A Solution to the Twin Paradox and a New Interpretation of Inertial and Non-Inertial Reference Frames

Henok Tadesse
Email: *entkidmt@yahoo.com*

01 April 2021

Abstract

In this paper we propose that the velocity in the Lorentz transformation (LT) equations and special relativity can only be *instantaneous* relative velocity between inertial reference frames. We will see that this interpretation will resolve the twin paradox and lead to a deeper understanding of inertial and non-inertial reference frames. Physicists failed to note that if the travelling twin suddenly decelerated from relativistic velocity to low velocity, all space and time *coordinates* and separations / *intervals* of events will also change suddenly, retroactively, in such a way *as if* the travelling twin had always been travelling at that low velocity. When the travelling twin returns to Earth and comes to rest relative to the stationary twin, all 'memory' of his/her past relativistic journey will be lost and time intervals of events will also change retroactively *as if* he/she had always been at rest relative to the stationary twin, *as if* he had never travelled at relativistic speeds. Only instantaneous velocity is relevant and all motion history of the twin is irrelevant in the Lorentz transformations. Therefore, the problem is *symmetrical* with respect to both twins. Intuitive and inconsistent application of time dilation (and length contraction), rather than rigorous and consistent application of Lorentz *coordinate* transformation of *events*, has caused the creation and persistence of the twin paradox. Lorentz transformation is fundamentally about *coordinate* transformation of events and time dilation and length contraction are only consequences of LT. The twin paradox is a result of a somehow mixed (relativistic and classical) views of time.

Introduction

The twin paradox of special relativity theory is usually presented as follows. We have twin brothers A and B on Earth. Twin B sets out on a round trip journey in a spaceship to a star twenty light years away from Earth at near the speed of light (0.99 c), while twin A stays on Earth. It takes twin B almost forty years to return back to Earth.

One of the predictions of special relativity theory is that 'moving' clocks slow down, and this is known as time dilation. Time passes slowly for the travelling twin as seen in the reference frame of twin A, which means twin B will age slowly compared to twin A. So when twin B returns back to Earth, she finds twin A to be older than herself. However, if all motion is relative, then twin B may also claim that she was the one who had been at rest the whole time, so it should be twin A who should be younger when they meet again.

Many solutions have been proposed to resolve this paradox. One of the most common explanations is the fact that twin B undergoes accelerations, so there will be no symmetry, hence

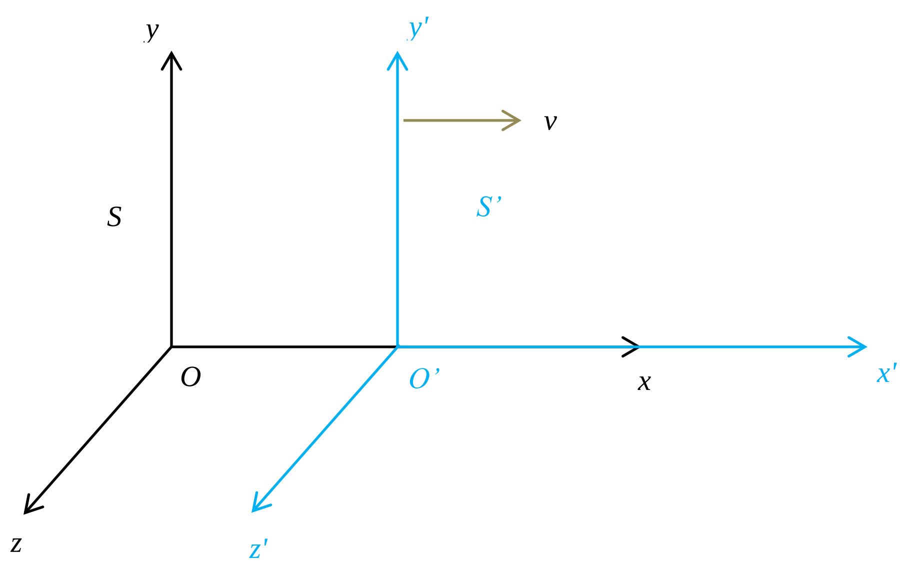
Electrical engineer, BSc, Debrezeit, Ethiopia Mobile: +251 910 751339
or +251 912 228639 Alternate email: *wchmar@gmail.com*

no paradox. However, no one has clearly shown *mathematically* the relationship between acceleration *a* and time dilation. Another explanation is that twin B is not in the same reference frame during the whole journey, while twin A is in the same reference frame. While it might be agreed that acceleration and frame switching/changing are somehow the ultimate cause of the supposed asymmetry, it is still not clear how *precisely* these resolve the paradox. But almost all proposed solutions agree that twin B should be the younger one. Only a few papers and articles propose otherwise. In this paper we propose a new solution to this paradox.

To set the stage for our arguments, let us first briefly review the Lorentz transformations.

