Schrödinger Equation of Hydrogen Atom in Atomic Unites, Theory of Chirality and the Territory of Modern Physics

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper is a subsequent paper to our previous papers "Chen's Formulas of the Fine-structure Constant" (viXra:2002.0203) and "Integrated Formulas of the Fine-structure Constant and Feigenbaum Constants" (viXra:2102.0162). In light of the basic concepts illustrated in the above stated previous papers that there are two values of the fine-structure constant (α_1 and α_2) and there are relationships such as $\alpha_2(\delta\alpha/\gamma_2)^2=1$ between the fine-structure constant α_2 and Feigenbaum constants (δ and α), in this paper we deduced Schrödinger equation of hydrogen atom in atomic unites in a concise, reasonable and beautiful form which otherwise couldn't be gained. In the end of this paper, all theories opened in our previous papers are named to be Theory of Chirality, and a picture of the territory of modern physics including Theory of Relativity, Quantum Theory, Chaos Theory and Theory of Chirality is given.

Keywords: Schrödinger equation; hydrogen atom; the fine-structure constant; Feigenbaum constants; Theory of Chirality, territory of modern physics.

1. Schrödinger Equation of Hydrogen Atom in Atomic Unites

In our previous paper[1], we gave the following deduction process.

Stationary Schrodinger Equation: $-\dfrac{\hbar^2}{2m}\nabla^2\psi + U\psi = E\psi,$

applied to hydron atom： $\nabla^2\psi + \dfrac{2m_e}{\hbar^2}(E + \dfrac{e^2}{4\pi\varepsilon_0 r})\psi = 0,\ E = -\dfrac{m_e e^4}{2n^2(4\pi\varepsilon_0)^2\hbar^2}$

With substitution and simlification：

$$\frac{2m_e}{\hbar^2}\left(\frac{m_e e^4}{2n^2(4\pi\varepsilon_0)^2\hbar^2}-\frac{e^2}{4\pi\varepsilon_0 r}\right)\psi=\nabla^2\psi,$$

$$\left[\frac{1}{n^2}\left(\frac{m_e e^2}{4\pi\varepsilon_0\hbar^2}\right)^2-\frac{2}{r}\frac{m_e e^2}{4\pi\varepsilon_0\hbar^2}\right]\psi=\nabla^2\psi,$$

$$\left[\frac{1}{n^2}\left(\frac{e^2}{4\pi\varepsilon_0\hbar c}\frac{m_e c}{\hbar}\right)^2-\frac{2}{r}\frac{e^2}{4\pi\varepsilon_0\hbar c}\frac{m_e c}{\hbar}\right]\psi=\nabla^2\psi,$$

As $\sqrt{\alpha_1\alpha_2}=\frac{v_e}{c}=\frac{e^2}{4\pi\varepsilon_0\hbar c}$, $\lambda_e=\frac{h}{m_e c}$ and $\alpha_1=\frac{\lambda_e}{2\pi a_0}$:

$$\left[\frac{1}{n^2}\left(\sqrt{\alpha_1\alpha_2}\frac{2\pi}{\lambda_e}\right)^2-\frac{2}{r}\sqrt{\alpha_1\alpha_2}\frac{2\pi}{\lambda_e}\right]\psi=\nabla^2\psi,$$

$$\left[\frac{1}{n^2(\lambda_e/2\pi/\sqrt{\alpha_1\alpha_2})^2}-\frac{2}{(\lambda_e/2\pi/\sqrt{\alpha_1\alpha_2})r}\right]\psi=\nabla^2\psi,$$

$$\left[\frac{1}{n^2 a_0{}^2(\alpha_1/\alpha_2)}-\frac{2}{a_0 r\sqrt{\alpha_1/\alpha_2}}\right]\psi=\nabla^2\psi$$

