometric functions. Specifically,they are the inverses of the sine,cosine,tangent,cotangent,secant,and cosecant functions,and are used to obtain an angle from any of the angle's trigonometric ratios. Inverse trigonometric functions are widely used in engineering,navigation,physics,and geometry. We apply the function arccosine to both sides of the equation:

$\arccos[\cos(\mu\cdot \alpha^{-1})]=\arccos[-(13\cdot 2^{-1}\cdot 10^{-1})^2]$

$\cos^{-1}[\cos(\mu\cdot \alpha^{-1})]=\cos^{-1}[-(13\cdot 2^{-1}\cdot 10^{-1})^2]^1$

$\mu\cdot \alpha^{-1}=\cos^{-1}[-2^{-1}\cdot(13\cdot 10^{-1})^2]$

All the possible solutions of Equation is:

$\mu\cdot \alpha^{-1}=2\cdot k\cdot \pi+\cos^{-1}[-2^{-1}\cdot(13\cdot 10^{-1})^2]$

$\mu\cdot \alpha^{-1}=2\cdot k\cdot \pi-\cos^{-1}[-2^{-1}\cdot(13\cdot 10^{-1})^2]$

Being k all the integer numbers.

$\mu\cdot \alpha^{-1}=2\cdot k\cdot \pi+2{,}5773614938622113648$

$\mu\cdot\alpha^{-1}=2\cdot k\cdot\pi-2{,}5773614938622113648$

That for k=40.046,we have:

$\mu\cdot\alpha^{-1}=2\cdot 40.046\cdot\pi+2{,}5773614938622113648$

$\mu\cdot\alpha^{-1}=25.1619{,}0161728075822663$ (41)

So the product $\mu\cdot\alpha^{-1}$ is one of the roots of the trigonometric equation (1).

6. Comparison of the Root of the trigonometric equation with the Measured Value of the Fine Structure-Constant and the Proton to Electron Mass Ratio

We have calculated that the roots of the equation:

$2\cos(\mu/\alpha)+(13/10)^2=0$

$2^3\cdot\cos(\mu\cdot\alpha^{-1})+(13/5)^2=0$

$\cos(\mu\cdot\alpha^{-1})=-(13\cdot 2^{-1}\cdot 10^{-1})^2$

can be found using the following:

$\mu\cdot\alpha^{-1}=2\cdot k\cdot\pi+\cos^{-1}[-2^{-1}\cdot(13\cdot 10^{-1})^2]$

For the specific case when k=40.046,we get the root:

$\mu\cdot\alpha^{-1}=2\cdot 40.046\cdot\pi+2{,}5773614938622113648$

From the 2.018 CODATA recommended value of the Proton to Electron Mass Ratio $\mu=1.836{,}15267343$ and the 2.018 CODATA recommended value of $\alpha^{-1}=137{,}035999084$ we have:

$\mu\cdot\alpha^{-1}=25.1619{,}01607423763113812$ (42)

If we calculate the difference between the value (41) and the value (42) we have:

$\text{Difference}=[\mu\cdot\alpha^{-1}(42)-\mu\cdot\alpha^{-1}(41)]/\mu\cdot\alpha^{-1}(42)=3{,}92\cdot 10^{-10}$

This difference is so small that we consider the equation (1) to be exact.

7. Exponential Form of the Equation

In mathematics,the trigonometric functions are real functions which relate an angle of a right-angled triangle to ratios of two side lengths. They are widely used in all sciences that are related to geometry,such as navigation,solid mechanics,celestial mechanics,geodesy,and many others. They are among the simplest periodic functions,and as such are also widely used for studying periodic phenomena through Fourier analysis.

The cosine function is one of the basic functions encountered in trigonometry. The cosine function is simply denoted as cosx,where x is the angle. The ratio of the lengths of the side adjacent to the angle and the hypotenuse of a right-angled triangle is called the cosine function which varies as the angle varies. It is defined in the context of a right-angled triangle for acute angles. The cosine function is used to model many real-life scenarios–radio waves,tides,sound waves,musical tones,electrical currents. The cosine function cosx is a periodic function and has a period of $2\cdot\pi$. Euler's formula relates sine and cosine to the exponential functions:

$e^{iz}=\cos(z)+i\cdot\sin(z)$

$e^{-iz}=\cos(z)-i\cdot\sin(z)$

where e is the base of the natural logarithm and i is the imaginary number.
So:

$e^{i\mu/\alpha}=\cos(\mu\cdot\alpha^{-1})+i\cdot\sin(\mu\cdot\alpha^{-1})$

Solving this linear system in sine and cosine,one can express them in terms of the exponential function:

$\sin(z)=(e^{iz}-e^{-iz})/2\cdot i$

$\cos(z)=(e^{iz}+e^{-iz})/2$

So:

$\cos(\mu\cdot\alpha^{-1})=(e^{i\mu/\alpha}+e^{-i\mu/\alpha})/2$

$\cos(\mu\cdot\alpha^{-1})=-2^{-1}\cdot(13\cdot10^{-1})^2$

$(e^{i\mu/\alpha}+e^{-i\mu/\alpha})/2=-2^{-1}\cdot(13\cdot10^{-1})^2$

$e^{i\mu/\alpha}+e^{-i\mu/\alpha}=-2^{1}\cdot2^{-1}\cdot(13\cdot10^{-1})^2$

$e^{i\mu/\alpha}+e^{-i\mu/\alpha}=-(13\cdot10^{-1})^2$

$e^{i\mu/\alpha}+e^{-i\mu/\alpha}+(13\cdot10^{-1})^2=0$

$e^{i\mu/\alpha}+e^{-i\mu/\alpha}+(13/10)^2=0$

$$10^2\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^2=0 \qquad (43)$$

This exponential Form has the same root with the equation (1). The expressions (1) and (2) are very important because they show us that the Fine Structure constant α depends on the Proton to Electron Mass Ratio μ.
Also this unity Formula can also be written in the form:

$$10\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})^{1/2}=13\cdot i \qquad (44)$$

8. Unity Formula that connect the Fine Structure constant,the Proton to Electron Mass Ratio and the Fifth Power of the Golden Mean:

The ancient Greek mathematicians studied for the first time the golden ratio φ,due to its frequent appearance in geometry. The golden ratio is an omnipresent number in nature,found in the architecture of living creatures as well as human buildings, music, finance, medicine, philosophy, and of course in physics and mathematics including quantum computation. It is the most irrational number known with the simplest continued fraction representation at all and a number-theoretical chameleon with a self-similarity property. All these properties render it to be suitable for quantum computer application.

The number 2 can also be written in the form:

$2=5\cdot\varphi^{-2}+\varphi^{-5}$

From (9) the number 13 can also be written in the form:

$13=5\cdot\varphi^{2}-\varphi^{-5}$

So from the equation (2) we have:

$10^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^{2}=0$

$5^{2}\cdot 2^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^{2}=0$

So the beautiful unity formula that connect the Fine Structure constant,the Proton to Electron Mass Ratio and the Fifth Power of the Golden Mean is:

$$5^{2}\cdot(5\cdot\varphi^{-2}+\varphi^{-5})^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+(5\cdot\varphi^{2}-\varphi^{-5})^{2}=0 \tag{45}$$

9. Unity Formula that connect the Fine Structure constant,the Proton to Electron Mass Ratio and Mathematical constants π,φ,e,i

The most important Mathematical constants are the Archimedes constant π,the golden ratio φ,the Euler's number e and the imaginary number i. The following expression connects six basic mathematical constants,the number 0,the number 1,golden ratio φ,Archimedes constant π,Euler's number e and imaginary unit i:

$$e^{i\pi/1+\varphi}+e^{-i\pi/1+\varphi}+e^{i\pi/\varphi}+e^{-i\pi/\varphi}=0 \tag{46}$$

From the equation (2) we have:

$10^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^{2}=0$

$10^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})=-13^{2}$

$$10^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})=13^{2}\cdot e^{i\pi} \tag{47}$$

So the unity Formula that connect the Fine Structure constant,the Proton to Electron Mass Ratio and Mathematical constants π,φ,e,i is:

$$10^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})=(5\cdot\varphi^{2}-\varphi^{-5})^{2}\cdot e^{i\pi} \tag{48}$$

Also this unity Formula can also be written in the form:

$$[5\cdot(5\cdot\varphi^{-2}+\varphi^{-5})]^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})=(5\cdot\varphi^{2}-\varphi^{-5})^{2}\cdot e^{i\pi} \tag{49}$$

10. The radio of the speed of light and the speed of sound

Sound or light waves are disturbances that transfer energy from one place to another. Sound waves can travel in different media,such as air or water,and move at different speeds depending on the medium in which they travel. The speed of sound is lower in gases,higher in liquids and even higher in solids. For example,the speed of sound in air is about 340 m/s,in water about 1.500 m/s and in iron over 5.000 m/s. These differences are due to the way a wave is disturbed during its propagation by atoms or molecules of the elastic medium. The more elastic the bonds between the molecules of the medium,the slower the speed of sound propagation. Therefore in solids the propagation velocity is greater than the diffusion velocity in liquids. In solids the propagation velocity is proportional to the square root of the measure of elasticity Y to the density of the medium p. The absolute speed limit at which a wave can travel,which is the speed of light in vacuum,is approximately equal to 300.000 km/s. However it was not known whether sound waves also have a higher speed limit when traveling through solids or liquids.

