

# Exact mathematical Formula that connect 6 dimensionless physical constants

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## Abstract

In this paper are a new formula for the Planck length  $l_p$  and a new formula for the Avogadro number  $N_A$ . Also 9 Mathematical formulas that connect dimensionless physical constants. The 6 dimensionless physical constants are the Proton to Electron Mass Ratio  $\mu$ , the Fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro number  $N_A$ , the Gravitational coupling constant  $\alpha_G$  for the electron and the gravitational coupling constant  $\alpha_G(p)$  of proton. After a new formula for Gravitational Constant  $G$  and new exact formula for the Avogadro number  $N_A$ . Finally 8 exact Mathematical formulas that connect 6 dimensionless physical constants and a new exact formula for Gravitational Constant  $G$ .

## 1. Introduction

In physics, a dimensionless physical constant is a physical constant that is dimensionless, a pure number having no units attached and having a numerical value that is independent of whatever system of units may be used. The term fundamental physical constant is used to refer to some universal dimensionless constants. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values.

In the 1.920s and 1.930s, Arthur Eddington embarked upon extensive mathematical investigation into the relations between the fundamental quantities in basic physical theories, later used as part of his effort to construct an overarching theory unifying quantum mechanics and cosmological physics. The mathematician Simon Plouffe has made an extensive search of computer databases of mathematical formulas, seeking formulas for the mass ratios of the fundamental particles. An empirical relation between the masses of the electron, muon and tau has been discovered by physicist Yoshio Koide, but this formula remains unexplained.

## 2. Measurement of the 6 dimensionless physical constants

### 2.1. Measurement of the Fine Structure constant

The 2.018 CODATA recommended value of  $\alpha$  is:

$$\alpha = \frac{q_e^2}{4 \cdot \pi \cdot \epsilon_0 \cdot \hbar \cdot c} = 0.0072973525693(11)$$

With standard Uncertainty  $0,000000011 \times 10^{-3}$  and Relative Standard Uncertainty  $1,5 \times 10^{10}$ . For reasons of convenience, historically the value of the reciprocal of the fine-structure constant is often specified. The 2.018 CODATA recommended value is given by:

$$\alpha^{-1} = 137,035999084(21)$$

With standard Uncertainty  $0,000000021 \times 10^{-3}$  and Relative Standard Uncertainty  $1,5 \times 10^{10}$ . There is general agreement for the value of  $\alpha$ , as measured by these different methods. The preferred methods in 2.019 are measurements of electron anomalous magnetic moments and of photon recoil in atom interferometry. The most precise value of  $\alpha$  obtained experimentally (as of 2.012) is based on a measurement of  $g$  using a one-electron

so-called "quantum cyclotron" apparatus, together with a calculation via the theory of QED that involved 12.672 tenth-order Feynman diagrams:

$$\alpha^{-1} = 137,035999174(35)$$

This measurement of  $\alpha$  has a relative standard uncertainty of  $2,5 \times 10^{-10}$ . This value and uncertainty are about the same as the latest experimental results. Further refinement of this work were published by the end of 2.020, giving the value:

$$\alpha^{-1} = 137,035999206(11)$$

with a relative accuracy of 81 parts per trillion.

## 2.2. Measurement of the Proton to Electron Mass Ratio

The 2.018 CODATA recommended value of the Proton to Electron Mass Ratio  $\mu$  is:

$$\mu = 1.836,15267343$$

With standard Uncertainty 0,00000011 and Relative Standard Uncertainty  $6,0 \times 10^{-11}$ . The value of  $\mu$  is known at about 0,1 parts per billion. The value of  $\mu$  is a solution of the equation:

$$3 \cdot \mu^4 - 5.508 \cdot \mu^3 - 841 \cdot \mu^2 + 10 \cdot \mu - 2.111 = 0$$

The 2.018 CODATA recommended value of  $\mu^{-1}$  is:

$$\mu^{-1} = m_e/m_p = 0,000544617021487$$

With standard Uncertainty 0,000000000000033 and Relative Standard Uncertainty  $6,0 \times 10^{-11}$ .

