Modeling that matches, augments, and unites data about physics properties, elementary particles, cosmology, and astrophysics

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Abstract

This essay shows and uses modeling that matches and predicts data. Our work addresses five facets of physics. One facet is properties of objects. The modeling catalogs properties. The modeling suggests a new property – isomer. An isomer is a near copy of a set of most elementary particles. The set includes the photon. An isomer's photons scarcely interact with other isomers. A second facet is elementary particles. The modeling matches all known elementary particles. The modeling suggests new elementary particles. New particles include zero-charge quark-like particles, an inflaton, and other particles. The properties and the particles seem to suffice to explain dark matter. Most dark matter has bases in five new isomers of the Standard Model elementary particles. The modeling describes a graviton. Some models split gravitational fields in ways similar to the splitting of electromagnetic fields into electric fields and magnetic fields. A third facet is cosmology. The modeling suggests bases for five eras in the evolution of the universe. Two eras would precede inflation. The modeling points to a basis for the size of recent increases in the rate of expansion of the universe. More than eighty percent of dark matter is cold dark matter. Some dark matter has similarities to ordinary matter. A fourth facet is astrophysics. The modeling matches data about ratios of dark matter to ordinary matter. Some of that data pertains to densities of the universe. Some pertains to galaxy clusters. Some pertains to galaxies. One datum pertains to depletion of cosmic microwave background radiation. The modeling seems to offer insight about galaxy formation. That our work seems to explain facet three data and facet four data might confirm some of our work regarding facets one and two. A fifth facet is physics modeling. Our work augments and does not disturb centuries of useful physics. Our modeling has roots in discrete mathematics. Our modeling unites itself and widely-used physics modeling.

Keywords: Beyond the Standard Model, Dark matter, Galaxy evolution, Rate of expansion of the universe, Inflation, Quantum gravity

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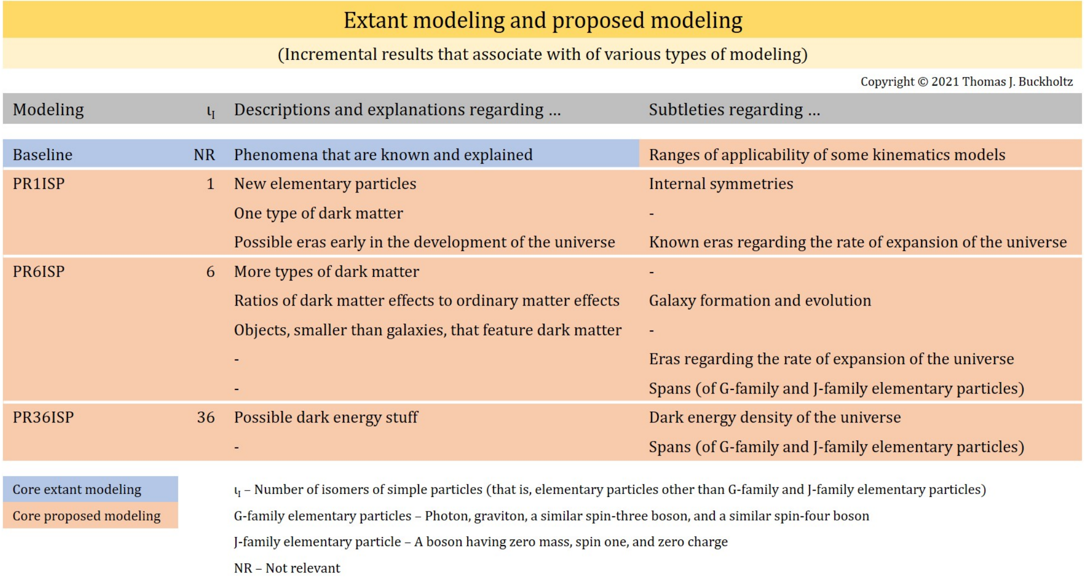
1. Introduction

1.1. Overview

This essay suggests advances regarding two sets of physics challenges. One set features describing elementary particles that people have yet to find and describing dark matter. One set features explaining cosmology and astrophysics data that people have yet to explain and predicting cosmology and astrophysics data that people have yet to obtain.

Our explanations regarding cosmology and astrophysics data offer the possibility that our descriptions of new particles and dark matter have merit.