Lorentz transformation equations

Consider two inertial reference frames S and S'. S' moves relative to S in the +x direction with velocity v. The origins of S and S' , which are O and O' respectively, coincide at $t = t' = 0$. An event observed in S' has coordinates (x', y', z', t'). The same event observed in S has coordinates (x, y, z, t) [3].



The Lorentz transformation specifies that these coordinates are related in the following way:

$$t' = \gamma \left(t - \frac{vx}{c^2} \right)$$

$$x' = \gamma (x - vt)$$

$$y' = y$$

$$z' = z$$

where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Writing the Lorentz transformation and its inverse in terms of coordinate differences, where for instance, one event (Event 1) has coordinates (x_1 , t_1) and (x_1' , t_1') , another event (Event 2) has coordinates (x_2 , t_2) and (x_2' , t_2') , and the differences are defined as:

$$\Delta x' = x_2' - x_1' \quad , \quad \Delta x = x_2 - x_1$$

$$\Delta t' = t_2' - t_1' \quad , \quad \Delta t = t_2 - t_1$$

we get

$$\Delta x' = \gamma \ (\ \Delta x - v \, \Delta t \ ) \quad , \quad \Delta x = \gamma \ (\Delta x' + v \, \Delta t')$$

$$\Delta t' = \gamma \left(\Delta t - \frac{v\Delta x}{c^2} \right) \quad , \quad \Delta t = \gamma \left(\Delta t' + \frac{v\Delta x'}{c^2} \right)$$

A solution to the twin paradox

Let the twins synchronize their clocks just before twin B starts moving. They both set their clocks to zero. We will assume that twin B continuously accelerates to attain a speed of 0.99c , then moves for several years at this constant velocity, and then decelerates to low, non-relativistic velocity to turn around near the star, then accelerates to 0.99c on her way back to Earth and travels at this velocity for several years, and then decelerates to land on Earth (Fig.1).

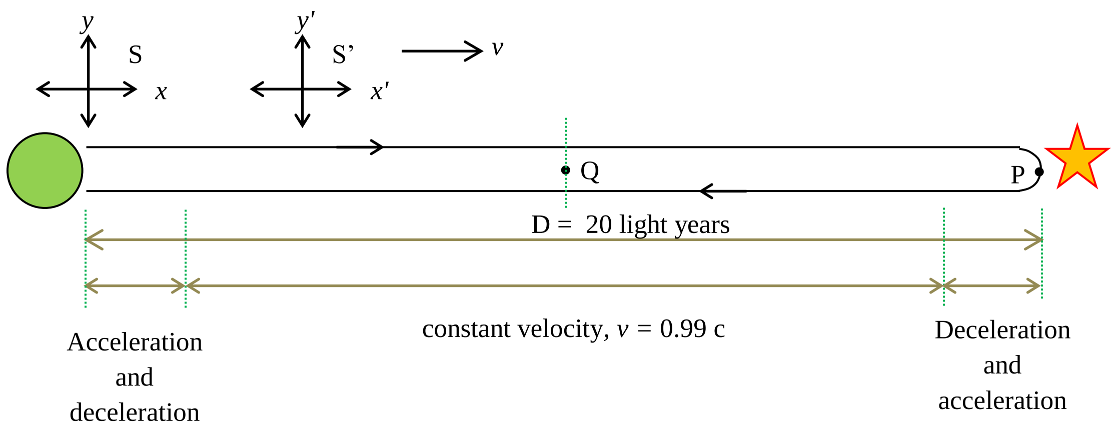


Fig.1

Let S be the reference frame of twin A and S' be the reference frame of twin B, the travelling twin. Let us identify two main events, one event just before twin B leaves the Earth, another event just after twin B lands on Earth. These events could be anything happening at these times, for example it could be blinking of eyes or heart beats of twin B. But let us just consider leaving and arrival of twin B as events. We will identify other intermediate events to see the ageing of twin B during the whole journey.

Event 1: twin B just leaving Earth, has coordinates (x_1, t_1)
Event 2: twin B at point Q (midway between Earth and star), on her way to star. (x_2, t_2)
Event 3: twin B turning around near the star. (x_3, t_3)
Event 4: twin B at point Q, on her way back to Earth (x_4,t_4)
Event 5: twin B landing on Earth. (x_5, t_5)

In twin A's reference frame (S), it takes twin B almost forty years to return back to Earth. Now the problem is to find how much time elapses for twin B, as seen by twin A, that is the time difference between Event 5 and Event 1.

We will consider the time elapsed:

- between Event 2 and Event 1
- between Event 3 and Event 1
- between Event 4 and Event 1
- between Event 5 and Event 1

It should be noted that the twin paradox requires us to determine only the time elapsed between Event 5 and Event 1. We don't need the time differences of intermediate events listed above in order to determine the time difference between Event 5 and Event 1. We analyze the other events only to clarify the new interpretation that only instantaneous velocity is relevant in Lorentz transformation and to get a more complete picture of the ageing of twin B during the whole journey.