## 2.3. Measurement of the Gravitational coupling constant $\alpha_G$ for the electron

In physics, the gravitational coupling constant  $\alpha_G$  is a constant that characterizes the gravitational pull between a given pair of elementary particles. For the electron pair this constant is denoted by  $\alpha_G$ . The choice of units of measurement, but only with the choice of particles. The gravitational coupling constant  $\alpha_G$  is a scaling ratio that can be used to compare similar unit values from different scaling systems (Planck scale, atomic scale, and cosmological scale). The gravitational coupling constant can be used for comparison of length, range and force values. The gravitational coupling constant  $\alpha_G$  is defined as:

$$\alpha_G = G \cdot m_e^2 / \hbar \cdot c$$

There is so far no known way to measure  $\alpha_G$  directly. The value of the constant gravitational coupling  $\alpha_G$  is only known in four significant digits. The approximate value of the constant gravitational coupling  $\alpha_G$  is:

$$\alpha_G = 1,751809945728109 \times 10^{-45}$$

## 2.4. Measurement of the Gravitational coupling constant $\alpha_G(p)$ for the electron

The gravitational coupling constant for the proton is produced similar to the electron, but replaces the mass of electrons with the mass of the protons. The gravitational coupling constant of the proton  $\alpha_G(p)$  is defined as:

$$\alpha_{G(p)} = G \cdot m_p^2 / \hbar \cdot c$$

The approximate value of the constant gravitational coupling of the proton  $\alpha_{G(p)}$  is:

$$\alpha_{G(p)} = 5,906151273795571 \times 10^{-39}$$

## 2.5. Measurement of the ratio N1 of electric force to gravitational force between electron and proton

The enormous value of the ratio of electric force to gravitational force was first pointed out by Benjamin Franklin in 1751. But Weyl and Eddington suggested that the number was about  $10^{40}$  and was related to cosmological quantities. The electric force  $F_c$  between electron and proton is defined as:

$$F_c = q_e^2 / 4 \cdot \pi \cdot \epsilon_0 \cdot r^2$$

The gravitational force  $F_g$  between electron and proton is defined as:

$$F_g = G \cdot m_e \cdot m_p / r^2$$

So:

$$N_1 = F_c / F_g$$

$$N_1 = q_e^2 / 4 \cdot \pi \cdot \epsilon_0 \cdot G \cdot m_e \cdot m_p$$

$$N_1 = k_e \cdot q_e^2 / G \cdot m_p \cdot m_e$$

$$N_1 = \alpha \cdot \hbar \cdot c / G \cdot m_p \cdot m_e$$

The approximate value of the ratio N1 of electric force to gravitational force between electron and proton is:

$$N_1 = 2,26866072471432 \times 10^{39}$$

## 2.6. Measurement of Avogadro number NA

The most accurate definition of the Avogadro number  $N_A$  value involves the change in molecular quantities and, in particular, the change in the value of an elementary charge. The exact value of the Avogadro number  $N_A$  is:

$$N_A = 6,02214076 \times 10^{23}$$

The value of the Avogadro number  $N_A$  can also be written in numerical expressions:

$$N_A = 2^{79} = 6,044629098 \times 10^{23}$$

$$N_A = 84.446.885^3 = 6,02214076 \times 10^{23}$$

## 3. New Formulas

### 3.1. New Formula for the Planck length $\ell_p$

The Bohr radius  $a_0$  is defined as:

$$a_0 = \hbar / \alpha \cdot m_e \cdot c$$

The reduced Planck constant  $\hbar$  is:

$$\hbar = \alpha \cdot m_e \cdot a_0 \cdot c$$

So:

$$\hbar^2 = \alpha^2 \cdot m_e^2 \cdot a_0^2 \cdot c^2$$

So:

$$(\hbar \cdot G / c^3) = \alpha^2 \cdot m_e^2 \cdot a_0^2 \cdot (G \cdot m_e^2 / \hbar \cdot c)$$