Extant modeling and proposed modeling

(Incremental results that associate with of various types of modeling)

Modeling	ι_I	Descriptions and explanations regarding ...	Subtleties regarding ...
Baseline	NR	Phenomena that are known and explained	Ranges of applicability of some kinematics models
PR1ISP	1	New elementary particles	Internal symmetries
		One type of dark matter	-
		Possible eras early in the development of the universe	Known eras regarding the rate of expansion of the universe
PR6ISP	6	More types of dark matter	-
		Ratios of dark matter effects to ordinary matter effects	Galaxy formation and evolution
		Objects, smaller than galaxies, that feature dark matter	-
		-	Eras regarding the rate of expansion of the universe
		-	Spans (of G-family and J-family elementary particles)
PR36ISP	36	Possible dark energy stuff	Dark energy density of the universe
		-	Spans (of G-family and J-family elementary particles)

Core extant modeling

Core proposed modeling

ι_I – Number of isomers of simple particles (that is, elementary particles other than G-family and J-family elementary particles)

G-family elementary particles – Photon, graviton, a similar spin-three boson, and a similar spin-four boson

J-family elementary particle – A boson having zero mass, spin one, and zero charge

NR – Not relevant

Figure 1: Extant modeling and proposed modeling

1.2. *Methods*

We blend two sets of work - extant modeling and proposed modeling.

We use the two-word term extant modeling to describe models developed by people other than us. We divide the models into two categories. We use the word core and the word unverified to discuss that division. The word core means that people have found that the models match data. The word unverified points to other extant modeling.

We use the two-word term proposed modeling to describe our work. We divide the models into two categories. We use the word core and the word supplementary to discuss that division. Core proposed modeling addresses properties of elementary particles and dark matter. Core proposed modeling also suggests explanations for cosmology and astrophysics data. Supplementary proposed modeling features suggested supplements to core extant modeling kinematics models.

This essay unites core extant modeling and core proposed modeling. Core extant modeling provides models for the motions of and changes to objects. Core proposed modeling suggests and interrelates properties of objects.

Proposed modeling augments core extant modeling. Proposed modeling does not disturb core extant modeling. Some relevant core extant modeling features the principle of stationary action and has bases in functions of continuous variables. Proposed modeling has bases in discrete mathematics. Proposed modeling has bases in a principle for which we use the term double-entry bookkeeping.

Some extant modeling uses space-time coordinates. Core proposed modeling has bases that do not use space-time coordinates. Core proposed modeling does not disturb core extant modeling that people might associate with notions of space-time.

Proposed modeling suggests limits regarding the usefulness of some extant modeling models.

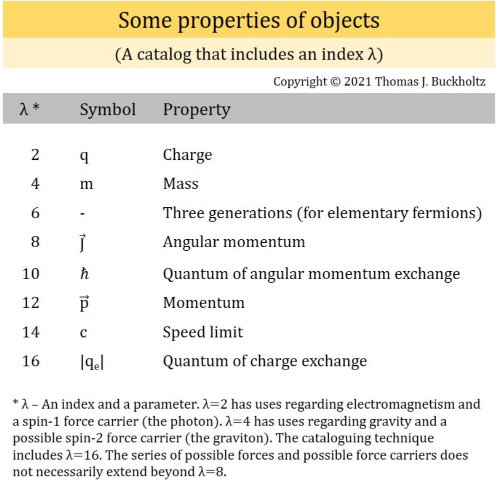
1.3. *Results*

We preview some results that this essay discusses.

Figure 1 shows physics results that core proposed modeling might add to physics results that associate with core extant modeling. Results accumulate downward. (Results that associate with a specific one of the four types of modeling include results that pertain for types of modeling that the figure shows above the specific type of modeling.) Figure 5 discusses the notion of isomer.

Proposed modeling catalogs some properties of objects. A catalog features an index λ. The notion of λ has uses beyond the use as an index. For example, for some modeling, $\lambda = 2$ pairs with electromagnetism and $\lambda = 4$ pairs with gravity.

Proposed modeling interrelates some properties of objects. For example, models regarding elementary bosons interrelate mass, spin, and charge.



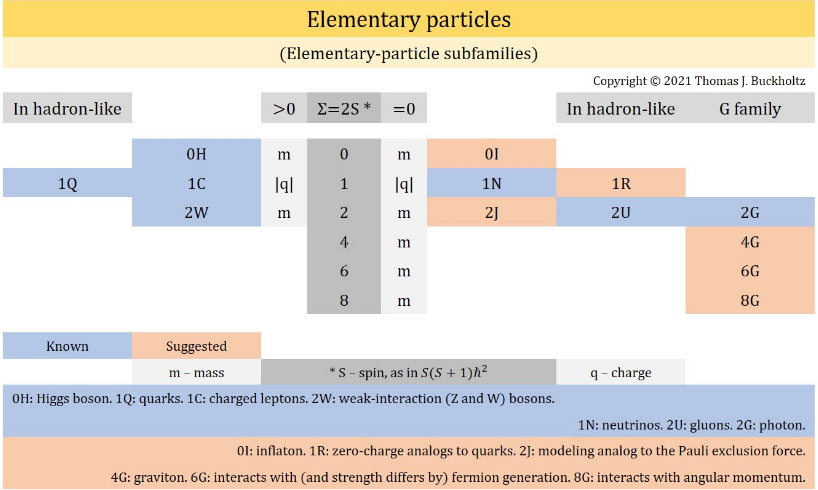
Some properties of objects

(A catalog that includes an index λ)