Researchers in the United Kingdom and Russia [14] have calculated that the maximum speed of sound is twice the speed of sound measured to date. The maximum velocity that sound can reach is about 36,1 km/s. This can happen in metallic hydrogen. The two basic physical constants without dimension,the Fine Structure constant α and the Proton to Electron Mass Ratio μ,play an important role in understanding our Universe. Among the fundamental constants,those that are dimensionless and do not depend on the choice of units play a special role in physics. Researchers show that a simple combination of α and μ leads to another dimensionless quantity that has an unexpected and specific effect on a basic property of concentrated phases,the speed at which waves travel in solids and liquids,the speed of sound:

$$\upsilon s/c=\alpha\cdot(1/2\cdot\mu)^{1/2} \tag{50}$$

c the speed of light in a vacuum and
υs the maximum speed of sound.

We pose:

$\omega=(c/\upsilon s)^2$

So from (49) we have:

$\upsilon s/c=\alpha\cdot(1/2\cdot\mu)^{1/2}$

$(\upsilon s/c)^2=\alpha^2/2\cdot\mu$

$(c/\upsilon s)^2=2\cdot\mu/\alpha^2$

$\omega=2\cdot\mu/\alpha^2$

$\mu\cdot\alpha^{-1}=\omega\cdot\alpha/2$

So from (1) we have:

$2\cdot10^2\cdot\cos(\mu\cdot\alpha^{-1})+13^2=0$

$$2\cdot10^2\cdot\cos(\omega\cdot\alpha/2)+13^2=0 \tag{51}$$

Also from (2) we have:

$10^2\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^2=0$

$$10^2\cdot(e^{i\omega\alpha/2}+e^{-i\omega\alpha/2})+13^2=0 \tag{52}$$

The expressions (51) and (52) are very important because they show us that the radio of the speed of light and the maximum speed of sound c/υs depends only from the Fine Structure constant α and not from the Proton to Electron Mass Ratio μ.

12. Conclusions

In this paper was presented the Unity Formula that connects the Fine Structure constant α and the Proton to Electron Mass Ratio μ. We suggest the exact formula for the Fine Structure constant α with the Golden Angle and the Fifth Power of the Golden Mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5}$$

Also we propose a simple and accurate expression for the Fine Structure constant α in terms of the Archimedes constant π:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\ln2\cdot\pi$$

After propose two exact mathematical expressions for the Proton to Electron Mass Ratio:

$$\mu^{32}=\varphi^{-42}\cdot F_5^{160}\cdot L_5^{47}\cdot L_{19}^{40/19}$$

$$\mu^{3}=7^{-1}\cdot(5\cdot13)^{3}\cdot[\ln(2\cdot5)]^{11}$$

It was presented the mathematical formulas that connects the Proton to Electron Mass Ratio μ,the Fine structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro number N_A,the Gravitational coupling constant α_G for the electron and the gravitational coupling constant of proton $\alpha_{G(p)}$:

$$\alpha=\mu\cdot N1\cdot\alpha G$$

$$\alpha\cdot\mu=N1\cdot\alpha G(p)$$

$$\alpha^{2}=N1^{2}\cdot\alpha G\cdot\alpha G(p)$$

$$1=2\cdot e\cdot(6\cdot7\cdot\varphi/5^{2}\cdot e)^{13/2}\cdot NA\cdot\alpha\cdot\alpha G^{1/2}\approxeq2\cdot e\cdot NA\cdot\alpha\cdot\alpha G^{1/2}$$

$$\mu\cdot N1=4\cdot e^{2}\cdot(6\cdot7\cdot\varphi/5^{2}\cdot e)^{13}\cdot\alpha^{3}\cdot NA^{2}\approxeq4\cdot e^{2}\cdot\alpha^{3}\cdot NA^{2}$$

$$1=2\cdot e\cdot(6\cdot7\cdot\varphi/5^{2}\cdot e)^{13/2}\cdot\alpha\cdot\mu\cdot NA\cdot\alpha G^{1/2}\approxeq2\cdot e\cdot\alpha\cdot\mu\cdot NA\cdot\alpha G^{1/2}$$