So:

$$(\hbar \cdot G / c^2) = \alpha^2 \cdot a_0^2 \cdot (G \cdot m_e^2 / \hbar \cdot c)$$

So:

$$\ell_{pl}^2 = \alpha^2 \cdot a_0^2 \cdot a_G$$

So the new formula for the Planck length  $\ell_{pl}$  is:

$$\ell_{pl} = \alpha \cdot a_0 \cdot a_G^{1/2} \quad (1)$$

### 3.2. New formula for the Avogadro number $N_A$

Jeff Yee in his paper «The Relationship of the Mole and Charge» [10], the mole and charge are related by deriving Avogadro's number from three constants, the Bohr radius, the Planck length and Euler's number. The Avogadro number  $N_A$  can be calculated from the Planck length  $\ell_{pl}$ , the Bohr radius  $a_0$  and Euler's number  $e$ :

$$N_A = a_0 / 2 \cdot e \cdot \ell_{pl}$$

So:

$$N_A = a_0 / 2 \cdot e \cdot \alpha \cdot a_0 \cdot a_G^{1/2}$$

So the new Formula for the Avogadro number  $N_A$  is:

$$N_A = (2 \cdot e \cdot \alpha \cdot a_G^{1/2})^{-1} \quad (2)$$

So:

$$2 \cdot e \cdot N_A \cdot \alpha \cdot a_G^{1/2} = 1 \quad (3)$$

### 4. Mathematical Formulas that connects dimensionless physical constants

#### 4.1. Mathematical Formulas that connects 3 dimensionless physical constants

The exact mathematical formula that connects the mass ratio of proton to electron, the fine-structure constant  $\alpha$  and the proton-proton gravitational coupling constant  $\alpha G(pp)$  is:

$$\alpha^7 = \mu^7 \cdot [\alpha G(pp) \cdot \log_2(2 \cdot \pi)]$$

The exact mathematical formula that connects the mass ratio of proton to electron, the fine-structure constant  $\alpha$  and the proton-electron gravitational coupling constant  $\alpha G(pe)$  is:

$$\alpha^7 = \mu^8 \cdot [\alpha G(pp) \cdot \log_2(2 \cdot \pi)]$$

The exact mathematical formula that connects the mass ratio of proton to electron, the fine-structure constant  $\alpha$  and the gravitational coupling constant of electrons-electrons  $\alpha G(ee)$  is:

$$\alpha^7 = \mu^9 \cdot [\alpha G(pp) \cdot \log_2(2 \cdot \pi)]$$

The exact mathematical formula that connects the Proton to Electron Mass Ratio  $\mu$ , the fine-structure constant  $\alpha$ , the Gravitational coupling constant  $\alpha G$  for the electron and the gravitational coupling constant of proton  $\alpha G(p)$  is:

$$\alpha G(p) = \mu^2 \cdot \alpha G$$

The mathematical formula that connects the Proton to Electron Mass Ratio  $\mu$ , the ratio  $N1$  of electric force to gravitational force between electron and proton, the Avogadro number  $N_A$ , the Gravitational coupling constant  $\alpha G$  for the electron and the gravitational coupling constant of proton  $\alpha G(p)$  is:

$$2 \cdot e \cdot N_A \cdot \alpha \cdot \alpha G^{1/2} = 1 \quad (4)$$

#### 4.2. Mathematical Formulas that connects 4 dimensionless physical constants

The exact mathematical formula that connects the Proton to Electron Mass Ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N1$  of electric force to gravitational force between electron and proton and the Gravitational coupling constant  $\alpha G$  for the electron is:

$$\alpha = \mu \cdot N1 \cdot \alpha G \quad (5)$$

The exact mathematical formula that connects the Proton to Electron Mass Ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N1$  of electric force to gravitational force between electron and proton and the gravitational coupling constant of proton  $\alpha G(p)$  is:

$$\alpha \cdot \mu = N1 \cdot \alpha G(p) \quad (6)$$

The exact mathematical formula that connects the fine-structure constant  $\alpha$ , the ratio  $N1$  of electric force to gravitational force between electron and proton, the Gravitational coupling constant  $\alpha G$  for the electron and the gravitational coupling constant of proton  $\alpha G(p)$  is:

$$\alpha^2 = N1^2 \cdot \alpha G \cdot \alpha G(p) \quad (7)$$

The exact mathematical formula that connects the Proton to Electron Mass Ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N1$  of electric force to gravitational force between electron and proton and the Avogadro number  $N_A$  is:

$$\mu \cdot N1 = 4 \cdot e^2 \cdot a^3 \cdot NA^2 \quad (8)$$

The exact mathematical formula that connects the Proton to Electron Mass Ratio  $\mu$ , the fine-structure constant  $a$ , the Avogadro number  $NA$  and the Gravitational coupling constant  $aG$  for the electron is:

$$2 \cdot e \cdot a \cdot \mu \cdot NA \cdot aG^{1/2} = 1 \quad (9)$$

#### 4.3. Mathematical Formula that connects 5 dimensionless physical constants

The exact mathematical formula that connects the Proton to Electron Mass Ratio  $\mu$ , the fine-structure constant  $a$ , the ratio  $N1$  of electric force to gravitational force between electron and proton, the Avogadro number  $NA$  and the Gravitational coupling constant  $aG$  for the electron is:

$$4 \cdot e^2 \cdot a \cdot \mu \cdot aG^2 \cdot NA^2 \cdot N1 = 1 \quad (10)$$

The exact mathematical formula that connects the Proton to Electron Mass Ratio  $\mu$ , the fine-structure constant  $a$ , the ratio  $N1$  of electric force to gravitational force between electron and proton, the Avogadro number  $NA$  and the gravitational coupling constant of proton  $aG(p)$  is:

$$\mu^3 = 4 \cdot e^2 \cdot a \cdot aG(p)^2 \cdot NA^2 \cdot N1 \quad (11)$$

The exact mathematical formula that connects the Proton to Electron Mass Ratio  $\mu$ , the ratio  $N1$  of electric force to gravitational force between electron and proton, the Avogadro number  $NA$ , the Gravitational coupling constant  $aG$  for the electron and the gravitational coupling constant of proton  $aG(p)$  is:

$$\mu = 2 \cdot e \cdot aG^{1/2} \cdot aG(p) \cdot NA \cdot N1 \quad (12)$$

#### 4.4. Mathematical formula that connects 6 dimensionless physical constants

The mathematical Formula that connects the Proton to Electron Mass Ratio  $\mu$ , the fine-structure constant  $a$ , the ratio  $N1$  of electric force to gravitational force between electron and proton, the Avogadro number  $NA$ , the Gravitational coupling constant  $aG$  for the electron and the gravitational coupling constant of proton  $aG(p)$  is:

$$\mu = 4 \cdot e^2 \cdot a \cdot aG \cdot aG(p) \cdot NA^2 \cdot N1 \quad (13)$$

### 5. New formula

#### 5. New formula for Gravitational Constant G

The 2.018 CODATA recommended value of Gravitational Constant  $G$  is:

$$G = 6,67430 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$$

With standard Uncertainty  $0,00015 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$  and Relative Standard Uncertainty  $2,2 \times 10^{-5}$ . The Gravitational Constant  $G$  is:

$$G = aG \cdot \hbar \cdot c / m_e^2$$

From (5):

$$G=(a_G \cdot a_{G(p)} \cdot N_1/\mu \cdot a) \cdot (\hbar \cdot c/me^2)$$

From (13):

$$G=\hbar \cdot c/4 \cdot e^2 \cdot a^2 \cdot NA^2 \cdot me^2$$

So the new formula for Gravitational Constant G is:

$$G=(4 \cdot e^2 \cdot a^2 \cdot NA^2)]^{-1} \cdot (\hbar \cdot c/me^2) \quad (14)$$