λ *	Symbol	Property
2	q	Charge
4	m	Mass
6	-	Three generations (for elementary fermions)
8	$\vec{\mathrm{j}}$	Angular momentum
10	$\hbar$	Quantum of angular momentum exchange
12	$\vec{\mathrm{p}}$	Momentum
14	c	Speed limit
16	$\lvert q_e \rvert$	Quantum of charge exchange

* λ – An index and a parameter. λ=2 has uses regarding electromagnetism and a spin-1 force carrier (the photon). λ=4 has uses regarding gravity and a possible spin-2 force carrier (the graviton). The cataloguing technique includes λ=16. The series of possible forces and possible force carriers does not necessarily extend beyond λ=8.

Figure 2: A catalog of some properties of objects



Elementary particles

(Elementary-particle subfamilies)

In hadron-like		>0	Σ=2S *	=0		In hadron-like	G family
	0H	m	0	m	0I		
1Q	1C	|q|	1	|q|	1N	1R	
	2W	m	2	m	2J	2U	2G
			4	m			4G
			6	m			6G
			8	m			8G

Known 🟦 Suggested 🟧

m – mass

* S – spin, as in $S(S+1)\hbar^2$

q – charge

0H: Higgs boson. 1Q: quarks. 1C: charged leptons. 2W: weak-interaction (Z and W) bosons.

1N: neutrinos. 2U: gluons. 2G: photon.

0I: inflaton. 1R: zero-charge analogs to quarks. 2J: modeling analog to the Pauli exclusion force.

4G: graviton. 6G: interacts with (and strength differs by) fermion generation. 8G: interacts with angular momentum.

Figure 3: Subfamilies of elementary particles

Figure 2 shows a catalog of some properties of objects. Each of $\lambda = 4$ and $\lambda = 6$ relates to mass. Each of $\lambda = 8$ and $\lambda = 10$ relates to angular momentum. Each of $\lambda = 12$ and $\lambda = 14$ relates to momentum. Each of $\lambda = 2$ and $\lambda = 16$ relates to charge.

Proposed modeling matches all known elementary particles. Proposed modeling suggests elementary particles that people have yet to find.

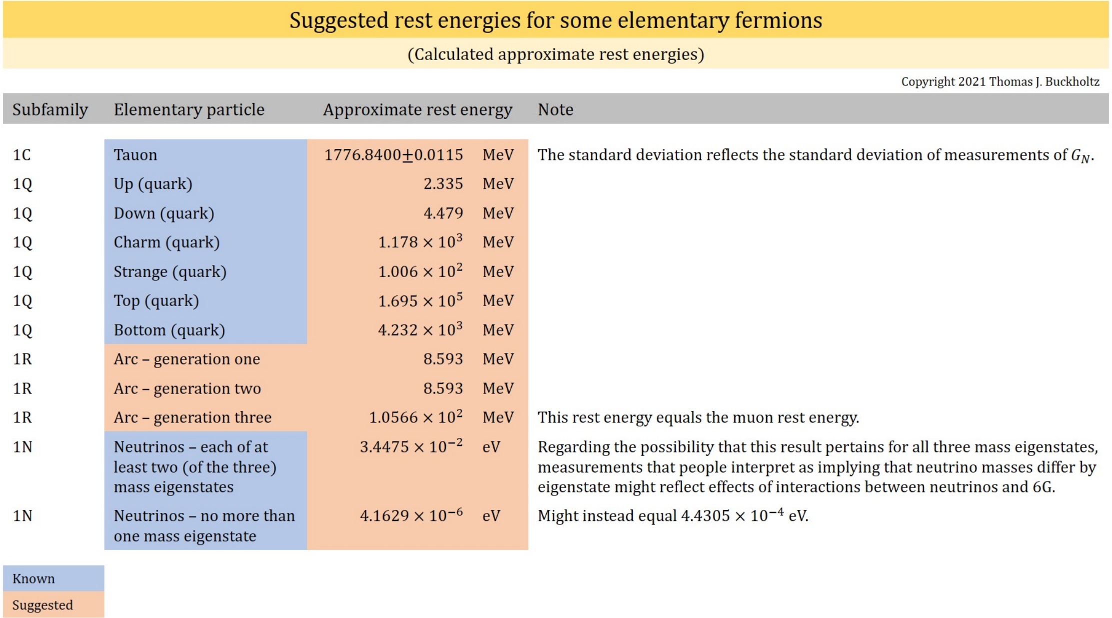
Figure 3 summarizes some information about elementary particles. The figure alludes to all known elementary particles. The figure alludes to elementary particles that proposed modeling suggests and that people have yet to find. Each row discusses one value of Σ. The symbol Σ equals $2S$. The symbol S denotes spin as per the extant modeling expression $S(S+1)\hbar^2$ regarding angular momentum.

