Expansion, Topology and Entropy

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Abstract

Object of this work is, to determine, if objects observed more distant are moving away faster than less distant ones. The escape velocity Hr is defined by the HUBBLE-Parameter H, locally H_0, which is proportional to the reciprocal of the age T. The calculations are based on the model published in viXra:1310.0189. The idea stems from Cornelius LANCZOS, outlined at a lecture on the occasion of the Einstein-Symposium 1965 in Berlin. The model defines the expansion of the universe as a consequence of the existence of a metric wave field. That field also should be the reason for all relativistic effects, both SR and GR. In the context of this work the propagation function of that wave field is determined. Its phase rate is equal to the reciprocal of PLANCK's smallest increment r_0. Even the other PLANCK-units set up the basis of the model being functions of space and time.With it, the model leads to a quantization of the universe into single line-elements with the size of r_0. Thus, a kind of finite-element-method becomes possible, at which point the single elements are explicitly defined by the wave function. As per definition, objects in the free fall aren't moving either with respect to the metrics and are carried-with during expansion. With the help of the propagation function it's possible to calculate the HUBBLE-Parameter H even for greater distances. Furthermore the entropy of the universe as a whole is determined considering the special topology of the universe. German version available in viXra. „Expansion, Topologie und Entropie"

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1. Preamble

Object of this work is, to determine, if objects observed more distant than 0.01R (world radius) are moving away faster than less distant ones. Mostly astronomers and cosmologists are interested in that question. The escape velocity Hr is just defined by the HUBBLE-Parameter H, locally H_0, which is proportional to the reciprocal of the age T. Hence it's not about a constant either. Therefore I'm intentionally using the word parameter. Furthermore should be examined, if it's possible, to calculate the entropy of the universe as a whole, and in which regard we have to consider its special topology (4D).

The calculations are based on the model published in [1]. The idea stems from Cornelius LANCZOS [2], outlined at a lecture on the occasion of the Einstein-Symposium 1965 in Berlin. This lecture is also prefixed to [1]. The model defines the expansion of the universe as a consequence of the existence of a four-legged wave field. That field also should be the reason for all relativistic effects, both SR and GR. The temporal function of that field is based on the Hankel function, consisting of the sum of two Bessel functions (J_0 and Y_0). The special properties of the Bessel functions lead to an increase of wave length, defined by the distance between two zero-crossings. The propagation velocity c_M of the field depends on space and time being in the range between $1.09 \cdot 10^{-22} ms^{-1}$ (nowadays) at the local observer up to 0.851661c at the particle horizon.

That involves, that the wave length λ_0 and the phase rate β_0 of the propagation function are having different values. Its phase rate is equal to the reciprocal of PLANCK's smallest increment r_0. Even the other PLANCK-units set up the basis of the model being functions of space and time. In the distance r_0 in the form of a cubic face-centered space-lattice (fc) particular vortices are collocated. LANCZOS called them „MINKOWSKIan line elements, which are only approximately MINKOWSKIan“, here abbreviated as MLE. Thus it's rather about a physical object and not about that, the MINKOWSKIan line element is actually defined. I nominated the whole wave field as metric wave field (metrics).

With it, the model leads to a quantization of the universe into single line-elements with the size of r_0. Thus, a kind of *finite-element-method* becomes possible, at which point the single elements are explicitly defined by the wave function. The wave length λ_0 and r_0 are increasing over time. As per definition, objects in the free fall, aren't moving either with respect to the metrics and are carried-with during expansion. With the help of the propagation function it's possible to calculate the HUBBLE-Parameter H even for greater distances. Farther away we just observe a greater local H_0, because H was greater in the old days. Summarized, with greater distance even a greater H should turn out, which the calculation confirms.

Because the entropy of wave fields can be calculated, it will be determined too. But we have to consider special circumstances at this point. It allows a foresight into the far future of our universe. Finally, the work deals with the different kinds of distance vectors and the question is answered, why vectors greater than cT are possible.

A special feature of the model is, that the so called *subspace*, that's the space, the metric wave field propagates in, disposes of a third property among μ_0 and ε_0. That's the specific conductivity κ_0 in the size of $1.23879 \cdot 10^{93}\, Sm^{-1}$, the cause of expansion. Whether and how it doesn't lead to contradictions with the propagation of „normal“ EM-waves, is *not* subject of the work on hand. According to the model they propagate as overlaid interferences of the metric wave field. See [1] for more detailed information. There you will find even a special section dedicated to the unexpected results of the SN-1a-Cosmology-Experiment.

2. Fundamentals and hypotheses

Before we get to the actual calculation, it's necessary, to define certain base items of the model, mostly without derivation. You read about this in [1]. The PLANCK-units, furthermore the base items of the theoretical electro-technics play a very special role in this connection. For this reason, as usual there, I'm using the letter j instead of i or ⅈ as usual in mathematics.

2.1. Definition of base items

At first the base items of the theoretical electro-technics. They apply independently from the model (1). Beneath (2) the most important PLANCK-units are shown. The introduction of the specific conductivity of the vacuum turns out to be the *missing link* among each other and even to other values.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \Big| \quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{L_0}{C_0}} = \frac{\varphi_0}{q_0} = \frac{\mathbf{E}}{\mathbf{H}} \quad \Big| \quad L_0 = \mu_0 r_0 \quad C_0 = \varepsilon_0 r_0 \quad R_0 = 1/(\kappa_0 r_0) \tag{1}$$

$$r_0 = \sqrt{\frac{G\hbar}{c^3}} = \sqrt{\frac{2t}{\mu_0 \kappa_0}} \quad \Big| \quad m_0 = \sqrt{\frac{\hbar c}{G}} = \frac{\mu_0 \kappa_0 \varphi_0^2}{Z_0} \quad \Big| \quad \varphi_0 = \sqrt{\hbar Z_0} \quad q_0 = \sqrt{\hbar / Z_0} \tag{2}$$

One single line-element can be specified by the model of a lossy Schwingkreises mit oscillating circuit. One special property *of that model only* is, that the Q-factor of the circuit equals the phase angle $2\omega_0 t$ of the Bessel function. It applies $Q_0 = 2\omega_0 t$. The value ω_0 corresponds to the PLANCK-frequency in this connection.

$$\omega_0 = \sqrt{\frac{c^5}{G\hbar}} = \sqrt{\frac{\kappa_0}{2\varepsilon_0 t}} = \frac{1}{\sqrt{L_0 C_0}} = \frac{c}{r_0} \quad \Big| \quad t_0 = \sqrt{\frac{G\hbar}{c^5}} = \sqrt{\frac{\varepsilon_0 t}{2\kappa_0}} \tag{3}$$

$$Q_0 = 2\omega_0 t = \kappa_0 r_0 Z_0 = \frac{\hbar R_0}{\varphi_0^2} = \frac{R_0}{Z_0} = \left(\frac{c}{c_M + v}\right)^2 = \sqrt{\frac{2\kappa_0 t}{\varepsilon_0}} \tag{4}$$

$$H_0 = \frac{\dot{r}_0}{r_0} = \frac{1}{R_0 C_0} = \frac{\varepsilon_0}{\kappa_0} \frac{1}{L_0 C_0} = \frac{1}{\kappa_0 \mu_0 r_0^2} = \frac{\varepsilon_0 \omega_0^2}{\kappa_0} = \frac{1}{2T} = \frac{\omega_0}{Q_0} \tag{5}$$

The numeric value of Q_0 according to table 1 is about $7.5419 \cdot 10^{60}$ and depends on the real value of H_0. Except for the quantities of subspace μ_0, ε_0, κ_0 and c all other ones are functions of space, time and even of the velocity v with respect to the metric wave field. The reason is, that the spatiotemporal function of the metric wave field should emulate the relativistic effects. The GR-dependencies aren't furthermore considered here.

That makes the PLANCK units depend on the frame of reference, which is even defined by them. And all of them are bound by the phase angle Q_0. But the variations mostly cancel each other creating the impression, that the values are constant. Reference-frame-dependent values are marked with a swung dash e.g. $\tilde{Q}_0$ being constants by character.

Still important are the values with a phase angle $Q_1 = 1$. They describe the conditions directly at the particle horizon. They are constants too, because they are defined only by quantities of subspace. Thus, they are mostly qualified for reference-frame-independent conversions of certain values, so-called couplings. An example is the conversion of the magnetic flux φ_1 to the magnetic field strength $H_1 = \varphi_1/(\mu_0 r_1^2)$ as basis of a temporal function containing reference-frame-dependent elements (r_0). r_1 would be the so-called coupling-length then.

Expression (8) shows the relations to the PLANCK-units and to the values of the universe as a whole.

$$r_1 = \frac{1}{\kappa_0 Z_0} \quad \Big| \quad M_1 = \mu_0 \kappa_0 \hbar \quad \Big| \quad t_1 = \frac{1}{2}\frac{\varepsilon_0}{\kappa_0} \quad \Big| \quad \omega_1 = \frac{\kappa_0}{\varepsilon_0} = \frac{1}{2t_1} \tag{6}$$

$$R = Q_0 r_0 = Q_0^2 r_1 \quad \Big| \quad M_1 = Q_0 m_0 \quad \Big| \quad T = Q_0 t_0 = Q_0^2 t_1 \quad \Big| \quad \omega_1 = Q_0 \omega_0 = Q_0^2 H_0 \tag{7}$$

$$\varphi_1 = \sqrt{\hbar_1 Z_0} \quad \Big| \quad q_1 = \sqrt{\hbar_1 / Z_0} \quad \Big| \quad \hbar_1 = \hbar Q_0 \quad \Big| \quad \kappa_0 = \frac{c^3}{\mu_0 G \hbar H_0} \tag{8}$$

The action quantum $\hbar_1$ and $\hat{\hbar}_1$ is not a quantity of subspace, but the initial action, our universe „got" in the early beginning. That value is the only one „set-screw", with which „one" could exert influence on the future appearance of the universe. All other values are „hard-wired" with Q_0 depending on space and time. There is no „fine-tuning" either. With expression (2) right-hand and (8) it's about an effective value, i.e. $\hbar$, φ_0 and q_0 are temporal functions too. For section 3.2.1. still the definition of NEWTON's gravitational constant:

$$G = \frac{c^3}{\mu_0 \kappa_0 \hbar H} = \frac{2c^3 t}{\mu_0 \kappa_0 \hbar} = c^2 \frac{R}{M_1} = c^2 \frac{r_0}{m_0} \tag{695 [1]}$$

2.2. Temporal function

We get the exact temporal function for the magnetic flux φ_0 by solving the differential equation (9). It is based on a lossy oscillating circuit *with expansion*, i.e. the single components R_0, L_0 and C_0 are changing with increasing r_0. Expression (9) mainly differs from a normal oscillating circuit without expansion, with harmonic solution by the factor before $\dot{\varphi}_0$, 1 with expansion, ½ without.

$$\ddot{\varphi}_0 t + \dot{\varphi}_0 + \frac{1}{2}\frac{\kappa_0}{\varepsilon_0}\varphi_0 = 0 \tag{9}$$

In contrast to the expression without expansion there is no drop-down in the resonance frequency ω_0 with (9), normally caused by the influence of the loss-resistance R_0. But we obtain another as solution:

$$y = a_0\, {}_0F_1(;1;-Bx) \qquad \text{with} \quad a_0 = \hat{\varphi}_i/2 \quad B = \frac{1}{2}\frac{\kappa_0}{\varepsilon_0} \quad x = t \tag{10}$$

According to [4] applies

$${}_0F_1(;b;x) = \Gamma(b)(jx)^{b-1} J_{b-1}(j2x^{\frac{1}{2}}) \qquad \text{Hypergeometric function } {}_0F_1 \tag{11}$$

J_n is the Bessel function of n^{th} order, thus

$${}_0F_1(;1;-Bx) = \Gamma(1)(jBx)^0 J_0(\sqrt{4Bx}) \tag{12}$$

$$y = a_0 J_0(\sqrt{4Bx}) \tag{13}$$

$$\varphi_0 = a_0 J_0\left(\sqrt{\frac{2\kappa_0 t}{\varepsilon_0}}\right) = a_0 J_0(Q_0) \tag{14}$$

Since it's about a differential equation of 2^{nd} order and the grade of the Bessel function is integer, the general solution is:

$$\varphi_0 = \hat{\varphi}_i (c_1 J_0(2\omega_0 t) + c_2 Y_0(2\omega_0 t)) \tag{15}$$

The factors c_1 and c_2 may be imaginary or complex even here. According to [5] it's more favourable, if we consider both Hankel functions:

$$H_0^{(1)}(x) = J_0(x) + Y_0(x) \quad \text{and} \tag{16}$$

$$H_0^{(2)}(x) = J_0(x) - Y_0(x) \tag{17}$$

as linearly independent solutions composing the general solution

$$y(x) = c_1 H_0^{(1)}(x) + c_2 H_0^{(2)}(x) \tag{18}$$

with it. Then, the general solution (15) reads then:

$$\varphi_0 = \hat{\varphi}_i (H_0^{(1)}(2\omega_0 t) + H_0^{(2)}(2\omega_0 t)) \tag{19}$$

For our further examinations, we set c_1 and c_2 in (19) equal to 1 for the moment. Then we get as specific solution (20) and for approximation, envelope curve and effective value:

$$\varphi_0 = \hat{\varphi}_i J_0(2\omega_0 t) = \hat{\varphi}_i \operatorname{Re}(H_0^{(1)}(2\omega_0 t)) \qquad \varphi_0 = \hat{\varphi}_i J_0\left(\sqrt{\frac{2\kappa_0 t}{\varepsilon_0}}\right) \tag{20}$$

$$\varphi_0 = \sqrt{\frac{2}{\pi}} \frac{1}{\sqrt{2\omega_0 t}} \cos\left(2\omega_0 t - \frac{\pi}{4}\right) \qquad \text{Approximation} \tag{21}$$

$$\hat{\varphi}_0 = \sqrt{\frac{2}{\pi}} \frac{\hat{\varphi}_i}{\sqrt{2\omega_0 t}} \qquad \text{Envelope curve} \tag{22}$$

$$\varphi_0 = \frac{\varphi_1}{\sqrt{2\omega_0 t}} \qquad \varphi_0 \sim q_0 \sim Q_0^{-\frac{1}{2}} \;\Big|\; \hbar = \varphi_0 q_0 \sim Q_0^{-1} \qquad \text{Effective value} \tag{23}$$

The exact course of φ_0 (20), as well as of the approximate function of the envelope curve (21) and of the effective value (22) is shown in figure 1. Also depicted are the original Bessel functions, which you can't see however, because they are completely covered by the approximation.

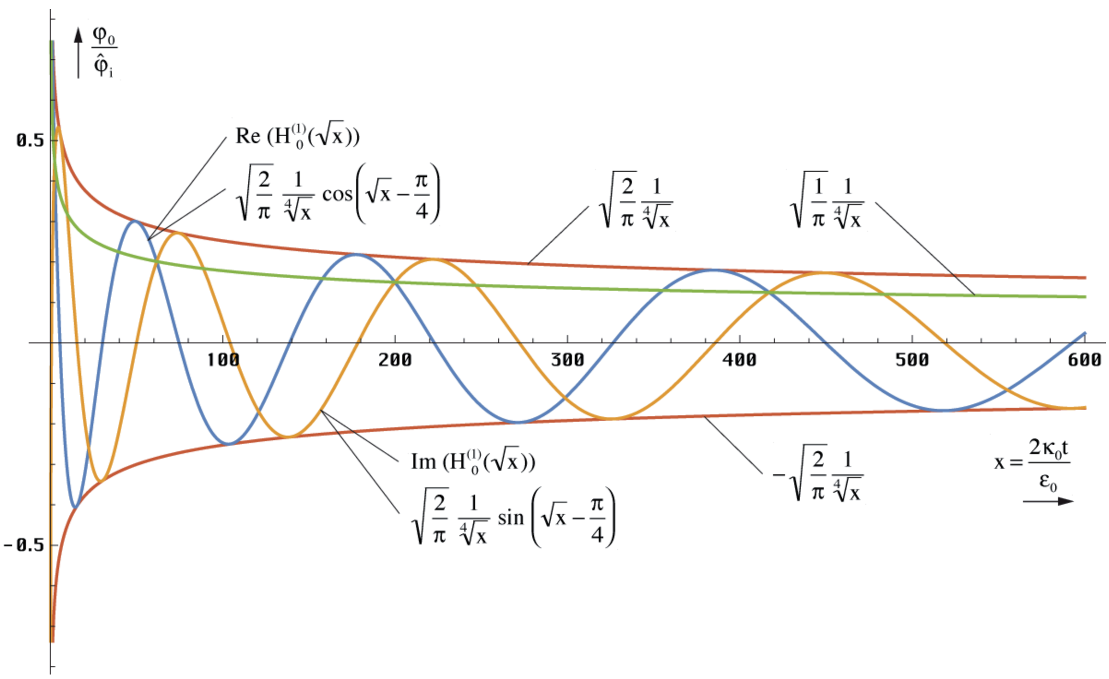

Figure 1
Course of magnetic flux as well as of approximation- and envelope-functions across a greater time period

Thus, with greater arguments, no differences are statable, neither in the amplitude, nor in the phase. Most important for the quality of the approximation is the course in the striking distance of t=0. It is shown in figure 2 and it turns out to be very good until the particle horizon at $Q_0=1$. All data so far are summarized. See [1] for details and the exact derivation.