## 6. New exact formulas

### 6.1. Correction number

We need to find a correction number to have more accurate equations. We propose:

$$2 \cdot e=2 \cdot e \cdot (6 \cdot 7 \cdot \phi/5^2 \cdot e)^{13/2} \quad (15)$$

### 6.2. New exact formula for the Avogadro number NA

From (2) and (14) the new exact formula for the Avogadro number NA is:

$$NA=[(6 \cdot 7 \cdot \phi/5^2 \cdot e)^{13/2} \cdot a \cdot a_G^{1/2}]^{-1} \quad (16)$$

### 6.3. Mathematical formulas that connects dimensionless physical constants

From (4) and (15):

$$2 \cdot e \cdot (6 \cdot 7 \cdot \phi/5^2 \cdot e)^{13/2} \cdot NA \cdot a \cdot a_G^{1/2}=1 \quad (17)$$

From (8) and (15):

$$\mu \cdot N_1=4 \cdot e^2 \cdot (6 \cdot 7 \cdot \phi/5^2 \cdot e)^{13} \cdot a^3 \cdot NA^2 \quad (18)$$

From (9) and (15) the new exact formula for the Avogadro number NA is:

$$2 \cdot e \cdot (6 \cdot 7 \cdot \phi/5^2 \cdot e)^{13/2} \cdot a \cdot \mu \cdot NA \cdot a_G^{1/2}=1 \quad (19)$$

From (10) and (15) the new exact formula for the Avogadro number NA is:

$$4 \cdot e^2 \cdot (6 \cdot 7 \cdot \phi/5^2 \cdot e)^{13} \cdot a \cdot \mu \cdot a_G^2 \cdot NA^2 \cdot N_1=1 \quad (20)$$

From (11) and (15) the new exact formula for the Avogadro number NA is:

$$\mu^3=4 \cdot e^2 \cdot (6 \cdot 7 \cdot \phi/5^2 \cdot e)^{13} \cdot a \cdot a_{G(p)}^2 \cdot NA^2 \cdot N_1 \quad (21)$$

From (12) and (15) the new exact formula for the Avogadro number NA is:

$$\mu = 2 \cdot e \cdot (6 \cdot 7 \cdot \varphi / 5^2 \cdot e)^{13/2} \cdot aG^{1/2} \cdot aG(p) \cdot NA \cdot N1 \quad (22)$$

#### 6.4. Exact mathematical formula that connects 6 dimensionless physical constants

From (13) and (15) the exact mathematical formula that connects 6 dimensionless physical constants is:

$$\mu = 4 \cdot e^2 \cdot (6 \cdot 7 \cdot \varphi / 5^2 \cdot e)^{13} \cdot a \cdot aG \cdot aG(p) \cdot NA^2 \cdot N1 \quad (23)$$

So:

$$\mu = 2^{15} \cdot 3^{13} \cdot 5^{-26} \cdot 7^{13} \cdot \varphi^{13} \cdot e^{-11} \cdot a \cdot aG \cdot aG(p) \cdot NA^2 \cdot N1 \quad (24)$$

So:

$$a \cdot \mu^{-1} \cdot aG \cdot aG(p) \cdot NA^2 \cdot N1 = 2^{-15} \cdot 3^{-13} \cdot 5^{26} \cdot 7^{-13} \cdot \varphi^{-13} \cdot e^{11} \quad (25)$$

#### 6.5. New exact Formula for Gravitational Constant G

From (14) and (15) the new exact formula for Gravitational Constant G is:

$$G = [4 \cdot e^2 \cdot (6 \cdot 7 \cdot \varphi / 5^2 \cdot e)^{13} \cdot a^2 \cdot NA^2]^{-1} \cdot (\hbar \cdot c / m_e^2) \quad (26)$$

