# The Accurate Values of Bohr Radius, the Classical Electron Radius, Electron Mass and Positron Mass

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his $70^{th}$ birthday

**Abstract**

Based on the exact values of the fine-structure constant (α) and the speed of light in atomic units ($c_{au}$) determined by our corresponding formulas and CODATA current (2018) recommended Rydberg constant ($R_\infty$ or $R_H$), we calculated out much more accurate values of Bohr radius ($a_0$= 5.2917721093848(101)×$10^{-11}$m), the classical electron radius ($r_e$=2.8179403267674(54)×$10^{-15}$ m) and electron mass ($m_e$=9.1093837003120(175)×$10^{-31}$ kg) than CODATA current (2018) recommended values which are $a_0$=5.29177210903(80)×$10^{-11}$ m, $r_e$=2.817940 3262(13)×$10^{-15}$ m and $m_e$=9.1093837015(28)×$10^{-31}$ kg respectively. In addition, we also calculated out accurate value of the positron mass ($m_{e+}$=9.1093836486490(174)×$10^{-31}$ kg) which was supposed to be slightly less than that of electron mass. In the end, we suggest more reasonable definitions of atomic units and predict Rydberg constant for anti-hydrogen atom ($R_{\infty/e+}$=10973731.505923(21) $m^{-1}$) which is a little different to Rydberg constant for ordinary hydrogen atom ($R_\infty$=10973731.568160(21) $m^{-1}$).

**Keywords:** Bohr radius; the classical electron radius; electron mass; positron mass; the fine-structure constant; Rydberg constant; atomic unit of time; atomic units.

## 1. Introduction

In our previous papers[1-11], we gave formulas of the fine structure constant (α) and their applications or relevant developments. Definitions and some typical formulas of the fine-structure constant are listed as follows.

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} \quad \alpha_2 = \frac{2\pi r_e}{\lambda_e} \quad \alpha_c = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{v_e}{c} = \sqrt{\frac{r_e}{a_0}} = \sqrt{\alpha_1 \alpha_2}$$

$$c_{au} = \frac{1}{\alpha_c} = \frac{c}{v_e} = \sqrt{\frac{a_0}{r_e}} = \frac{1}{\sqrt{\alpha_1 \alpha_2}}$$ ($c_{au}$ means the speed of light in atomic units)

$2\pi - e$ formula: $$(2\pi)_{Chen\text{-}k} = \left(\frac{e}{e^{\gamma_{c-k}}}\right)^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots \frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

$$\alpha_1 = \frac{36}{7 \cdot (2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{13 \cdot (2\pi)_{Chen-278}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818$$

$$\alpha_1 = \frac{1}{56 + 81 + \frac{1}{28 - \frac{13 \cdot (2 \cdot 56 \cdot 11 - 1)}{3 \cdot 5 \cdot (2 \cdot 56 \cdot 43 + 1)}}} = 1/137.035999037435$$

$$\alpha_2 = \frac{1}{56 + 81 + \frac{1}{28 - \frac{2 \cdot (16 \cdot 27 - 1)}{3 \cdot (16 \cdot 81 + 1)}}} = 1/137.035999111818$$

the key element and its isotopes: ${}^{136,137,138}_{56}Ba_{80,81,82}$

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = \sqrt{137.035999037435 \times 137.035999111818}$$

$$= \sqrt{112 \times (168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{14 \cdot 112 \cdot (2 \cdot 173 + 1) + \frac{1}{3}})}$$

$$= 56 + 81 + \frac{1}{28 - \frac{2 \cdot 27 \cdot 173 - 1}{2 \cdot 5 \cdot (8 \cdot 17 \cdot 31 + 1)}} = 56 + 81 + \frac{1}{27 + \frac{3 \cdot 31 \cdot (32 \cdot 11 + 1)}{2 \cdot 5 \cdot (8 \cdot 17 \cdot 31 + 1)}}$$

$$= 137.035999074627$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{59}_{27}Cr_{32}$ ${}^{58,60,62}_{28}Ni_{30,32,34}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{78}_{34}Se_{44}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{100}_{44}Ru_{56}$

${}^{107,109}_{47}Ag_{60,62}$ ${}^{112}_{48}Cd_{64}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{150}_{62}Sm_{88}$ ${}^{157}_{64}Gd_{93}$ ${}^{162}_{66}Dy_{96}$ ${}^{168}_{68}Er_{100}$ ${}^{173}_{70}Yb_{103}$

${}^{4\cdot 47}_{76}Os_{112}$ ${}^{203,205}_{81}Tl_{122,124}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{237}_{93}Np^{*}_{144}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{285}_{112}Cn^{*}_{173}$