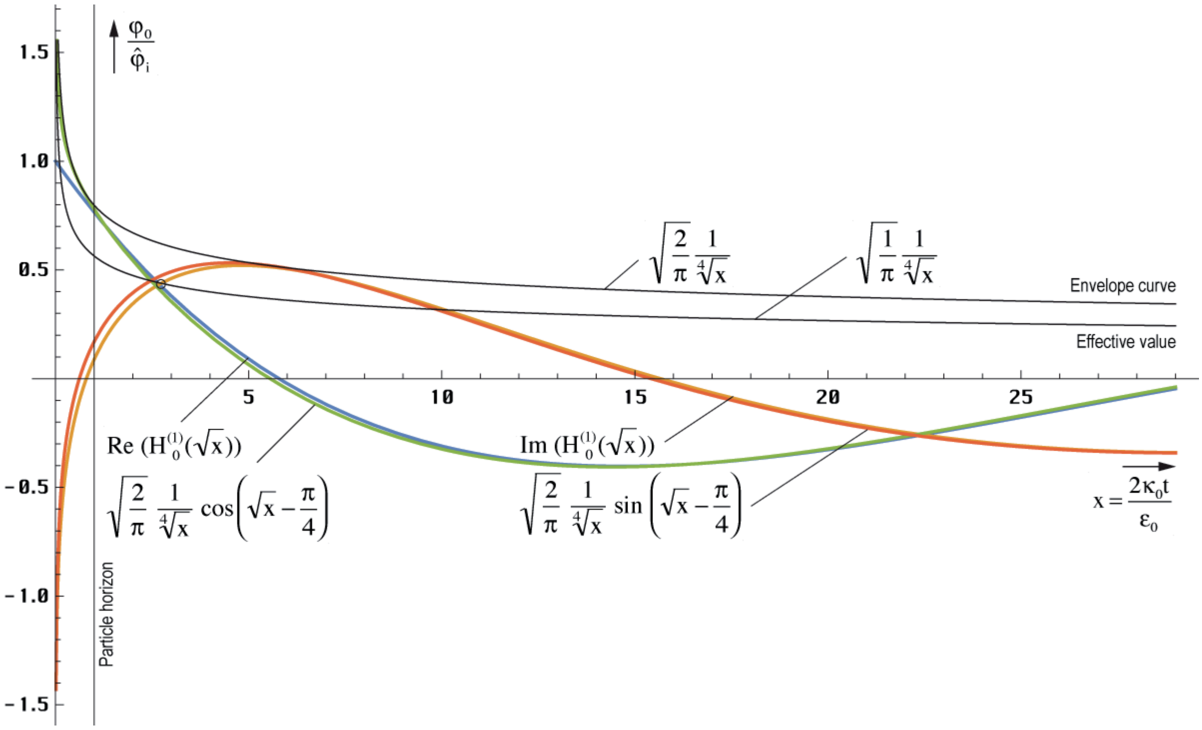

Figure 2
Course of flux as well as of the approximate- and envelope-functions nearby the singularity

2.3. Propagation function

2.3.1. Exact solution

2.3.1.1. Temporal function

In contrast to MAXWELL, which used the first term of the harmonic solution (108 [1]) $e^{j\omega t}$ as ansatz, we now choose the first term of expression (19), obtained as an independent solution of the differential equation (9). It's about the temporal function of the magnetic flux φ_0 there, relating to one single MLE, from which the charge q_0 can be derived. For the propagation function however we need the magnetic and electric field strength **H** and **E**. The relation:

$$\varphi = \int_A \mathbf{B}\,dA \qquad \text{with } \mathbf{B}=\mu_0\mathbf{H} \qquad \text{leads to} \qquad |\mathbf{H}| = \frac{\hat{\varphi}_0}{\mu_0 r_0^2} \tag{24}$$

Because of r_0 indeed the right-hand expression depends on the frame of reference. Moreover we are rather looking for the starting value at T=0. The temporal function is just known. Hence, we must carry out a reference-frame-independent coupling only. The coupling-length r_k is not arbitrary in this case. Because the imaginary part of the Hankel function is coming from infinity, the starting value φ_0 is defined at the point $2\omega_0 t=Q_0=1$. The coupling-length at this point is r_1 as already predicted more above. This value is denominated as $\mathbf{H_1}$ resp. $\mathbf{E_1}$. With respect to the fact, that (23) is an effective value, we obtain the following relations:

$$\mathbf{E_1} = \frac{q_1}{\varepsilon_0 r_1^2}\sqrt{2} = \frac{1}{Z_0}\frac{\varphi_0}{\varepsilon_0 r_0^2}\sqrt{2} \qquad \mathbf{H_1} = \frac{\varphi_0}{\mu_0 r_0^2}\sqrt{2} \tag{25}$$

$$\underline{\mathbf{E}} = \mathbf{E_1}\, H_0^{(1)}(2\omega_0 t) \qquad \underline{\mathbf{H}} = \mathbf{H_1}\, H_0^{(1)}(2\omega_0 t) \tag{26}$$

Here again, the real part of the vector corresponds to an orientation in y-, the imaginary one in z-direction, x is the poropagation direction. As already stated, there is an analogy between the exponential function $e^{j2\omega t}$ and the Hankel function. Both are transcendent complex functions and periodic resp. almost periodic. Of course, there is also a solution of the MAXWELL equations for (26). The detailed derivation can be read in [1] once again. Important is the complex wave propagation velocity $\underline{c}$ and the field wave impedance $\underline{Z}_F$:

$$\underline{c} = \frac{c}{j\omega_0 t}\frac{1}{\sqrt{1-\left(\frac{H_2^{(1)}(2\omega_0 t)}{H_0^{(1)}(2\omega_0 t)}\right)^2}} \qquad \text{with} \qquad \Theta = \frac{H_2^{(1)}(2\omega_0 t)}{H_0^{(1)}(2\omega_0 t)} \tag{27}$$

$$\underline{c} = \frac{c}{j\omega_0 t}\frac{1}{\sqrt{1-\Theta^2}} \qquad \underline{Z}_F = \frac{Z_0}{j\omega_0 t}\frac{1}{\sqrt{1-\Theta^2}} \tag{28}$$

One can see, the propagation velocity tends to zero for greater t. The same applies even to the field wave impedance. We have to do with a quasi-stationary wave field (standing wave), which fulfils the requirements, made on a metrics, very well. The propagation velocity is complex again. A split into real- and imaginary part proves to be quite difficult, but it's mathematically possible. The solution for $\underline{c}$ reads:

$$\underline{c} = -\frac{\sqrt{2}}{\rho_0}\frac{c}{2\omega_0 t}\left(\sqrt{1-\frac{1}{\sqrt{1+\theta^2}}} - j\sqrt{1+\frac{1}{\sqrt{1+\theta^2}}}\right) \qquad \text{Ambiguous!} \qquad \text{with} \tag{29}$$

$$A = \frac{J_0(2\omega_0 t)J_2(2\omega_0 t) + Y_0(2\omega_0 t)Y_2(2\omega_0 t)}{J_0^2(2\omega_0 t) + Y_0^2(2\omega_0 t)} \qquad \rho_0 = \sqrt[4]{(1-A^2+B^2)^2 + (2AB)^2}$$

$$B = \frac{J_2(2\omega_0 t)Y_0(2\omega_0 t) - J_0(2\omega_0 t)Y_2(2\omega_0 t)}{J_0^2(2\omega_0 t) + Y_0^2(2\omega_0 t)} \qquad \theta = \frac{2AB}{1-A^2+B^2} \tag{30}$$

An altogether quite complex expression turns out, that can still be simplified someway however (31). A starts at $+\infty$ converging to -1. The course resembles the function $1/A^2-1$ approximately, which cannot be used well as approximation however. B has a course like $1/B^2$ and is converging to zero. The same is applied even to θ then. The bracketed expression converges to 1 with it. $1/\rho_0$ is the value-function converging to $\frac{1}{2}\sqrt{2}$.

$$\underline{c} = -\frac{2}{\rho_0}\frac{c}{2\omega_0 t}\left(\sin\frac{1}{2}\arctan\theta + j\sin\frac{1}{2}\arctan\theta\right) = \frac{2}{\rho_0}\frac{c}{2\omega_0 t}e^{-j\frac{1}{2}(\arctan\theta+\pi)} \tag{31}$$

Unfortunately (31) cannot be transformed into an expression similar to (179 [1]) with area-functions, so that the ambiguity of the arctan-function leads to a partially wrong result. We should better calculate with the following substitution therefore:

$$\arctan\theta = \arg\left(\left(1-A^2+B^2\right) + j2AB\right) \qquad \arg\underline{c} = \frac{1}{2}\operatorname{arccot}\theta - \frac{\pi}{4} \tag{32}$$

While the real part of $\underline{c}$ is defined as the velocity in propagation direction, the imaginary part can be interpreted as a velocity rectangular thereto. The appearance of an imaginary part in $\underline{c}$ means also that there is an attenuation anywhere (refer to figure 4). A numerical handling of (27) even can be processed with »Mathematica« resulting in the course figured in figure 3. Since the Hankel functions, with larger arguments, can be expressed well by other analytic functions, we will try to declare approximative solutions later.

We have to do with a case of inversion here. This manifests by the fact that the propagation-velocity first ascends from zero to an amount of 0.851661c (at 0.748514 t_1) in order to re-descend asymptotically to zero.

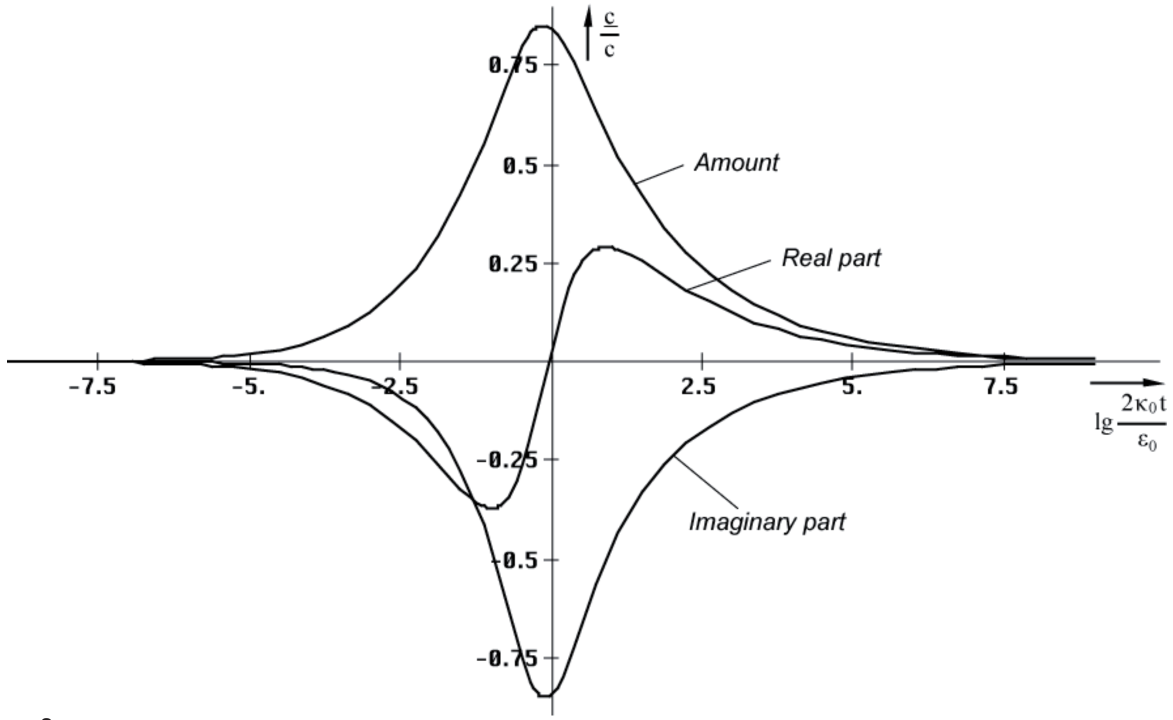

Figure 3
Propagation-velocity
in dependence on time (logarithmic time-scale)

With it, the world-radius (wave-front) of this model doesn't expand with c but with 0.851661c only, which figures no violation of the SRT anyway. However, a contradiction arises to the usual definition R=cT, which will be solved at the end.

2.3.1.2. Propagation rate

To specify the propagation-function we need both, the temporal function and the propagation rate $\underline{\gamma}=\alpha+j\beta$. The normal form of the propagation function is given by:

$$\underline{\mathbf{E}} = \mathbf{E}\, e^{j\omega\left(t-\frac{x}{\underline{c}}\right)} = \mathbf{E}\, e^{j\omega t-\underline{\gamma}x} = \mathbf{E}\, e^{j\left(\omega t+j\underline{\gamma}x\right)} \qquad (33)$$

Contrary to (33) the argument in the case with expansion is real. Strictly speaking, namely it's not the Hankel function but the modified Hankel function $Z_0^{(2)} = I_0(z)-jK_0(z)$ being the equivalent of the exponential-function. It is valid for $I_0(z)=J_0(jz)$ however only for pure imaginary arguments. With complex arguments, the real part cannot be drawn to a position ahead of the Hankel function as usual with the exponential-function, since the power rules aren't applied to Hankel functions anyway. It's possible first with larger arguments z. In general the modified Hankel function isn't used however. Therefore, we use for the base the „ordinary“ Hankel function adapting the propagation-function accordingly. To avoid contradictions with the classic definition of propagation rate — real-part equals attenuation rate, imaginary-part equals phase-rate — the propagation-function should read as follows then (analogously for $\underline{\mathbf{H}}$):

$$\underline{\mathbf{E}} = \mathbf{E}\, H_0^{(1)}\left(2\omega_0\left(t - \frac{x}{\underline{c}}\right)\right) = \mathbf{E}\, H_0^{(1)}(2\omega_0 t - j\underline{\gamma} x) \tag{34}$$

This is not quite the classic expression for a propagation-function. Attention should be paid to the factor 2 which can be assigned both to the frequency, as well as the time-constant. With the definition of propagation rate $\underline{\gamma} = \alpha + j\beta$ it obviously belongs to the frequency since $\underline{\gamma}$ depends on phase velocity dx/dt, but not on the half of dx/(2dt). By equating both arguments of (34) one gets then:

$$\underline{\gamma} = -\frac{2\omega_0}{\underline{c}} = j\kappa_0 Z_0 \sqrt{1-\Theta^2} \tag{35}$$

From (31) the reciprocal of $\underline{c}$ can be determined very easily. Then we get for $\underline{\gamma}$:

$$\frac{1}{\underline{c}} = -\frac{\omega_0 t\, \rho_0}{c}\left(\cos\frac{1}{2}\arctan\theta - j\sin\frac{1}{2}\arctan\theta\right) \tag{36}$$

$$\underline{\gamma} = \alpha + j\beta = -2\omega_0/\underline{c} = \frac{2\omega_0^2 t\, \rho_0}{c}\left(\cos\frac{1}{2}\arctan\theta - j\sin\frac{1}{2}\arctan\theta\right) \tag{37}$$

$$\underline{\gamma} = \rho_0 \kappa_0 Z_0\left(\cos\frac{1}{2}\arctan\theta - j\sin\frac{1}{2}\arctan\theta\right) \tag{38}$$

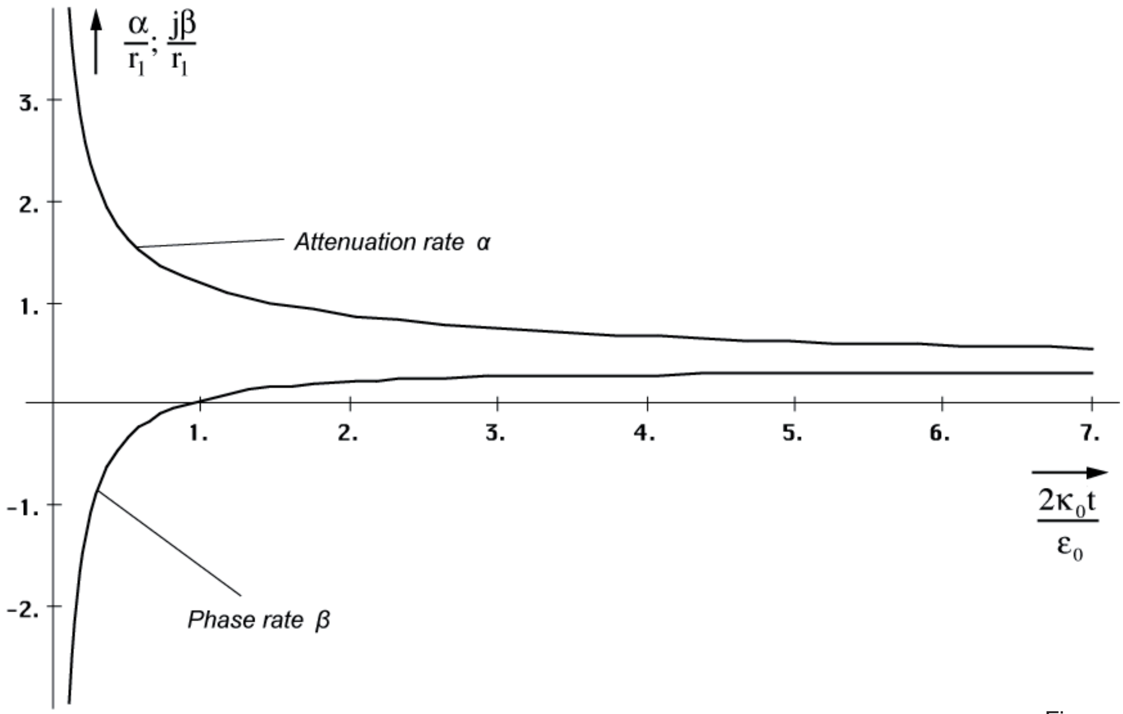

Figure 4
Phase-rate and attenuation rate
in dependence on time (linear scale)

With accurate contemplation one recognizes that α and β, evaluated by its action, are exchanged in fact (α = phase-rate, β = attenuation rate). This is caused thereby that a rotation of about 90° (j) occurs during propagation (figure 7). x turns into y and y into –x. The attenuation α, starting at the point of time t=0, starting off infinity, is decreasing exponentially. To the present point of time, one can say that there is basically no attenuation anyway. This doesn't apply however considering cosmologic time periods.

