The Free Fall of the String

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Abstract

We consider the motion of string in free fall in gravity. The solutions are not identical with the string accelerated kinetically by acceleration a. So, we distinguish between non-inertial field and the gravity field and we discuss the principle of equivalence. In conclusion we suggest to drop the charged objects from the very high tower Burj Khalifa in order to say crucial words on the principle of equivalence.

1 Introduction

It is well known that Galileo performed experiment in Pisa - later the famous experiment - with the result that the every falling body is falling with a uniform acceleration, the resistance of the medium being through which it was falling remained negligible. He also derived the correct kinematic law for the distance traveled during a uniform acceleration starting from rest, namely, that it is proportional to the square of the elapsed time. Galileo expressed the time-squared law using geometrical constructions and mathematically precise words.

We here repeat the Galileo experiment in the virtual mathematical form. Namely, with the string. We discuss the motion of the string with accelerated boundary conditions, and by gravity, and we discover substantial differences leading to the adequate philosophy of the principle of equivalence.

It is possible to show that the uniformly accelerated string of the length l, where the left end and the right end is accelerated by constant acceleration a, forms the mathematical problem described by the wave equation (Koshlyakov, et al., 1962)

$$u_{tt} = c^2 u_{xx} + g(x,t), \tag{1}$$

where $g(x,t) = \frac{1}{\varrho}p(x,t)$ and $p(x,t)$ is the external force, and the boundary conditions are

$$u(x=0) = \frac{1}{2}at^2; \quad u(x=l) = \frac{1}{2}at^2 + l \tag{2}$$

and the initial conditions being

$$u(t=0) = f(x); \quad u_t(t=0) = F(x). \tag{3}$$

The solution of the system is well known , $u = v + w$ (Koshlyakov et al., 1962), where w is the solution of the homogenous equation (1) (g=0) with the initial conditions

$$w(x=0) = 0; \quad w_t(l=0) = 0 \tag{4}$$

and with the boundary conditions

$$w(l=0) = f(x); \quad w_t(l=0) = F(x). \tag{5}$$

The solution v is derived in the final form (Koshlyakov et al., 1962):

$$v(x,t) = \sum_{k=1}^{\infty} T_k \sin\left(\frac{k\pi x}{l}\right), \tag{6}$$

where

$$T_k(t) = \frac{2}{l\omega_k}\int_0^t d\tau \int_0^l G(\xi,\tau)\sin\omega_k(t-\tau)\sin\left(\frac{k\pi\xi}{l}\right)d\xi, \tag{7}$$

with

$$\omega_k = \frac{k\pi c}{l}; \quad G(\xi,\tau) = g - a. \tag{8}$$

2 The Free fall of the string in gravity

Now, let us consider the string with length l, the upper end is hanged in the gravity with the acceleration g and the second end is free at time $t = 0$. So the mathematical formulation of the problem is as follows (Koshlyakov, et al., 1962):

$$u_{tt} = c^2 u_{xx} + g \tag{9}$$

with

$$u(x=0) = 0; \quad u_x(x=l) = 0 \tag{10}$$

$$u(t=0) = 0; \quad u_t(t=0) = 0. \tag{11}$$

Putting $u = v + w$, we get for w the obligate system of equations:

$$w_{tt} = c^2 w_{xx} \tag{12}$$

with the boundary conditions

$$w(x=0) = 0; \quad w_x(x=l) = 0 \tag{13}$$

and the initial conditions

$$w(t=0) = -v(t=0); \quad w_t(t=0) = -v_t(t=0). \tag{14}$$

It is possible to show (Koshlyakov, et al., 1962) that

$$v = \frac{gx(2l-x)}{2c^2}. \tag{15}$$

So, we can write

$$f(x) = \frac{gx(x-2l)}{2c^2}; \quad F(x) = 0. \tag{16}$$

Then, by the standard method of integration, we get

$$u(x,t) = \frac{gx(2l-x)}{2c^2} -$$

$$\frac{16gl^2}{\pi^3c^2}\sum_{k=1}^{\infty}\frac{1}{(2k+1)^3}\cos\left(\frac{(2k+1)\pi at}{2l}\right)\sin\left(\frac{(2k+1)\pi x}{2l}\right) \tag{17}$$

and

$$u(x=l) = \frac{gl^2}{2c^2} - \frac{16gl^2}{\pi^3c^2}\sum_{k=1}^{\infty}\frac{(-1)^k}{(2k+1)^3}\cos\left(\frac{(2k+1)\pi at}{2l}\right). \tag{18}$$

