Modification of Cantor Sets With Potential Infinities

Anders Lindman
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Abstract

In continuous Euclidean space all lines have an infinite number of points, e.g. a line A = 10 cm has the same number of points as a line B = 5 cm. In this paper a new set theory (MST for modified set theory) is defined so that lines of different lengths always contain different numbers of points. Instead of allowing several actual infinities only one actual infinity is defined. All other sets are either finite or have potential infinite cardinality. This makes the logic of sets more straightforward than with Georg Cantor's transfinite sets.

1. Actual infinity vs potential infinities

The term actual infinity here means the same as Geog Cantor's absolute infinity (sometimes translated as actual infinity). There is only one actual infinity since having more than one would make it inconsistent with the idea of preventing Euclidean space from having objects of different sizes all consisting of an infinite number of points.

Potential infinity is larger than finite yet less than actual infinity. An example of potential infinity is the set of natural numbers N = {0, 1, 2, 3, 4, 5, …} which has no largest number yet the natural numbers always have a finite number of digits.

This is similar to how Aristotle described it: “He distinguished between actual and potential infinity. Actual infinity is completed and definite, and consists of infinitely many elements.” - Wikipedia

> **Definition 1:** *Actual infinity* is the set A of all sets excluding itself.

Unlike the universal set, actually infinity *A* cannot contain itself because that would make non-discrete Euclidean space able to have actual infinities within actual infinities.

> **Definition 2:** *Potential infinity* is the sets with a cardinality larger than for all finite sets and less than actual infinity.

The difference between Cantor’s transfinite sets and potential infinite sets is how cardinality is defined in MST.

2. Cardinality in MST

For finite sets the cardinality in MST is the same as the cardinality for finite sets, e.g. |{a, b, c}| = 3. Potential infinite sets have a cardinality related to the unit cardinality U.

> **Definition 3:** The *unit cardinality U* is the cardinality for the set of natural numbers N = {0, 1, 2, 3, 4, 5, …}. U = |N|.

This makes the set of for instance the even numbers in MST have a cardinality less than the unity cardinality U. |{0, 2, 4, …}| = 0.5 U. Adding the cardinality of the odd numbers |{1, 3, 5, …} results in the unit cardinality: 0.5 U + 0.5 U = U.

More examples:

- The cardinality of the set of integers Z is 2U
- The cardinality of the set of rational numbers Q is $4U^2$
- The cardinality of the powerset of N is 2^U
- The cardinality of the powerset of the set of even numbers is $2^{0.5U}$

The cardinality of a potential infinite set P plus the cardinality of a finite set F that is not a subset of P is: $|P \cup F| = |P| + e$, where *e* is an infinitesimal number.

Actual infinity has no cardinality that can be expressed in terms of the unit cardinality U. Therefore the cardinality of actual infinity is defined as: |A| = infinity.

3. Real Numbers

The set of real numbers is called MR (modified real numbers) in MST and consists of all rational numbers and all irrational numbers. And the irrational numbers in MST are defined as potential infinities.

The cardinality of MR is $8U^2$. The irrational numbers are in MST defined as the set D of the Dedekind cuts between all rational numbers. And then the cardinality of the set D of those cuts is the same as the cardinality for the set of the rational numbers Q. $|MR| = |D \cup Q| = 4U^2 + 4U^2 = 8U^2$.

For example the irrational number Pi = 3.14159265… has an infinite number of decimals in R and only a potential infinite number of decimals in MR.