Proposed modeling predicts masses for some elementary particles. Formulas for masses of elementary particles include aspects that reflect charge and spin.

For the Higgs boson and the weak interaction bosons, proposed modeling suggests that the ratios of squares of masses $(m_{\mathrm{Higgs}})^2 : (\mathrm{m_Z})^2 : (m_{\mathrm{W}})^2$ are $17 : 9 : 7$. Details include the following. Start from $17 = 4^2 + 1$ for the Higgs boson and $10 = 3^2 + 1$ for the weak interaction bosons. If $S = 1$, subtract one. If the magnitude of the charge is $|q_e|$, subtract two. The symbol q_e denotes the charge of the electron.

Proposed modeling suggests a formula for the masses of the elementary fermions. The formula yields values of $\log(m/m_e)$. The symbol m_e denotes the mass of the electron. The fine-structure constant - α or $((q_e)^2/(4\pi\varepsilon_0))/(\hbar c)$ - appears in the formula. An aspect - $\hbar$ - related to spin appears in α. An aspect - q_e - related to charge appears in α.

Figure 4 shows rest energies that proposed modeling suggests for some elementary fermions. Unverified



Suggested rest energies for some elementary fermions

(Calculated approximate rest energies)

Subfamily	Elementary particle	Approximate rest energy		Note
1C	Tauon	1776.8400±0.0115	MeV	The standard deviation reflects the standard deviation of measurements of G_N.
1Q	Up (quark)	2.335	MeV	
1Q	Down (quark)	4.479	MeV	
1Q	Charm (quark)	1.178×10^3	MeV	
1Q	Strange (quark)	1.006×10^2	MeV	
1Q	Top (quark)	1.695×10^5	MeV	
1Q	Bottom (quark)	4.232×10^3	MeV	
1R	Arc – generation one	8.593	MeV	
1R	Arc – generation two	8.593	MeV	
1R	Arc – generation three	1.0566×10^2	MeV	This rest energy equals the muon rest energy.
1N	Neutrinos – each of at least two (of the three) mass eigenstates	3.4475×10^{-2}	eV	Regarding the possibility that this result pertains for all three mass eigenstates, measurements that people interpret as implying that neutrino masses differ by eigenstate might reflect effects of interactions between neutrinos and 6G.
1N	Neutrinos – no more than one mass eigenstate	4.1629×10^{-6}	eV	Might instead equal 4.4305×10^{-4} eV.

Known

Suggested

Figure 4: Suggested rest energies for some elementary fermions

extant modeling suggests that measurements show indirectly that at least one neutrino rest energy differs from the rest energies of the other two neutrinos. Proposed modeling can comport with the notion of unequal neutrino rest energies. Proposed modeling might also comport with the notion that the three neutrino rest energies equal each other. For this case, proposed modeling suggests that the measurements reflect interactions between neutrinos and 6G.

Proposed modeling suggests that most dark matter has bases in isomers of most - but not all - elementary particles.

Proposed modeling suggests that nature includes six isomers of a set of elementary particles. (Here, we discuss PR6ISP modeling. See figure 1. We postpone discussing PR36ISP modeling.) Proposed modeling calls the isomers isomer zero, isomer one, . . ., and isomer five. Stuff that measures as ordinary matter is most of - but not all of - the stuff that has bases in isomer zero.

Regarding each isomer, the set of elementary particles includes all elementary particles except G-family elementary particles and the jay (or, 2J) boson. Except for charged leptons, the elementary particles in one isomer might be identical to the elementary particles in each other isomer. For charged leptons, pairings of rest energy and generation can differ between isomers. We provide an example. For isomer zero, the electron is a charged lepton that associates with generation one. For isomer zero, the muon is a charged lepton that associates with generation two. For isomer one, a charged lepton that has the mass of the isomer zero electron associates with generation three. For isomer one, a charged lepton that has the mass of the isomer zero muon associates with generation one.

Proposed modeling suggests that - in the early universe - jay bosons catalyze roughly equal - across isomers - populations of stuff. Aside from the jay boson, only G-family bosons can intermediate interactions between isomers.

Each isomer has its own analog of the extant modeling notion of the photon. (We postpone discussing the proposed modeling notion that 2G intermediates some interactions between isomers.)