${}^{310}_{124}Ch^{ie}_{186}$ ${}^{2\cdot 157}_{126}Ch^{ie}_{4\cdot 47}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{14\cdot 31,435}_{173}Ch^{ie}_{261,262}$

related nuclides for the last formula: 2021/8/26

Based on calculations with the speed of light in atomic units ($c_{au}$) and Rydberg constant ($R_\infty$ or $R_H$), in this paper we give much more accurate values of Bohr radius ($a_0$), the classical electron radius ($r_e$) and electron mass ($m_e$) than the CODATA current (2018) recommended values. In addition, we also give accurate value of the positron mass ($m_{e+}$) which was supposed to be slightly less than that of electron mass in our previous paper[3]. In the end, we suggest more reasonable definitions of atomic units and predict the value of Rydberg constant for anti-hydrogen atom which is a little different to Rydberg constant for ordinary hydrogen atom.

## 2. Rydberg Constant and Hartree Energy

The Rydberg formula for hydrogen atom is part of the Bohr model of hydrogen atom. It describes the wavelength of the photon emitted or absorbed by an electron when making a jump between orbitals in hydrogen atom. It has the following forms.

$$\frac{1}{\lambda}=\frac{1}{R_H}(\frac{1}{n_1^2}-\frac{1}{n_2^2})$$

$$\frac{1}{\lambda_{1-\infty}}=\frac{1}{R_\infty}=\frac{1}{R_H}(\frac{1}{1^2}-\frac{1}{\infty^2})$$

$$R_\infty = R_H = 10973731.568160(21)\ \text{m}^{-1}$$

2018 CODATA recommended value

The Rydberg constant ($R_\infty$) is a physical constant relating to the electromagnetic spectra of a hydrogen atom. The constant is used to express the limiting value of the highest wave number of any photon that can be emitted from a hydrogen atom, or the wave number of the lowest-energy photon capable of ionizing a hydrogen atom from its ground state.

The Hartree energy ($E_H$ or $H_a$) is the atomic unit of energy often used in atomic physics and computational chemistry. It is equal to the electric potential energy of the hydrogen atom in its ground state. The Hartree energy is defined based on the Rydberg constant, the Planck constant, and the speed of light as follows.

Hartree Energy: $E_h = H_a = 2R_\infty hc$

$R_\infty = 10973731.568160(21)\ \text{m}^{-1}$

$h = 6.62607015\times10^{-34}$ J s (exact)   $c = 299792458\ \text{m s}^{-1}$ (exact)

$$E_h = H_a = 2\times10973731.568160(21)\times6.62607015\times10^{-34}\times299792458$$

$$= 4.3597447222072(83)\times10^{-18}\ \text{J}$$

2018 CODATA recommended value: $E_h = H_a = 4.3597447222071(85)\times10^{-18}$ J

## 3. Atomic Unit of Time

Atomic unit of time is a derived unit of time in atomic units, here we denote it as $t_{au}$. Definition of 1 atomic unit of time is $t_{au}=a_0/(\alpha_c c)= a_0/v_e$, it means to take the linear speed of electron in hydrogen atom's ground orbital in Bohr model ($v_e$) as one unit of speed and to take Bohr radius ($a_0$) as one unit of length and then to calculate the time unit ($t_{au}$). This is the smallest meaningful time unit under which an electron takes a circle on the first Bohr pitch divided by 2π. Atomic unit of time is much smaller than second in human world, but it seems like second in hydrogen atom and it should be a scientific definition of time unit.

Atomic units (a.u.):

$m_{e/au} = e_{au} = \hbar_{au} = 4\pi\varepsilon_0 = 1$

Derived units in atomic units: $a_{0/au} = 1,\ v_{e/au} = 1,\ \mathrm{t}_{au} = 1$

Express $t_{au}$ in the international system of units $(\mathrm{SI})$:

atomic unit of time: $\mathrm{t}_{au} = \dfrac{\hbar}{E_h} = \dfrac{\hbar}{2R_\infty hc} = \dfrac{1}{4\pi R_\infty c}$

$E_h = H_a = 4.3597447222072(83) \times 10^{-18}$ J

$h = 6.62607015 \times 10^{-34}$ J s (exact) $\hbar = \dfrac{h}{2\pi}$

$$\mathrm{t}_{au} = \frac{6.62607015 \times 10^{-34}}{2\pi \times 4.3597447222072(83) \times 10^{-18}} = 2.4188843265857(46) \times 10^{-17}\ \mathrm{s}^{-1}$$

2018 CODATA recommended value: $\mathrm{t}_{au} = 2.4188843265857(47) \times 10^{-17}\ \mathrm{s}^{-1}$

## 4. More Accurate Value of Bohr Radius

Bohr radius ($a_0$) was calculated from the speed of light in atomic units ($c_{au}$) and Rydberg constant ($R_\infty$) as follows.