The trick that has eluded physicists so far regards what relative velocity v to use in the Lorentz transformation equations. All treatments of the paradox so far vaguely assume the velocity 0.99c at which the spaceship is travelling for most of the journey to calculate time dilation. This is the fallacy that has led to the creation of the paradox in the first place and for its continued persistence to this date.

In this paper, we propose that the velocity v is in fact the ***instantaneous*** velocity. We will return to the implications of this interpretation later on, but let us compute the age of stationary twin A as seen by travelling twin B using this interpretation of relative velocity. Note that the usual approach is to calculate the age of the travelling twin as seen by the stationary twin. Here we use the reverse approach because we can easily and clearly determine the space and time coordinates of events in frame S. However, both approaches are equivalent, according to this paper.

Although twin B is in a continuously changing or switching reference frame, since only *instantaneous* velocity is relevant, the problem is completely *symmetrical* with respect to both twins.

Time difference between Event 2 and Event 1

In frame S

Event 1: twin B leaving Earth

$$x_1 = 0 \ , \quad t_1 = 0$$

Event 2: twin B mid-point between Earth and star, at point Q, on the way to star

Since the star is 20 light years away, we assume that it takes 10 years for twin B to reach mid-point between Earth and star in frame S, on the way to star.

$$x_2 = \frac{D}{2} = 10 \ light \ years \ , \quad t_2 = 10 \ years$$

$$\Delta t = t_2 - t_1 = 10 \ years$$

$$\Delta x = x_2 - x_1 = 10 \ light \ years$$

In frame S'

The ***instantaneous*** velocity of twin B at mid-point between Earth and star is 0.99*c*, and this is the velocity we use in the Lorentz transformation equations.

$$\gamma = \frac{1}{\sqrt{1 - \frac{(0.99c)^2}{c^2}}} = 7.0888$$

Using the Lorentz transformation equation:

$$\Delta t' = \gamma \left(\Delta t - \frac{v \Delta x}{c^2} \right)$$

$$\Rightarrow \Delta t' = 7.0888 * \left(10 \text{ years} - \frac{0.99c * 10 \ light \ years}{c^2} \right) = 0.70888 \ years$$

Therefore, when twin B is moving with speed 0.99*c* at point Q, by symmetry, each twin sees the other as aged by only 0.70888 years, and each twin sees his/her own age as 10 years.

Time difference between Event 3 and Event 1

Let us assume that the spaceship is moving with velocity (in frame S) of 10 km/s on its curved path while turning around near the star. This means that it has decelerated from 0.99c to 10 km/s. Let us consider the age of twin A , as seen by travelling twin B, when the spaceship is just at the mid-point, at point P, of this curved path. Note that the spaceship velocity is in the minus *y*-direction at this point and has no velocity component in the *x*-directions, therefore we will use the corresponding Lorentz transformation equations using this *instantaneous* velocity.

In frame S

Event 1: twin B leaving Earth

$$x_1 = 0 \quad , \quad y_1 = 0 \ , \quad t_1 = 0$$

Event 3: twin B turning around near the star, at point P

$$x_3 = 20 \ light \ years \quad y_3 = 0 \ , \quad t_3 = 20 \ years$$

$$v_x = 0 \quad , \quad v_y = 10 \ \frac{km}{s}$$

$$\Delta t = t_3 - t_1 = 20 \ years$$

$$\Delta x = x_3 - x_1 = 20 \text{ light years}$$

$$\Delta y = y_3 - y_1 = 0 - 0 = 0$$

In frame S'

$$\gamma = \frac{1}{\sqrt{1 - \frac{{v_y}^2}{c^2}}} = 1.000000000555560 \approx 1$$

Using Lorentz transformation equation (the plus sign is because the velocity in this cases is in the –*y* direction):

$$\Delta t' = \gamma \left(\Delta t + \frac{v\Delta y}{c^2} \right)$$

$$\Longrightarrow \Delta t' = 1 * \left(20 \text{ years} + \frac{10 \ km/s \ * \ 0}{c^2} \right) = 20 \ years$$

We see that at middle of the turning curve, each twin sees the other to bethe same age as himself/herself, both 20 years old.