With exact value:

$$G = 6,67430 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$$

### 7. Conclusions

In this paper are a total of 22 new formulas. A new formula for the Planck length  $\ell_{pl}$ :

$$\ell_{pl} = a \cdot a0 \cdot aG^{1/2}$$

A new formula for the Avogadro number NA:

$$NA = (2 \cdot e \cdot a \cdot aG^{1/2})^{-1}$$

9 Mathematical formulas that connect dimensionless physical constants:

$$a = \mu \cdot N1 \cdot aG$$

$$a \cdot \mu = N1 \cdot aG(p)$$

$$a^2 = N1^2 \cdot aG \cdot aG(p)$$

$$\mu \cdot N1 = 4 \cdot e^2 \cdot a^3 \cdot NA^2$$

$$2 \cdot e \cdot a \cdot \mu \cdot NA \cdot aG^{1/2} = 1$$

$$4 \cdot e^2 \cdot a \cdot \mu \cdot aG^2 \cdot NA^2 \cdot N1 = 1$$



$$\mu^3 = 4 \cdot e^2 \cdot a \cdot aG(p)^2 \cdot NA^2 \cdot N1$$

$$\mu = 2 \cdot e \cdot aG^{1/2} \cdot aG(p) \cdot NA \cdot N1$$

$$\mu = 4 \cdot e^2 \cdot a \cdot aG \cdot aG(p) \cdot NA^2 \cdot N1$$

A new formula for Gravitational Constant G:

$$G = (4 \cdot e^2 \cdot a^2 \cdot NA^2)^{-1} \cdot (\hbar \cdot c / me^2)$$

A new exact formula for the Avogadro number NA:

$$NA = [(6 \cdot 7 \cdot \phi / 5^2 \cdot e)^{13/2} \cdot a \cdot aG^{1/2}]^{-1}$$

8 exact Mathematical formulas that connect 6 dimensionless physical constants

$$2 \cdot e \cdot (6 \cdot 7 \cdot \phi / 5^2 \cdot e)^{13/2} \cdot NA \cdot a \cdot aG^{1/2} = 1$$

$$\mu \cdot N1 = 4 \cdot e^2 \cdot (6 \cdot 7 \cdot \phi / 5^2 \cdot e)^{13} \cdot a^3 \cdot NA^2$$

$$2 \cdot e \cdot (6 \cdot 7 \cdot \phi / 5^2 \cdot e)^{13/2} \cdot a \cdot \mu \cdot NA \cdot aG^{1/2} = 1$$

$$4 \cdot e^2 \cdot (6 \cdot 7 \cdot \phi / 5^2 \cdot e)^{13} \cdot a \cdot \mu \cdot aG^2 \cdot NA^2 \cdot N1 = 1$$

$$\mu^3 = 4 \cdot e^2 \cdot (6 \cdot 7 \cdot \phi / 5^2 \cdot e)^{13} \cdot a \cdot aG(p)^2 \cdot NA^2 \cdot N1$$

$$\mu = 2 \cdot e \cdot (6 \cdot 7 \cdot \phi / 5^2 \cdot e)^{13/2} \cdot aG^{1/2} \cdot aG(p) \cdot NA \cdot N1$$

$$\mu = 4 \cdot e^2 \cdot (6 \cdot 7 \cdot \phi / 5^2 \cdot e)^{13} \cdot a \cdot aG \cdot aG(p) \cdot NA^2 \cdot N1$$

$$a \cdot \mu^{-1} \cdot aG \cdot aG(p) \cdot NA^2 \cdot N1 = 2^{-15} \cdot 3^{-13} \cdot 5^{26} \cdot 7^{-13} \cdot \phi^{-13} \cdot e^{11}$$

A new exact formula for Gravitational Constant G:

$$G = [4 \cdot e^2 \cdot (6 \cdot 7 \cdot \phi / 5^2 \cdot e)^{13} \cdot a^2 \cdot NA^2]^{-1} \cdot (\hbar \cdot c / me^2)$$

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