Exact mathematical Formula that connect 6 dimensionless physical constants

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Abstract

In this paper are a new formula for the Planck length $\ell_{p\ell}$ and a new formula for the Avogadro number N_A. Also 9 Mathematical formulas that connect dimensionless physical constants. The 6 dimensionless physical constants are the Proton to Electron Mass Ratio μ,the Fine-structure constant α,the ratio N_1 of electric force to gravitational force between electron and proton,the Avogadro number N_A,the Gravitational coupling constant α_G for the electron and the gravitational coupling constant $\alpha_{G(p)}$ of proton. After a new formula for Gravitational Constant G and new exact formula for the Avogadro number N_A. Finally 8 exact Mathematical formulas that connect 6 dimensionless physical constants and a new exact formula for Gravitational Constant G.

1. Introduction

In physics,a dimensionless physical constant is a physical constant that is dimensionless,a pure number having no units attached and having a numerical value that is independent of whatever system of units may be used. The term fundamental physical constant is used to refer to some universal dimensionless constants. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values.

In the 1.920s and 1.930s,Arthur Eddington embarked upon extensive mathematical investigation into the relations between the fundamental quantities in basic physical theories,later used as part of his effort to construct an overarching theory unifying quantum mechanics and cosmological physics. The mathematician Simon Plouffe has made an extensive search of computer databases of mathematical formulas,seeking formulas for the mass ratios of the fundamental particles. An empirical relation between the masses of the electron,muon and tau has been discovered by physicist Yoshio Koide,but this formula remains unexplained.

2. Measurement of the 6 dimensionless physical constants

2.1. Measurement of the Fine Structure constant

The 2.018 CODATA recommended value of α is:

$$\alpha = q_e^2/4\cdot\pi\cdot\varepsilon_0\cdot\hbar\cdot c = 0.0072973525693(11)$$

With standard Uncertainty $0,0000000011\times10^{-3}$ and Relative Standard Uncertainty $1,5\times10^{10}$. For reasons of convenience,historically the value of the reciprocal of the fine-structure constant is often specified. The 2.018 CODATA recommended value is given by:

$$\alpha^{-1} = 137,035999084(21)$$

With standard Uncertainty $0,000000021\times10^{-3}$ and Relative Standard Uncertainty $1,5\times10^{10}$. There is general agreement for the value of α,as measured by these different methods. The preferred methods in 2.019 are measurements of electron anomalous magnetic moments and of photon recoil in atom interferometry. The most precise value of α obtained experimentally (as of 2.012) is based on a measurement of g using a one-electron so-called "quantum cyclotron" apparatus,together with a calculation via the theory of QED that involved 12.672 tenth-order Feynman diagrams:

α^{-1}=137,035999174(35)

This measurement of α has a relative standard uncertainty of $2,5\times10^{-10}$. This value and uncertainty are about the same as the latest experimental results. Further refinement of this work were published by the end of 2.020,giving the value:

α^{-1}=137,035999206(11)

with a relative accuracy of 81 parts per trillion.

2.2. Measurement of the Proton to Electron Mass Ratio

The 2.018 CODATA recommended value of the Proton to Electron Mass Ratio μ is:

μ=1.836,15267343

With standard Uncertainty 0,00000011 and Relative Standard Uncertainty $6,0\times10^{-11}$. The value of μ is known at about 0,1 parts per billion. The value of μ is a solution of the equation:

$3\cdot\mu^4-5.508\cdot\mu^3-841\cdot\mu^2+10\cdot\mu-2.111=0$

The 2.018 CODATA recommended value of μ^{-1} is:

μ^{-1}=me/mp=0,000544617021487

With standard Uncertainty 0,000000000000033 and Relative Standard Uncertainty $6,0\times10^{-11}$.

2.3. Measurement of the Gravitational coupling constant αG for the electron

In physics,the gravitational coupling constant αG is a constant that characterizes the gravitational pull between a given pair of elementary particles. For the electron pair this constant is denoted by αG. The choice of units of measurement,but only with the choice of particles. The gravitational coupling constant αG is a scaling ratio that can be used to compare similar unit values from different scaling systems (Planck scale,atomic scale,and cosmological scale). The gravitational coupling constant can be used for comparison of length,range and force values. The gravitational coupling constant αG is defined as:

$\alpha G=G\cdot me^2/\hbar\cdot c$

There is so far no known way to measure αG directly. The value of the constant gravitational coupling αG is only known in four significant digits. The approximate value of the constant gravitational coupling αG is:

$\alpha G \simeq 1,751809945728109\times10^{-45}$

2.4. Measurement of the Gravitational coupling constant αG(p) for the electron

The gravitational coupling constant for the proton is produced similar to the electron,but replaces the mass of electrons with the mass of the protons. The gravitational coupling constant of the proton αG(p) is defined as:

$\alpha_{G(p)}=G\cdot m_p^2/\hbar\cdot c$

The approximate value of the constant gravitational coupling of the proton $\alpha_{G(p)}$ is:

$\alpha_{G(p)}\approx 5{,}906151273795571\text{x}10^{-39}$

2.5. Measurement of the ratio N1 of electric force to gravitational force between electron and proton

The enormous value of the ratio of electric force to gravitational force was first pointed out by Bergen Davis in 1.904. But Weyl and Eddington suggested that the number was about 10^{40} and was related to cosmological quantities. The electric force F_c between electron and proton is defined as:

$F_c=q_e^2/4\cdot\pi\cdot\varepsilon_0\cdot r^2$

The gravitational force F_g between electron and proton is defined as:

$F_g=G\cdot m_e\cdot m_p/r^2$

So:

$N_1=F_c/F_g$

$N_1=q_e^2/4\cdot\pi\cdot\varepsilon_0\cdot G\cdot m_e\cdot m_p$

$N_1=k_e\cdot q_e^2/G\cdot m_p\cdot m_e$

$N_1=\alpha\cdot\hbar\cdot c/G\cdot m_p\cdot m_e$

The approximate value of the ratio N1 of electric force to gravitational force between electron and proton is:

$N_1\approx 2{,}26866072471432\text{x}10^{39}$

2.6. Measurement of Avogadro number NA

The most accurate definition of the Avogadro number N_A value involves the change in molecular quantities and,in particular,the change in the value of an elementary charge. The exact value of the Avogadro number N_A is:

$N_A=6{,}02214076\times 10^{23}$

The value of the Avogadro number N_A can also be written in numerical expressions:

$N_A\approx 2^{79}=6{,}044629098\times 10^{23}$

$N_A=84.446.885^3=6{,}02214076\times 10^{23}$

3. New Formulas

3.1. New Formula for the Planck length $\ell_{p\ell}$

The Bohr radius a_0 is defined as:

$\alpha_0 = \hbar / \alpha \cdot m_e \cdot c$

The reduced Planck constant ħ is:

$\hbar = \alpha \cdot m_e \cdot \alpha_0 \cdot c$

So:

$\hbar^2 = \alpha^2 \cdot m_e^2 \cdot \alpha_0^2 \cdot c^2$

So:

$(\hbar \cdot G/c^3) = \alpha^2 \cdot m_e^2 \cdot \alpha_0^2 \cdot (G \cdot m_e^2/\hbar \cdot c)$

So:

$(\hbar \cdot G/c^2) = \alpha^2 \cdot \alpha_0^2 \cdot (G \cdot m_e^2/\hbar \cdot c)$

So:

$\ell_{p\ell}^2 = \alpha^2 \cdot \alpha_0^2 \cdot \alpha_G$

So the new formula for the Planck length $\ell_{p\ell}$ is:

$$\ell_{p\ell} = \alpha \cdot \alpha_0 \cdot \alpha_G^{1/2} \tag{1}$$

3.2. New formula for the Avogadro number N_A

Jeff Yee in his paper «The Relationship of the Mole and Charge» [10],the mole and charge are related by deriving Avogadro's number from three constants,the Bohr radius,the Planck length and Euler's number. The Avogadro number N_A can be calculated from the Planck length $\ell_{p\ell}$,the Bohr radius α_0 and Euler's number e:

$N_A = \alpha_0 / 2 \cdot e \cdot \ell_{p\ell}$

So:

$N_A = \alpha_0 / 2 \cdot e \cdot \alpha \cdot \alpha_0 \cdot \alpha_G^{1/2}$

So the new Formula for the Avogadro number N_A is:

$$N_A = (2 \cdot e \cdot \alpha \cdot \alpha_G^{1/2})^{-1} \tag{2}$$

So:

$$2 \cdot e \cdot N_A \cdot \alpha \cdot \alpha_G^{1/2} = 1 \tag{3}$$

4. Mathematical Formulas that connects dimensionless physical constants

4.1. Mathematical Formulas that connects 3 dimensionless physical constants

The exact mathematical formula that connects the mass ratio of proton to electron,the fine-structure constant α and the proton-proton gravitational coupling constant $\alpha G(pp)$ is:

$\alpha^7=\mu^7\cdot[\alpha G(pp)\cdot \log 2(2\cdot\pi)]$

The exact mathematical formula that connects the mass ratio of proton to electron,the fine-structure constant α and the proton-electron gravitational coupling constant $\alpha G(pe)$ is:

$\alpha^7=\mu^8\cdot[\alpha G(pp)\cdot \log 2(2\cdot\pi)]$

The exact mathematical formula that connects the mass ratio of proton to electron,the fine-structure constant α and the gravitational coupling constant of electrons-electrons $\alpha G(ee)$ is:

$\alpha^7=\mu^9\cdot[\alpha G(pp)\cdot \log 2(2\cdot\pi)]$

The exact mathematical formula that connects the Proton to Electron Mass Ratio μ,the fine-structure constant α,the Gravitational coupling constant αG for the electron and the gravitational coupling constant of proton $\alpha G(p)$ is:

$\alpha G(p)=\mu^2\cdot\alpha G$

The mathematical formula that connects the Proton to Electron Mass Ratio μ,the ratio N1 of electric force to gravitational force between electron and proton,the Avogadro number N_A,the Gravitational coupling constant αG for the electron and the gravitational coupling constant of proton $\alpha G(p)$ is:

$$2\cdot e\cdot N_A\cdot\alpha\cdot\alpha G^{1/2}=1 \quad (4)$$

4.2. Mathematical Formulas that connects 4 dimensionless physical constants

The exact mathematical formula that connects the Proton to Electron Mass Ratio μ,the fine-structure constant α,the ratio N1 of electric force to gravitational force between electron and proton and the Gravitational coupling constant αG for the electron is:

$$\alpha=\mu\cdot N1\cdot\alpha G \quad (5)$$

The exact mathematical formula that connects the Proton to Electron Mass Ratio μ,the fine-structure constant α,the ratio N1 of electric force to gravitational force between electron and proton and the gravitational coupling constant of proton $\alpha G(p)$ is:

$$\alpha\cdot\mu=N1\cdot\alpha G(p) \quad (6)$$

The exact mathematical formula that connects the fine-structure constant α,the ratio N1 of electric force to gravitational force between electron and proton,the Gravitational coupling constant αG for the electron and the gravitational coupling constant of proton $\alpha G(p)$ is:

$$\alpha^2=N1^2\cdot\alpha G\cdot\alpha G(p) \quad (7)$$

The exact mathematical formula that connects the Proton to Electron Mass Ratio μ,the fine-structure constant α,the ratio N1 of electric force to gravitational force between electron and proton and the Avogadro number N_A is:

$$\mu \cdot N1 = 4 \cdot e^2 \cdot \alpha^3 \cdot NA^2 \tag{8}$$

4.3. Mathematical Formula that connects 5 dimensionless physical constants

The exact mathematical formula that connects the Proton to Electron Mass Ratio μ,the fine-structure constant α,the ratio N1 of electric force to gravitational force between electron and proton,the Avogadro number NA and the Gravitational coupling constant αG for the electron is:

$$4 \cdot e^2 \cdot \alpha \cdot \mu \cdot \alpha G^2 \cdot NA^2 \cdot N1 = 1 \tag{9}$$

The exact mathematical formula that connects the Proton to Electron Mass Ratio μ,the fine-structure constant α,the ratio N1 of electric force to gravitational force between electron and proton,the Avogadro number NA and the gravitational coupling constant of proton αG(p) is:

$$\mu^3 = 4 \cdot e^2 \cdot \alpha \cdot \alpha G(p)^2 \cdot NA^2 \cdot N1 \tag{10}$$

The exact mathematical formula that connects the Proton to Electron Mass Ratio μ,the ratio N1 of electric force to gravitational force between electron and proton,the Avogadro number NA,the Gravitational coupling constant αG for the electron and the gravitational coupling constant of proton αG(p) is:

$$\mu = 2 \cdot e \cdot \alpha G^{1/2} \cdot \alpha G(p) \cdot NA \cdot N1 \tag{11}$$

4.4. Mathematical formula that connects 6 dimensionless physical constants

The mathematical Formula that connects the Proton to Electron Mass Ratio μ,the fine-structure constant α,the ratio N1 of electric force to gravitational force between electron and proton,the Avogadro number NA,the Gravitational coupling constant αG for the electron and the gravitational coupling constant of proton αG(p) is:

$$\mu = 4 \cdot e^2 \cdot \alpha \cdot \alpha G \cdot \alpha G(p) \cdot NA^2 \cdot N1 \tag{12}$$