$$t_{au} = \frac{1}{4\pi R_\infty c} = 2.4188843265857(46) \times 10^{-18}\ \mathrm{s}^{-1}$$

$t_{au} = \dfrac{a_0}{v_e}$ ($v_e$ is the linear speed of electron in the ground state in a hydrogen atom)

$$c_{au} = \frac{c}{v_e} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = 137.035999074627 \text{ (exact)}$$

$c = 299792458\ \mathrm{m\ s}^{-1}$ (exact)

$$v_e = \alpha_c c = \frac{299792458}{137.035999074627} = 2187691.26378784\ \mathrm{m\ s}^{-1} \text{ (exact)}$$

$$a_0 = v_e t_{au} = \alpha_c c t_{au} = \frac{c}{c_{au}} t_{au} = \frac{1}{4\pi R_\infty c_{au}}$$

$= 2187691.26378784 \times 2.4188843265857(46) \times 10^{-18}$

$= 5.2917721093848(101) \times 10^{-11}$ m

2018 CODATA recommended value: $a_0 = 5.29177210903(80) \times 10^{-11}$ m

about 80 times more accurate than CODATA recomended value

## 5. More Accurate Value of the Classic Electron Radius

The classic electro radius ($r_e$) was calculated from the speed of light in atomic units ($c_{au}$) and Bohr radius ($a_0$) as follows.

$$c_{au}^{2}=\frac{1}{\alpha_c^{2}}=\frac{1}{\alpha_1\alpha_2}=\frac{a_0}{r_e}$$

$c_{au}=137.035999074627$ (exact)

$a_0=5.2917721093848(101)\times10^{-11}$ m

$$r_e=\frac{a_0}{c_{au}^{2}}=\frac{5.2917721093848(101)\times10^{-11}}{137.035999074627^{2}}$$

$=2.8179403267674(54)\times10^{-15}$ m

2018 CODATA recommended value: $r_e=2.8169403262(13)\times10^{-11}$ m

about 240 times more accurate than CODATA recomended value

## 6. More Accurate Value of the Electron Mass

The electron mass ($m_e$) was calculated from the speed of light in atomic units ($c_{au}$) and Rydberg constant ($R_\infty$) or Bohr radius ($a_0$) as follows.

Consider an electron becoming a photon with frequency of $\nu_{e/au}$,

$\nu_{e/au}$ could be called electron-photon relation frequency in atomic units,

according to Einstein's equation of $E=mc^2$ and $E=h\nu$, there should be:

$$E_{e/au}=h_{au}\nu_{e/au}=m_{e/au}c_{au}^{2}\ \Rightarrow\ \frac{2\pi\hbar_{au}\nu_{e/au}}{m_{e/au}}=c_{au}^{2}\ \Rightarrow\ \frac{\hbar_{au}\nu_{e/au}}{m_{e/au}}=\frac{c_{au}^{2}}{2\pi}$$

in atomic units, $\hbar_{au}=1,\ m_{e/au}=1,$ so:

$$\nu_{e/au}=\frac{c_{au}^{2}}{2\pi}=\frac{136.035999074627^{2}}{2\cdot3.14159265358979}=2988.74919715056\ t_{au}^{-1}$$

Consider an electron becoming a photon with frequency of $\nu_e$ in SI:

$$E_e=h\nu_e=m_ec^2$$

$$t_{au}=\frac{1}{4\pi R_\infty c}=\frac{a_0}{c/c_{au}}$$

$$\nu_e=\frac{\nu_{e/au}}{t_{au}}=2R_\infty cc_{au}^{2}=\frac{c_{au}c}{2\pi a_0}$$

$$m_e=\frac{h\nu_e}{c^2}=\frac{2hR_\infty c_{au}^{2}}{c}=\frac{\hbar}{a_0c/c_{au}}$$

$$=\frac{2\times6.62607015\times10^{-34}\times10973731.568160(21)\times137.035999074627^{2}}{299792458}$$

$=9.1093837003120(175)\times10^{-31}$ kg

CODATA recommended value: $9.1093837015(28)\times10^{-31}$ kg

about 165 times more accurate than CODATA recomended value

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## 7. Accurate Value of Positron Mass

In our previous paper[3], we supposed that the masses of electron and positron were different a little but had proportional relationships, and hence we could calculate positron mass as follows.