At the point of time 0.897 t_1 (Q = 0.947), the function β has a zero-passage. This supplies the somewhat particular course in logarithmic presentation (figure 5). It's about a phase-jump of 180° in this case. From the point of time 100 t_1 on we are able to declare, referring to figure 4, the following approximation:

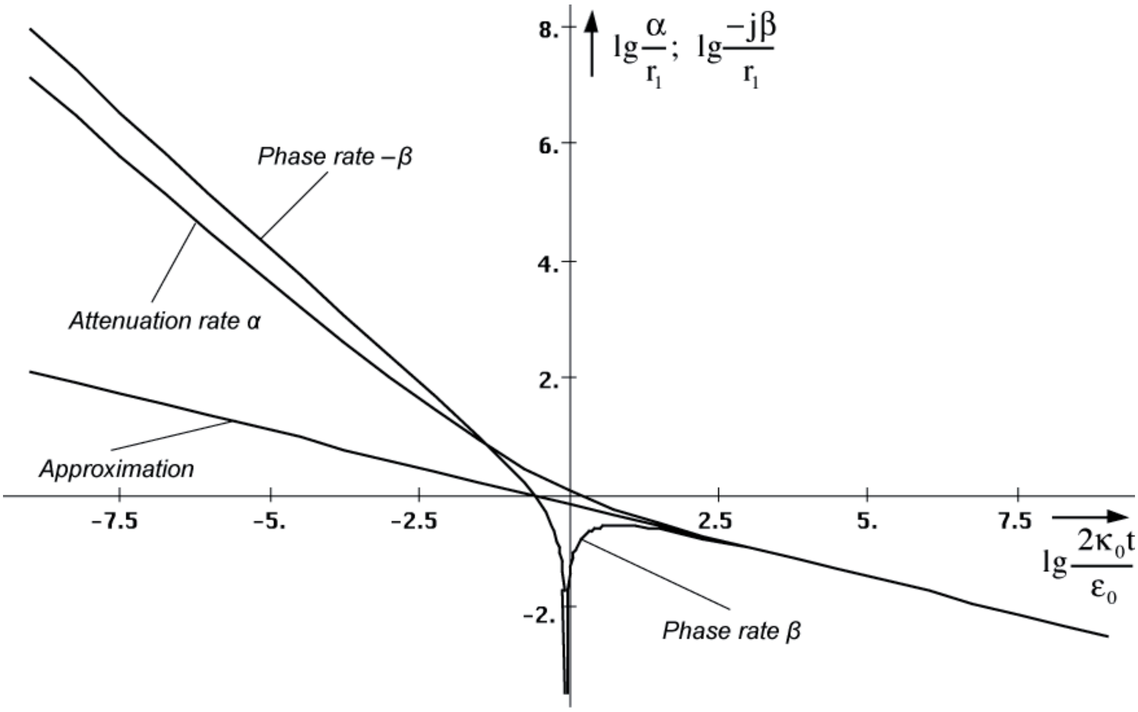

Figure 5
Phase rate and attenuation rate
in dependence on time (logarithmic)

$$\underline{\gamma} \approx (1+j)\kappa_0 Z_0 \sqrt[4]{\frac{\varepsilon_0}{2\kappa_0 t}} \qquad \underline{\gamma} \approx (1+j)\frac{\kappa_0 Z_0}{\sqrt{2\omega_0 t}} \tag{39}$$

These relationships can be derived as well graphically from figure 4, as explicitly using (35) by application of (42). However, it's necessary to multiply (35) with j, in order to take account of the 90° turning (figure 7). Then, to the approximation $\underline{\gamma} = 2\omega_0/\underline{c}$ is applied. Phase rate and attenuation rate are the same from 100 t_1 on approximately. This is the behaviour of an ideal conductor.

2.3.2. Asymptotic approximation

In [6] an asymptotic formula for the Hankel function is declared. It reads:

$$H_\nu^{(1)}(z) = \sqrt{\frac{2}{\pi z}}\, e^{j\left(z - \frac{\pi}{2}\nu - \frac{\pi}{4}\right)}\left[1 + O\left(z^{-1}\right)\right] \qquad \text{for } 0 < z < \infty \tag{40}$$

Put into (27), one sees that nearly all expressions can be reduced. The root-expression *R* converges to a value of:

$$R = \sqrt{1-\left[1+O_2(t^{-1/2})-O_0(t^{-1/2})\right]^2} \quad \approx \quad \sqrt{2O_2(t^{-1/2})-2O_0(t^{-1/2})} \tag{41}$$

The root-expression just only depends on the remainder terms which is tending to zero as well. Therefore, this base is not suitable for our purposes.

For $\underline{\gamma}$, we have already found an approximation, still remain $\underline{c}$ and $\underline{Z}_F$. In figure 3 we have already figured the course of $\underline{c}$. To the graphic determination of an approximation, we require the double logarithmic representation however (figure 6). To be considered, is the fact that the imaginary part is actually negative.

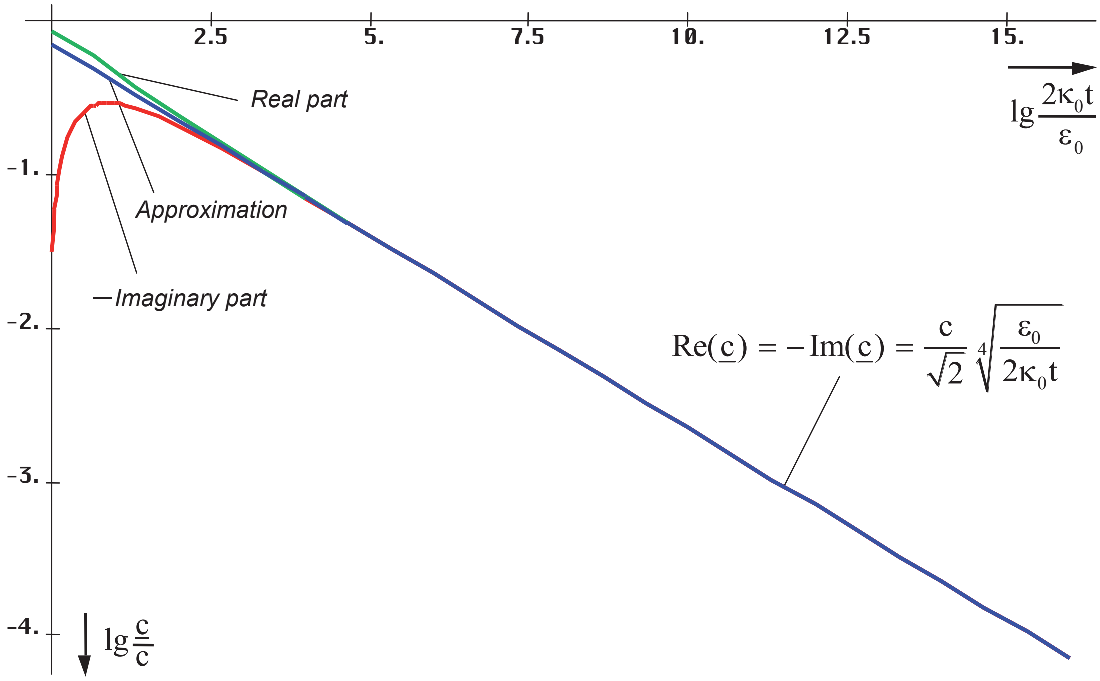

Figure 6
Propagation-velocity
in dependence on time (double logarithmic)

$$\underline{c} = \frac{1-j}{\sqrt{2}}\, c \sqrt[4]{\frac{\varepsilon_0}{2\kappa_0 t}} \qquad \underline{c} = \frac{1-j}{2}\frac{c}{\sqrt{\omega_0 t}} \tag{42}$$

$$|\underline{c}| = c \sqrt[4]{\frac{\varepsilon_0}{2\kappa_0 t}} \qquad |\underline{c}| = \frac{c}{\sqrt{2\omega_0 t}} \qquad (1.0916\cdot 10^{-22}\ \mathrm{ms}^{-1}) \tag{43}$$

$$\underline{Z}_F = \frac{1-j}{\sqrt{2}} Z_0 \sqrt[4]{\frac{\varepsilon_0}{2\kappa_0 t}} \qquad \underline{Z}_F = \frac{1-j}{2}\frac{Z_0}{\sqrt{\omega_0 t}} \tag{44}$$

2.3.3. Expansion curve

At the world-radius, the universe expands with the maximum velocity of 0.851661c, in the inside with a velocity decreasing more and more. Since the wave count in the interior of a sphere with defined radius r(c,t) is decreasing, the deficit is balanced by an increase of wavelength. Outside the wave count ascends continuously due to propagation.

For greater t the expansion of the wavefront proceeds nearly rectilinear with an angle of −45° proportionally $t^{3/4}$. But the behaviour looks somewhat different near the singularity. In

The track-course of a single sector of wave front near the singularity is shown in figure 7. We see a kind of parabola, with greater t a hyperbola. And there is a rotation in propagation direction about an angle of 90°.

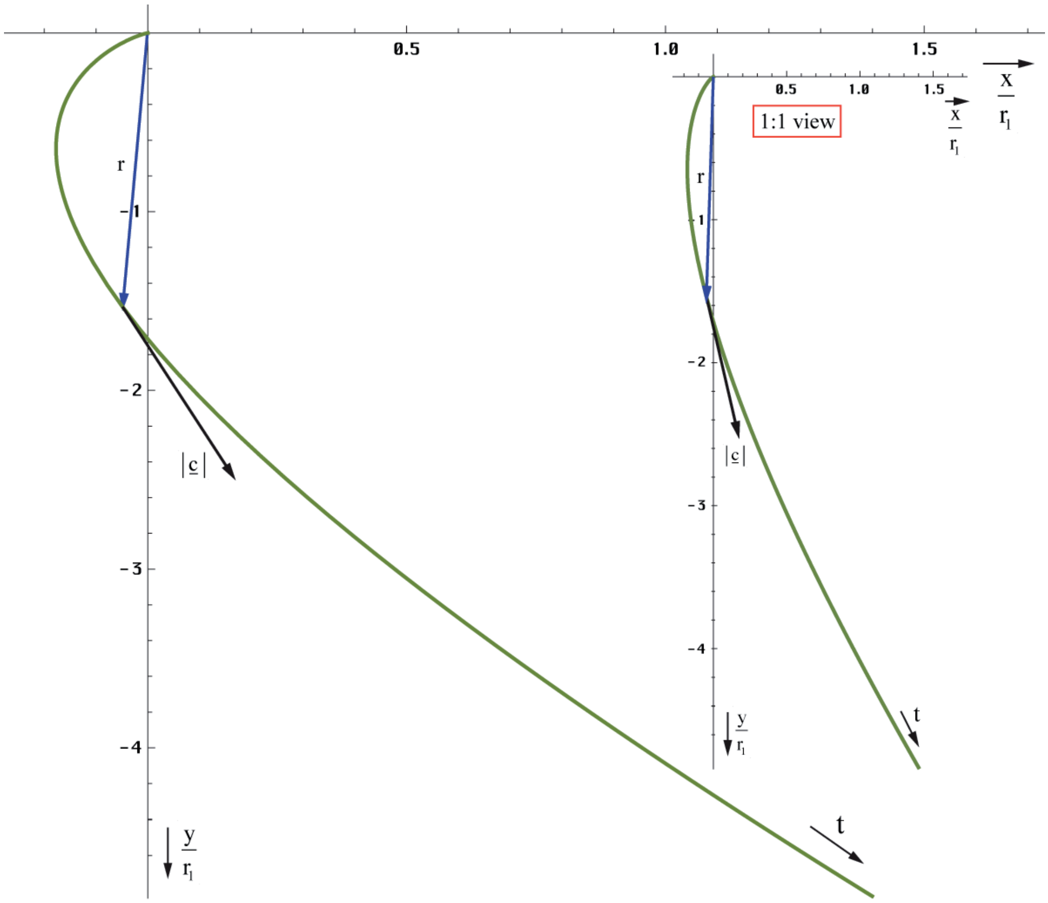

Figure 7
Track-curve near the singularity
in dependence on time

2.3.4. Approximative solution

Now we want to set-up an approximation for the propagation function. The normal form is $\mathbf{E}=\hat{\mathbf{E}}\,e^{j\omega t-\underline{\gamma}x}$ with $\underline{\gamma}=\alpha+j\beta$. But with the exact solution (39) there is a case on hand, with which α and β contain both damping- and phase-information and the wave function isn't harmonic either. That way we aren't able to form a reasonable propagation function at all.

In the case $t \gg t_1$ phase- and attenuation rate are of the same size. Thus, the model behaves similar to a metal.There α does not stand for a damping, but for a rotation, namely as long as, with vertical incidence, a value of π is reached so that the wave exits the metal in the opposite direction after a minimal intrusion. The depth of penetration depends on the material properties, the wave length and the angle of incidence. In case of this model the material properties aren't constant either, $\underline{\gamma}$ decreases with t and x. Hence it suffices to a rotation of 90° only and the wave remains in the medium (vacuum). In any case, there is a rotation too.

To cope with it, we do a rotation of the coordinate system about π/4. That corresponds to a Multiplikation with $\sqrt{j}$ and we get a purely imaginary solution. So becomes α=0 and γ=jβ and the exponentially related attenuation vanishes. Indeed, we still have to multiply the result with $\sqrt{2}$ and to replace x by r. Despite α=0 the amplitudes of **E** and **H** are decreasing continuously. That's caused by the Hankel function alone, resp. by the radical expression in

(45). With it amplitude and phase are firmly interlinked (minimum phase system). Now the rotation angle in space is equal to $\theta+\pi/4$. But a separation of phase- and damping-information isn't possible yet. But we can work with very high precision using the approximation equations in this case. To the general Hankel function $H_0^{(1)}(\omega t-\beta x)$ the following approximation applies (analogously for $\underline{\mathbf{H}}$):

$$\underline{\mathbf{E}} = \hat{\mathbf{E}}\, H_0^{(1)}(\omega t-\beta x) \approx \hat{\mathbf{E}} \sqrt{\frac{2}{\pi(\omega t-\beta x)}}\, e^{j(\omega t-\frac{\pi}{4}-\beta x)} \tag{45}$$

Instead of $\underline{\gamma}x$ only the product βx with the phase rate appears in the exponent, since the amplitude rate is already emulated by the radical expression. With $t \gg 0$ the angle $\pi/4$ can be omitted. After rotation and transition $x \to r$ and $\omega \to 2\omega_0$ turns out:

$$\underline{\mathbf{E}} = \hat{\mathbf{E}}\, H_0^{(1)}(2\omega_0 t-2\beta_0 r) \approx \frac{2\mathbf{E}_1}{\sqrt{2\omega_0 t-2\beta_0 r}}\, e^{j(2\omega_0 t-\frac{\pi}{4}-2\beta_0 r)} \qquad H_1 = \frac{\varphi_1}{\mu_0 r_1^2} \quad E_1 = \frac{q_1}{\varepsilon_0 r_1^2} = \frac{1}{Z_0}\frac{\varphi_1}{\varepsilon_0 r_1^2} \tag{46}$$

$\mathbf{E}_1$ is the peak value of $\mathbf{E}$ with $Q_0=1$. Indeed are both $\omega=2\omega_0$ and $\beta=2\beta_0$ (with double frequency even the phase rate must be doubled) no constants at all. That means, they depend on t and r at the same time, limiting the manageability of the approximation very much. You can see that also with the phase velocity v_{ph}. It is defined in the following manner:

$$v_{ph} = \frac{2\omega_0}{\beta} = \frac{2c}{\sqrt{2\omega_0 t}} = 2|\underline{c}| \qquad \text{for } t \gg 0 \tag{47}$$

Thus, the phase velocity is equal to the double absolute value of propagation velocity. That's caused by the factor 2, since phasing with double frequency propagates with double velocity too. For interest, also the group velocity should be stated here:

$$v_{gr} = \frac{1}{d\beta/d\omega_0} = -2|\underline{c}| \qquad \text{for } t \gg 0 \tag{48}$$