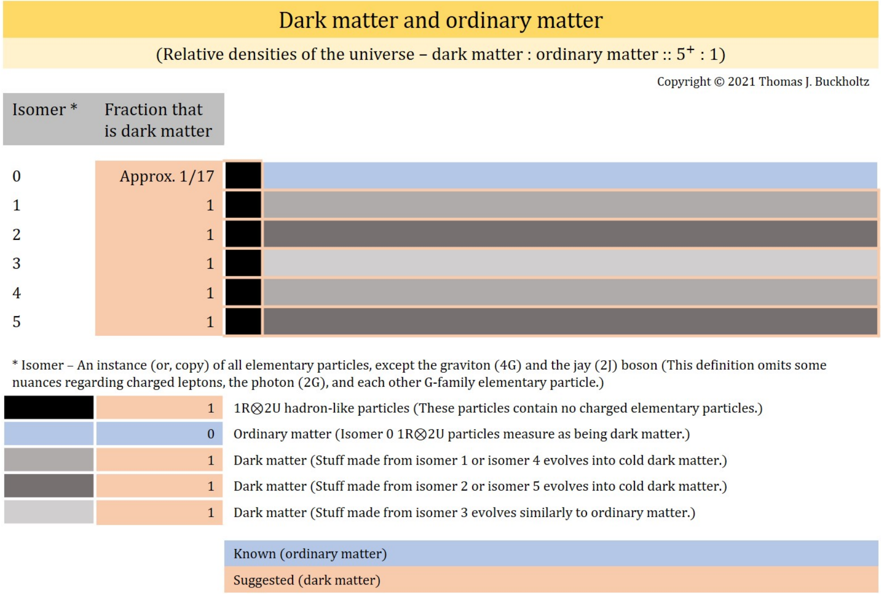
Each isomer forms, based on the isomer's arcs (or, 1R elementary fermions) and gluons, hadron-like particles. We use the symbol 1R⊗2U to denote these hadron-like particles. These hadron-like particles have no (non-virtual) charged components. Isomer zero 1R⊗2U hadron-like particles do not interact with isomer zero photons. Isomer zero 1R⊗2U hadron-like particles measure as being dark matter.

Figure 5 shows a proposed modeling explanation for the inferred ratio - five-plus to one - of dark matter density of the universe to ordinary matter density of the universe.

Proposed modeling suggests insight regarding eras in the evolution of the universe.

Proposed modeling suggests phenomena that govern changes in the rate of expansion of the universe.

Proposed modeling models include a decomposition of the gravitational field that an object produces. The components of gravity (or, 4G) have parallels to components that extant modeling (for example, Maxwell's equations) attributes to electromagnetic fields. For a stationary object, extant modeling points


Dark matter and ordinary matter

(Relative densities of the universe – dark matter : ordinary matter :: 5^{+} : 1)

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Isomer *	Fraction that is dark matter
0	Approx. 1/17
1	1
2	1
3	1
4	1
5	1

* Isomer – An instance (or, copy) of all elementary particles, except the graviton (4G) and the jay (2J) boson (This definition omits some nuances regarding charged leptons, the photon (2G), and each other G-family elementary particle.)

■	1	1R⊗2U hadron-like particles (These particles contain no charged elementary particles.)
▨	0	Ordinary matter (Isomer 0 1R⊗2U particles measure as being dark matter.)
▨	1	Dark matter (Stuff made from isomer 1 or isomer 4 evolves into cold dark matter.)
▨	1	Dark matter (Stuff made from isomer 2 or isomer 5 evolves into cold dark matter.)
▨	1	Dark matter (Stuff made from isomer 3 evolves similarly to ordinary matter.)

Known (ordinary matter)

Suggested (dark matter)

Figure 5: Dark matter and ordinary matter

to a spatial monopole component - of 2G - that reflects the charge of the object. A spatial dipole component reflects the magnetic dipole moment of the object.