$$\frac{m_e}{m_{e^+}} = \frac{1+(\alpha_1\alpha_2)^2}{1-(\alpha_1\alpha_2)^2} = \frac{1+{\alpha_c}^2}{1-{\alpha_c}^2} = \frac{1+\dfrac{1}{{c_{au}}^4}}{1-\dfrac{1}{{c_{au}}^4}}$$

$$E_e = h\nu_e = m_e c^2 \;\Rightarrow\; m_e = \frac{h\nu_e}{c^2} = \frac{h(\nu_{e/au}/t_{au})}{c^2} \propto \nu_{e/au}$$

$$E_{e^+} = h\nu_{e^+} = m_{e^+} c^2 \;\Rightarrow\; m_{e^+} = \frac{h\nu_{e^+}}{c^2} = \frac{h(\nu_{e^+/au}/t_{au})}{c^2} \propto \nu_{e/au}$$

$$\frac{\nu_{e/au}}{\nu_{e/au}} = \frac{1+(\alpha_1\alpha_2)^2}{1-(\alpha_1\alpha_2)^2} = \frac{1+{\alpha_c}^2}{1-{\alpha_c}^2} = \frac{1+\dfrac{1}{{c_{au}}^4}}{1-\dfrac{1}{{c_{au}}^4}}$$

$$\nu_{e/au} = \frac{{c_{au}}^2}{2\pi}$$

$$\nu_{e^*/au} = \frac{{c_{au}}^2}{2\pi} \times \frac{1-\dfrac{1}{{c_{au}}^4}}{1+\dfrac{1}{{c_{au}}^4}} = \frac{{c_{au}}^2}{2\pi} \times \frac{1-\dfrac{1}{137.035999074627^4}}{1+\dfrac{1}{137.035999074627^4}} = 2988.74918020013\ {t_{au}}^{-1}$$

$$\nu_{e^+} = \frac{\nu_{e^+/au}}{t_{au}} = \frac{2988.74918020013}{2.4188843265857(44)\times 10^{-17}} = 1.2355899566387(24)\times 10^{20}\ \mathrm{s}^{-1}$$

$$m_{e^+} = \frac{h\nu_{e^+}}{c^2} = \frac{6.62607015\times 10^{-34}\ \mathrm{J\ s} \times 1.2355899566387(24)\times 10^{20}\ \mathrm{s}^{-1}}{(299792458\ \mathrm{m\ s}^{-1})^2}$$

$$= 9.1093836486490(174)\times 10^{-31}\ \mathrm{kg}$$

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## 8. Meanings of $\nu_{e/au}$ and $\nu_{e+/au}$

Atomic unit of time ($t_{au}$) should be a scientific time unit in the universe (maybe the unique scientific unit of time), in comparison, the second in human world is not. So the electron-photon relation frequency in atomic units ($\nu_{e/au}$), the positron-photon relation frequency in atomic units and their average/plus/minus values should also have scientific meanings, and here we relate them to nuclides. They should relate to some critical elements such as $_{43}Ti^*$, $_{56}Ba$, $_{61}Pm^*$, $_{83}Bi^*$, $_{112}Cn^*$, $_{137}Fy^{ie}$ and $_{139}Ch^{ie}$.

$$\nu_{e/au}=\frac{c_{au}{}^{2}}{2\pi}=\frac{136.035999074627^{2}}{2\pi}=2988.74919715056\ \mathrm{t}_{au}{}^{-1}$$

$$\nu_{e/au}=36\cdot 83+1-\frac{1}{3}+\frac{1}{12}-\frac{1}{3\cdot 5\cdot 83+\frac{5\cdot 47}{3\cdot 139}}=2988.749197150558337\ \mathrm{t}_{au}{}^{-1}$$

$^{83}_{36}Kr_{47}$ $^{107,108}_{47}Ag_{60,62}$ $^{139}_{57}La_{82}$ $^{140,142}_{58}Ce_{82,84}$ $^{143}_{60}Nd_{83}$ $^{209}_{83}Bi^{*}_{126}$ $^{5\cdot 47}_{92}U^{*}_{143}$ $^{346}_{137}Fy^{ie}_{209}$ $^{348}_{139}Ch^{ie}_{209}$

$$\nu_{e/au}=36\cdot 83+1-\frac{1}{3}+\frac{1}{12}-\frac{1}{3\cdot 5\cdot 83}+\frac{1}{2\cdot 9\cdot 7\cdot(16\cdot 3\cdot 5\cdot 7\cdot 13-1)+\frac{8}{9}}$$

$$=36\cdot 83+1-\frac{1}{3}+\frac{1}{12}-\frac{1}{3\cdot 5\cdot 83}+\frac{1}{2\cdot 9\cdot 7\cdot(2\cdot 61\cdot 179+1)+\frac{8}{9}}$$

