

# Exact mathematical expressions of the Proton to Electron Mass Ratio with symmetrical shape and physical meaning

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## Abstract

We present 26 exact mathematical expressions of the proton to electron mass ratio with exact value  $\mu=1.836,15267343$ . We propose an exact mathematical expression using Fibonacci and Lucas numbers:

$$\mu^{32}=\varphi^{-42}\cdot F_5^{160}\cdot L_5^{47}\cdot L_{19}^{40/19}$$

Also we present the exact mathematical expressions between the proton to electron mass ratio with the mathematical constants and also the exact mathematical expressions between the proton to electron mass ratio with the fine-structure constant.

## 1. Introduction

The proton to electron mass ratio  $\mu$  is a ratio of like-dimensioned physical quantities, it is a dimensionless quantity, a function of the dimensionless physical constants, and has numerical value independent of the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious mass ratio between the proton and electron. The numerical challenge of the mass ratio of proton to electron in the field of elementary particle physics began with the discovery of the electron by JJ Thomson in 1.897, and with the identification of the point nature of the proton by E. Rutherford in 1.911. These two particles have electric charges that are identical in size but opposite charges.

## 2. Measurement of the Proton to electron mass ratio

The 2.018 CODATA recommended value of proton to electron mass ratio  $\mu$  is:

$$\mu=m_p/m_e=1.836,15267343$$

With standard uncertainty 0,00000011 and relative standard uncertainty  $6,0\times 10^{-11}$ . The value of  $\mu$  is known at about 0,1 parts per billion. The value of  $\mu$  is a solution of the equation:

$$3\cdot\mu^4-5.508\cdot\mu^3-841\cdot\mu^2+10\cdot\mu-2.111=0$$

The 2.018 CODATA recommended value of  $\mu^{-1}$  is:

$$\mu^{-1}=m_e/m_p=0,000544617021487$$

With standard uncertainty 0,000000000000033 and relative standard uncertainty  $6,0\times 10^{-11}$ .

## 3. Background of the search for mathematical expression

The search for mathematical expression for this dimensionless number motivated many serious scientists. First Peirles in 1.928 proposed the mathematical expression:

$$\mu=2\cdot(\pi-1)\cdot\pi/a$$

A year later Reinhold Furth in 1.929 assumed that proton to electron mass ratio  $\mu$  could be derived from the quadratic equation containing the fine-structure constant  $\alpha$ :

$$\mu=64\cdot\pi/15\cdot\alpha$$

Later in 1.935, A. Eddington, who accepted some of Furth's ideas, presented in his book «New Pathways in Science» the equation for the the proton to electron mass ratio  $\mu$ :

$$10\cdot\mu^2-136\cdot\mu+1=0$$

However both approaches can not be used nowadays as they give very high deviation from the currently known experimental value of  $\mu$ . Haas in 1.938 presented the expression:

$$\mu=3\cdot\sqrt{2}\cdot\pi/a$$

Later in 1.951 Lenz noted that  $\mu$  can be approximated with the formula:

$$\mu=6\cdot\pi^5$$

In 1.990, I.J. Good, a British mathematician assembled eight conjectures of numerology for the ratio of the rest masses of the proton and the electron. Recently the professional approach to mathematically decode  $\mu$  was done by Simon Plouffe. He used a large database of mathematical constants and specialized programs to directly find an expression. Along with his main remarkable result for the expression for  $\mu$  via Fibonacci and Lucas numbers and golden ratio he also noted that expression for  $\mu$  using  $\pi$  can be improved as:

$$\mu=6\cdot\pi^5+328/\pi^8$$

#### 4. Exact mathematical expression using Fibonacci and Lucas numbers

Simon Plouffe in his work [9] «A search for a mathematical expression for mass ratios using a large database» proposed the mathematical expression:

$$\mu=F_{10}\cdot F_5^{3/2}\cdot L_5^{15/32}\cdot\varphi^{-1/16}$$

So:

$$\mu=55\cdot 5^{3/2}\cdot 11^{15/32}\cdot\varphi^{-1/16}$$

With numerical value  $\mu=1.836,15267481714\dots$  and relative standard uncertainty  $1\times 10^{-9}$ . We propose the exact mathematical expression for the proton to electron mass ratio:

$$\mu=11^{47/32}\cdot 5^{5/2}\cdot 9.349^{5/76}\cdot\varphi^{-21/16} \tag{1}$$

With exact numerical value  $\mu=1.836,15267343$ . However:

$$(2\cdot\varphi-1)^2=5, \varphi^5-\varphi^{-5}=11, (\varphi^{19}-\varphi^{-19})=9.349$$

So:

$$\mu^{32}=(\varphi^5-\varphi^{-5})^{47}\cdot(2\cdot\varphi-1)^{160}\cdot(\varphi^{19}-\varphi^{-19})^{40/19}\cdot\varphi^{-42} \quad (2)$$

Also:

$$\mu^{32}=\varphi^{-42}\cdot F_5^{160}\cdot L_5^{47}\cdot L_{19}^{40/19} \quad (3)$$

The formula has an exact value, a symmetrical shape and a greater physical meaning than all types. It seems to be the formula of the universe.