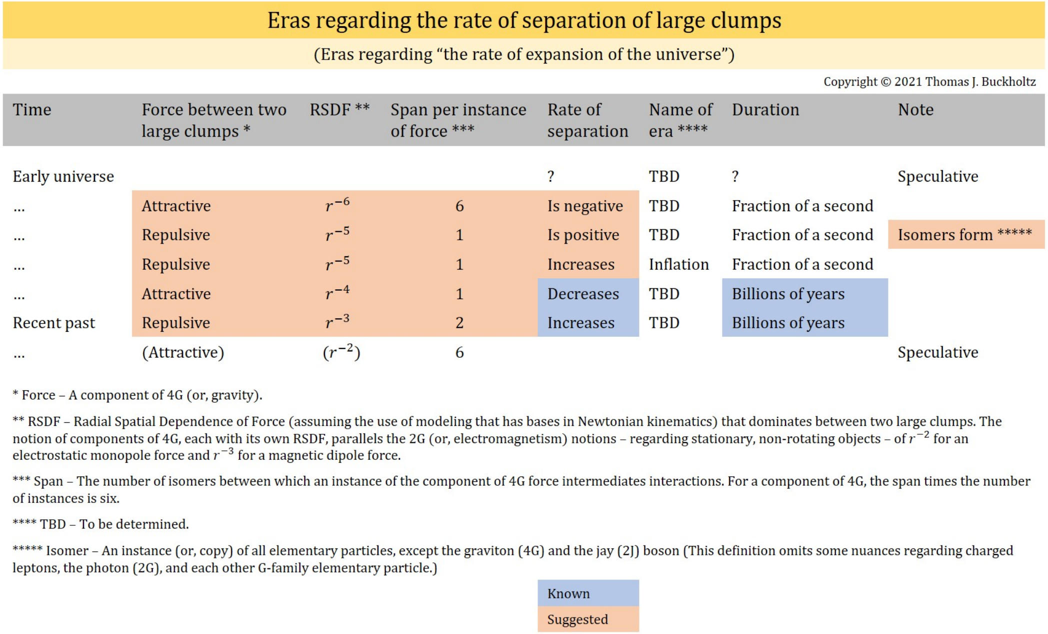
We explore the proposed modeling decomposition of gravity. The rest energy of an object is non-negative. For 4G, proposed modeling points, as people might expect, to an attractive spatial monopole component of gravity. A dipole component dilutes overall attraction. (The dipole component has similarities to some aspects of the extant modeling general relativity notion of frame dragging.) For objects that are adequately massive and adequately close to each other, dipole repulsion can exceed monopole attraction. Modeling for 4G also includes - at least - an attractive quadrupole component and two repulsive octupole components. The monopole component of 4G intermediates attractive interactions between elementary particles in any one of the six isomers and elementary particles in any of the six isomers. We say that the monopole component has a span of six isomers. The quadrupole component of 4G intermediates attractive interactions between stuff in any isomer and stuff in (only) the same isomer. We say that the quadrupole component has a span of one isomer. (In effect, each isomer has its own quadrupole component.) Each of the two octupole components of 4G intermediates repulsive interactions between stuff in any isomer and stuff in (only) the same isomer. We say that each octupole component has a span of one isomer. (In effect, each isomer has its own pair of octupole components.) The dipole component of 4G intermediates repulsive interactions between stuff in any isomer and stuff in (only) the same isomer and one other isomer. We say that the dipole component has a span of two isomers. (Among the six isomers, three instances of the dipole component exist.)

Proposed modeling might resolve seeming inabilities of extant modeling to explain unexpectedly large increases in the rate of expansion of the universe during the most recent some billions of years. The proposed modeling explanation has bases in the notion of isomers and in the notion of the repulsive dipole component of 4G.

The difference between span one for the quadrupole component of 4G and span two for the dipole component of 4G might resolve the following seeming problem regarding unverified extant modeling. People develop extant modeling for kinematics of large clumps and for equations of state for large regions. (Large clumps might include filaments and galaxy clusters.) People tune models to account for phenomena during the multi-billion-year period during which the rate of expansion decreases. People say that applying the models to the current era of increasing rate of expansion underestimates current increases in the rate. Proposed modeling suggests that such extant modeling models underestimate the dominant repulsive effect by - in effect - a factor of two. The factor of two reflects the ratio of the span of the dipole component of 4G to the span of the quadrupole component of 4G.

Proposed modeling suggests insight regarding the early universe.

Unverified extant modeling suggests an era that people call inflation and a related elementary particle - the inflaton. The proposed modeling list of elementary particles includes a candidate - the aye (or, 0I)



Eras regarding the rate of separation of large clumps

(Eras regarding “the rate of expansion of the universe”)

Time	Force between two large clumps *	RSDF **	Span per instance of force ***	Rate of separation	Name of era ****	Duration	Note
Early universe				?	TBD	?	Speculative
...	Attractive	r^{-6}	6	Is negative	TBD	Fraction of a second	
...	Repulsive	r^{-5}	1	Is positive	TBD	Fraction of a second	Isomers form *****
...	Repulsive	r^{-5}	1	Increases	Inflation	Fraction of a second	
...	Attractive	r^{-4}	1	Decreases	TBD	Billions of years	
Recent past	Repulsive	r^{-3}	2	Increases	TBD	Billions of years	
...	(Attractive)	(r^{-2})	6				Speculative

* Force – A component of 4G (or, gravity).

** RSDF – Radial Spatial Dependence of Force (assuming the use of modeling that has bases in Newtonian kinematics) that dominates between two large clumps. The notion of components of 4G, each with its own RSDF, parallels the 2G (or, electromagnetism) notions – regarding stationary, non-rotating objects – of r^{-2} for an electrostatic monopole force and r^{-3} for a magnetic dipole force.

*** Span – The number of isomers between which an instance of the component of 4G force intermediates interactions. For a component of 4G, the span times the number of instances is six.

**** TBD – To be determined.

***** Isomer – An instance (or, copy) of all elementary particles, except the graviton (4G) and the jay (2J) boson (This definition omits some nuances regarding charged leptons, the photon (2G), and each other G-family elementary particle.)

Known
Suggested

Figure 6: Eras regarding the evolution of the universe

particle - for the inflaton.

Proposed modeling suggests insight regarding two possible eras that would precede inflation.

Figure 6 catalogs eras regarding the evolution of the universe. Proposed modeling suggests aspects regarding each of five eras.