$$=2988.749197150558337\ \mathrm{t}_{au}{}^{-1}$$

$^{27}_{13}Al_{14}$ $^{31}_{15}P_{16}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{83,84}_{36}Kr_{47,48}$ $^{107,109}_{47}Ag_{60,62}$ $^{126}_{52}Te_{74}$ $^{179,180}_{72}Hf_{107,108}$ $^{209}_{83}Bi^{*}_{126}$

$$\nu_{e/au}=36\cdot 83+\frac{3}{4+\frac{4\cdot 139}{2\cdot 5\cdot 7\cdot 17\cdot 109+1}}=36\cdot 83+\frac{3\cdot(2\cdot 5\cdot 7\cdot 17\cdot 109+1)}{8\cdot 25\cdot 49\cdot 53}$$

$$=2988.749197150558337\ \mathrm{t}_{au}{}^{-1}$$

$^{35,37}_{17}Cl_{18,20}$ $^{53}_{24}Cr_{29}$ $^{83}_{36}Kr_{47}$ $^{107,109}_{47}Ag_{60,62}$ $^{7\cdot 17}_{50}Sn_{69}$ $^{178}_{72}Hf_{106}$ $^{209}_{83}Bi^{*}_{126}$ $^{2\cdot 139}_{109}Mt^{*}_{169}$ $^{348}_{139}Ch^{ie}_{209}$

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$$\nu_{e^{*}/au}=\frac{\nu_{e/au}(1-\alpha_c{}^{4})}{1+\alpha_c{}^{4}}=\frac{c_{au}{}^{2}}{2\pi}\times\frac{1-1/137.035999074627^{4}}{1+1/137.035999074627^{4}}=2988.74918020013\ \mathrm{t}_{au}{}^{-1}$$

$$\nu_{e^{*}/au}=49\cdot 61-\frac{1}{3}+\frac{1}{12}-\frac{1}{4\cdot 5\cdot 61-\frac{73}{128\cdot 3}}=2988.749180200125105\ \mathrm{t}_{au}{}^{-1}$$

$^{83,84,86}_{36}Kr_{47,48,50}$ $^{85,87}_{37}Rb_{48,50}$ $^{107,109}_{47}Ag_{60,62}$ $^{113,115}_{49}In_{64,66}$ $^{145,2\cdot 73,3\cdot 49}_{61}Pm^{*}_{84,85,86}$ $^{180,181}_{73}Ta_{107,108}$

$$\nu_{e^{*}/au}=49\cdot 61-\frac{1}{3}+\frac{1}{12}-\frac{1}{23\cdot 53}+\frac{1}{25\cdot 23\cdot 31\cdot 103-\frac{29}{2\cdot 49}}$$

$$=2988.749180200125105\ \mathrm{t}_{au}{}^{-1}$$

$^{23}_{11}Na_{12}$ $^{24,25,26}_{12}Mg_{12,13,14}$ $^{31}_{15}P_{16}$ $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{50,51}_{23}V_{27,28}$ $^{53}_{24}Cr_{29}$ $^{55}_{25}Mn_{30}$

$^{63,65}_{29}Cu_{34,36}$ $^{69,71}_{31}Ga_{38,40}$ $^{82,83,84,85}_{36}Kr_{46,47,48,50}$ $^{85,87}_{37}Rb_{48,50}$ $^{95}_{42}Mo_{53}$ $^{103}_{45}Rh_{58}$ $^{113,115}_{49}In_{64,66}$

$^{127}_{53}I_{74}$ $^{173}_{70}Yb_{103}$ $^{262}_{103}Lr^{*}_{158}$ $^{15\cdot 29}_{173}Ch^{ie}_{262}$

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$$\nu_{e^{+}/au}=49\cdot 61-\frac{1}{4-\frac{2^{11}\cdot 3}{(4\cdot 67+1)\cdot(4\cdot 19\cdot 23-1)}}=49\cdot 61-\frac{(4\cdot 67+1)\cdot(4\cdot 19\cdot 23-1)}{4\cdot 19\cdot 89\cdot(4\cdot 3\cdot 23+1)}$$

$$=2988.749180200125105\ \mathrm{t}_{au}{}^{-1}$$

$^{39,40,41}_{19}K_{20,21,22}$ $^{89}_{39}Y_{50}$ $^{113,115}_{49}In_{64,66}$ $^{145,146,3\cdot 49}_{61}Pm^{*}_{84,85,86}$ $^{151,153}_{63}Eu_{88,90}$ $^{165}_{67}He_{2\cdot 49}$ $^{227}_{89}Ac^{*}_{138}$