As $\alpha_1/\alpha_2\approx1$, simplyfied to: $\left[\frac{1}{n^2 a_0{}^2}-\frac{2}{a_0 r}\right]\psi=\nabla^2\psi$

factor 2 seems not beautiful

In atomic units (au: $e=m_e=\hbar=1$ and $\varepsilon_0=\frac{1}{4\pi}$),

$$a_{0/au}=\frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}=1,\quad v_{e/au}=\frac{e^2}{4\pi\varepsilon_0\hbar}=1,\quad c_{au}=\frac{v_{e/au}}{\alpha_c}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_1\alpha_2}}$$

$$\left[\frac{1}{n^2(\alpha_1/\alpha_2)}-\frac{2}{r_{au}\sqrt{\alpha_1/\alpha_2}}\right]\psi=\nabla_{au}{}^2\psi$$

$$\left(\frac{c_{au}{}^2}{\alpha_1{}^2 n^2}-\frac{2c_{au}}{\alpha_1 r_{au}}\right)\psi=\nabla_{au}{}^2\psi$$

the above equation could be called Schrodinger eqution of hydrogen atom in atomic unites, the later form of the equation shows factor 2 is still reasonable and beautiful.

As $\alpha_1/\alpha_2\approx1$，simplyfied to：$\left[\frac{1}{n^2}-\frac{2}{r_{au}}\right]\psi=\nabla_{au}{}^2\psi$

Discover: 2018/4-6; Revise: 2019/12/13 (add au form)

$$\alpha_1/\alpha_2=\frac{137.035999111818}{137.035999037435}=1.0000000005428=1+\frac{23\cdot59}{25\cdot10^{11}}=\left(1+\frac{23\cdot59}{50\cdot10^{11}}\right)^2$$

$$\sqrt{\alpha_1/\alpha_2}=1+\frac{23\cdot59}{50\cdot10^{11}}=1.0000000002714$$

Relations to nuclides: ${}^{23}_{11}Na_{12}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{99,100}_{44}\mathrm{Ru}_{55,56}$ ${}^{105}_{46}Pd_{59}$ ${}^{119}_{50}Sn_{69}$
${}^{137}_{56}Ba_{81}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{169}_{69}Tm_{100}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{226}_{88}Ra^{*}_{138}$

2019/8/28 – 29

However, in the above deduction, the later form of Schödinger equation of hydrogen atom in atomic unites is not fully correct, and needs correction as follows.

$$(\frac{c_{au}{}^2}{\alpha_1{}^2 n^2} - \frac{2c_{au}}{\alpha_1 r_{au}})\psi = \nabla_{au}{}^2\psi$$

above eqution is not fully correct, and is revised to be:

$$(\frac{\alpha_2{}^2 c_{au}{}^2}{n^2} - \frac{2\alpha_2 c_{au}}{r_{au}})\psi = \nabla_{au}{}^2\psi$$

2021/2/27

2. Integrated Formulas of the Fine-structure Constant and Feigenbaum Constants

In our previous papers[5,6], some integrated formulas of the fine-structure constant and Feigenbaum constants were given, the following are their summary.

$$2\pi = 2\cdot 3.14159265358979$$

$$(2\pi)_{Chen-k} = e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

The Fine-structure Constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7\cdot(2\pi)_{Chen-112}}\frac{1}{112+\frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13\cdot(2\pi)_{Chen-278}}{100}\frac{1}{112-\frac{1}{64\cdot 3\cdot 29}} = 1/137.035999111818$$

Feigenbaum Constants: $\delta = 4.66920160910299$, $\alpha = 2.50290787509589$

$$\alpha_1(\delta/\gamma_1)^2(2\pi) = 1$$

$$\gamma_1 = 1 - \frac{1}{47\cdot 109} + \frac{1}{27\cdot 7\cdot(3\cdot 8\cdot(3\cdot 8\cdot(4\cdot 137-1)-1)-1)} = 0.999804818668238$$

$$\frac{1}{\gamma_1} = 1 + \frac{1}{2\cdot 13\cdot 197} - \frac{16\cdot 7\cdot 17\cdot(16\cdot 3\cdot 23-1)}{125\cdot 10^{12}} = 1.00019521943495$$