The maximal quantity u_{max} is at point $t = 2l/c$ and so we get

$$u_{max} = \frac{gl^2}{2c^2} + \frac{16gl^2}{\pi^3c^2}\sum_{k=1}^{\infty}\frac{(-1)^k}{(2k+1)^3}. \tag{19}$$

With regard to the mathematical formula

$$\sum_{k=1}^{\infty}\frac{(-1)^k}{(2k+1)^3} = \frac{\pi^3}{32}, \tag{20}$$

we get

$$u_{max} = \frac{gl^2}{c^2}. \tag{21}$$

So, the length of the string (rod) is in the interval $(l, l + \frac{gl^2}{c^2})$.

3 The string in the Einstein theory

The Einstein Gravity is based on the Einstein-Hilbert field equations (EHFE). They are the space-time geometry equations for the determining of the metric tensor of space-time for a given arrangement of stressenergy in the space-time. They are the non-linear partial differential equations and the solutions of the EHFE are the components of the metric tensor.

The inertial trajectories of particles are geodesics in the resulting geometry calculated using the geodesic equation.

EHFE obeying local energy-momentum conservation, they reduce to Newtons law of gravitation where the gravitational field is weak and velocities are much less than the speed of light.

There is the simple derivation of the EHFE given by Fock (1964). The similar derivation was performed by Chandrasekhar (1972), Kenyon (1996), Landau et al. (1987), Rindler (2003) and others. Source theory derivation of Einstein equations was performed by Schwinger (1970).

It is well known that the gravity mass M_G of some body is equal to the its inertial mass M_I, where gravity mass is a measure of a massive body to create the gravity field (or, gravity force) and the inertial mass of a massive body is a measure of the ability of the resistance of the body when it is accelerated. At present time we know, that if components of elementary particles have the same gravity and inertial masses, the body composed with such elementary particles has the identical gravity and inertial mass. There is no need to perform experimental verification. So, particle physics brilliantly confirms the identity of the inertial and gravity masses.

According to the Newton theory, the gravity potential is given by the equation

$$U(r) = -\kappa \frac{M}{r}, \tag{22}$$

where r is a distance from the center of mass of a body, κ is the gravitational constant and its numerical value is in SI units $6.67430(15)10^{-11} m^3.kg^{-1}.s^{-2}$ (CODATA, 2018).

The potential U is as it is well known the solution of the Poisson equation:

$$\Delta U(r) = -4\pi\kappa\varrho, \tag{23}$$

where ϱ is the density of the distributed masses.

The problem is, what is the geometrical formulation of gravity equation (23) following from the space-time element ds, which has the specific form in case of the special theory of relativity.

Let us postulate that the motion of a body moving in the g-field is determined by the variational principle

$$\delta \int ds = 0. \tag{24}$$

In order to get the Newton equation of motion, we are forced to perform the following identity:

$$g_{00} = c^2 - 2U = -4\pi\kappa\varrho. \tag{25}$$

The second mathematical requirement, which has also the physical meaning is the covariance of the derived equation. It means that the necessary mathematical operation are the following replacing of original symbols:

$$U \to g_{\mu\nu} \tag{26}$$

with

$$\Delta U \to Tensor\ equation \tag{27}$$

and

$$\varrho \to T_{\mu\nu}, \tag{28}$$

where $T_{\mu\nu}$ is the tensor of energy and momentum.

In order to get the tensor generalization of eq. (23) it is necessary to construct new tensor $R_{\mu\nu}$, which is linear combination of the more complicated tensor $R_{\alpha\beta,\mu\nu}$, or

$$R_{\mu\nu} = g^{\alpha\beta} R_{\mu\alpha,\beta\nu} \tag{29}$$

and the scalar quantity R, which is defined by equation

$$R = g^{\lambda\mu} R_{\lambda\mu} \tag{30}$$

and construct the combination tensor $G_{\lambda\mu}$ of the form

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \tag{31}$$

which has the mathematical property, that the covariant divergence of this tensor is zero, or,

$$\nabla^{\lambda} G_{\lambda\mu} = 0. \tag{32}$$

.