2021/8/30

$$\nu_{(e+e^*)/2/au}=\frac{\nu_{e/au}}{1+\alpha_c{}^4}=36\cdot83+1-\frac{1}{3}+\frac{1}{12}-\cfrac{1}{16\cdot7\cdot11+\cfrac{25\cdot11}{2\cdot3\cdot83}}$$

$$=36\cdot83+1-\frac{1}{3}+\frac{1}{12}-\cfrac{1}{9\cdot137-\cfrac{223}{2\cdot3\cdot83}}$$

$$=36\cdot83+1-\frac{1}{3}+\frac{1}{12}-\frac{1}{16\cdot7\cdot11}+\cfrac{1}{7\cdot83\cdot(4\cdot7\cdot13^2+1)+\cfrac{7}{25}}$$

$$=2988.749188675341433\ \mathrm{t}_{au}{}^{-1}$$

${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{83,84}_{36}Kr_{47,48}$

${}^{99,100}_{44}Ru_{55,56}$ ${}^{112}_{48}Cd_{64}$ ${}^{126}_{52}Te_{74}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{169}_{69}Tm_{100}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{188}_{76}Os_{112}$ ${}^{209}_{83}Bi^*_{126}$

${}^{209}_{84}Po^*_{125}$ ${}^{223}_{87}Fr^*_{136}$ ${}^{226}_{88}Ra^*_{138}$ ${}^{278}_{109}Mt^*_{169}$ ${}^{285}_{112}Cn^*_{173}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{314}_{126}Ch^{ie}_{188}$ ${}^{8\cdot43,346,348}_{136,137,138}Fy^{ie}_{16\cdot13,209,210}$

2021/8/25

$$\nu_{(e+e^*)/2/au}=36\cdot83+\cfrac{3}{4+\cfrac{32\cdot83}{29\cdot(24\cdot(18\cdot49-1)-1)}}=36\cdot83+\frac{3\cdot29\cdot(24\cdot(18\cdot49-1)-1)}{4\cdot11\cdot41\cdot(16\cdot5\cdot17+1)}$$

$$=2988.749188675341432\ \mathrm{t}_{au}{}^{-1}$$

${}^{53}_{24}Cr_{29}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{166}_{68}Er_{98}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^*_{126}$ ${}^{223}_{87}Fr^*_{136}$

2021/8/30

$$\nu_{e+e^*/au}=\frac{2\nu_{e/au}}{1+\alpha_c{}^4}=43\cdot139+\frac{1}{2}-\frac{1}{8\cdot7\cdot11}+\cfrac{1}{8\cdot3\cdot59\cdot(4\cdot3^5-1)+\cfrac{37}{2\cdot29}}$$

$$=5977.498377350682866$$

${}^{85,87}_{37}Rb_{48,50}$ ${}^{99}_{43}Tc^*_{56}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$

${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{184,186}_{74}W_{110,112}$ ${}^{6\cdot37}_{86}Rn^*_{136}$ ${}^{226}_{88}Ra^*_{138}$ ${}^{278}_{109}Mt^*_{169}$ ${}^{12\cdot29}_{139}Ch^{ie}_{209}$ ${}^{400}_{157}Ch^{ie}_{243}$

2021/8/29

$$\nu_{e+e^*/au}=\frac{2\nu_{e/au}}{1+\alpha_c{}^4}=43\cdot139+\cfrac{1}{2+\cfrac{16\cdot3\cdot83}{13\cdot(2\cdot3\cdot7\cdot59-1)}}=43\cdot139+\frac{13\cdot(2\cdot3\cdot7\cdot59-1)}{2\cdot11\cdot41\cdot(16\cdot5\cdot17+1)}$$

$$=5977.498377350682865$$

${}^{27}_{13}Al_{14}$ ${}^{59}_{27}Co_{32}$ ${}^{99}_{43}Tc^*_{56}$ ${}^{118}_{50}Sn_{68}$ ${}^{139}_{57}La_{82}$ ${}^{141}_{59}Pr_{82}$ ${}^{166,168}_{68}Er_{98,100}$ ${}^{209}_{83}Bi^*_{126}$ ${}^{278}_{109}Mt^*_{169}$ ${}^{348}_{139}Ch^{ie}_{209}$

2021/8/30

$$\nu_{e-e^*/au}=\frac{2\alpha_c{}^4\nu_{e/au}}{1+\alpha_c{}^4}=\cfrac{1}{27\cdot5\cdot19\cdot23+\cfrac{7\cdot11}{2\cdot71}}=\cfrac{1}{4\cdot7^3\cdot43-\cfrac{5\cdot13}{2\cdot71}}=0.000016950433233$$

${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{77}_{34}Se_{43}$ ${}^{99}_{43}Tc^*_{56}$ ${}^{98,99,100,101,104}_{44}Ru_{54,55,56,57,60}$