Time difference between Event 4 and Event 1

In frame S

Event 1: twin B leaving Earth

$$x_1 = 0 \;, \quad t_1 = 0$$

Event 4: twin B at mid-point between star and Earth, at point Q, on the way back to Earth

We assume that it takes 30 years for twin B to reach mid-point between star and Earth in frame S, on the way back to Earth.

$$x_4 = \frac{D}{2} = 10\; light\; years \;, \quad t_4 = 30\; years$$

$$\Delta t = t_4 - t_1 = 30\; years$$

$$\Delta x = x_4 - x_1 = 10\; light\; years$$

In frame S'

The ***instantaneous*** velocity of twin B at mid-point between Earth and star is 0.99*c*, and this is the velocity we use in the Lorentz transformation equations.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(0.99c)^2}{c^2}}} = 7.0888$$

Using the Lorentz transformation equation (the plus sign is because the velocity in this cases is in the –*x* direction):

$$\Delta t' = \gamma \left(\Delta t + \frac{v \Delta x}{c^2} \right)$$

$$\Rightarrow \Delta t' = 7.0888 * \left(30 \text{ years} + \frac{0.99c \; * \; 10\; light\; years}{c^2} \right) = 282.843\; years$$

When twin B is moving with velocity 0.99*c* at point Q on her way back home, each twin sees the other twin's age to be 282.843 years, and his/her own age as 30 years.

Time difference between Event 5 and Event 1

This is the time difference that directly concerns the twin paradox.

In frame S

Event 1: twin B leaving Earth

$$x_1 = 0 \ , \quad t_1 = 0$$

Event 5: twin B returning back and just landing on Earth

$$x_5 = 0 \ , \quad t_5 = 40 \, years$$

$$\Delta t = t_5 - t_1 = 40 \, years$$

$$\Delta x = x_5 - x_1 = 0 - 0 = 0$$

In frame S'

Now that we have obtained the coordinates of the events in frame S, we only need to use Lorentz transformation to find the corresponding coordinates in frame S'.

Since we have specified that Event 1 and Event 5 are just before departure and just after returning, respectively, in both cases $v = 0$, from which $\gamma = 1$.

Therefore, using the Lorentz transformation equation:

$$\Delta t' = \gamma \left(\Delta t - \frac{v \Delta x}{c^2} \right)$$

$$\Longrightarrow \Delta t' = 1 * \left(40 - \frac{0 * 0}{c^2} \right) = 40 \, years = \Delta t$$

We can see that each twin sees the other to be the same age as himself/herself, 40 years and therefore there is no paradox.

Note again that, unconventionally, all the other intermediate events we analyzed so far are irrelevant to determine the time difference between Event 5 and Event 1. Classically, the total time taken by a series of events is the sum of the time differences between intermediate events. As we have seen, this is not the case in special relativity and Lorentz transformations. One may say that the twin paradox is somehow a result of such mixed (classical and relativistic) views.

The fact that twin B was moving near the speed of light for most of the journey is irrelevant in determining the age of twin A and twin B (as seen by the other) just as she is landing on Earth. Only the velocity of twin B while landing (almost zero compared to the speed of light, hence γ = 1) is relevant in the Lorentz transformation.

In summary:

	Gamma γ	Proper time on Earth	age of Twin A as seen by twin B	age of twin B as seen by twin A
Twin B, just before departure	1	0 years	0 years	0 years
Twin B , mid-point between Earth and star, on her way to star	7.0888	10 years	0.70888 years	0.70888 years
Twin B, at mid-point of curve while turning around, near star	1	20 years	20 years	20 years
Twin B, mid-point between star and Earth, on her way back to Earth	7.0888	30 years	282.843 years	282.843 years
Twin B , just after returning to Earth	1	40 years	40 years	40 years

We can see that twin B is 282.843 years old at the mid-point between star and Earth, on her way back to Earth. Then she becomes younger as she decelerates to land on Earth. The age of twin B (as seen by twin A) depends on the instantaneous velocity of twin B in frame S, and not on any motion *history* of twin B.

Only *instantaneous* velocity is relevant, and all motion history is irrelevant, and this makes the problem completely symmetrical. For example, at the mid-point between the star and the Earth, on the way back to Earth, we calculated the age of twin A to be 282.843 years in frame S'. Suppose that, just at that point, the spaceship of twin B suddenly decelerated to zero velocity, making gamma (γ = 1) . Now the age of twin A will also change instantly, retroactively.

$$\Delta t' \;=\; \gamma \;\left(\; \Delta t + \;\frac{v\Delta x}{c^2}\;\right)$$

$$\Rightarrow\; \Delta t' \;=\; 1 * \;\left(\; 30 \text{ years} + \;\frac{0 \;*\; 10\; light\; years}{c^2}\;\right) = 30\; years$$

Each twin will instantly start to see the other to be the same age as himself/ herself, 30 years old.

New interpretation of inertial and non-inertial reference frames

It has always been assumed that the special relativity theory does not apply to accelerating observers. In this paper, we propose that Lorentz transformation equations and special relativity theory apply to all observers, in uniform or accelerated motion, based on a novel idea introduced in this paper, which I adopt from my previous papers[1][2].

Let S be an inertial reference frame (Fig.2). Consider an accelerating observer A who is continuously changing his/her velocity, moving along the curved path shown.