With regard to the fact that also the energy-momentum tensor $T_{\mu\nu}$ has the zero divergence, we can identify eq. (31) with the tensor $T_{\mu\nu}$, or

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi\kappa}{c^2} T_{\mu\nu}, \tag{33}$$

where the appeared constant in the last equation is introduce to get the classical limit of the equation.

The approximate solution of the last equation is as follows

$$ds^2 = (c^2 - 2U)dt^2 - \left(1 + \frac{2U}{c^2}(dx^2 + dy^2 + dz^2)\right). \tag{34}$$

The space-time element (34) is able to explain the shift of the frequency of light in gravitational field and the deflection of light in the gravitational field of massive body with mass M.

So, we have seen that the basic mathematical form of the Einstein general relativity is the Riemann manifold specified by the metric with the physical meaning. The crucial principle is the equality of the inertial and gravitational masses.

While the derivation of the EHFE is elementary, Feynman wrote that the derivation of EHFE by Einstein is difficult to understand. Namely:

Einstein himself, of course, arrived at the same Lagrangian but without the help of a developed field theory, and I must admit that I have no idea how he guessed the final result. We have had troubles enough arriving at the theory - but I feel as though he had done it while swimming underwater, blindfolded, and with his hands tied behind his back! (Feynman et al., 1995).

Now the question arises, what is the equation of motion of the string in a gravitational field. The general solution is beyond of the possibility of mathematical physics and the specific case is of no easy solution. Namely, the force acting on the point moving is the homogenous gravitational field was calculated in the 3-form as follows (Landau, et al., 1988):

$$\mathbf{f} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} \left\{ \mathbf{grad} \ln \sqrt{h} + \sqrt{h} \left[\frac{\mathbf{v}}{c} \mathbf{rot}\ \mathbf{g} \right] \right\}, \tag{35}$$

with (Landau, et al., 1988).

$$h = 1 + \frac{2\varphi}{c^2}, \tag{36}$$

where φ s gravitational potential generating the acceleration $\mathbf{g}$.

So, we see that if we perform the application of the last formula on the string motion, the problem is beyond of the problems of the university physics. So, we have decided for the classical solution in the framework of the equations of mathematical physics.

4 Discussion

We have seen how to calculate the internal motion of the uniformly accelerated non-relativistic of the length l by the gravity force which is the analogue of Galileo experiment with dropping objects from the leaning tower of Pisa. Galileo have used two bodies made of the same material, differing only in size. The effects of air friction were ignored. The two bodies reached the ground at the same time. So, he supported the conclusion that the every falling body is falling with a uniform acceleration, the resistance of the medium being negligible. Galileo experimentation represented the kernel of scientific investigation and Galileo was keen to point this out (Frova et al., 2006).

Galileo experiment inspired Einstein in formulation of the equivalence principle with two reference frames, K and K'. K is a uniform gravitational field, whereas K' has no gravitational field but is uniformly accelerated in such a way that objects in the two frames experience identical forces. According to Einstein systems K and K' are physically exactly equivalent. (Einstein, 1911).

Or, in other words: *Inertia and gravity are identical; hence and from the results of special relativity theory it inevitably follows that the symmetric fundamental tensor $g_{\mu\nu}$ determines the metric properties of space, of the motion of bodies due to inertia in it, and, also, the influence of gravity* (Einstein, 1918).

According to Fock (1964), *principle of equivalence is understood to be the statement that in some sense a field of acceleration is equivalent to a gravitational field. It means that by introducing a suitable system of coordinates (which is usually interpreted as an accelerated frame of reference) one can so transform the equations of motion of a mass point in a gravitational field that in this new system they will have the appearance of equations of motion of a free mass point. Thus a gravitational field can, so to speak, be replaced, or rather imitated, by a field of acceleration. Owing to the equality of inertial and gravitational mass such a transformation is the same for any value of the mass of the particle. But it will succeed in its purpose only in an infinitesimal region of space, i.e. it will be strictly local. In the general case the transformation described corresponds mathematically to passing to a locally geodesic system of coordinates.*

The principle of equivalence states that it is impossible to distinguish between the action on a particle of matter of a constant acceleration, or, of static support in a gravitational field (Lyle, 2008).