$$\gamma_1{}^2 = 1 - \frac{1}{13\cdot 197} + \frac{1}{2\cdot 7\cdot 41\cdot(4\cdot 5\cdot 19\cdot 31-1)} = 0.99960967543223$$

$$\frac{1}{\gamma_1{}^2} = 1 + \frac{1}{512\cdot 5} - \frac{1}{4\cdot 9\cdot 7\cdot 17\cdot 19\cdot 83} = 1.00039047698053$$

$$\alpha_1(\delta/\gamma_{1-Chen-25\cdot 17})^2(2\pi)_{Chen-25\cdot 17} = 1$$

$$(2\pi)_{Chen-25\cdot 17} = e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot 3\cdot 71}{25\cdot 17})^{23\cdot 37}} = 6.28564399787948$$

$$\gamma_{1-Chen-25\cdot17} = 1 + \frac{1}{2\cdot79\cdot109\cdot139 - \frac{3}{25}} = 1.00000041773574$$

$$\frac{1}{\gamma_{1-Chen-25\cdot17}} = 1 - \frac{1}{2\cdot79\cdot109\cdot139 + \frac{22}{25}} = 0.999999582264432$$

$$\gamma_{1-Chen-25\cdot17}{}^{2} = 1 + \frac{1}{128\cdot9\cdot(2\cdot3\cdot173+1) + \frac{7}{10}} = 1.0000008354717$$

$$\frac{1}{\gamma_{1-Chen-25\cdot17}{}^{2}} = 1 - \frac{1}{2\cdot5\cdot7\cdot(2\cdot83\cdot103+1) - \frac{9}{29}} = 0.999999164529037$$

$\alpha_2(\delta\alpha/\gamma_2)^2 = 1$

$$\gamma_2 = 1 - \frac{1}{5\cdot7\cdot17} + \frac{1}{4\cdot3\cdot17\cdot23\cdot137 - \frac{11}{59}} = 0.998320883415699$$

$$\frac{1}{\gamma_2} = 1 + \frac{1}{2\cdot27\cdot11} - \frac{1}{5\cdot17\cdot(16\cdot3\cdot157+1) + \frac{16}{17}} = 1.00168194075892$$

$$\gamma_2{}^{2} = 1 - \frac{1}{2\cdot149} + \frac{1}{29\cdot31\cdot(2\cdot3\cdot49\cdot13+1) - \frac{16}{19}} = 0.996644586263908$$

$$\frac{1}{\gamma_2{}^{2}} = 1 + \frac{1}{27\cdot11} - \frac{1}{2\cdot3\cdot25\cdot11\cdot(4\cdot11\cdot47+1) - \frac{2}{5}} = 1.00336671044256$$

$\frac{2\pi}{(\alpha\gamma)^2} = 1$

$$\gamma = 1 + \frac{1}{32\cdot3\cdot7} - \frac{1}{23\cdot151\cdot173 + \frac{9}{4\cdot7}} = 1.00148643087192$$

$$\frac{1}{\gamma} = 1 - \frac{1}{32\cdot3\cdot7+1} + \frac{1}{2\cdot41\cdot(2\cdot3\cdot25\cdot49-1) - \frac{1}{6}} = 0.99851577532546$$

$$\gamma^2 = 1 + \frac{1}{16\cdot3\cdot7} - \frac{1}{3\cdot13\cdot31\cdot(2\cdot9\cdot41+1) - \frac{1}{22}} = 1.0029750712205$$

$$\frac{1}{\gamma^2} = 1 - \frac{1}{16\cdot3\cdot7+1} + \frac{1}{8\cdot81\cdot19\cdot73 + \frac{1}{3\cdot13}} = 0.997033753573803$$

3. The Final Form of Schrödinger Equation of Hydrogen Atom in Atomic Unites

As we have:

$$\left(\frac{\alpha_2{}^2 c_{au}{}^2}{n^2} - \frac{2\alpha_2 c_{au}}{r_{au}}\right)\psi = \nabla_{au}{}^2\psi$$

$$\alpha_1(\delta/\gamma_1)^2(2\pi) = 1,\quad \alpha_2(\alpha\delta/\gamma_2)^2 = 1$$