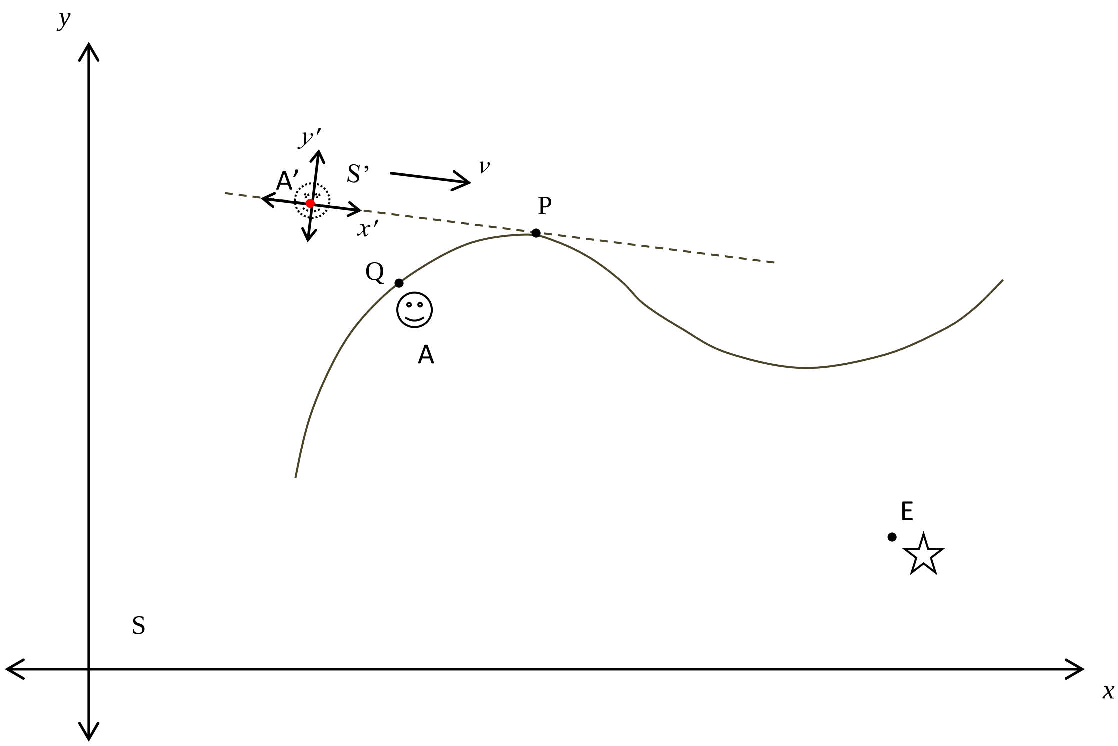


Fig.2

The light source emits a short light pulse from point E, at $t = 0$, in reference frame S. At the instant of light emission, the observer A is at point Q. The problem is first to determine the space and time coordinates of light emission and light detection events in the inertial frame S and then in the reference frame of the accelerating observer A.

Event 1 : emission of light
Event 2: detection of light

Determination of the coordinates of the events in frame S is straightforward. Provided that the position of observer A along the curved path is known at every instant of time, the point P where observer A will detect the light pulse is determined from:

$$\frac{length\ of\ curved\ path\ QP}{average\ velocity\ of\ observer\ A\ between\ Q\ and\ P} = \frac{length\ of\ light\ path\ EP}{speed\ of\ light}$$

However, determining the space and time coordinates of the events in the reference frame of observer A has always been difficult to figure out. This is because observer A is in continuously changing reference frame (due to acceleration) and it has never been clear how special relativity theory applies to such observer or if special relativity applies to such observer at all.

The novel idea proposed in this paper, adopted from my previous papers[1][2], is as follows. Once the point P is determined in frame S, a tangent line is drawn at point P. We introduce ***an imaginary inertial observer A' who will arrive at point P simultaneously with real observer A and whose velocity is equal to the instantaneous velocity of observer A at point P.***

We propose a novel idea as follows:

Two observers that are at the same point of space simultaneously at a given instant of time and moving with equal velocities at that point will observe identical, the same phenomena (for example, interference fringe position) at that point of space and at that instant of time.

Once we have determined point P, we know the instantaneous velocity *v* of observer A at that point (from complete knowledge of motion of observer A), which will also be the velocity of the imaginary inertial observer A'. We also know the time taken (Δt) for observer A to move from point Q to point P. From knowledge of Δt and from knowledge of *v*, we determine the position of imaginary inertial observer A' at the instant of light emission.

Now we can answer the question:
what are the space and time coordinates of the events in the reference frame of observer A ?
Since accelerating observer A is in continuously changing reference frames, which of these reference frames is considered as " the reference frame of observer A" ?

The answer to this question is :
"the reference frame of the imaginary inertial observer A' is also the *preferred* reference frame of observer A for this particular problem. "

Therefore, once we have determined the position and velocity of the imaginary inertial observer A' at the instant of light emission, we simply use the Lorentz transformation equations from inertial frame S to inertial frame S' of observer A'. Note that we have to use the generalized Lorentz transformation equations (see APPENDIX) because the velocity of S' is at an angle relative to the +*x*-axis. The velocity to be used in the Lorentz transformation equations is the *instantaneous* velocity of observer A at point P.