5. New formula

5. New formula for Gravitational Constant G

The 2.018 CODATA recommended value of Gravitational Constant G is:

$G = 6{,}67430 \times 10^{-11}\ m^3/kg \cdot s^2$

With standard Uncertainty $0{,}00015 \times 10^{-11}\ m^3/kg \cdot s^2$ and Relative Standard Uncertainty $2{,}2 x 10^{-5}$. The Gravitational Constant G is:

$G = \alpha G \cdot ħ \cdot c/me^2$

From (5):

$G = (\alpha G \cdot \alpha G(p) \cdot N1/\mu \cdot \alpha) \cdot (ħ \cdot c/me^2)$

From (13):

$G = ħ \cdot c/4 \cdot e^2 \cdot \alpha^2 \cdot NA^2 \cdot me^2$

So the new formula for Gravitational Constant G is:

$$G=(4\cdot e^{2}\cdot \alpha^{2}\cdot N_A^{2})]^{-1}\cdot(\hbar\cdot c/m_e^{2}) \tag{13}$$

6. New exact formulas

6.1. Correction number

We need to find a correction number to have more accurate equations. We propose:

$$2\cdot e\approx 2\cdot e\cdot(6\cdot 7\cdot \varphi/5^{2}\cdot e)^{13/2} \tag{14}$$

6.2. New exact formula for the Avogadro number N_A

From (2) and (14) the new exact formula for the Avogadro number N_A is:

$$N_A=[(6\cdot 7\cdot \varphi/5^{2}\cdot e)^{13/2}\cdot \alpha\cdot \alpha_G^{1/2})]^{-1} \tag{15}$$

6.3. Mathematical formulas that connects dimensionless physical constants

From (4) and (14):

$$2\cdot e\cdot(6\cdot 7\cdot \varphi/5^{2}\cdot e)^{13/2}\cdot N_A\cdot \alpha\cdot \alpha_G^{1/2}=1 \tag{16}$$

From (8) and (14):

$$\mu\cdot N_1=4\cdot e^{2}\cdot(6\cdot 7\cdot \varphi/5^{2}\cdot e)^{13}\cdot \alpha^{3}\cdot N_A^{2} \tag{17}$$

From (9) and (14):

$$4\cdot e^{2}\cdot(6\cdot 7\cdot \varphi/5^{2}\cdot e)^{13}\cdot \alpha\cdot \mu\cdot \alpha_G^{2}\cdot N_A^{2}\cdot N_1=1 \tag{18}$$

From (10) and (14):

$$\mu^{3}=4\cdot e^{2}\cdot(6\cdot 7\cdot \varphi/5^{2}\cdot e)^{13}\cdot \alpha\cdot \alpha_{G(p)}^{2}\cdot N_A^{2}\cdot N_1 \tag{19}$$

From (11) and (14):

$$\mu=2\cdot e\cdot(6\cdot 7\cdot \varphi/5^{2}\cdot e)^{13/2}\cdot \alpha_G^{1/2}\cdot \alpha_{G(p)}\cdot N_A\cdot N_1 \tag{20}$$

6.4. Exact mathematical formula that connects 6 dimensionless physical constants

From (12) and (14) the exact mathematical formula that connects 6 dimensionless physical constants is:

$$\mu=4\cdot e^{2}\cdot(6\cdot 7\cdot \varphi/5^{2}\cdot e)^{13}\cdot \alpha\cdot \alpha_G\cdot \alpha_{G(p)}\cdot N_A^{2}\cdot N_1 \tag{21}$$

So:

$$\mu=2^{15}\cdot 3^{13}\cdot 5^{-26}\cdot 7^{13}\cdot \varphi^{13}\cdot e^{-11}\cdot \alpha\cdot \alpha G\cdot \alpha G(p)\cdot NA^{2}\cdot N1 \tag{22}$$

So:

$$\alpha\cdot \mu^{-1}\cdot \alpha G\cdot \alpha G(p)\cdot NA^{2}\cdot N1=2^{-15}\cdot 3^{-13}\cdot 5^{26}\cdot 7^{-13}\cdot \varphi^{-13}\cdot e^{11} \tag{23}$$

6.5. New exact Formula for Gravitational Constant G

From (13) and (14) the new exact formula for Gravitational Constant G is:

$$G=[4\cdot e^{2}\cdot (6\cdot 7\cdot \varphi/5^{2}\cdot e)^{13}\cdot \alpha^{2}\cdot NA^{2}]^{-1}\cdot (\hbar\cdot c/me^{2}) \tag{24 }$$