So: $\left(\frac{c_{au}{}^2}{(\alpha\delta/\gamma_2)^4 n^2} - \frac{2c_{au}}{(\alpha\delta/\gamma_2)^2 r_{au}}\right)\psi = \nabla_{au}{}^2\psi$

2021/2/27

Define:

$$c_{au} = \frac{c}{v_e} = \frac{1}{\sqrt{\alpha_1\alpha_2}} = \frac{1}{\sqrt{\frac{\lambda_e}{2\pi a_0}\frac{2\pi r_e}{\lambda_e}}} = \sqrt{\frac{a_0}{r_e}} = \sqrt{2\pi(\delta/\gamma_1)^2(\alpha\delta/\gamma_2)^2} = \frac{\delta^2\alpha}{\gamma_1\gamma_2}\sqrt{2\pi}$$

c_{au}: the speed of ligth in atomic unites

v_e: the line speed of electron in ground state in hydrogen atom in Bohr model

$$c_{au-Ch} = \sqrt{\frac{\alpha_2}{\alpha_1}} = \alpha_2 c_{au} = \frac{c_{au}}{(\alpha\delta/\gamma_2)^2} = \frac{\sqrt{2\pi}}{\alpha\gamma_1/\gamma_2} = \frac{2\pi\sqrt{r_e a_0}}{\lambda_e}$$

$$= 1 - \frac{23\cdot 59}{50\cdot 10^{11}} = 0.9999999994572$$

Relationships with nuclides: ${}^{50,51}_{23}V_{27,28}$ ${}^{105}_{46}Pd_{59}$ ${}^{119}_{50}Sn_{69}$ ${}^{169}_{69}Tm_{100}$

c_{au-Ch} could be called chaotic speed of light in atomic unites or Chen speed

So: $\left(\frac{c_{au-Ch}{}^2}{n^2} - \frac{2c_{au-Ch}}{r_{au}}\right)\psi = \nabla_{au}{}^2\psi$

the above last formula could be called:

the final form of Schrodinger equation of hydrogen atom in atomic unites

2021/2/28

Note that if $\alpha_1=\alpha_2$, the above equation will be simplified to:

$$\left(\frac{1}{n^2} - \frac{2}{r_{au}}\right)\psi = \nabla_{au}{}^2\psi$$

But this simplest equation is not reasonable and beautiful because the factor 2 is very strange and in lack of beauty and reasonability. On the other hand, the consistence of $c_{au\text{-}Ch}{}^2$ and $2c_{au\text{-}Ch}$ is very good, reasonable and beautiful. So our basic concept that there are two values of the fine-structure constant, i.e., α_1 and α_2, is critical for Schrödinger equation of hydrogen atom to be a reasonable and beautiful equation. This also demonstrates all theories in our previous papers should be correct.

4. Theory of Chirality

All theories opened in our previous papers[1-6], registered as copyrights[7-9] and other unopened as manuscripts (such as “New Theory of Chemical Bonds” written by us in 2009) are hereby named to be Theory of Chirality. The main principles and conclusions of Theory of Chirality are listed as follows.

$$\text{Chirality} = \pm 2\pi$$

$$(2\pi)_{Chen-k} = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots \frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7\cdot(2\pi)_{Chen-112}} \frac{1}{112+\frac{1}{75^2}} = 1/137.035999037435$$

$$1/\alpha_1 = 56+81+\frac{1}{28-\frac{13\cdot(2\cdot56\cdot11-1)}{3\cdot5\cdot(2\cdot56\cdot43+1)}} = 137.035999037435$$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13\cdot(2\pi)_{Chen-278}}{100} \frac{1}{112-\frac{1}{64\cdot3\cdot29}} = 1/137.035999111818$$

$$1/\alpha_2 = 56+81+\frac{1}{28-\frac{2\cdot(16\cdot27-1)}{3\cdot(16\cdot81+1)}} = 137.035999111818$$