Proposed modeling suggests insight regarding various inferred ratios of dark matter to ordinary matter.

Based on notions of isomers and spans, proposed modeling suggests details regarding galaxy formation scenarios and galaxy evolution scenarios. Details suggest that galaxies tend to evolve toward some specific ratios of dark matter stuff to ordinary matter stuff.

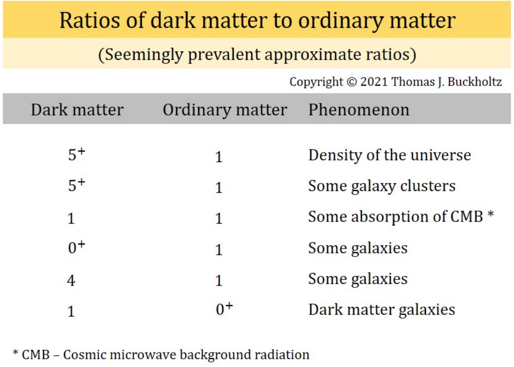
Figure 7 lists some seemingly prevalent inferred ratios of dark matter to ordinary matter. We use the word seemingly because we are aware of at least one set - but not necessarily many sets - of measurements that yield each ratio. Generally, we are not aware of measurements that produce seemingly as-significant other ratios. Ratios regarding galaxy clusters seem to reflect ratios regarding densities of the universe. The one-to-one ratio regarding some absorption of CMB (or, cosmic microwave background radiation) might confirm aspects regarding the notion of isomers and the notion of spans for components of electromagnetism (or, 2G). People attribute half of the absorption to ordinary matter hydrogen atoms. A seemingly relevant component of 2G has a span of two. Hydrogen atom like objects in one isomer that does not feature ordinary matter would account for the other half of the absorption. Proposed modeling galaxy evolution scenarios suggest explanations for numbers - in figure 7 - that pertain for individual galaxies. Proposed modeling galaxy evolution scenarios reflect notions of isomers and notions of spans for components of gravity (or, 4G).

Proposed modeling suggests insight regarding physics modeling techniques.

Figure 8 suggests that proposed modeling provides a framework for cataloging, comparing, and uniting aspects of proposed modeling and aspects of extant modeling. Figure 8 uses and extends notions that figure 2 shows.

Figure 8 points to distinctions and pairings regarding some aspects of physics modeling. The figure pairs classical physics electromagnetism ($\lambda = 2$) and quantum physics electromagnetism ($\lambda = 16$). The figure pairs classical physics gravitation ($\lambda = 4$) and possible quantum physics gravitation ($\lambda = 6$). The figure pairs classical physics angular momentum ($\lambda = 8$) and quantum physics angular momentum ($\lambda = 10$).

Figure 8 points to possible reasons why unverified extant modeling seems to have difficulties developing the notion of quantum gravity. One extant modeling approach features trying to quantize aspects of general relativity. The $\lambda = 4$ row points to the notion that these approaches do not consider isomers. If nature includes more than one isomer, quantizing general relativity might be problematic. One extant modeling approach features trying to extend the Standard Model to include a quantized spin two force.



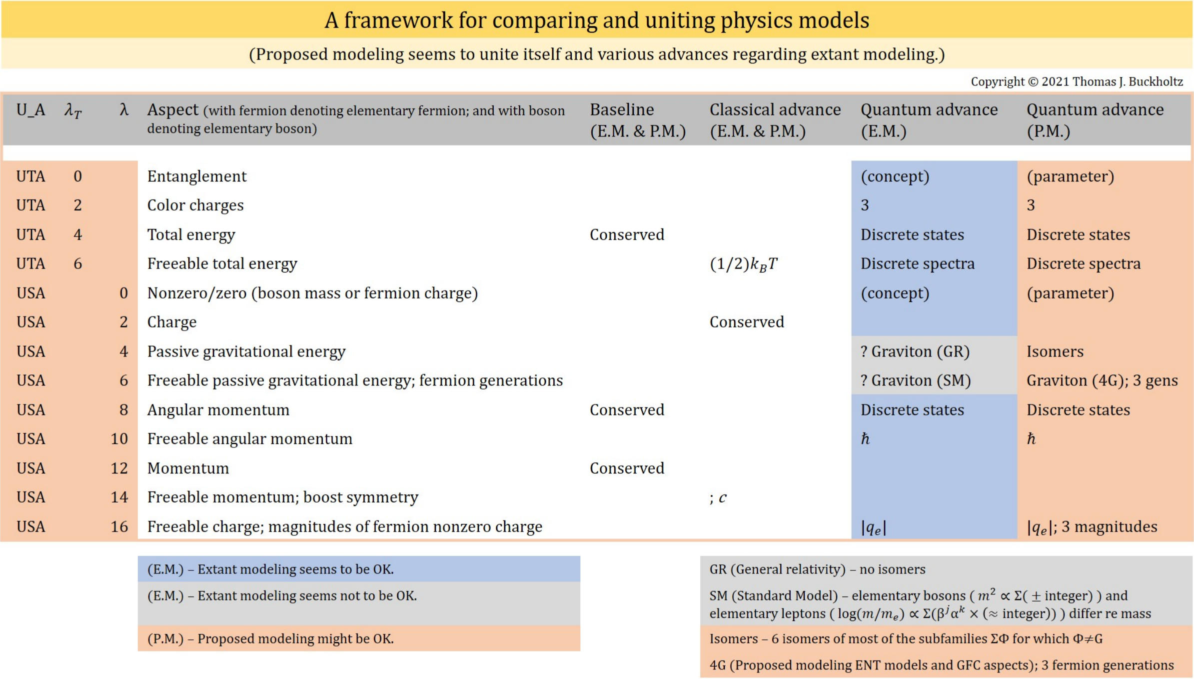
Ratios of dark matter to ordinary matter