## 5. Exact mathematical expressions between mathematical constants

The exact mathematical expressions for the proton to electron mass ratio  $\mu$  are:

$$\mu=165\cdot[(\ln 10)^{11}/7]^{1/3} \quad (4)$$

$$7\cdot\mu^3=(5\cdot 13)^3\cdot[\ln(2\cdot 5)]^{11} \quad (5)$$

$$\mu=1.836+[2\cdot\sqrt{77-11}]^{-1} \quad (6)$$

$$\mu=2^{10}+2^9+2^8+2^5+2^3+2^2+2^{-3}+2^{-6}+2^{-7}+2^{-8}+2^{-12}+2^{-14}+2^{-16}+2^{-17}+2^{-20}+2^{-21}+2^{-23}+2^{-24}+2^{-27} \quad (7)$$

$$\mu=2\cdot 5^4+4\cdot 5^3+3\cdot 5^2+2\cdot 5^1+4\cdot 5^{-1}+8\cdot 5^{-2}+4\cdot 5^{-3}+2\cdot 5^{-5}+2\cdot 5^{-7}+3\cdot 5^{-8}+5^{-10}+2\cdot 5^{-11}+2\cdot 5^{-12} \quad (8)$$

$$\mu=1.836+(2^5\cdot 3\cdot 29/5\cdot 7\cdot 521) \quad (9)$$

The exact mathematical expressions between the proton to electron mass ratio  $\mu$  and the Archimedes's constant  $\pi$  are:

$$\mu=64\cdot\pi^3-48\cdot\pi+8\cdot\pi^{-1}+2\cdot\pi^{-7}+8\cdot\pi^{-9}+\pi^{-11}+6\cdot\pi^{-15}+\pi^{-17} \quad (10)$$

$$\mu=(826\cdot\pi)-(4.610/3\cdot\pi)-(809/3) \quad (11)$$

$$\mu=6\cdot\pi^5+\pi^{-3}+2\cdot\pi^{-6}+2\cdot\pi^{-8}+2\cdot\pi^{-10}+2\cdot\pi^{-13}+\pi^{-15} \quad (12)$$

The exact mathematical expression between the proton to electron mass ratio  $\mu$  and the golden ratio  $\varphi$  is:

$$\mu=\varphi^{15}+\varphi^{12}+\varphi^{10}+2\cdot\varphi^5+\varphi^3+\varphi^{-1}+\varphi^{-3}+\varphi^{-7}+\varphi^{-12}+\varphi^{-15}+\varphi^{-17}+\varphi^{-26}+\varphi^{-31}+\varphi^{-34} \quad (13)$$

The exact mathematical expression between the proton to electron mass ratio  $\mu$  and the Euler's number  $e$  is:

$$\mu=e^7+e^6+2\cdot e^5+e^3+2\cdot e^2+e^1+4\cdot e^{-1}+e^{-2}+e^{-3}+e^{-4}+e^{-5}+2\cdot e^{-8}+2\cdot e^{-10}+e^{-11}+2\cdot e^{-16} \quad (14)$$

The exact mathematical expression between the proton to electron mass ratio  $\mu$ , the Archimedes's constant  $\pi$  and the golden ratio  $\varphi$  is:

$$\mu=[\pi^9-(3.981/40)]/10\cdot\varphi \quad (15)$$

The exact mathematical expression between the proton to electron mass ratio  $\mu$ , the golden ratio  $\varphi$  and the Euler's number  $e$  is:

$$\mu=3\cdot(90\cdot e+37)/(-407\cdot\phi+659) \quad (16)$$

The exact mathematical expression between the proton to electron mass ratio  $\mu$ , the Archimedes's constant  $\pi$  and the Euler's number  $e$  is:

$$\mu=3\cdot(e^\pi)^2+9\cdot(e^\pi)+495\cdot(e^\pi)^{-1}+11\cdot(e^\pi)^{-2}+11\cdot(e^\pi)^{-4}+22\cdot(e^\pi)^{-5}+11\cdot(e^\pi)^{-6}+8\cdot(e^\pi)^{-7} \quad (17)$$