It turns out that although observer A is in accelerated motion before reaching point P, it is *as if* observer A had been moving inertially along the tangent line indefinitely. The motion history of observer A (initial position, velocity and acceleration) is used only to determine point P. Once point P is determined, the motion history of observer A will be irrelevant and it is *as if* observer A had been moving indefinitely inertially along the tangent line.

The reference frame of imaginary inertial observer A' is the only inertial reference frame momentarily coinciding with the accelerating (but not rotating) reference frame of observer A that allows correct evaluation of the space and time coordinates of *both* events (emission of light and detection of light) within a *single* frame of reference, for this particular problem. Therefore, the reference frame of imaginary inertial observer A' is the only *preferred* reference frame to correctly analyze this particular experiment in the reference frame of the accelerating observer. The accelerating observer is in continuously changing (inertial) reference frames and, of all these frames, only the reference frame of imaginary inertial observer A' is the preferred one, because only analysis relative to this frame will give the correct value of observables, for example interference fringe positions if this was a light interference experiment.

This is a new interpretation of Einstein's relativity theory. Ever since the theory of special relativity was formulated by Einstein in 1905, there has been a universal assumption that special relativity theory cannot be applied to accelerating observer/reference frames. This paper has clearly shown that Lorentz transformations and special relativity applies to an accelerating observer, and that there is no fundamental difference between an inertial observer and an accelerating observer with regard to special relativity theory. The only 'difference' is that it is much easier to compute the future positions of an inertial observer and hence to compute the point where a light beam will meet the inertial observer. In the case of an accelerated (non-inertial) observer, this problem will only become more involved. There is no fundamental difference between an inertial observer and a non-inertial observer in this new interpretation of special relativity theory and Lorentz transformation.

We can also determine the space and time coordinates of the events (Event 1: emission of light by the source at point E , Event 2 : Detection of light by observer A) in any other arbitrarily moving, accelerating reference frame (Fig.3).

Suppose that an observer B with arbitrary accelerated motion is at point M at the instant of light emission. The problem is to determine the space and time coordinates of the events (Event 1: emission of light from point E, Event 2: detection of light be observer A) in the reference frame of accelerating observer B.

The space and time coordinates of the events in the reference frame of accelerating observer B is determined basically by the same procedure we followed for observer A. Suppose that observer B is at point M at the instant of light emission, in frame S. Since we have already determined the time interval Δt between light emission by the source and light detection by observer A, in frame S, we can determine (from complete knowledge of motion of observer B) the position of observer B after time interval Δt , that is point N . This means that observer B is at point N when observer A is detecting light at point P. Once we have determined point N, we know the instantaneous velocity (magnitude and direction) of observer B at point N. We draw a line tangent to the path of observer B at point N. Then, as before, we assume an imaginary inertial observer B' whose velocity is equal to the instantaneous velocity u of observer B at point N. From knowledge of Δt and u , we can determine the point where the imaginary inertial observer B' was at the instant of light emission, along the tangent line. The space and time coordinates of the events in the reference frame of the real accelerating observer B is the same as the coordinates of the events in the reference frame of imaginary inertial observer B', which is determined by using the generalized Lorentz transformation from reference frame S to the reference frame of B', using the velocity u and taking into account the direction of velocity u relative to the $+x$ axis of frame S. However, we are usually interested only in the time and space intervals/separations of events rather than points (coordinates) in time and space of events, so once we have obtained u , we can calculate the time and space intervals/separations between the events in the reference frame of B' by using the differential form of the generalized Lorentz transformation equations (see APPENDIX). (Note that in the discussions so far, we have assumed the non-inertial reference frames to be only accelerating, not rotating.)