${}^{113,115}_{49}In_{64,66}$ ${}^{115,119}_{50}Sn_{65,69}$ ${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{169}_{69}Tm_{100}$ ${}^{175,176}_{71}Lu_{104,105}$

${}^{191,193}_{77}Ir_{114,116}$ ${}^{226}_{88}Ra^*_{138}$ ${}^{232}_{90}Th^*_{142}$ ${}^{4\cdot71,286}_{113}Nh^{ie}_{171,173}$ ${}^{289}_{114}Fl^{ie}_{175}$ ${}^{289}_{115}Mc^{ie}_{174}$ ${}^{8\cdot43,346,348}_{136,137,138}Fy^{ie}_{16\cdot13,209,210}$

2021/8/29

## 9. Meanings of Rydberg Constant

The Rydberg constant times Bohr radius gives a dimensionless constant, it should have some scientific meanings, and here we relate it to nuclides.

$$R_\infty = 10973731.568160(21)\ \mathrm{m}^{-1}$$

$$a_0 = 5.291772109385(10)\times 10^{-11}\ \mathrm{m}$$

$$R_{Chen} = \frac{1}{R_\infty a_0} = \frac{1}{R_\infty v_e t_{au}} = \frac{1}{R_\infty \alpha_c c t_{au}} = \frac{4\pi}{\alpha_c} = \frac{4\pi}{\sqrt{\alpha_1 \alpha_2}} = 4\pi c_{au}$$

$$= \frac{1}{10973731.568160 \times 5.291772109385 \times 10^{-11}}$$

$$= 4\pi \times 137.035999074627 = 1722.04515188077$$

$$R_{Chen} = 2\cdot 3\cdot 7\cdot 41 + \frac{1}{2\cdot 11} - \frac{1}{8\cdot 7\cdot 59 - \frac{1}{11\cdot 13}} = 1722.04515188074$$

$$R_{Chen} = 2\cdot 3\cdot 7\cdot 41 + \frac{1}{2\cdot 11} - \frac{1}{9\cdot(16\cdot 23 - 1) + \frac{3\cdot 47}{11\cdot 13}} = 1722.04515188074$$

$^{23}_{11}Na_{12}$ $^{27}_{13}Al_{14}$ $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{59}_{27}Co_{32}$ $^{75}_{33}As_{42}$ $^{82,83,84}_{36}Kr_{46,47,48}$ $^{105}_{46}Pd_{59}$

$^{107,109}_{47}Ag_{60,62}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ $^{208}_{82}Pd_{126}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{226}_{88}Ra^{*}_{138}$

$^{5\cdot 47}_{92}U^{*}_{143}$ $^{314}_{126}Ch^{ie}_{4\cdot 47}$ $^{346}_{137}Fy^{ie}_{209}$

2021/8/24

## 10. More Reasonable Atomic Units

As the masses of electron and positron are supposed to be different a little by us[3], the definitions of atomic units could be revised more reasonably.

Current atomic units (au):

$$m_{e/au} = e_{au} = \hbar_{au} = 4\pi\varepsilon_0 = 1$$

More reasonable atomic units (au):

$$e_{au} = \hbar_{au} = 4\pi\varepsilon_0 = 1,\ (m_{e/au} + m_{e^+/au})/2 = 1$$

$$m_{e/au} = 1 + (\alpha_1\alpha_2)^2 = 1 + \alpha_c{}^2 = 1 + \frac{1}{c_{au}{}^4} \quad m_{e^+/au} = 1 - (\alpha_1\alpha_2)^2 = 1 - \alpha_c{}^2 = 1 - \frac{1}{c_{au}{}^4}$$

$$c_{au} = \sqrt{112\times(168 - \frac{1}{3} + \frac{1}{12\cdot 47} - \frac{1}{14\cdot 112\cdot(2\cdot 173+1) + \frac{1}{3}})}$$

$$= 56 + 81 + \frac{1}{28 - \frac{2\cdot 27\cdot 173 - 1}{2\cdot 5\cdot(8\cdot 17\cdot 31 + 1)}} = 137.035999074627$$

2021/8/27

## 11. Rydberg Constant in Anti-Hydrogen Atom

The Rydberg constant for anti-hydrogen atom should be different a little with that for ordinary hydrogen atom as follows.