The exact mathematical expression between the proton to electron mass ratio  $\mu$ , the Archimedes's constant  $\pi$ , the golden ratio  $\phi$  and the Euler's number  $e$  is:

$$\mu=544\cdot\pi+493\cdot\phi-463\cdot e+588 \quad (18)$$

## 6. Exact mathematical expressions between the proton to electron mass ratio, the fine-structure constant and mathematical constants

The exact mathematical expressions between the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$  and mathematical constants are:

$$\mu=(398\cdot\alpha^{-1}+528)/(7\cdot\pi+8) \quad (19)$$

$$\mu=(5/7)\cdot(1.524\cdot\alpha+1.287\cdot\pi-917\cdot\phi) \quad (20)$$

$$\mu=(1/9)\cdot(119\cdot\alpha^{-1}+5\cdot\phi+210) \quad (21)$$

$$\mu=6\cdot\alpha^{-1}+360\cdot\phi-165\cdot\pi+345\cdot e+12 \quad (22)$$

$$\mu=182\cdot\alpha+141\cdot\phi+495\cdot\pi-66\cdot e+231 \quad (23)$$

$$\mu=807\cdot\alpha+1.205\cdot\pi-518\cdot\phi-411\cdot e \quad (24)$$

$$\mu=15\cdot\alpha^{-1}-3\cdot A+9\cdot S-11\cdot K-28\cdot\pi-23\cdot\phi+e-30 \quad (25)$$

$$\mu=14\cdot\alpha^{-1}+10\cdot QA+4\cdot A-5\cdot S-K-17\cdot\phi-12\cdot\pi-3 \quad (26)$$

where:

K the polygon circumscribing constant with value  $K=8,7000366252\dots$

S the silver constant with value  $S=2+2\cdot\cos(2\cdot\pi/7)=3,246979603717\dots$

A the Golden Apex with value  $A=e^\pi-7\cdot\pi-1=0,14954405765\dots$

QA the Aristotle's Quintessence with value  $QA=1,0191134319\dots$

## 7. Conclusions

All 26 mathematical expressions of the proton to electron mass ratio have exact values  $\mu=1.836,15267343$ . They have been analyzed in terms of their simplicity and numerical significance. Especially we propose these mathematical expressions:

$$\mu^{32}=\phi^{-42}\cdot F_5^{160}\cdot L_5^{47}\cdot L_{19}^{40/19}$$

$$\mu^{32}=(\phi^5-\phi^{-5})^{47}\cdot(2\cdot\phi-1)^{160}\cdot(\phi^{19}-\phi^{-19})^{40/19}\cdot\phi^{-42}$$

$$7 \cdot \mu^3 = (5 \cdot 13)^3 \cdot [(\ln 2 \cdot 5)]^{11}$$

$$\mu = 6 \cdot \pi^5 + \pi^{-3} + 2 \cdot \pi^{-6} + 2 \cdot \pi^{-8} + 2 \cdot \pi^{-10} + 2 \cdot \pi^{-13} + \pi^{-15}$$

$$\mu = \varphi^{15} + \varphi^{12} + \varphi^{10} + 2 \cdot \varphi^5 + \varphi^3 + \varphi^{-1} + \varphi^{-3} + \varphi^{-7} + \varphi^{-12} + \varphi^{-15} + \varphi^{-17} + \varphi^{-26} + \varphi^{-31} + \varphi^{-34}$$

$$\mu = 544 \cdot \pi + 493 \cdot \varphi - 463 \cdot e + 588$$

$$\mu = (1/9) \cdot (119 \cdot a^{-1} + 5 \cdot \varphi + 210)$$

$$\mu = 6 \cdot a^{-1} + 360 \cdot \varphi - 165 \cdot \pi + 345 \cdot e + 12$$

$$\mu = 807 \cdot a + 1.205 \cdot \pi - 518 \cdot \varphi - 411 \cdot e$$

$$\mu = 15 \cdot a^{-1} - 3 \cdot A + 9 \cdot S - 11 \cdot K - 28 \cdot \pi - 23 \cdot \varphi + e - 30$$

$$\mu = 14 \cdot a^{-1} + 10 \cdot Q_A + 4 \cdot A - 5 \cdot S - K - 17 \cdot \varphi - 12 \cdot \pi - 3$$

## References

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