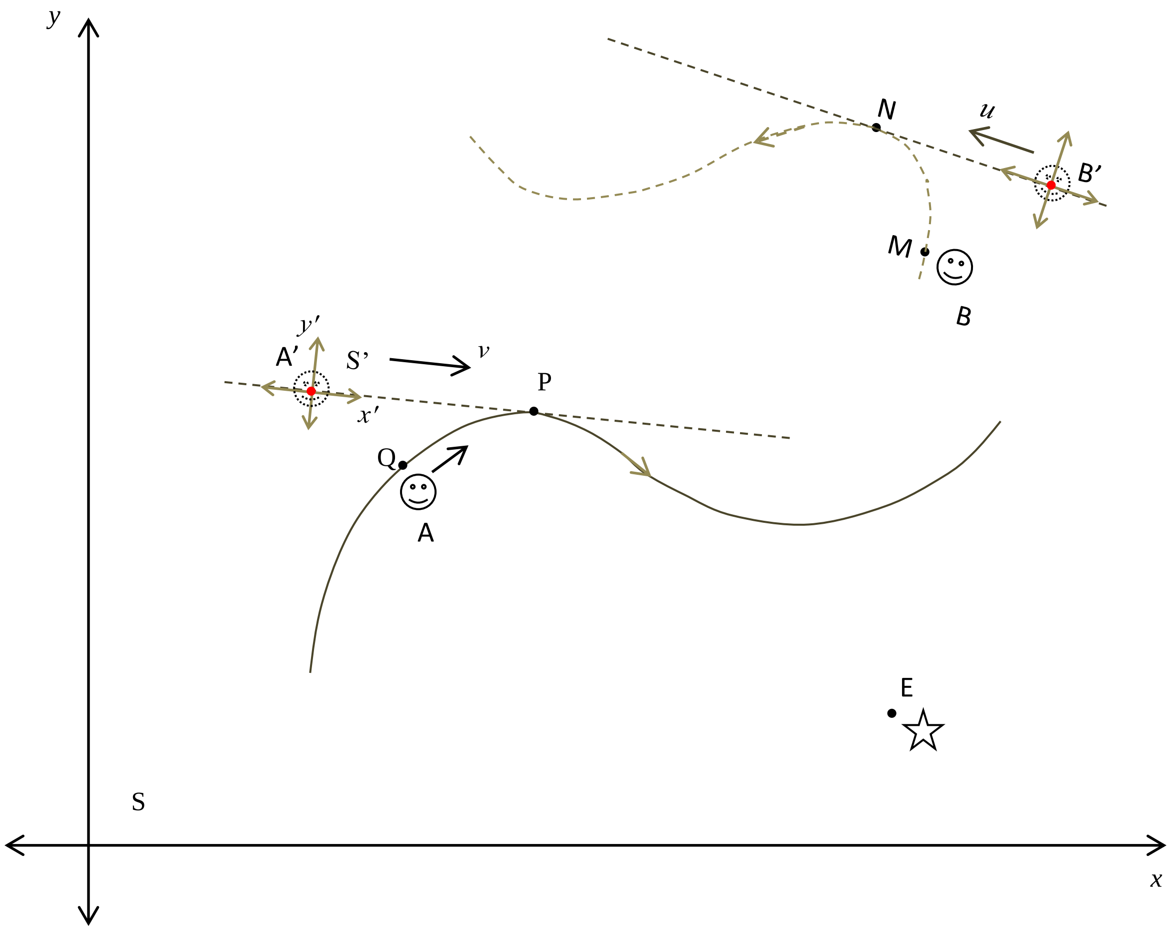


Fig.3

An experiment to test time-dilation

We propose a thought (or physical) experiment to test time-dilation as follows.

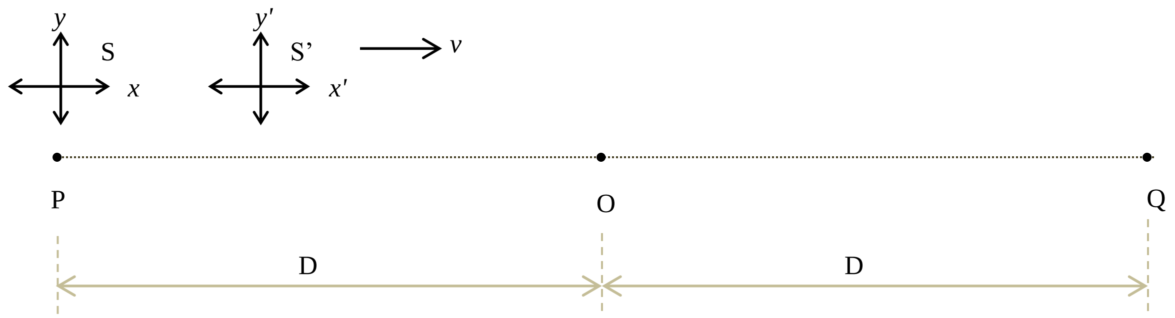


Fig.4

In the figure above, point O is the mid-point between points P and Q.

A light source moves with velocity *v/c* = 0.8 past points P, O and Q. S' is the reference frame of the moving light source, and the source is at the origin of reference frame S'. An onboard clock moving together with the light source automatically resets to zero just as it passes point P and starts counting. The light source is initially set to emit a very short light pulse at :

$$t = \frac{D}{c}$$

when the light source and the clock were initially at rest in frame S.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.8^2}} = 1.6667$$

Now, if there is time dilation, the moving source will emit the light pulse at time (in frame S):

$$t = \gamma \left(t' - \frac{vx'}{c^2} \right)$$

Since $x' = 0$

$$t = \gamma t' \quad \Longrightarrow \quad t = 1.6667 * \frac{D}{c}$$

Therefore, in reference frame S, the source will emit the light pulse at a distance of:

$$v\,t \;=\; (0.8c\,)\left(1.6667\,\frac{D}{c}\right) = 1.333\,D$$

from point P.

