On the Inverse Fourier Transform of the Planck-Einstein law

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Abstract

After proposing a natural metric for the space in which particles spin which implements the principle of maximum frequency, $E = hf$ is generalised and its inverse Fourier transform is calculated.

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There is a problem at the foundations of quantum mechanics that is neglected and deserves much more attention: Taking Planck-Einstein relation $E = \hbar\omega$ as a fundamental law of nature and abiding by the principle that *a fundamental law of nature must not depend on our choice of basis for the function space (Fourier basis)*, we expect the Inverse Fourier Transform of $E = \hbar\omega$ to yield the energy of electromagnetic field as a function of time, viz.

$$E(t) = \frac{\hbar}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{i\omega t}\omega \; d\omega,$$

but this integral is divergent, and I maintain that *this is the root of the problem of infinities (renormalisation) in Quantum Field Theory.* It is not hard to see that this problem is inherited by quantum mechanics and QFT as Schrödinger and Klein-Gordon equations are derived from the application of the operatorial form of Planck-Einstein-de Broglie law $p^\mu = \hbar k^\mu$ to the law of conservation of energy. Accordingly the resolution

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of this problem right at the beginning might help to overcome the problem of infinities. It is evident that if we are to get a proper inverse Fourier transform of $E = \hbar\omega$ we must correct it, and that is in fact what recent experiments suggest [1]. Mathematically however there are many ways to do so and without a firm physical motivation one can easily be lead astray. To find such a firm physical ground our starting point is the following comparison

$$E = \frac{1}{2}m\mathbf{v}^2 \quad \text{versus,} \quad E = \frac{1}{2}I\boldsymbol{\omega}^2$$

By analogy between $\mathbf{v}$ and $\boldsymbol{\omega}$, *if there is a maximum for $\boldsymbol{v}$ then there must also be a maximum for $\boldsymbol{\omega}$.* This analogy leads us to the following

Principle 1 (Principle of Maximum Angular Frequency)**.** *All angular frequencies of rotation are bounded above by ω_P where*

$$\omega_P := 2\pi f_P = 2\pi\sqrt{\frac{c^5}{\hbar G}}$$

As pointed earlier, contrary to the mainstream view that sees spin of elementary particles unrelated to any kind of rotational spin, I conjecture that it is the spin (rotational) motion of elementary particles that creates de Broglie matterwaves hence we must demand $E = \frac{1}{2}I\boldsymbol{\omega}^2$ to yield $E = \hbar\omega$. To that end *assuming that the spin (rotational) motion of an elementary particle is completely independent from its translational motion*, we conclude that the spin motions must occur in an ontologically distinct space which we hereafter refer to as the space of *angles-time.* We postulate that the distances in this angles-time space are measured by the following metric

$$d\Theta^2 = \omega_P^2 dt^2 \pm d\boldsymbol{\theta}^2, \tag{1}$$

which naturally implements the principle of maximum frequency. Another motivation for introducing a whole new independent space is to get close to explaining how particles which are separeted by space-like distances can still be correlated (quantum entanglement), for a new space with a new metric means that this space has a new independent notion of *locality*, independent of the locality in spacetime. Regarding the sign of this metric, it is something that must be singled out by experiments therefore we shall consider both cases. As a result of the principle of maximum frequency the rotational energy of a particle is given by

$$E = \frac{I\omega_P^2}{\sqrt{1-(\omega/\omega_P)^2}} \tag{2}$$

or

$$E = -\frac{I\omega_P^2}{\sqrt{1+(\omega/\omega_P)^2}} \tag{3}$$

depending on the sign of the metric (1). By analogy with the gamma factor of special relativity we expect the $\pm I\omega_P^2$ to yield the inertial energy of rotation but that would only be the case if I was a constant, but it turns out that if we are to recover $E = \hbar\omega$ from (2) or (3), I is not a constant and depends on the frequency of rotation. To see this, observe that the second term in the Taylor expansions of (2) and (3) is

$$\frac{1}{2}I\omega^2$$

showing the expected compatibility with classical mechanics. But to get $E = \hbar\omega$ we need to use the definition of I in terms of spin angular momentum S and angular frequency ω

$$I = \frac{S}{\omega}$$

which yields

$$E = \frac{1}{2}S\omega \tag{4}$$

which can be interpreted as meaning that wave-particle duality is a classical fact and it is in the quantisation of spin that quantum mechanics enters: to get $E = \hbar\omega$ we now only need to let

$$S = 2\hbar$$

in (4). This $S = 2\hbar$ is nothing but an 'old' quantum-mechanical quantisation rule. Putting all these considerations together, complete exact form of Planck–Einstein relation is given by

$$\boxed{E(\omega) = \frac{2\hbar\omega_P^2}{\omega}\left(\frac{1}{\sqrt{1-(\omega/\omega_P)^2}} - 1\right)} \tag{5}$$

or

$$E(\omega) = \frac{2\hbar\omega_P^2}{\omega}\left(1 - \frac{1}{\sqrt{1+(\omega/\omega_P)^2}}\right) \tag{6}$$

Note that by construction $E = \hbar\omega$ is an approximation to the above equation, by being the first term in the Taylor expansion of the angles-time space factor. Unlike $E = \hbar\omega$ however, our proposed generalisation of Planck–Einstein relation is not plagued by infinities and it is perfectly possible to take its inverse Fourier transform. To this purpose, recall that

$$\mathcal{F}_\omega^{-1}[\frac{1}{\sqrt{1+(\omega/\omega_P)^2}}] = \omega_P\sqrt{\frac{2}{\pi}}K_0\big(\omega_P t\big) \tag{7}$$

where $K_0(t)$ is the zeroth order modified Bessel function of the second kind. And

$$\mathcal{F}_\omega^{-1}[\frac{1}{\sqrt{1-(\omega/\omega_P)^2}}] = -i\omega_P\sqrt{\frac{2}{\pi}}K_0\big(-i\omega_P t\big). \tag{8}$$

Now observe that

$$i\omega E(\omega) = 2i\hbar\omega_P^2\left(1 - \frac{1}{\sqrt{1+(\omega/\omega_P)^2}}\right),$$

and

$$i\omega E(\omega) = 2i\hbar\omega_P^2 \left(\frac{1}{\sqrt{1-(\omega/\omega_P)^2}} - 1 \right),$$

which is

$$\mathcal{F}[\frac{d}{dt}E(t)] = 2i\hbar\omega_P^2 \mathcal{F}[\delta(t) - \omega_P \sqrt{\frac{2}{\pi}} K_0\big(\omega_P t\big)],$$

and

$$\mathcal{F}[\frac{d}{dt}E(t)] = -2i\hbar\omega_P^2 \mathcal{F}[i\omega_P \sqrt{\frac{2}{\pi}} K_0\big(-i\omega_P t\big) + \delta(t)],$$

therefore

$$\boxed{E(t) = 2i\hbar\omega_P^2 \left(1 - \omega_P \sqrt{\frac{2}{\pi}} \int_0^t K_0\big(\omega_P \tau\big) d\tau \right)} \tag{9}$$

or

$$E(t) = -2i\hbar\omega_P^2 \left(1 + i\omega_P \sqrt{\frac{2}{\pi}} \int_0^t K_0\big(-i\omega_P \tau\big) d\tau \right). \tag{10}$$

References

[1] Dakotah Thompson et al. Hundred-fold enhancement in far-field radiative heat transfer over the blackbody limit. *Nature*, 561:216–221, 2018.