Except for the algebraic sign both results are equal. That means, the propagation takes place free from any bias. Further to the approximation. With (22) in section 2.2. we had already found a very good approximation, almost exact, for the same temporal function.s

$$\underline{\mathbf{E}} \approx \hat{\mathbf{E}} \sqrt{\frac{2}{\pi}}\, \frac{e^{j(2\omega_0 t+2\beta_0 x)}}{\sqrt{2\omega_0 t+2\beta_0 x}} = 2\mathbf{E}_1 \frac{e^{j2(\omega_0 t+\beta_0 r)}}{\sqrt{2\omega_0 t+2\beta_0 r}} \qquad \text{with} \quad \beta_0 = \frac{\kappa_0 Z_0}{\sqrt{2\omega_0 t}} \tag{49}$$

Now, expression (49) enables to define an equivalent-$\alpha=\alpha_0$ and, with it, even an equivalent-$\underline{\gamma}_0=\alpha_0+j2\beta_0$, in order to get it up to the normal form for propagation functions.

$$\underline{\mathbf{E}} \approx 2\mathbf{E}_1\, e^{j2\omega_0 t-\underline{\gamma}_0 r} \qquad \text{with} \quad \underline{\gamma}_0 = \frac{1}{2r}\ln\left(2\omega_0 t+\frac{2\kappa_0 Z_0}{\sqrt{2\omega_0 t}}r\right)+j\frac{2\kappa_0 Z_0}{\sqrt{2\omega_0 t}} \tag{50}$$

That's already a big step forward. Unfortunately, both ω_0 and $\underline{\gamma}_0$ depend on time. It's not critical for $2\omega_0 t$, because it's multiplied by t anyway. Else with $\underline{\gamma}_0$, it should depend on r only. To the substitution of t in (49ff) we firstly put (43) left-hand into $t=r/|\underline{c}|$. The real propagation velocity becomes effective here and not v_{ph} or v_{gr}. Then we rearrange after t. Putting into (49) right-hand we get:

$$t = \frac{r}{c}\sqrt[4]{\frac{2\kappa_0 t}{\varepsilon_0}} \qquad t^{\not{4}3} = \frac{r^4}{c^4}\frac{2\kappa_0 \not{t}}{\varepsilon_0} = 2r^4\mu_0^2\varepsilon_0\kappa_0 \tag{51}$$

$$\beta_0^{12} = \frac{1}{8}\kappa_0^{\cancel{12}\,8} Z_0^{\cancel{12}\,8} \frac{\cancel{\varepsilon_0^3}}{\cancel{\kappa_0^3}} \cdot \frac{1}{2r^4 \cancel{\mu_0^2}\, \cancel{\varepsilon_0}\, \cancel{\kappa_0}} = \frac{\kappa_0^8 Z_0^8}{2^4 r^4} \qquad \Bigg| \qquad \beta_0 = \sqrt[3]{\frac{1}{2rr_1^2}} \tag{52}$$

With it,we obtain for $\underline{\gamma}_0$ and the product $\underline{\gamma}_0 r$ the following expressions:

$$\underline{\gamma}_0 = \frac{1}{2r} \ln\left(2\omega_0 t + \left(\frac{2r}{r_1}\right)^{\frac{2}{3}}\right) + j\left(\frac{2}{rr_1^2}\right)^{\frac{1}{3}} \qquad \text{for } t \gg 0 \tag{53}$$

$$\underline{\gamma}_0 r = \frac{1}{2} \ln\left(2\omega_0 t + \left(\frac{2r}{r_1}\right)^{\frac{2}{3}}\right) + j\left(\frac{2r}{r_1}\right)^{\frac{2}{3}} \qquad \text{for } t \gg 0 \tag{54}$$

Last but not least the time t can be completely eleminated. The value $\underline{\gamma}_0$ is proportional to $r^{-1/3}$ and, even more important, the product $\underline{\gamma}_0 r$ is proportional to $r^{2/3}$. Unfortunately, as already said, we can explicitely state $\underline{\gamma}_0(r)$ by approximation only. With the exact function (38) a separation, especially from t is impossible. But generally speaking, an exact solution is not required at all, since the approximation yields very good results until a striking distance to the particle horizon at $Q_0=1$, see figure 2. Therefore, we will not follow up that matter at this point.

All hitherto stated approximations are based on the 4D-expansion-centre $\{r_1,r_1,r_1,t_1\}$. But it‘s more practicable to find a function, related to another centre. Most suitable seems to be the point, where we are, the „point being“. At first we substitute the time according to $t \to \tilde{T}+t$. The swung dash stands for the initial value at the point t=0 (nowadays) describing an inertial system. Hence it’s about a constant. Because of $\tilde{T}=t_1\tilde{Q}_0^2$ we are able to factor out $\tilde{Q}_0$. The direction of time doesn’t change. To the temporal part applies:

$$2\omega_0 t = \tilde{Q}_0\left(1+\frac{t}{\tilde{T}}\right)^{\frac{1}{2}} \tag{55}$$

For the spatial part β_0 we build up the inertial system once again using the substitution $r_1 \to \tilde{R}$. Because of $\tilde{R}=r_1\tilde{Q}_0^2$, as well as $\tilde{r}\,\tilde{Q}_0=-r$, now we are measuring from the other end, we can write for $2\beta_0$:

Exactly →

$$2\beta_0 = \tilde{Q}_0\left|\frac{2}{\tilde{r}\tilde{Q}_0\,\tilde{r}_1^2\tilde{Q}_0^2}\right|^{\frac{1}{3}} = -\tilde{Q}_0\left|\frac{2}{r\tilde{R}^2}\right|^{\frac{1}{3}} \qquad 2\beta_0 r = -\tilde{Q}_0\left|\frac{2r-\tilde{r}_0}{\tilde{R}}\right|^{\frac{2}{3}} = -\tilde{Q}_0\left|\frac{2r}{\tilde{R}}-\frac{1}{\tilde{Q}_0}\right|^{\frac{2}{3}} \tag{56}$$

Actually I should have to write $\tilde{r}$ instead of r. But because it’s the argument of the function the tilde has been omitted. The right-hand expression considers the fact, that r_0 as smallest increment never can be underrun. The value α_0 is definitely determined by the envelope curve of the Hankel function, else it would be equal to zero. With it, we obtain for $\underline{\gamma}_0$ and the product $\underline{\gamma}_0 r$:

$$\underline{\gamma}_0 = \frac{1}{2r} \ln\tilde{Q}_0\left(\left(1+\frac{t}{\tilde{T}}\right)^{\frac{1}{2}} - \left(\frac{2r}{\tilde{R}}\right)^{\frac{2}{3}}\right) + j\tilde{Q}_0\left(\frac{2}{r\tilde{R}^2}\right)^{\frac{1}{3}} \tag{57}$$

$$\underline{\gamma}_0 r = \frac{1}{2} \ln\tilde{Q}_0\left(\left(1+\frac{t}{\tilde{T}}\right)^{\frac{1}{2}} - \left(\frac{2r}{\tilde{R}}\right)^{\frac{2}{3}}\right) + j\tilde{Q}_0\left(\frac{2r}{\tilde{R}}\right)^{\frac{2}{3}} \tag{58}$$

With r_0 we have already found one elementary length. But LANCZOS speaks about another one [1]. That’s the wave length of the metric wave field $\lambda_0=2\pi/\beta$. The approximation of λ_0 must be divided by 2 once again, due to the double phase velocity. Hence $\lambda_0=2\pi/\beta_0$ applies. To the comparison the expression for r_0 once again:

$$\lambda_0 = \frac{2\pi}{\rho_0(2\omega_0 t)\kappa_0 Z_0}\operatorname{cosec}\frac{1}{2}\arctan\theta(2\omega_0 t) \tag{59}$$

$$\lambda_0 = \frac{\pi}{\kappa_0 Z_0}\sqrt[4]{\frac{2\kappa_0 t}{\varepsilon_0}} = \frac{\pi}{\kappa_0 Z_0}\sqrt{2\omega_0 t} \qquad \text{for } \omega_0 t \gg 0 \tag{60}$$

$$r_0 = \frac{1}{\kappa_0 Z_0}\sqrt{\frac{2\kappa_0 t}{\varepsilon_0}} = \frac{2\omega_0 t}{\kappa_0 Z_0} = \sqrt{\frac{2t}{\kappa_0 \mu_0}} \tag{61}$$

Though λ_0 is smaller than r_0 and not identical to HEISENBERG's elementary length with it. λ_0 now is in the range of 10^{-68}m. Thus, LANCZOS was wrong in that point. But it only has been a guess on his part. In fact, it's about the wave length of the wave function forming the metric lattice itself. Expression (59) until (61) only represent the temporal functions. Then, the functions of time and space read as follows.

$$\lambda_0 = \frac{2\pi}{\rho_0(2\omega_0 t - \underline{\gamma}_0 r)\kappa_0 Z_0}\operatorname{cosec}\frac{1}{2}\arctan\theta(2\omega_0 t - \underline{\gamma}_0 r) \tag{62}$$

$$\lambda_0 = \pi r_0 \tilde{Q}_0^{-\frac{1}{2}}\left(\left(1+\frac{t}{\tilde{T}}\right)^{\frac{1}{2}} - \left(\frac{2r}{\tilde{R}}\right)^{\frac{2}{3}}\right)^{\frac{1}{2}} = \frac{\pi}{\kappa_0 Z_0}\sqrt{2\omega_0 t - 2\beta_0 r} \tag{63}$$

$$r_0 = dr = \tilde{r}_0\left(\left(1+\frac{t}{\tilde{T}}\right)^{\frac{1}{2}} - \left(\frac{2r}{\tilde{R}}\right)^{\frac{2}{3}}\right) = \frac{2\omega_0 t - 2\beta_0 r}{\kappa_0 Z_0} \tag{64}$$

The wave length λ_0 of the metrics is irrelevant for the further contemplations of this work, only β_0 matters. The double-bracketed expression in (64) is called *Navigational Gradient* in future. It is the essential expression I was looking for.

We only know the local age T, which results from the local HUBBLE-parameter (65). It quasi represents the temporal distance to the expansion centre. But we are able to determine the spatial distance to the world radius R. This forms a spatial singularity (event horizon) with it.

$$2\omega_0 t - \beta_0 r = \frac{\omega_0(H)}{H} \qquad \text{bei } r = 0 \qquad T = \frac{1}{2H} \tag{65}$$

$$R = -\frac{\omega_0(H)}{\beta_0 H} = -\frac{\omega_0 r_0}{H} = -2ct \qquad \text{bei } 2\omega_0 t = 0 \tag{66}$$

$$\beta_0 = \kappa_0 Z_0 \sqrt[4]{\frac{\varepsilon_0 H}{\kappa_0}} = \sqrt{\frac{c^3}{G\hbar}} = \frac{1}{r_0} \tag{67}$$

Thus we can get the value of $\beta_0 = 1/r_0$ even from (39), in that we replace the time by the HUBBLE-Parameter. For R turns out:

$$R = -\frac{c}{H} = -1.2188\cdot 10^{26}\text{m} = -1.2918\cdot 10^{10}\text{Lj} = -3.950\ \text{Gpc} \tag{68}$$

That's about 12 billion light years (according to table 1). The local age amounts only to the half, namely 6,6 billion years, the local world radius is equal to cT. Longer time-like vectors up to 2cT are possible because of the expansion and wave propagation of the metric wave field. Full particulars in the next sections.

3. Expansion, topology and entropy

In section 2.3.4. we found with (64) an expression for the temporal and spatial dependence of PLANCK's elementary-length r_0, figuring at least locally a scale for the proportions (distance). On this occasion I refer once again to the fact that this is *also* applied to the size of material bodies, which is changing in the same measure as r_0. Otherwise we could not observe any expansion either.

Just particularly is this a matter of the mutual distances of material bodies. These follow a function, which differ with the considered distance, since quantity and expansion-velocity of the PLANCK elementary-length is changing with ascending distance to the coordinate-origin. But only distances with their starting-point in the origin should will be considered here. Of considerable importance for deeper contemplations is even the number of line elements (MLEs) along an imagined line with the length r (wave count vector **Λ**). We distinguish two cases in this connection: Wave count vector with constant r and r with constant wave count vector. More final case to the best fits the existing circumstances, since we can assume that no point is distinguished to other points in the cosmos. The average relative velocity against the metrics at the coordinate-origin is equal to zero at free fall. This should be so everywhere then. With it, the expansion of the universe can be traced back to the expansion of the metrics alone. This corresponds to the case of a constant wave count vector.

3.1. Expansion

3.1.1. Constant distance

Because of the *real lattice constant* r_0 the wave count vector **Λ** for smaller distances r is defined in the following manner:

$$\mathbf{\Lambda} = \frac{r}{r_0}\mathbf{e}_r \tag{69}$$

$\mathbf{e_r}$ is the unit-vector. In the following, we consider only the figure Λ however. For larger distances, we have to replace Λ by dΛ and r by dr using the corresponding expression (64) for r_0:

$$d\Lambda = \frac{1}{\tilde{r}_0}\,\frac{dr}{(1+t')^{\frac{1}{2}} - \left(\frac{2r}{\tilde{R}}\right)^{\frac{2}{3}}} \qquad \text{with} \quad t' = \frac{t}{\tilde{T}} \tag{70}$$

To the solution we replace as follows (it applies $\tilde{R}/\tilde{r}_0 = \tilde{Q}_0$):

$$d\Lambda = \frac{3}{2}\frac{\tilde{R}}{\tilde{r}_0}\frac{r'^2}{a^2-r'^2}dr' \qquad \text{mit} \quad r' = \left(\frac{2r}{\tilde{R}}\right)^{\frac{1}{3}} \;\Bigg|\; a^2 = (1+t')^{\frac{1}{2}} \;\Bigg|\; dr = \frac{3}{2}\tilde{R}r'^2 dr' \tag{71}$$

$$\Lambda = \frac{3}{2}\tilde{Q}_0\int\frac{r'^2}{a^2-r'^2}dr' = \frac{3}{2}\tilde{Q}_0\left(a\,\text{artanh}^{*)}\frac{r'}{a} - r'\right) \tag{72}$$

*) arcoth for | r′ | > a (behind the particle horizon)

$$\Lambda = \frac{3}{2}\tilde{Q}_0\left(\left(1+\frac{t}{\tilde{T}}\right)^{\frac{1}{4}}\text{artanh}\frac{\left(\frac{2r}{\tilde{R}}\right)^{\frac{1}{3}}}{\left(1+\frac{t}{\tilde{T}}\right)^{\frac{1}{4}}} - \left(\frac{2r}{\tilde{R}}\right)^{\frac{1}{3}}\right) \qquad \text{def}\ \Lambda_0 = \frac{R}{2r_0} = \frac{Q_0}{2} \tag{73}$$

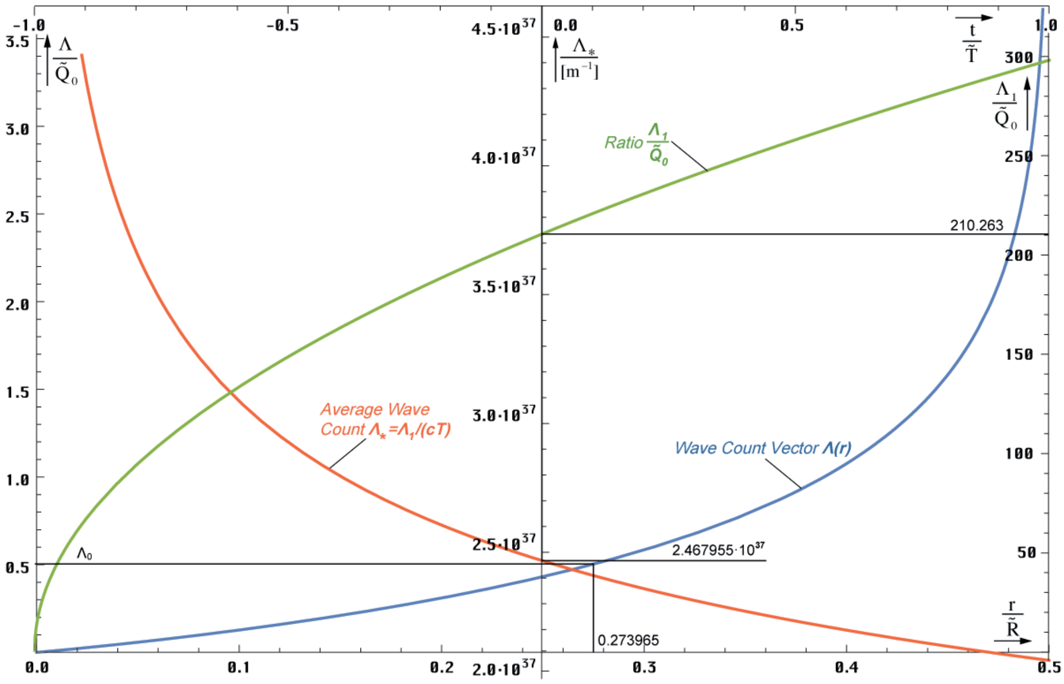

Figure 8
Wave count vector as function of distance r and t

The wave count Λ follows the blue function depicted in figure 8. Approaching to half the world radius (R/2), it seems to be, that Λ strives towards infinity. If we want to define a finite wave count Λ_0, we take only a certain part of the world radius to calculate the wave count for it. Because of $R/(2r_0)=Q_0/2$ we opt for that value. The value amounts to 0.273965R, that is 54.79% of the distance to the particle horizon (cT). In total however an infinite value will not be reached, since r_0 becomes smaller and smaller going to r_1. Out there, at Q=1 is the back of beyond, we reached the particle horizon.