With exact value:

$G=6,67430\times 10^{-11}\ m^{3}/kg\cdot s^{2}$

7. Conclusions

In this paper are a total of 22 new formulas. A new formula for the Planck length $\ell_{p\ell}$:

$$\ell_{p\ell}=\alpha\cdot \alpha 0\cdot \alpha G^{1/2}$$

A new formula for the Avogadro number NA:

$$NA=(2\cdot e\cdot \alpha\cdot \alpha G^{1/2})^{-1}$$

9 Mathematical formulas that connect dimensionless physical constants:

$$\alpha=\mu\cdot N1\cdot \alpha G$$

$$\alpha\cdot \mu=N1\cdot \alpha G(p)$$

$$\alpha^{2}=N1^{2}\cdot \alpha G\cdot \alpha G(p)$$

$$\mu\cdot N1=4\cdot e^{2}\cdot \alpha^{3}\cdot NA^{2}$$

$$2\cdot e\cdot \alpha\cdot NA\cdot \alpha G^{1/2}=1$$

$$4\cdot e^{2}\cdot \alpha\cdot \mu\cdot \alpha G^{2}\cdot NA^{2}\cdot N1=1$$

$$\mu^{3}=4\cdot e^{2}\cdot \alpha\cdot \alpha G(p)^{2}\cdot NA^{2}\cdot N1$$

$$\mu=2\cdot e\cdot \alpha G^{1/2}\cdot \alpha G(p)\cdot NA\cdot N1$$

$$\mu=4\cdot e^{2}\cdot \alpha\cdot \alpha G\cdot \alpha G(p)\cdot NA^{2}\cdot N1$$

A new formula for Gravitational Constant G:

$$G=(4 \cdot e^2 \cdot \alpha^2 \cdot NA^2)]^{-1} \cdot (\hbar \cdot c/me^2)$$

A new exact formula for the Avogadro number NA:

$$NA=[(6 \cdot 7 \cdot \varphi/5^2 \cdot e)^{13/2} \cdot \alpha \cdot \alpha G^{1/2})]^{-1}$$

8 exact Mathematical formulas that connect 6 dimensionless physical constants:

$$2 \cdot e \cdot (6 \cdot 7 \cdot \varphi/5^2 \cdot e)^{13/2} \cdot NA \cdot \alpha \cdot \alpha G^{1/2}=1$$

$$\mu \cdot N1=4 \cdot e^2 \cdot (6 \cdot 7 \cdot \varphi/5^2 \cdot e)^{13} \cdot \alpha^3 \cdot NA^2$$

$$2 \cdot e \cdot (6 \cdot 7 \cdot \varphi/5^2 \cdot e)^{13/2} \cdot \alpha \cdot \mu \cdot NA \cdot \alpha G^{1/2}=1$$

$$4 \cdot e^2 \cdot (6 \cdot 7 \cdot \varphi/5^2 \cdot e)^{13} \cdot \alpha \cdot \mu \cdot \alpha G^2 \cdot NA^2 \cdot N1=1$$

$$\mu^3=4 \cdot e^2 \cdot (6 \cdot 7 \cdot \varphi/5^2 \cdot e)^{13} \cdot \alpha \cdot \alpha G(p)^2 \cdot NA^2 \cdot N1$$

$$\mu=2 \cdot e \cdot (6 \cdot 7 \cdot \varphi/5^2 \cdot e)^{13/2} \cdot \alpha G^{1/2} \cdot \alpha G(p) \cdot NA \cdot N1$$

$$\mu=4 \cdot e^2 \cdot (6 \cdot 7 \cdot \varphi/5^2 \cdot e)^{13} \cdot \alpha \cdot \alpha G \cdot \alpha G(p) \cdot NA^2 \cdot N1$$

$$\alpha \cdot \mu^{-1} \cdot \alpha G \cdot \alpha G(p) \cdot NA^2 \cdot N1=2^{-15} \cdot 3^{-13} \cdot 5^{26} \cdot 7^{-13} \cdot \varphi^{-13} \cdot e^{11}$$

A new exact formula for Gravitational Constant G:

$$G=[4 \cdot e^2 \cdot (6 \cdot 7 \cdot \varphi/5^2 \cdot e)^{13} \cdot \alpha^2 \cdot NA^2]^{-1} \cdot (\hbar \cdot c/me^2)$$

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