We have seen that the motion of the accelerated string by the non-gravity forces differs from the motion of the string caused by the gravity with the acceleration g.

The controversions between different opinions can be easily solved with regard to the physical definition of gravity and inertia. Namely: gravity is form of matter in the physical vacuum. And inertia is the result of the interaction of the massive body with quantum vacuum being the physical medium.

It is well known that synchrotron radiation influences the motion of the electron in accelerators. The corresponding equation which describes the classical motion is so called the Lorentz-Dirac equation, which differs from the the so called Lorentz equation

$$mc\frac{du_\mu}{ds} = \frac{e}{c}F_{\mu\nu}u^\nu \tag{37}$$

only by the additional term which describes the radiative corrections. So, the equation with the radiative term is as follows (Landau et al., 1988):

$$mc\frac{du_\mu}{ds} = \frac{e}{c}F_{\mu\nu}u^\nu + g_\mu, \tag{38}$$

where u_μ is the four-velocity and the radiative term was derived by Landau et al. in the form (Landau et al., 1988):

$$g_\mu = \frac{2e^3}{3mc^3}\frac{\partial F_{\mu\nu}}{\partial x^\alpha}u^\nu u^\alpha - \frac{2e^4}{3m^2c^5}F_{\mu\alpha}F^{\beta\alpha}u_\beta + \frac{2e^4}{3m^2c^5}\left(F_{\alpha\beta}u^\beta\right)\left(F^{\alpha\gamma}u_\gamma\right)u_\mu. \tag{39}$$

The last equation can be easily converted into equation for charged particle moving in gravity. However, the term describing the radiation caused by gravity is not present (Landau, et al., 1988).

It was proved by author (Pardy, 2009) that synchrotron radiation influences the spin motion of the electron in accelerators. The corresponding equation which describes the classical spin motion is so called the Bargman-Michel-Telegdi-Pardy and is of the form (Pardy, 2009):

$$\frac{da_\mu}{ds} = 2\mu F_{\mu\nu}a^\nu - 2\mu' u_\mu F^{\nu\lambda}u_\nu a_\lambda +$$

$$\Lambda u_\mu \left\{ \frac{2e^3}{3mc^3}\frac{\partial F_{\lambda\nu}}{\partial x^\alpha}u^\nu u^\alpha - \right.$$

$$\left. \frac{2e^4}{3m^2c^5}F_{\lambda\alpha}F^{\beta\alpha}u_\beta + \frac{2e^4}{3m^2c^5}\left(F_{\alpha\beta}u^\beta\right)\left(F^{\alpha\gamma}u_\gamma\right)u_\lambda \right\} a^\lambda \qquad (40)$$

where Λ is the bremsstrahlung constant.

Let us remark that the conversion of this equation to the situation where the interaction with the gravitational field is present, was not still derived.

We know, that free the fall law of the positronium is of the same law as the free fall of an electron, or, positron apart. Also, free fall of the protonium is of the same law as the free fall of the proton, or, antiproton apart. It was experimentally verified. It means that the charge interaction with gravity is zero. Gravity interact only with mass and the result of such interaction is the free fall with emission of gravitons. In case of the binary system it was confirmed by NASA and the spectral formula of the emission of gravitons by the binary was calculated by author (Pardy, 1983; 1994a; 1994b; 2011; 2018; 2019). In case of the existence of the gravitational index of refraction, the gravitational Cherenkov radiation is possible (Pardy, 1994c; 1994d).

While Galileo dropped objects from the leaning tower of Pisa, now, we have possibility to drop charged objects from the very high tower Burj Khalifa, in order to confirm the law that charged objects accelerated by the gravitational field do not radiate the electromagnetic energy. It is not excluded that such experiment with the adequate title Galileo-Pardy-Burj Khalifa project will be realized sooner, or, later. The project is cheaper than LHC.

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