$$1=4\cdot e^{2}\cdot(6\cdot7\cdot\varphi/5^{2}\cdot e)^{13}\cdot\alpha\cdot\mu\cdot\alpha G^{2}\cdot NA^{2}\cdot N1\approxeq4\cdot e^{2}\cdot\alpha\cdot\mu\cdot\alpha G^{2}\cdot NA^{2}\cdot N1$$

$$\mu^{3}=4\cdot e^{2}\cdot(6\cdot7\cdot\varphi/5^{2}\cdot e)^{13}\cdot\alpha\cdot\alpha G(p)^{2}\cdot NA^{2}\cdot N1\approxeq4\cdot e^{2}\cdot\alpha\cdot\alpha G(p)^{2}\cdot NA^{2}\cdot N1$$

$$\mu=2\cdot e\cdot(6\cdot7\cdot\varphi/5^{2}\cdot e)^{13/2}\cdot\alpha G^{1/2}\cdot\alpha G(p)\cdot NA\cdot N1\approxeq2\cdot e\cdot\alpha G^{1/2}\cdot\alpha G(p)\cdot NA\cdot N1$$

Also the exact mathematical formula that connects 6 dimensionless physical constants is:

$$\mu=4\cdot e^{2}\cdot(6\cdot7\cdot\varphi/5^{2}\cdot e)^{13}\cdot\alpha\cdot\alpha G\cdot\alpha G(p)\cdot NA^{2}\cdot N1\approxeq4\cdot e^{2}\cdot\alpha\cdot\alpha G\cdot\alpha G(p)\cdot NA^{2}\cdot N1$$

So:

$$\alpha\cdot\mu^{-1}\cdot\alpha G\cdot\alpha G(p)\cdot NA^{2}\cdot N1=2^{-15}\cdot3^{-13}\cdot5^{26}\cdot7^{-13}\cdot\varphi^{-13}\cdot e^{11}$$

We observe that in expression appears the Fibonacci numbers 5 and 13. A new formula for the Planck length $\ell_{p\ell}$:

$$\ell_{p\ell}=\alpha\cdot\alpha0\cdot\alpha G^{1/2}$$

New formulas for the Avogadro number NA:

$$NA=[2\cdot e\cdot(6\cdot 7\cdot \varphi/5^2\cdot e)^{13/2}\cdot \alpha\cdot \alpha G^{1/2})]^{-1}\approx(2\cdot e\cdot \alpha\cdot \alpha G^{1/2})^{-1}$$

New formulas for Gravitational Constant G:

$$G=[4\cdot e^2\cdot(6\cdot 7\cdot \varphi/5^2\cdot e)^{13}\cdot \alpha^2\cdot NA^2]^{-1}\cdot(\hbar\cdot c/me^2)\approx[4\cdot e\cdot \alpha^2\cdot NA^2]^{-1}\cdot(\hbar\cdot c/me^2)$$

It was explained that the $\mu\cdot\alpha^{-1}$ is one of the roots of the following trigonometric equation:

$$2\cdot 10^2\cdot \cos(\mu\cdot\alpha^{-1})+13^2=0$$

The exponential form of this equation is:

$$10^2\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^2=0$$

This exponential form can also be written with the beautiful form:

$$10^2\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})=13^2\cdot e^{i\pi}$$

Also this unity formula can also be written in the form:

$$10\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})^{1/2}=13\cdot i$$

So other beautiful formula that connect the Fine Structure constant,the Proton to Electron Mass Ratio and the Fifth Power of the Golden Mean is:

$$5^2\cdot(5\cdot\varphi^{-2}+\varphi^{-5})^2\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+(5\cdot\varphi^2-\varphi^{-5})^2=0$$

Also the formula that connect the Fine Structure constant,the Proton to Electron Mass Ratio and Mathematical constants π,φ,e,i is:

$$10^2\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})=(5\cdot\varphi^2-\varphi^{-5})^2\cdot e^{i\pi}$$

Also this unity Formula can also be written in the form:

$$10\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})^{1/2}=13\cdot i$$

The expressions are very important because they show us that the Fine Structure constant α depends on the Proton to Electron Mass Ratio μ.

Finally we showed that the radio $\omega=(c/us)^2$ of the speed of light and the maximum speed of sound depends only from the Fine Structure constant α and not from the Proton to Electron Mass Ratio μ:

$$10^2\cdot(e^{i\omega\alpha/2}+e^{-i\omega\alpha/2})+13^2=0$$

All these equations are simple,elegant and symmetrical in a great physical meaning.

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