$$\nu_e = \frac{\nu_{e/au}}{t_{au}} = \frac{\frac{c_{au}^{\ 2}}{2\pi}}{\frac{1}{4\pi R_\infty c}} = 2R_\infty c c_{au}^{\ 2}$$

$$m_e = \frac{h\nu_e}{c^2} = \frac{2hR_\infty c c_{au}^{\ 2}}{c^2} = \frac{2hR_\infty c_{au}^{\ 2}}{c}$$

$$\frac{m_e}{m_{e^+}} = \frac{R_{\infty/e}}{R_{\infty/e^+}} = \frac{1+\frac{1}{c_{au}^{\ 4}}}{1-\frac{1}{c_{au}^{\ 4}}} = 1.00000000567141$$

$$R_{\infty/e^+} = R_{\infty/e}\frac{1-\frac{1}{c_{au}^{\ 4}}}{1+\frac{1}{c_{au}^{\ 4}}} = \frac{10973731.568160(21)}{1.00000000567141} = 10973731.505923(21)$$

2021/8/27

## 12. Meanings of $c_{au}/(1+1/c_{au}^4)^{1/2}$ and $c_{au}[(1-1/c_{au}^4)/(1+1/c_{au}^4)]^{1/2}$

Considering the mass difference between electron and positron, it seems that the speed of light in atomic units ($c_{au}$) could be modified and has corresponding meanings as follows. This implies the speed of light in anti-matter would be different a little.

Consider an electron becoming a photon with frequency of $\nu_{e/au}$,

$\nu_{e/au}$ is defined as electron-photon relation frequency in atomic units,

$$\nu_{e/au} = \frac{c_{au}^{\ 2}}{2\pi}$$

Consider a positron becoming a photon with frequency of $\nu_{e^+/au}$,

$$\nu_{e^+/au} = \frac{c_{au}^{\ 2}}{2\pi}\frac{1-\frac{1}{c_{au}^{\ 4}}}{1+\frac{1}{c_{au}^{\ 4}}} = \frac{1}{2\pi}[c_{au}(\frac{1-\frac{1}{c_{au}^{\ 4}}}{1+\frac{1}{c_{au}^{\ 4}}})^{1/2}]^2$$

So the following modified $c_{au}$ should have some meanings:

$$c_{au/e^+} = c_{au}(\frac{1-\frac{1}{c_{au}^{\ 4}}}{1+\frac{1}{c_{au}^{\ 4}}})^{1/2} = 137.035999074627\times\frac{(1-\frac{1}{137.035999074627^4})^{1/2}}{(1+\frac{1}{137.035999074627^4})^{1/2}}$$

$$= 137.035998686033$$

$$c_{au/e^+} = c_{au}\left(\frac{1-\frac{1}{c_{au}^{\ 4}}}{1+\frac{1}{c_{au}^{\ 4}}}\right)^{1/2} = \sqrt{112\times(168-\frac{1}{3}+\frac{1}{12\cdot 47}-\frac{1}{2\cdot(2\cdot 3\cdot(8\cdot 9\cdot 5\cdot 83+1)+1)-\frac{1}{6}})}$$

$$=137.035998686033$$

${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{98,100}_{44}Ru_{54,56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112}_{48}Cd_{64}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{143}_{60}Nd_{83}$
${}^{157,158}_{64}Gd_{93,94}$ ${}^{166,168}_{68}Er_{98,100}$ ${}^{4\cdot 47}_{76}Os_{112}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$
${}^{227}_{89}Ac^{*}_{138}$ ${}^{5\cdot 47}_{92}U^{*}_{143}$ ${}^{244}_{94}Pu^{*}_{150}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{314}_{126}Ch^{ie}_{4\cdot 47}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

2021/8/27

$$c_{au/ee^+} = \frac{c_{au}}{(1+\frac{1}{c_{au}^{\ 4}})^{1/2}} = \frac{137.035999074627}{(1+\frac{1}{137.035999074627^4})^{1/2}} = 137.035999268924$$

$$c_{au/ee^+} = \frac{c_{au}}{(1+\frac{1}{c_{au}^{\ 4}})^{1/2}} = \sqrt{112\times(168-\frac{1}{3}+\frac{1}{12\cdot 47}-\frac{1}{4\cdot(2\cdot 3\cdot 47-1)\cdot(2\cdot 3\cdot 109-1)-\frac{1}{5}})}$$

$$=137.035999268924$$

${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{98,100}_{44}Ru_{54,56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112}_{48}Cd_{64}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{143}_{60}Nd_{83}$
${}^{157,158}_{64}Gd_{93,94}$ ${}^{166,168}_{68}Er_{98,100}$ ${}^{183}_{74}W_{109}$ ${}^{4\cdot 47}_{76}Os_{112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$
${}^{227}_{89}Ac^{*}_{138}$ ${}^{5\cdot 47}_{92}U^{*}_{143}$ ${}^{244}_{94}Pu^{*}_{150}$ ${}^{278}_{109}Mt^{*}_{169}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{314}_{126}Ch^{ie}_{4\cdot 47}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

2021/8/31

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