To test this experimentally, theoretically, an array of light detectors can be placed along the path of the light source. The detector that detects the light pulse first will be the detector that is at a distance of 1.333*D* from point P. This will confirm time-dilation of special relativity. This experiment is basically the same as the muon time-dilation experiment.

Can this experiment be practical? The high velocity of the source required makes this experimental impractical. But let us check if it is possible.

For example, if v = 10 km/s, such as that of a missile,

$$\frac{v}{c} \;=\; 0.0000333333333333333$$

$$\gamma \;=\; \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \;=\; 1.00000000055556$$

$$t \;=\; \gamma\,t' \quad\Longrightarrow\quad t \;=\; 1.00000000055556 * \frac{D}{c}$$

$$v\,t \;=\; (0.0000333333333333333\;c\,)\;(\,1.00000000055556\;D/c\,)$$

$$vt \;=\; 0.0000333333333518519\;\;D$$

If we take *D* = 50 km

$$vt \;=\; 0.0000333333333518519\;*50000 = \;\;1.6667\,m$$

From an estimate of possible error margin of the experiment, it may be difficult to make it practical.

Conclusion

We have shown the fallacious understanding of the Lorentz transformations that led to the creation and persistence of the twin paradox. In the usual thought experiment of the twin paradox, one of the twins makes a round trip journey to a star light years away at near the speed of light (for example, 0.99*c*). We know that, although this twin is travelling near the speed of light for the most part of the journey, his/her velocity just before landing on Earth is almost zero, making gamma $\gamma = 1$. However, physicists always (vaguely) used the near light speed velocity when applying time dilation in order to determine the age of the travelling twin as seen from Earth. The comparison between the twins is made just after the twin has returned and landed on Earth, by which time his/her velocity has become zero and gamma has become one, but physicists wrongly use the 0.99c velocity to compare the ages of the twins when they are at rest relative to each other. The 0.99c velocity should be used to calculate the age of the travelling twin only when the relative velocity of the twins is 0.99c. Only *instantaneous* velocity is relevant and all motion history is irrelevant in the Lorentz transformations. This leads to the conclusion that the problem is completely *symmetrical* with respect to both twins. Intuitive and inconsistent application of time dilation (and length contraction), rather than rigorous and consistent application of Lorentz *coordinate* transformation of *events*, has caused the creation and persistence of the twin paradox. Lorentz transformation is fundamentally about *coordinate* transformation of *events* and time dilation and length contraction are only consequences of LT. The twin paradox is a result of a somehow mixed (relativistic and classical) view of time.

This new interpretation has led to a profound, deeper understanding of inertial and non-inertial reference frames. The universal understanding is that special relativity theory cannot be applied to accelerating observers and accelerating reference frames. We have shown that there is no fundamental difference between inertial and non-inertial observers and the new interpretation of special relativity theory treats both kinds of observers basically in the same way.

Thanks to Almighty God Jesus Christ and His Mother Our Lady Saint Virgin Mary

References

1. *A New Theoretical Framework of Absolute and Relative Motion, the Speed of Light, Electromagnetism and Gravity*, by Henok Tadesse

https://vixra.org/abs/1906.0219

2. *Relativistic Analysis of Sagnac Effect in the Reference Frame of the Accelerating Observer*, by Henok Tadesse

https://vixra.org/abs/2103.0197

3. Twin paradox, Wikipedia

APPENDIX

The generalized Lorentz transformation equation is:

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = B(v) \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

where

$$\mathbf{B}(\boldsymbol{v}) = \begin{bmatrix} \gamma & -\gamma\beta_x & -\gamma\beta_y & -\gamma\beta_z \\ -\gamma\beta_x & 1+(\gamma-1)\frac{\beta_x{}^2}{\beta^2} & (\gamma-1)\frac{\beta_x\beta_y}{\beta^2} & (\gamma-1)\frac{\beta_x\beta_z}{\beta^2} \\ -\gamma\beta_y & (\gamma-1)\frac{\beta_x\beta_y}{\beta^2} & 1+(\gamma-1)\frac{\beta_y{}^2}{\beta^2} & (\gamma-1)\frac{\beta_y\beta_z}{\beta^2} \\ -\gamma\beta_z & (\gamma-1)\frac{\beta_x\beta_z}{\beta^2} & (\gamma-1)\frac{\beta_y\beta_z}{\beta^2} & 1+(\gamma-1)\frac{\beta_z{}^2}{\beta^2} \end{bmatrix}$$

where β_x , β_y , β_z and γ are :

$$\beta_x = \frac{v_x}{c} \qquad \beta_y = \frac{v_y}{c} \qquad \beta_z = \frac{v_z}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_x{}^2 + v_y{}^2 + v_z{}^2}{c^2}}}$$