At first I guessed the value to be $\Lambda_1=Q_0^2$, since even $R=r_1Q_0^2$ applies. But that's not the case. The little more ambitious calculation for $r= R/2-r_1 \rightarrow 1-10^{-120}$ under application of the power series for $(1-x)^{⅓}$, multiple substitutions up to the transformation of the function artanh→ arsinh→ ln, turns out $\Lambda_1= \frac{3}{2}\, Q_0 \ln Q_0 \approx 210\, Q_0 = 1.58461\cdot 10^{63}$ using the values from table 1. For Λ_1 applies $t'\equiv t\equiv 0$ i.e. a constant wave count vector. But by expansion and wave propagation „outwards" the phase angle $2\omega_0 T=Q_0 : t^{½}$ increases continuously. And because of (4) $\Lambda_1(T) = \frac{3}{2}\sqrt{bT}\,\ln\sqrt{bT}$ applies with $b = 2\kappa_0/\varepsilon_0$.

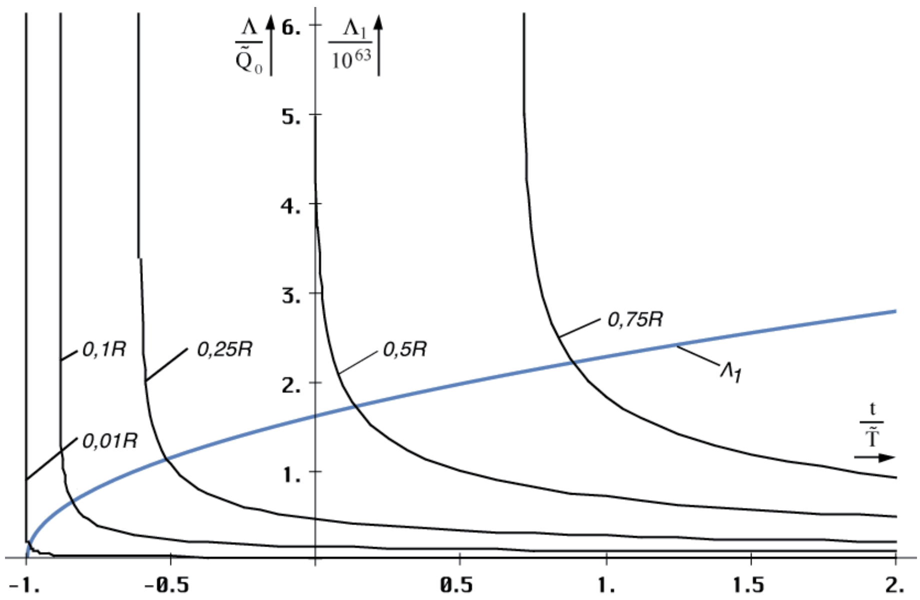

Figure 9
Temporal dependence of the wave count vector for several distances r

The temporal dependence for several initial distances r is shown in figure 9. The larger the considered length, the later on the point of time, the wave count vector is defined from. That's easy to understand, we can regard a length as existent only then, when the world-radius is larger or equal to. If the world-radius is smaller, so such a length doesn't exist. Therefore, lengths larger than 0.5R aren't defined at present and function (73) does not have a real solution before a value of e.g. t=0.75T is reached (t=0 is the present point of time). Altogether, the wave count decreases. That results from the fact that we are considering a constant length with expanding r_0. So it happens, that MLEs are permanently „scrolled out" at the „tail" leading to a degradation of the wave count vector at the same time.

3.1.2. Constant wave count vector

3.1.2.1. Solution

At first we start with the left expression of (73) for t=0 (a=1). It specifies the quantity of the wave count vector at the present point and at each point of time, if we want to assume it as constant. We just look for the function $F(a, \tilde{r}')$ being nothing other as the temporal dependence on a given length $\tilde{r}'$. See (71) for a(t).

$$\Lambda = \frac{3}{2}\tilde{Q}_0\left(\operatorname{artanh}\tilde{r}' - \tilde{r}'\right) = \frac{3}{2}\tilde{Q}_0\left(a\operatorname{artanh}\frac{\tilde{r}'F}{a} - \tilde{r}'F\right) = \text{const} \qquad (74)$$

An explicit reduction by differentiating and zero-setting (the left expression turns to zero on this occasion) leads to the trivial solution F=0. Otherwise, only an implicit solution can be found as solution of the equation:

$$a\operatorname{artanh}\frac{\tilde{r}'F}{a} - \operatorname{artanh}\tilde{r}' - \tilde{r}'(F-1) = 0 \qquad r(t) = \tilde{r}F^3(t) \qquad (75)$$

or in »Mathematica«-notation F1[t,r]:

```
Fa1=Function[a=FindRoot[#1*ArcTanh[#2/#1*x]-ArcTanh[#2]-
#2*(x-1)==0,{x,1}, MaxIterations->30]; (Round[(x/.a)*10^7]/10^7)^3];      (76)
F1=Function[Fa1[(1+#1)^.25,(2*#2)^(1/3)]];
```

In this connection we have to be particular about the method (tangent-method) and the initial value. There was a problem using secant method. The temporal course is shown in figure 10.

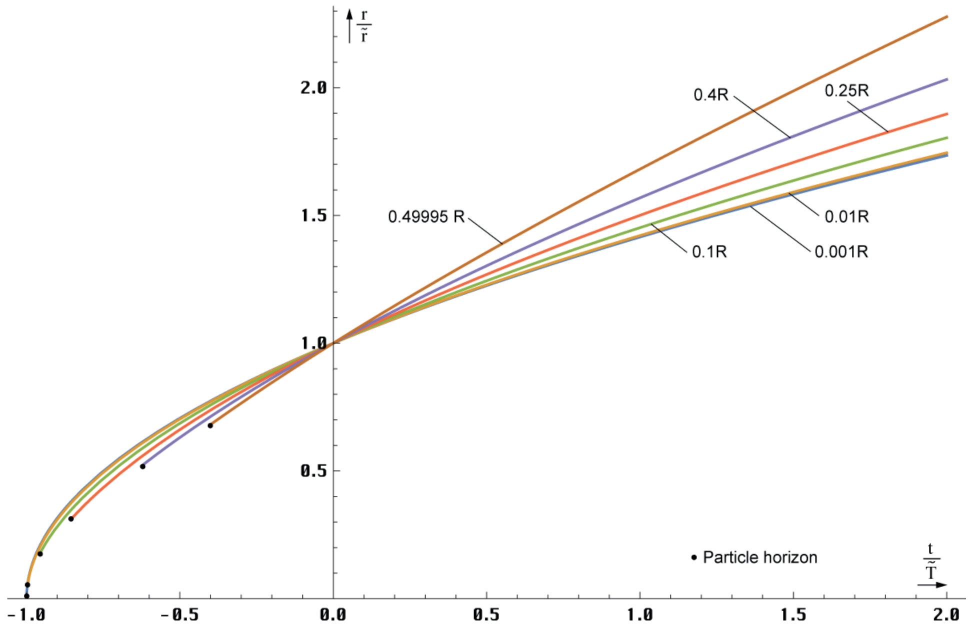

Figure 10
Temporal dependence
of a given distance r

There is only a limited definition-range for the solution. It is temporally bounded below by the spatial singularity, the considered length is greater than the world-radius and doesn't exist yet. The greater the considered length, the smaller the definition range. With world-radius the space-like vector R/2 = cT is meant.

3.1.2.2. Approximative solutions

A simple solution for small r explicitly arises from (75) under application of the two first terms of the TAYLOR series for the function artanh:

$$r \;\approx\; \tilde{r}\left(1+\frac{t}{\tilde{T}}\right)^{\frac{1}{2}} \;=\; 2\omega_0 t\frac{\tilde{r}}{\tilde{Q}_0} \qquad \text{for}\;\; \tilde{r}\le 0{,}01\,\tilde{R} \tag{77}$$

This exactly corresponds to the behaviour of PLANCK's elementary-length (MLE) and is valid until 0.01R approximately. For larger distances, the ascend is larger. First we examine the course in the proximity of t=0 (figure 11) as well as the ascend $\Delta r/\Delta t$ with $\Delta t=2\cdot 10^{-3}$. With root-functions the ascend (dr/dt) is equal to the exponent m in this point:

$$r \;=\; \tilde{r}\left(1+\frac{t}{\tilde{T}}\right)^{m} \tag{78}$$

This is shown in figure 11. It is in the range of 1/2…3/4. Using the function Fit[] with the help of (79) approximations of different precision for the exponent m can be found:

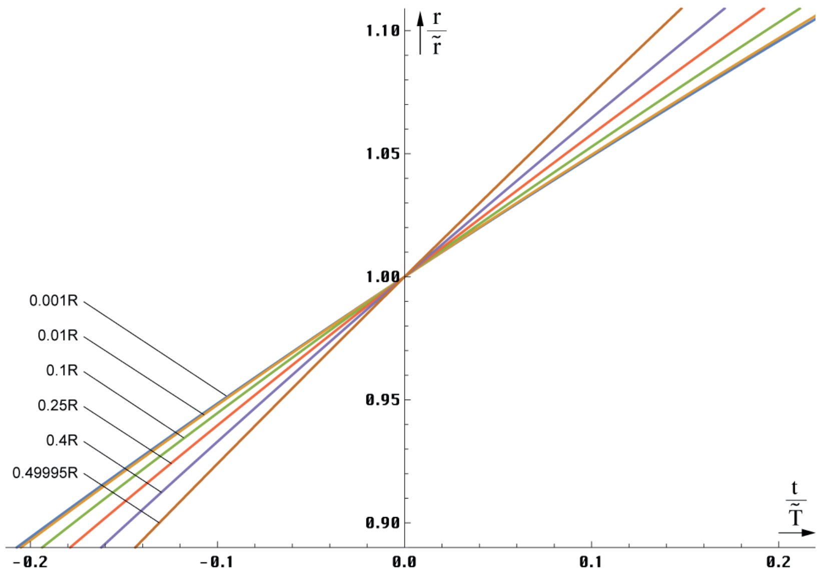

Figure 11
Ascend of several given distances in the proximity of t=0

```
mmm = {{0, .5}};
For[x = 0; i = 0, x < .499, (++i), x += 0.01;
AppendTo[mmm, {x, N[F1[0.0001, x] - F1[0, x]]/0.0001}]]
Fit[mmm, {1, m, m^2, m^3, ... }, m]
```
(79)

$$m \approx 0.513536 + 0.17937r + 0.490927r^2 \qquad \text{with}\;\; r = r/\tilde{R}$$

$$m \approx 0.500(822) + 0.50052r - 1.13082r^2 + 2.16233r^3 \tag{80}$$

$$m \approx 0.500(843) + 0.598206r - 3.45991r^2 + 18.3227r^3 - 42.6995r^4 + 38.0733r^5$$

The third equation of (80) is very exact and suitable even for calculations with more extreme demands. Indeed, there is a need to consider the restricted definition-range, which is not being co emulated automatically by the approximative solution. It is pointed out here once again that the distances and velocities, regarded in this section, are a matter of space-like vectors having nothing to do with the time-like vectors considered in section 4.3.4.4.6. of [1] Cosmologic red-shift.

3.1.2.3. The HUBBLE-parameter

Having defined the HUBBLE-parameter only for small lengths and PLANCK's elementary-length (r_0) until now, which are following the relationships for a radiation-cosmos (m=1/2), we have to correct our statements for larger distances. With m=m(r) the HUBBLE-parameter $H=\dot{r}/r$ becomes also a function of distance:

$$H = \frac{m}{\tilde{T}+t} \qquad\qquad H_0 = \frac{m}{\tilde{T}} \tag{81}$$

The course is shown in figure 12. The metrics examined by this model is a non-linear metrics. With it, the question has become unnecessary, whether our universe is a radiation- or dust-cosmos. The answer is — as well, as. It's a question of the dimensions of the considered area. For small lengths, the distance behaves like a radiation-cosmos, in the range between zero and 0.5R like a dust-cosmos, with 0.5R like photons overlaid the metrics.

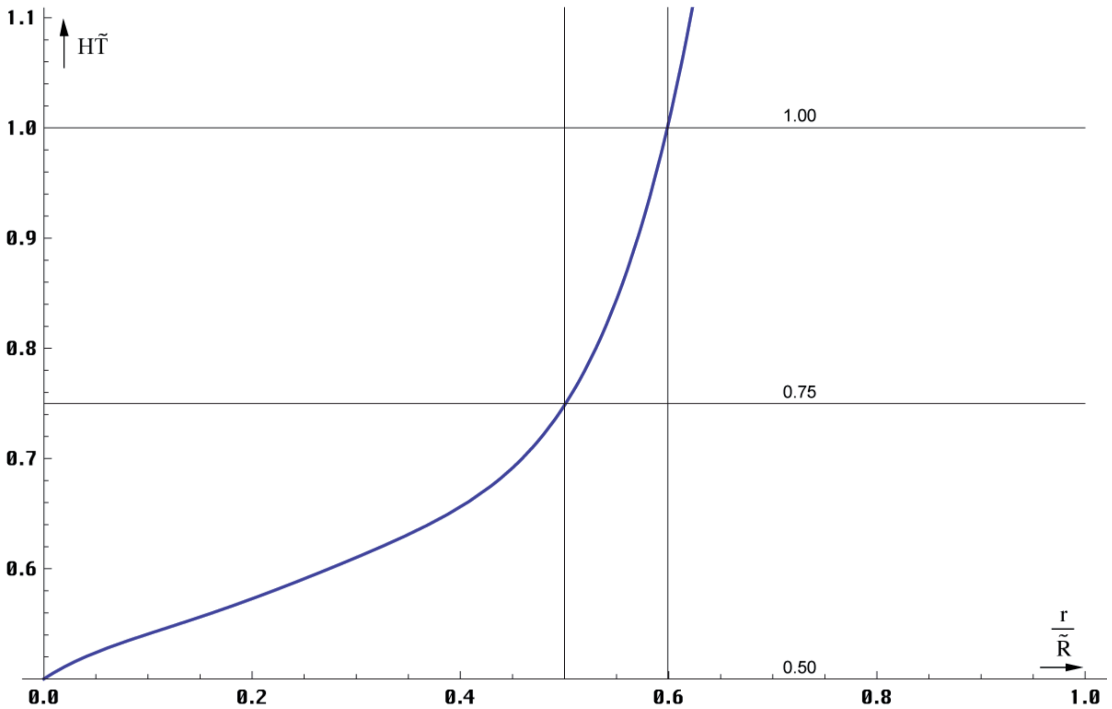

Figure 12
HUBBLE-parameter as a function of the distance for t=0, the values r>0.5R are extrapolated.

However, more latter distance is not an area of infinite red-shift as in other models. It shows with the dilatory-factor q very well The course is depicted in figure 13.

$$q = -\frac{r\ddot{r}}{\dot{r}^2} = \frac{1}{m} - 1 \tag{82}$$

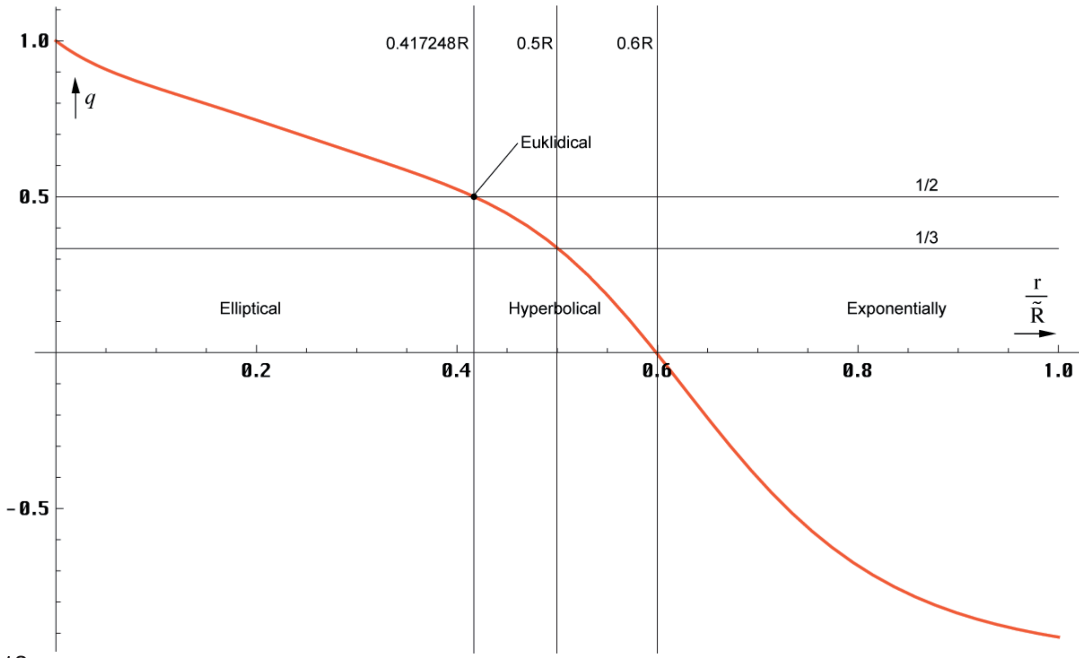

Figure 13
Dilatory-factor as a function of the
distance for t=0, the values r>0.5R are extrapolated

The expansion-velocity H_0r as a function of the distance is shown in figure 14. The speed of light is reached in an essentially minor distance as with the standard-models, but only on paper. While the size of r_0 at $0.5R=cT$ tends to r_1, the expansion speed along the time-like world line at this point is not infinite, rather it's smaller than c (0.75c). Otherwise we found out, that the maximum propagation speed $|\underline{c}_{max}|$ of the metric wave field only amounts to 0.851661c. But furthermore the world-radius should be cT, whereas time-like vectors with up to 2cT should be possible. So we have to do with four different distances resp. velocities, which all don't seem to fit together. But using this model it's possible to solve this conflict. Let's have a look on figure 44, which except for r_K, is a true-to-scale representation. As we can see, the wave front of the metric wave field propagates straight-forward with 0.851661c (propagation share). The part r_M of the world-radius caused by it amounts to 0.851661cT.