$$c_{au} = \frac{c}{v_e} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}} = \sqrt{112\times(168-\frac{1}{3}+\frac{1}{12\cdot47}-\frac{1}{14\cdot112\cdot(2\cdot173+1)})}$$
$$= 137.035999074626$$

$$c_{au} = 56+81+\frac{1}{28-\frac{5\cdot(4\cdot3\cdot7\cdot17-1)}{2\cdot5\cdot(4\cdot5\cdot7\cdot23+1)+1}} = 137.035999074626$$

$$\frac{1}{\delta} = \frac{1}{4.66920160910299}$$
$$= \frac{1}{4}-\frac{1}{27}+\frac{1}{4\cdot9\cdot23}-\frac{1}{2\cdot3\cdot7\cdot23\cdot(2\cdot3\cdot(4\cdot3\cdot11-1)+1)+\frac{2\cdot23}{3\cdot19}}$$

$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = \frac{1}{2}-\frac{1}{9}+\frac{1}{3\cdot31}-\frac{1}{23\cdot(8\cdot3\cdot17+1)}+\frac{1}{17\cdot23\cdot(8\cdot3\cdot11^4-1)}$$

$$\alpha_1(\delta/\gamma_1)^2(2\pi) = 1$$

$$\gamma_1 = 1-\frac{1}{47\cdot109}+\frac{1}{27\cdot7\cdot(3\cdot8\cdot(3\cdot8\cdot(4\cdot137-1)-1)-1)} = 0.999804818668238$$

$$\alpha_2(\alpha\delta/\gamma_2)^2 = 1,\quad \gamma_2 = 1-\frac{1}{5\cdot7\cdot17}+\frac{1}{4\cdot3\cdot17\cdot23\cdot137-\frac{11}{59}} = 0.998320883415699$$

$$\frac{2\pi}{(\alpha\gamma)^2}=1,\quad \gamma=1+\frac{1}{32\cdot 3\cdot 7}-\frac{1}{23\cdot 151\cdot 173+\frac{9}{4\cdot 7}}=1.00148643087192$$

$$\left(\frac{a_2{}^2 c_{au}{}^2}{n^2}-\frac{2a_2 c_{au}}{r_{au}}\right)\psi=\nabla_{au}{}^2\psi,\quad \left(\frac{c_{au-Ch}{}^2}{n^2}-\frac{2c_{au-Ch}}{r_{au}}\right)\psi=\nabla_{au}{}^2\psi$$

$$c_{au}=\frac{c}{v_e}=\frac{1}{\sqrt{\alpha_1\alpha_2}}=\frac{a_0}{r_e}=\frac{\delta^2\alpha}{\gamma_1\gamma_2}\sqrt{2\pi}$$

$$c_{au-Ch}=\sqrt{\frac{\alpha_2}{\alpha_1}}=\alpha_2 c_{au}=\frac{c_{au}}{(\alpha\delta/\gamma_2)^2}=\frac{\sqrt{2\pi}}{\alpha\gamma_1/\gamma_2}=\frac{2\pi\sqrt{r_e a_0}}{\lambda_e}=1-\frac{23\cdot 59}{50\cdot 10^{11}}$$

……

5. The Territory of Modern Physics

The territory of modern physics including Theory of Relativity, Quantum Theory, Chaos Theory and Theory of Chirality is shown as follows (**Fig. 1**).