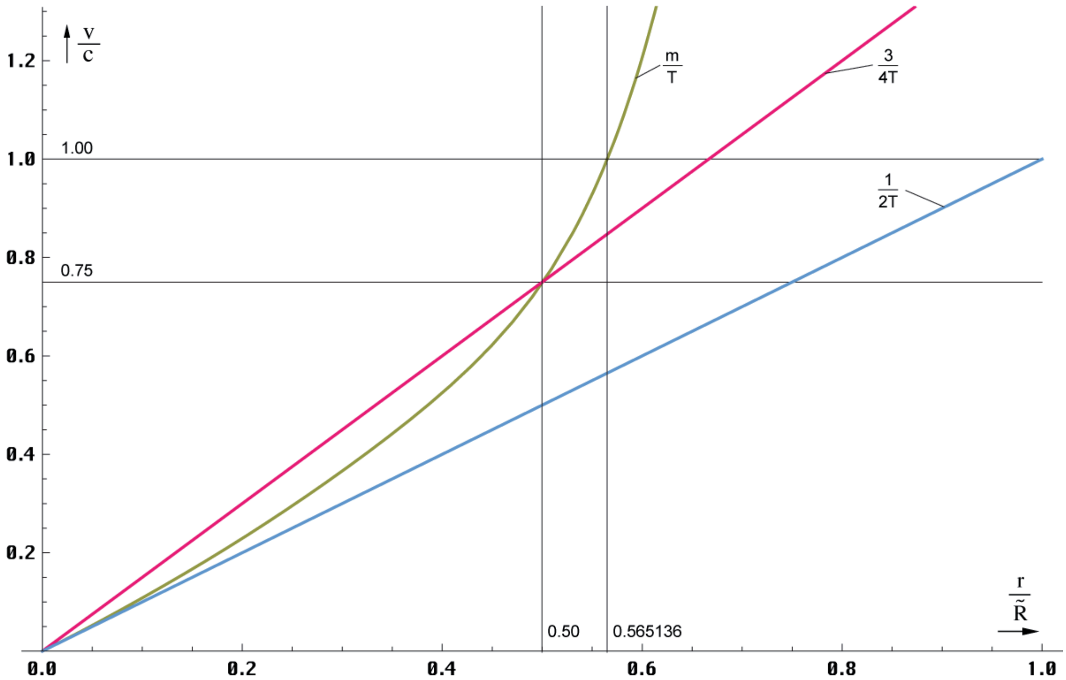

Figure 14
Expansion-velocity as a function of the
distance for t=0, the values r>0.5R are extrapolated

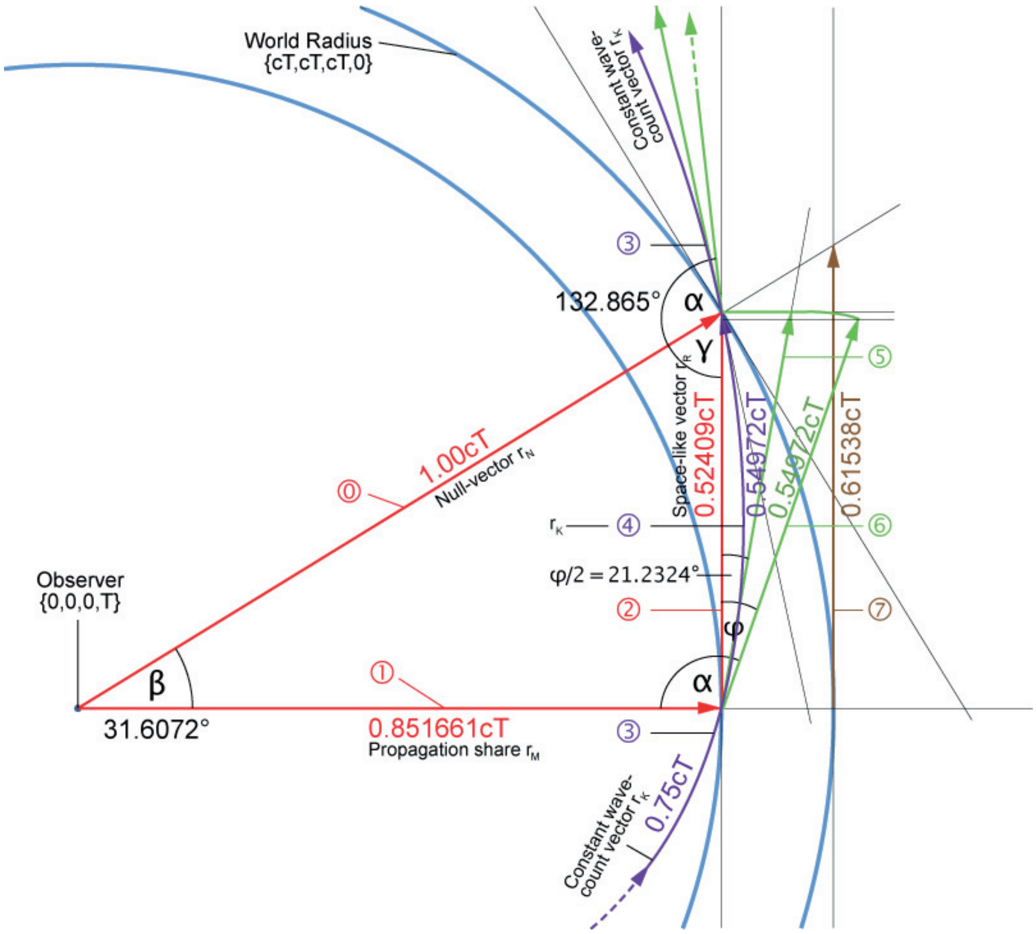

Figure 15
Expansion-velocity and
world-radius in the model

As noticed furthermore, the constant wave count vector r_K up to the vicinity of R/2 is running on the same track as the incoming time-like vector r_T with 0.75 c (arc length 0.75 cT). But it's tilted about the angle α_1, so that we have to sum geometrically. In addition the partial vector ④ is curved. But the object we are looking for is the space-like vector r_R (expansion share). Flattening the partial vector ④ by bending it up to ⑥ by the angle φ_1 of the metric wave function $\varphi = \arg \underline{c} = \alpha - \pi/2$ with $Q = 1$ we realize it to be too short. See expression (32) for the definition. There is a better fit with φ/2. I'm able to provide the following solutions with $\alpha_1 = 2.31893 \triangleq 132.865°$:

$$c = \sqrt{c_M^2 + c_R^2} = \sqrt{c_M^2 + c_K^2 \sin^2\alpha_1 \cos^2\varphi/2} = c\sqrt{0.851661^2 + 0.75^2 \cdot 0.5 \cdot \sin^2\alpha_1 \left(1 + \sin\alpha_1\right)}$$

$$c = c\sqrt{0.7253265 + 0.28125 \cdot 0.73296^2 \cdot 1.73296} = 0.99356c \qquad \Delta = -6.4 \cdot 10^{-3} \qquad (83)$$

$$c = \sqrt{c_M^2 + c_K^2 \cos^2\alpha_1} = c\sqrt{0.851661^2 + 0.680271^2} = 0.99279c \qquad \Delta = -7.2 \cdot 10^{-3} \qquad (84)$$

Expression (84) is based on the directional derivative ⑥ and is similar exact. Interestingly enough the following expression yields a good result too:

$$c = \frac{2}{\pi}(c_M + c_K) = \frac{2}{\pi}(0.851661 + 0.75)\,c = 1.01965c \qquad \Delta = +1.9 \cdot 10^{-2} \qquad (85)$$

maybe a fluke, perhaps a hidden principle? The differences are even smaller than the ones accepted in the QED. But because of the curvature an exact result would have surprised me. A further reason for the differences could be, that the maximum speed $|\underline{c}_{max}|$ isn't at $Q = 1$

really, but at Q=0.865167 exactly. But using that value we get yet an even greater deviation. More information about the time-like vector r_T you can find in section 5. The conclusions obtained here essentially carry weight on the calculation of the entropy of the metric wave field.

3.2. Energy and Entropy

3.2.1. Entropy

Now we will consider the discrete MLE and our model from the energetic point of view. Since entropy is much more important than energy for the thermodynamician, we will take it into account by examining entropy first. We want to mark entropy with S henceforth. In order to avoid confusions with the POYNTING-vector, we will always figure it bold as vector (**S**). If we write S, we always mean entropy and with **S** always the POYNTING-vector.

From the statistic point of view, the entropy of a system is defined by (86) where k is the BOLTZMANN-constant and N the number of all possible inner configurations.

$$S = k \ln N \tag{86}$$

With a single MLE (N=1) entropy would be equal to zero theoretically. That's wrong of course, since statistics necessitates a minimum number of N to be applied at all. With N=1 the result, mathematically can take on a whatever value without offending the „statistics". Therefore we want to try to find out, if there is another possibility to determine the entropy of this single MLE.

Strictly speaking the MLE is a matter of a ball-capacitor with the mass m_0 moving in its inherent magnetic field. We don't know what happens inside the capacitor. Basically it behaves like a (primordial) black hole. According to [7] the SCHWARZSCHILD-radius of such a BH is defined as:

$$r_s = \frac{2mG}{c^2} \tag{87}$$

Now let's substitute m with m_0 here (2). We get $r_S=2r_0$, substantiating our foregoing assumption. The surface of this black hole yields with it to $A=4\pi r_0^2$. It's interesting that the expression for the SCHWARZSCHILD-radius can be derived even without aid of the SRT or URT. Because both, SRT and URT according to this model are only emulated by the metric fundamental lattice. Such relationships must be basic qualities of the lattice itself. They apply as well microscopically as macroscopically then.

In [8] pp. 211 a method is figured to determine the entropy of a black hole. It is based on quantum physical considerations fitting our MLE very well. The author assumes the KERR-NEWMAN-solution of the EINSTEIN-vacuum-equations $R_{ik}=0$ with stationary rotating, electrically loaded source and external electromagnetic field (88) with $R \equiv r^2-2mr+a^2$ and $\rho^2 \equiv r^2+a^2\cos^2\vartheta$, $M=mGc^{-2}$ und $a=Lm^{-1}c^{-1}$; m is the mass and L the moment of momentum.

$$ds^2 = -\frac{R}{\rho^2}\left[c\,dt - a\sin^2\vartheta\, d\varphi\right]^2 + \frac{\rho^2}{R}dr^2 + \rho^2 d\vartheta^2 + \frac{\sin^2\vartheta}{\rho^2}\left[(r^2+a^2)\,d\varphi - a\,dt\right]^2 \tag{88}$$

We don't want to engross it here. The author finally comes to the following statements for the radius $r_\pm$ of the black hole and its surface A:

$$r_\pm = M \pm \sqrt{M^2-a^2} \qquad A = 8\pi\left[M^2 \pm M\sqrt{M^2-a^2}\right] \tag{89}$$

$$r_{\pm} = \sqrt{\frac{2t}{\mu_0\kappa_0}} \pm \sqrt{\frac{2t}{\mu_0\kappa_0} - \left(\frac{2t}{\mu_0\kappa_0}\right)_{L=\hbar}} \qquad r_{\pm} = r_0 \pm \sqrt{r_0^2 - (r_0^2)_{L=\hbar}} \qquad (90)$$

The result depends thereon, if the MLE disposes of a moment of momentum or not. With $m=m_0$ under application of (2), (4) and (695 [1]) we obtain the following values for the SCHWARZSCHILD-radius: Without moment of momentum (L=0) for $r_-=0$, $r_+=r_s=2r_0$ as well as $A=4\pi r_0^2$. With moment of momentum $L=\hbar$, here the brackets apply, we get two identical solutions $r_\pm=r_0$. The surface yields $A=\pi r_0^2$.

Furthermore, the author refers to a work of BEKENSTEIN (1973), according to which the entropy of a black hole should be proportionally to its surface. The exact proportionality-factor has been determined by HAWKING (1974) in a quantum physical manner to:

$$S_b = \frac{kc^3}{4G\hbar}A \quad = \quad k\frac{A}{4r_0^2} \quad = \quad k\frac{A}{(4)r_s^2} \qquad (91)$$

k is the BOLTZMANN-constant, the bracketed number applies to $L=\hbar$. Interestingly enough, the expression contains PLANCK's elementary-length and even with $\hbar$ according to our definition instead of h. If we now re-insert the values, we get:

$$S_b = 4\pi k \quad \text{for } L=0 \qquad \text{as well as} \qquad S_b = \pi k \quad \text{for } L=\hbar \qquad (92)$$

Now we want to examine, whether the MLE actually owns a moment of momentum. We are based on our model (effective-value) developed in section 3.3. of [1]. For the moment of momentum **L** applies generally:

$$\mathbf{L} = \mathbf{r}\times\mathbf{p} = m\cdot(\mathbf{r}\times\mathbf{v}) \qquad (93)$$

With $m=m_0$, $r=r_0$, $v=c$, $c\perp r$ we get after application of (2) for the amount L:

$$L = m_0 c r_0 = \hbar \qquad \text{and because of} \qquad c=\omega_0 r_0 \qquad (94)$$

$$W_0 = m_0 c^2 = \hbar\omega_0 \qquad (95)$$

Expression (95) is apparently right. With it, we have explicitly proven, that the MLE owns a moment of momentum. It’s equal to PLANCK's quantity of action i.e. as with a spin–2–particle or vice-versa:

> *The PLANCK's quantity of action is defined by the effective-value of the moment of momentum of the MINKOVSKIan line-element. The inherent moment of momentum (spin) is identical to the track moment of momentum.*

The last statement is justified by the fact that it's a matter of effective-value here. In reality, r_0, m_0 and the track- and inherent moment of momentum are temporally variable, nearly periodic functions. PLANCK's quantity of action is the sum of track- and inherent moment of momentum then. It’s equal to h, at which point one time the track-, the other time the inherent moment of momentum becomes zero. Such an interdependence even is called dualism. Naturally, PLANCK's quantity of action can be defined not only as moment of momentum. Another possibility is e.g. $q_0\varphi_0$. Because of GIBBS‘ fundamental equation the temperature of the MLE and with it of the whole metric wave field is equal to zero [8].

Going back to entropy. We see that the BOLTZMANN-constant figures an elementary quality of our metric fundamental lattice, as elementary as ε_0, μ_0 and κ_0. Here, someone may say, this cannot be correct, since k is a purely statistical constant. Just we can answer this interjection: »The BOLTZMANN-constant is so elementary because it’s statistical«. Even π allows to be defined statistically.

3.2.2. Topology

We have determined the entropy of one discrete MLE. How does it look with a larger length then again? Since the single-entropy is a multiple of the BOLTZMANN-constant, we can calculate-on with the already known statistical relationships (86). In this connection the (absolute) maximum number of possible inner configurations within a volume with the radius r is given by the number of MLE's contained in this volume. With a cubic-face-centred crystal-lattice, the number within a cube with the edge length d is defined as:

$$N = 4\left(\frac{d}{\rho}\right)^3 = 4\left(\frac{d}{r_0}\right)^3 \qquad (96)$$

ρ is the lattice constant in this case. The fc-cube just contains 4 elements in total. Then, within a ball with the diameter $d = \Lambda r_0$ and the volume $\pi/6\, d^3$ there are

$$N = \frac{2}{3}\pi\left(\frac{d}{\rho}\right)^3 = \frac{2}{3}\pi\left(\frac{\Lambda r_0}{r_0}\right)^3 = \frac{2}{3}\pi\Lambda^3 \qquad (97)$$

individual MLE's. As long as ρ is not too large, we can insert (69) for Λ, otherwise (73):

$$N = \pi\tilde{Q}_0^3\left(\left(1+\frac{t}{\tilde{T}}\right)^{\frac{1}{4}}\operatorname{artanh}\left(\left(1+\frac{t}{\tilde{T}}\right)^{-\frac{1}{4}}\left(\frac{2r}{\tilde{R}}\right)^{\frac{1}{3}}\right)-\left(\frac{2r}{\tilde{R}}\right)^{\frac{1}{3}}\right)^3 \quad \text{or} \qquad (98)$$

$$N = \pi\tilde{Q}_0^3\left(t^{\frac{1}{4}}\operatorname{artanh}\left(t^{-\frac{1}{4}}(2K_1 r)^{\frac{1}{3}}\right)-(2K_1 r)^{\frac{1}{3}}\right)^3 \quad \text{with } r = \mathrm{r}/\tilde{R} \text{ and } K_1 = 1 \qquad (99)$$

That‘s the number of elements within a sphere with the radius r. The course is shown in figure 16 curve ①. If we insert the expression $\Lambda_1 = \tfrac{3}{2} Q_0 \ln Q_0$ into (97), we obtain even a result for N_1. Here $t \equiv 0$ reapplies. Then, the whole universe would contain altogether $N_1 = \tfrac{9}{4}\pi\, Q_0^3 \ln^3 Q_0 = 8.35202\cdot 10^{189}$ elements. Because of the propagation of the metric wave field this value is increasing continuously too (see figure 18), and that according to $N_1(T) = \tfrac{9}{4}\pi\,(\sqrt{bT})^3 \ln^3 \sqrt{bT}$ with $b = 2\,\kappa_0/\varepsilon_0$.

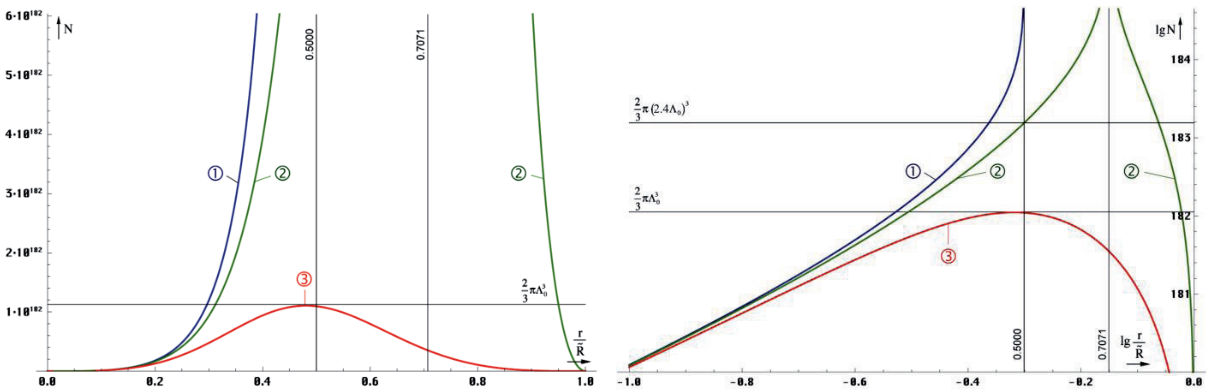

Figure 16
Number of MLE´s in dependence on the radius linear and logarithmic

But for the calculation of the entropy S these values are sparsely helpful. As is known S is about a statistical value and (99) violates a basic rule of the statistics: *A value must not be counted repeatedly.* The relations (96ff) namely apply for a „normal“ 3D-sphere only.

But at the universe we have to take into account the particular 4D-topology. An observer in the free fall only imagines to be located in the spatial centre of the universe. In reality he is situated at a temporally singularity, the event horizon {0,0,0,T}. He is unable to overcome it, because beyond there is the future. Indeed, it's not about a point, but about a hyper-surface. All other observers at their own 3D-locations reside widespread at the same surface. Since T proceeds steadily, the temporal radius increases too and the observers are quasi „surfing“ on the „time wave“. If one observer wants to visit another, he must accelerate. Thus, his temporally course is slowing down. Indeed, he does not travel to the past, but he is only „broken away“ from the unbraked time lapse. He suddenly finds himself inside the sphere. With $v=c$ the time stands still for him. Now he is situated at the real spatial centre, but only, because it came up to him.

That means, the spatial 4D-centre is not with the observer, but in the distance cT at the coordinates {cT,cT,cT,0}. More correct would be t_1 instead of zero here. With the spatial centre it's also about a hyper-surface, a spatial singularity, the particle horizon. We cannot overcome even that. Like the temporal radius it‘s expanding steadily. Altogether it's about a closed system.

If two observers could swap their positions, they would find the same conditions on both locations. Since overall in the universe the same physical laws apply. Interesting thereat is, that we *observe* different conditions in a definite distance r. The reason is the finite speed of light. The universe is *not* hot-wired, there is *no* instantaneous interconnection between whatever points (except for quantum entanglement). For all observers the universe consists of the local conditions plus all forces and signals resulting from prior states, delayed by $t \geq r/c$. The farther, the elder the condition, that caused the impact.

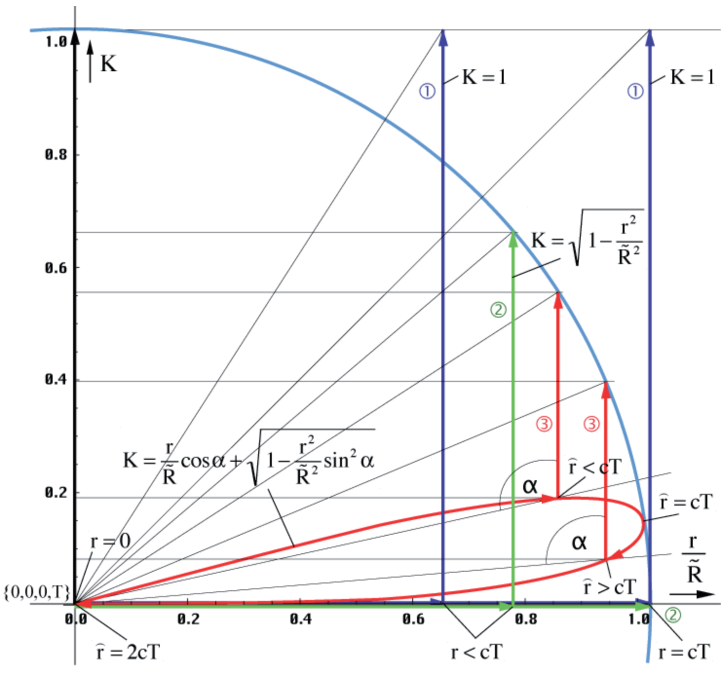

Figure 17
Factor K in dependence on the radius for the 3 solutions (schematic presentation)

And exactly that is the reason, why we cannot use expression (99). Approaching the distance cT, the MLE-density within Λ is increasing enormously indeed. But similarly, the universe in that distance, at that time has had an essentially smaller world radius, a smaller surface. That means, the cross section must be smaller than at solution ①. *The larger* the distance r, *the smaller* the surface A, the opposite way around, as with a „normal“ sphere.

Even e.g. the spherical shell in the distance $R/2-r_1$ namely consists of only one single element. If its condition changes, it has a *simultaneous* effect *on all* vectors coming from *all* directions. But we are allowed to count only one element.

In fact that‘s good for MACH's principle, spatial damping cancels out, the strongest force is coming from the „edge“, but not for the statistics. That's why we are forced to find a function, which considers these special conditions. In doing so the reference to the time *t* should not get lost. Because I'm not a topology-expert, I tried to find such a function, at least roughly by introduction of a correction factor K; the whole by trial and error. So it's not about a correct derivation here. With small r a possible solution should run similarly as with a 3D-sphere, likewise as solution ①. In the vicinity of R/2 it should flatten out however. Either the border R/2 should not be passed.

In addition to ① two more possible solutions are depicted in figure 17 to the correction of one single coordinate. With solution ② (100) I assumed the volume of the inverse sphere to decrease with *r*. Solution ③ (101) additionally considers the curvature in the vicinity of R/2 under consideration of the angle α.

$$\mathrm{N} = \pi\tilde{\mathrm{Q}}_0^3 \left(t^{\frac{1}{4}} \operatorname{artanh}\left(t^{-\frac{1}{4}} (2\mathrm{K}_2 r)^{\frac{1}{3}} \right) - (2\mathrm{K}_2 r)^{\frac{1}{3}} \right)^3 \quad \text{with } \mathrm{K}_2 = \sqrt{1-r^2} \tag{100}$$

$$\mathrm{N} = \pi\tilde{\mathrm{Q}}_0^3 \left(t^{\frac{1}{4}} \operatorname{artanh}\left(t^{-\frac{1}{4}} (2\mathrm{K}_3 r)^{\frac{1}{3}} \right) - (2\mathrm{K}_3 r)^{\frac{1}{3}} \right)^3 \quad \text{with } \mathrm{K}_3 = r\cos\alpha + \sqrt{1-r^2\sin^2\alpha} \tag{101}$$

The angle $\alpha(r)$ calculates as follows (*applies only in connection with (101)!!!*)

$$\alpha = \frac{\pi}{4} - \arg\left(-\mathrm{j}4r \left(1 - \left(\frac{\mathrm{H}_2^{(1)}(r^{-1}/2)}{\mathrm{H}_0^{(1)}(r^{-1}/2)} \right)^2 \right)^{-\frac{1}{2}} \right) \tag{102}$$

It’s even only a rule of thumb. The course of both functions is depicted in figure 17. As we can see, function (100) is less suitable, because it exceeds the R/2-border at $\mathrm{N}=2/3\pi(1.1955\cdot\mathrm{Q}_0)^3=2/3\pi(2.3909\cdot\Lambda_0)^3$ — a crooked value. There isn’t a flattening either, but a pole outside R/2.

Function (101) on the contrary fulfils all demands. It proceeds as with a 3D-sphere, like solution ① at small r and there is a flattening in the direct vicinity of R/2. Indeed, the function is defined beyond R/2, but without pole, and the value re-drops to zero at 2cT. That means, it’s about a time-like vector remaining inside the world radius. That’s easy to understand. When rushing through the 4D-centre {cT,cT,cT,0} or passing it within spitting distance, the vector re-approaches the observer and N has to decline again. The maximum is at the „magic“ value $\mathrm{N}_0=2/3\pi(\mathrm{Q}_0/2)^3=2/3\pi\Lambda_0^3=1.12308\cdot10^{182}$. The reason, why the func-tion hits its maximum already on the verge of R/2, is its curvature. The arc-length becomes effective here.

By the way, all time-like vectors with the length 2cT, regardless of continuous or discontinuous (virtual), are coming from a point with the coordinates $\{r_1/2, r_1/2, r_1/2, t_1/4\}$. That’s behind the particle horizon, previous to the phase jump at Q=1, from a time, at which event- and particle-horizon still overlapped each other (Q=1/2). The real world age is T, the length 2cT is the result of curvature, propagation and expansion (see figure 21).

Thus I’m sure, that (101) fits the actual conditions to the best. Then, N_0 would be identical to the total number of possible micro-states of the universe and candidate for the calculation of the entropy S_0. The temporal dependence of N according to (101) for several constant distances is depicted in figure 18. The course of $\mathrm{N}_0(\mathrm{T})$ and $\mathrm{N}_1(\mathrm{T})$ in the comparison is shown top right. The rule of N_1 has been scaled down about 10^8, because both values gape apart too much.

Needless to say, the temporal functions are defined from N_0 on only, above they are cropped. Solution ① proceeds similarly, but N_1 is orders of magnitude greater, so that the crop takes place much higher in a range running nearly vertical up, which can no longer be processed by the plot program. And there is another difference. Distances >R/2 aren’t postponed into future with solution ① and ② similar to the dashed blue line (not to scale). That’s correct. In contrast, solution ③ shows them, as if it’s about a distance <R/2, which is also correct. Of course, there is even such a line with solution ③ (example 0.8R′), but it’s not being emulated by expression (101). That’s correct too, since there is a nearly infinite number of solutions already in the example range 0.5…0.8R and beyond, depending on R′.

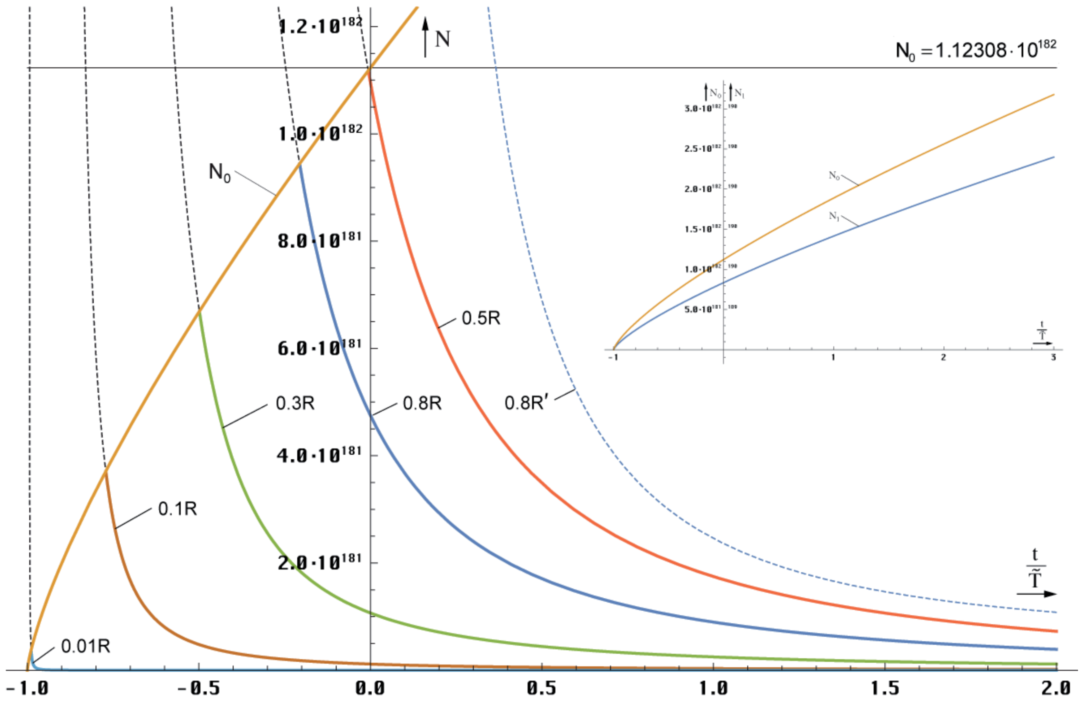

Figure 18
Number of MLEs in dependence on time
according to solution ③

3.2.3. Entropy

Now let's get down to the entropy. Generally (86) applies here. As determined more above, the entropy of the MLE calculates similar to that of a black hole according to (92) right (S_b). Thus, we have to multiply (86) with π. However, that applies to the metric wave field only and not to the CMBR. All other problems may be calculated with the conventional ansatz and (86). In doubt just divide the results by π.

The course of the entropy S in dependence on the radius is shown in figure 48. Starting with a value of $\pi k = 4.337465 \cdot 10^{-23} JK^{-1}$ with $r = r_0$ the entropy ① rises continuously with increasing r, runs through a phase of minor ascend and skyrockets towards infinite with $r \to cT$. But an infinite value will not be achieved, since the number of line elements until the edge is limited to $S_1(\Lambda_1)$.

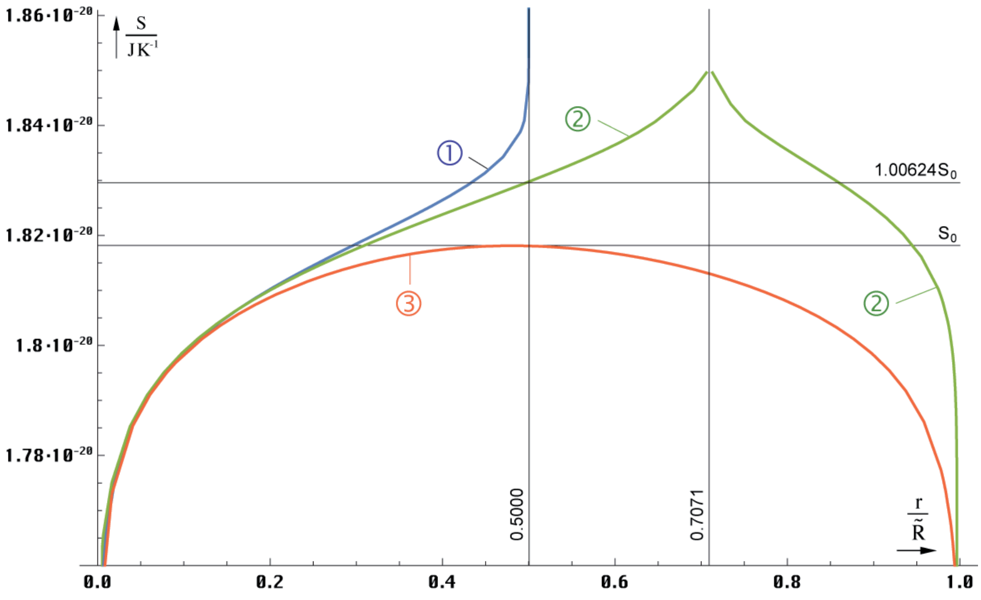

Figure 19
Entropy in dependence on the radius

Because of the pole solution ② is less suitable. For solution ① we obtain the huge value of $S_1 = 3\pi k(\frac{2}{3} + \ln Q_0 + \ln\ln Q_0) \approx 1312\pi k = 1.89701\cdot 10^{-20}\,JK^{-1}$. For solution ③ the entropy S_0 applies. It's defined as follows:

$$S_0 = \pi k \ln\left(\frac{2}{3}\pi\Lambda_0^3\right) = \pi k \ln\left(\frac{1}{12}\pi\tilde{Q}_0^3\right) = 1.81821\cdot 10^{-20}\,JK^{-1} \qquad (344)$$

The temporal dependence of S_0 for the case r=const is depicted in figure 20. Interestingly enough the values of regions with fixed size decrease steadily. Maybe that's the „motor" of the evolution from the lower to the higher. In the case constant wave count vector the entropy $S(r\neq R/2)$ remains constant across the whole definition range. It calculates according to (104) on the left. For S_0 the right expression applies:

$$S = \pi k \ln N \qquad S_0 = \tilde{S}_0 + 6\pi k \ln t = \tilde{S}_0 + 3\pi k \ln\left(1 + \frac{t}{\tilde{T}}\right) \qquad (104)$$

To calculate S_1 we advantageously substitute Q_0 with $\tilde{Q}_0 t^2$ in the expression in the paragraph below figure 48. The entropy with constant wave count vector isn't defined across all times for all radii either. Certain distances don't exist, until the radius of the expanding universe has reached that length. Then S gets the value S_0 resp. S_1 exactly on entry. It applies: The later the entry, the higher starting entropy. Curves are being cropped even here in turn. Solution ① looks similar like figure 20. The curve S_1 proceeds far beyond the plot however. Initial distances >R/2 are moved into future too, with solution ③ into the range <R/2, just like with N_1 and N_0.