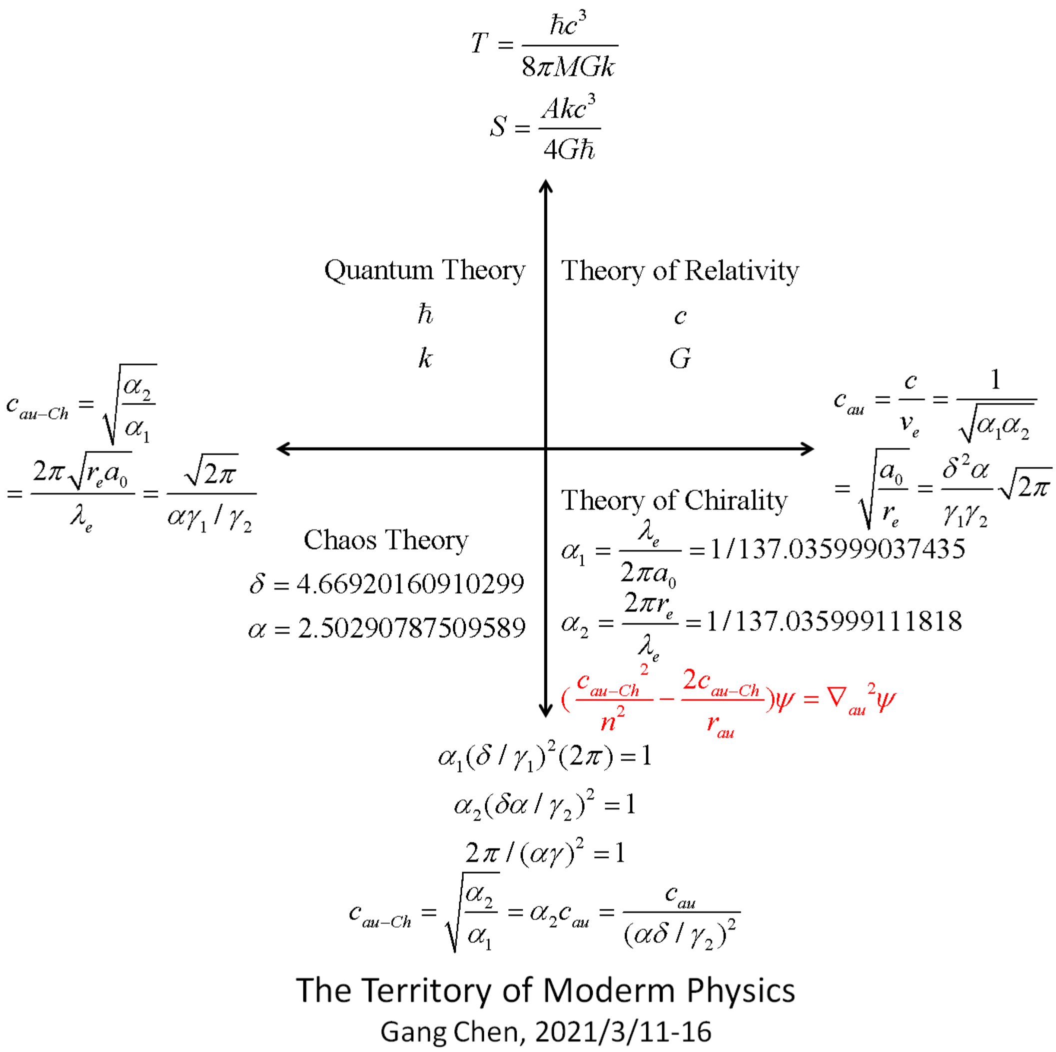

Fig. 1

References:

1. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2002.0203.
2. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2008.0020.
3. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2010.0252.
4. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2012.0107.
5. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2101.0187.
6. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2102.0162.
7. G. Chen and T-M. Chen, Copyright Registration, Chen's Periodic Table of Elements and Natural Group Theory, GuoZuoDengZi-2018-L-00472808.
8. G. Chen, T-M. Chen and T-Y. Chen, Copyright Registration, Chirality and Poetry Model of Atomic Nuclei, GuoZuoDengZi-2018-L-00421847.
9. G. Chen, T-M. Chen and T-Y. Chen, Copyright Registration, Chen's Theory of the Fine-structure Constant, GuoZuoDengZi-2018-L-00547467.

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Appendix I: Research History

Section	Page	Date	Location
1	1-3	2018/4-6	Chengdu
		2019/8/28-29	
		2019/12/13	
		2021/2/27	
2	3-4	2021/3/7-8	
3	5	2021/2/27-28	
4	6-7	2021/11-16	
5	7	2021/11-16	
Preparing this paper	1-9	2021/2/27-3/18	

Note: Date was recorded according to Beijing Time.