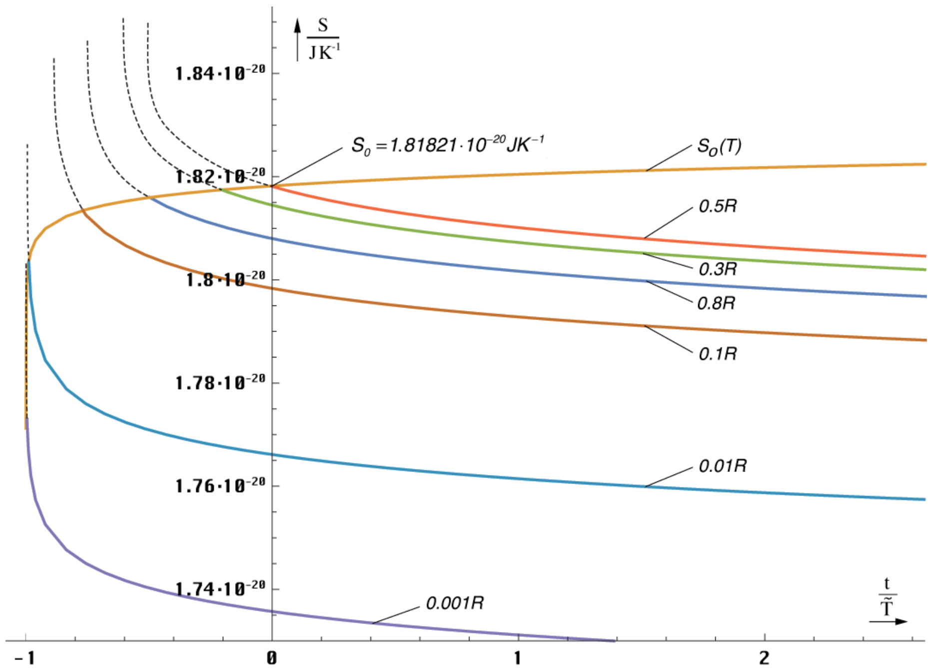

Figure 20
Temporal dependence of the entropy
for r=const (linear scale)

The temporal functions S_0 and S_1 are tending to ∞, as we can easily see by application of the limit theorems. Concerning the future of the universe we can say, that we don't have to fear a heat death. A thermodynamic equilibrium will never occur. The reason is the propagation of the metric wave field, as well as the expansion of the universe. That was a close shave!

4. Particle horizon

As shown in section 3.2.1. the MLE disposes of an inner SCHWARZSCHILD-radius with the value $r_\pm = r_0$. It has the property of a particle horizon. Because of the relations $R = r_0 Q_0$ and $r_1 = r_0/Q_0$ it may be possible, that such a particle horizon also exists on a macroscopic scale, for the cosmos as a whole. The HUBBLE-parameter $H_0 = \omega_0 Q_0^{-1}$ has the character of an angular frequency, just as $\omega_0 = \omega_1 Q_0^{-1}$. Thus, it may be possible, that even the whole universe owns an angular momentum in the amount of $\hbar_1 = \hbar Q_0$. The MLE with its spin 2 lets suppose, that the universe also owns a spin of the size 2. That would explain a lot of phenomena. Therefore, with this information, we want to try, to calculate such a hypothetic SCHWARZSCHILD-radius $R_\pm$ with ($L = \hbar_1 = \hbar Q_0$).

We start, in that we multiply (90) with Q_0 resetting the bracketed expression to the definition $a = \hbar m^{-1} c^{-1}$. The value M_1 is determined using the right-hand ansatz and (695 [1]):

$$R_\pm = Q_0 r_\pm = R \pm \sqrt{R^2 - \left(\frac{Q_0 \hbar_1}{2 M_1 c}\right)^2} \quad \text{with} \quad \frac{M_1 G}{c^2} = 2ct \quad \Big| \quad M_1 = m_0 Q_0 = \mu_0 \kappa_0 \hbar \qquad (105)$$

$$R_\pm = R \pm \sqrt{R^2 - Q_0^2 r_0^2} = R \pm \sqrt{R^2 - R^2} = R \qquad (106)$$

As result a double solution with $R_\pm = R$ turns out, exactly as with the MLE but on a larger scale. The universe inside is larger then outside apparently, maybe due to the curvature of the time-like vectors. Notably interesting is the value $M_1 = 1.73068 \cdot 10^{53}\,\text{kg}$ (Q_0 as per table 10). That's the total mass of the metric wave field and identical to MACH's counter mass. Dividing it by the volume $V_1 = \frac{4}{3}\pi R^3$ we obtain a value of $1.94676 \cdot 10^{-29}\,\text{kg}\,\text{dm}^{-3}$ for the density. This one is about 3/2 times greater than the value $G_{11}(R/2)$ calculated in section 7.2.7.2. of [1]. Well, we are living in a black hole actually and we can use nearly 100% thereof. Or is there yet an „outside" and the universe is nothing other than a huge line element?

5. Distance-vectors

Due to the progress in the technical domain taken place in the most recent time, the astronomers are able to look into the universe deeper and deeper and with it even farther back in time. The farther one looks however, all the more the structure of the universe becomes notably and must be taken into consideration on the interpretation of the measuring results. Otherwise the much money would have been poured down the drain.

But before expanding further, just let's have a look at a so simple quantity, like the distance respectively the spacing to a stellar object. The astronomer just sits in front of his telescope, observing an object and he tries to determine with different methods, how far away it is. And before he can determine the HUBBLE-parameter, he must determine the distance respectively the spacing to the object of course. And the first problem already appears here: What do we actually mean by distance as well as spacing? And what do we really want to determine?

In the close-up range this question can be answered relatively simply: The spacing is equal to the distance and the light from the object has covered this, when it has arrived at the observer. But if we leave the close-up range, looking at objects farther away, it's no longer like this. At first, we look at the object by means of photons, which have moved from the object into our direction. Thus, in reference to the metrics, it's about an (incoming) time-like vector (figure 21 and 22 r_T red pictured), a negative distance. We call it *time-like distance*. It corresponds to the constant wave count vector of the metrics. On this occasion, we how-ever actually observe the zero vector and not the time-like vector. With vanishing curvature both coincides indeed. As it looks like, when there is a curvature, will be presented later.

But the object, we observe nowadays, is already located at a completely different position, as our observation-data want to make believe, since these are already totally „outdated", when they reach us. One feature of this model is now, that this is not the case. Even when the signals are already very old, the object really resides in reference to the observer's R^4-

coordinate-system at that very position, where he observes it. The length of the vector from the object to the observer however cannot be influenced by him, because he is just only observer.

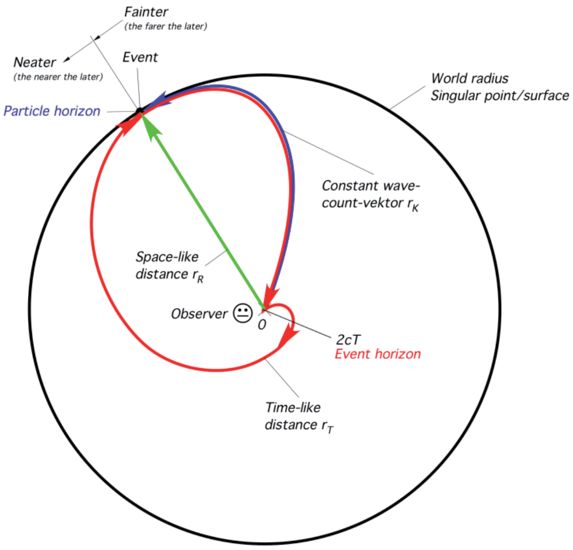

Figure 21
Distance-vectors with an object
at the edge of the universe (schematized)

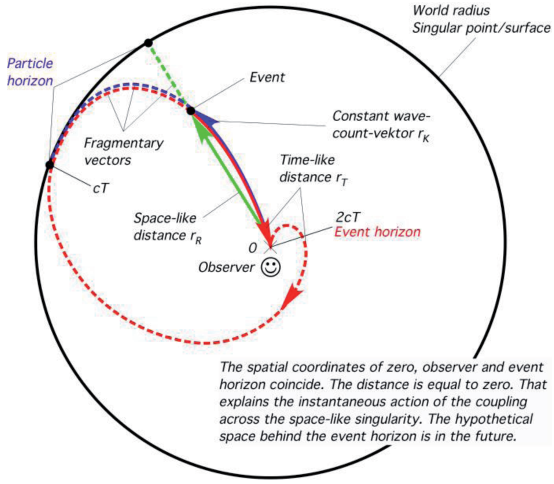

Figure 22
Distance-vectors with an object
in the close-up range of the observer (schematized)

But if the observer has the intent, to visit the object, that would be an (outgoing) space-like vector then, a positive distance/spacing, this cannot take place on the same way, which the ray of light has covered, because the observer would have to move with c thereto and each zero vector is unique. Now, another distance/spacing is applied to him.

To the difference between *distance* and *spacing*: These are (approximately) equal in the close-up range only. With larger distances, objects in the free fall remove themselves according to the distance-function with constant wave count vector. That would be the real *spacing* (r_K blue pictured). With it, also the definition of the *space-like distance* arises (r_R green pictured). This is the shortest way between the observer or better the traveller and the object. It is an imagined line and coincides with the coordinate r of the coordinate-system. Locally, it is equal to the space-like vector of the metrics.

1. The zero vector r_N is the way, a ray of light covers, at which point the velocity in reference to the subspace is c constantly. In the local range it is equal to the geometrical sum of space- and time-like vector.

2. The time-like distance r_T is the way, a ray of light, starting from the source, has covered, when it has been arrived at the observer. In the local range, it corresponds to the time-like vector of the metrics. But actually the zero vector r_N is observed.

3. The spacing r_K is the distance between two objects in the free fall. The vector proceeds along the field-lines of the gravitational-field and varies according to the spacing-function with constant wave count vector. It corresponds to the zero vector r_N of the metrics.

4. The space-like distance r_R is the shortest vector between a traveller and his destination. It's about an imagined line. It is identical to the coordinate r of the coordinate-system. In the local range, it corresponds to the space-like vector of the metrics. If one wants to travel along this line, permanent navigation (acceleration) is necessary.

But this way, the destination cannot be reached in the free fall, as an analogy from the navigation suggests, the difference between latitudinal and great-circle-distance. When start and destination are on the same latitude and if it's not exactly about the equator, the great-circle-distance is always smaller than the latitudinal-circle-distance. During great-circle-navigation however, the captain must change the course continually, just accelerate, whereas he could theoretically continue his journey without acceleration on the latitudinal circle, just in the free fall, when the water resistance would be zero. Thus, the voyager has the chance, to influence the distance, namely by means of navigation. To the better overview the definitions once again:

But let's descend to the *time-like distance* once again. This is the distance, the astronomer determines, when he analyzes incoming light- or radio-signals (zero vectors). They are subject to a red-shift according to the propagation-function in section 4.3.5.4.3. resp. 5.3.2. of [1]. The *time-like distance* is limited to the maximum *time-like distance*, which results from the total-age 2T. It applies $r_{Tmax} = R = 2cT$. In the course of this work, we had learned that the maximum *space-like distance* amounts to only the half of it: $r_{Rmax} = R/2 = cT$. The reason, that the maximum *time-like distance* may be greater, is the expansion of the universe, the propagation of the metric wave field and the curvature of the constant wave count vector. That all leads to the cosmologic red-shift, i.e. to a prolongation of the wave length and with it, to a prolongation of the vector as a whole.

All *time-like vectors* with the length 2cT are coming from the same point $\{r_1/2, r_1/2, r_1/2, t_1/4\}$ and are ending at all points of the hyper-surface $\{R,R,R,2T\}$ at the same time. Both are superimposed for any observer. The point $\{r_1/2, r_1/2, r_1/2, t_1/4\}$ is quasi „smeared“ across the whole universe, i.e. all points on the hyper-surface are interconnected via $\{r_1/2, r_1/2, r_1/2, t_1/4\}$ and, since photons are timeless, even instantaneously. That may be the cause for such effects like quantum entanglement etc.

6. Summary

In the course of this work, with the help of the model from [1], we succeeded in the definition of the propagation function of the metric wave field, postulated by LANCZOS. That, on the other hand, was the base for the determination of the HUBBLE-parameter for greater distances. It was shown, that this depends on the initial distance. The exact function could be determined. Furthermore the entropy of the metric wave field was determined — under consideration of the special 4D-topology of the universe. Its value will increase steadily even in future and there is no fear of a heath death anyway. The reason is the expansion of the universe, the propagation of the metric wave field and the curvature of the constant wave count vector in turn.

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8. Current values of the universal nature-constants

To the better comparison, it is opportune to depict an overview of all dependent and independent universal fundamental »constants« (table 1). Invariables are marked with the symbols (• ◦). One sees that there are actually only five universal fundamental (•) physical *constants* (μ_0, ε_0, κ_0, h_i and k).

The speed of light is also a genuine constant admittedly, however not fundamentally at all, since it can be combined from μ_0 and ε_0, just as r_1, ω_1 and t_1. The initial value of PLANCK's quantity of action h_i as well as some other values will be described later for the first time. These and all other ones are no genuine constants. They can be figured by combination of the five fundamental values as well as the corresponding space-time-coordinates.

Constant	Symbol	C	Value	Unit of measurement
Speed of light	c	◦	$2.99792458 \cdot 10^{8}$	$m\ s^{-1}$
Induction-constant	μ_0	•	$4\pi \cdot 10^{-7}$	$Vs\ A^{-1} m^{-1}$
Influence-constant	ε_0	•	$8.854187817 \cdot 10^{-12}$	$As\ V^{-1} m^{-1}$
Conductivity-constant	κ_0	•	$1.23879 \cdot 10^{93}$	$A\ V^{-1} m^{-1}$
Boltzmann-constant	k	•	$1.380658 \cdot 10^{-23}$	$J\ K^{-1}$
Planck's init. quant. of action	h_1	•	$7.95297 \cdot 10^{26}$	J s
Planck's quantity of action	h		$1.05457266 \cdot 10^{-34}$	J s
Gravitational-constant (init.)	G_1		$1.55558 \cdot 10^{-193}$	$m^3 kg^{-1} s^{-2}$
Gravitational-constant (Nwt.)	G		$6.67259 \cdot 10^{-11}$	$m^3 kg^{-1} s^{-2}$
Poynting-vector metrics (init.)	S_1		$3.3907 \cdot 10^{426}$	$W m^{-2}$
Poynting-vector metrics	S_0		$1.38959 \cdot 10^{122}$	$W m^{-2}$
Fine-structure-constant	α		$7.2973530 \cdot 10^{-3}$	1
Q-factor/phase metrics (g_{00}^{-1})	Q_0		$7.5419 \cdot 10^{60}$	1
Planck's mass	m_0		$2.17661 \cdot 10^{-8}$	kg
Planck's energy	W_0		$1.95624 \cdot 10^{9}$	J
Planck's length	r_0		$1.61612 \cdot 10^{-35}$	m
Planck's time-unit	t_0		$2.6954 \cdot 10^{-44}$	s
Circular frequency of metrics	ω_0		$1.85501 \cdot 10^{43}$	s^{-1}
Wave impedance vacuum	Z_0	◦	$376.73 \approx 2\pi \cdot 60$	Ω
Cut-off frequency vacuum	ω_1	◦	$1.3991 \cdot 10^{104}$	s^{-1}
Smallest time-unit vacuum	t_1	◦	$3.57372 \cdot 10^{-105}$	s
Smallest length vacuum	r_1	◦	$2.14127 \cdot 10^{-96}$	m
Hubble parameter	H		75.9 ± 4.4	$km\ s^{-1} Mpc^{-1}$
Hubble parameter	H_0 (ω_{-1})		$2.45972 \cdot 10^{-18}$	s^{-1}
Total age	$2T$		$1.291818 \cdot 10^{10}$	a
Local age	T		$6.45909 \cdot 10^{9}$	a
Local age	T (t_{-1})		$2.03275 \cdot 10^{17}$	s
Local world-radius	R		3.9500	Gpc
Local world-radius	R (r_{-1})		$1.21881 \cdot 10^{26}$	m

Table 2: Fundamental physical constants standard model