(Seemingly prevalent approximate ratios)

Dark matter	Ordinary matter	Phenomenon
5^+	1	Density of the universe
5^+	1	Some galaxy clusters
1	1	Some absorption of CMB *
0^+	1	Some galaxies
4	1	Some galaxies
1	0^+	Dark matter galaxies

* CMB – Cosmic microwave background radiation

Figure 7: Seemingly prevalent approximate ratios of dark matter to ordinary matter



A framework for comparing and uniting physics models

(Proposed modeling seems to unite itself and various advances regarding extant modeling.)

U_A	λ_T	λ	Aspect (with fermion denoting elementary fermion; and with boson denoting elementary boson)	Baseline (E.M. & P.M.)	Classical advance (E.M. & P.M.)	Quantum advance (E.M.)	Quantum advance (P.M.)
UTA	0		Entanglement			(concept)	(parameter)
UTA	2		Color charges			3	3
UTA	4		Total energy	Conserved		Discrete states	Discrete states
UTA	6		Freeable total energy		$(1/2)k_BT$	Discrete spectra	Discrete spectra
USA		0	Nonzero/zero (boson mass or fermion charge)			(concept)	(parameter)
USA		2	Charge		Conserved		
USA		4	Passive gravitational energy			? Graviton (GR)	Isomers
USA		6	Freeable passive gravitational energy; fermion generations			? Graviton (SM)	Graviton (4G); 3 gens
USA		8	Angular momentum	Conserved		Discrete states	Discrete states
USA		10	Freeable angular momentum			$\hbar$	$\hbar$
USA		12	Momentum	Conserved			
USA		14	Freeable momentum; boost symmetry		; c		
USA		16	Freeable charge; magnitudes of fermion nonzero charge			$\|q_e\|$	$\|q_e\|$; 3 magnitudes

(E.M.) – Extant modeling seems to be OK.

(E.M.) – Extant modeling seems not to be OK.

(P.M.) – Proposed modeling might be OK.

GR (General relativity) – no isomers

SM (Standard Model) – elementary bosons ($m^2 \propto \Sigma(\pm \text{integer})$) and elementary leptons ($\log(m/m_e) \propto \Sigma(\beta^j\alpha^k \times (\approx \text{integer}))$) differ re mass

Isomers – 6 isomers of most of the subfamilies ΣΦ for which Φ≠G

4G (Proposed modeling ENT models and GFC aspects); 3 fermion generations

Figure 8: A framework for cataloging, comparing, and uniting aspects of proposed modeling and aspects of extant modeling

The $\lambda = 6$ row points to differences regarding proposed modeling models for masses of elementary bosons and proposed modeling models for masses of elementary fermions. If the Standard Model does not offer a better - than current - treatment of mass, developing a notion of quantum gravity based on the Standard Model might be problematic.

Proposed modeling offers an approach to - and description of - quantum gravity that is as straightforward as the proposed modeling approach to electromagnetism. The $\lambda = 4$ row and the $\lambda = 6$ row - each in figure 8 - point to possible merit regarding the proposed modeling approach to quantum gravity.

Proposed modeling might provide insight about notions of modeling, objects, and entanglement. The $\lambda_T = 0$ (or, $UTA0$) row in figure 8 alludes to modeling for which a parameter distinguishes between essentially no entanglement regarding an object and significant entanglement regarding the object. Elementary particles in the subfamilies 0I, 1Q, 1R, 2J, and 2U model as entangled. (See figure 3.) Elementary particles in the subfamilies 0H, 1C, 1N, 2W, 2G, 4G, 6G, and 8G can model as not entangled. We use the two-word phrase can model because, for example, modeling for refracted photons does not necessarily feature a lack of entanglement.

2. Methods

This unit addresses the following opportunities. Motivate and develop methods that proposed modeling uses. Use the methods. Develop and show results from using the methods. Discuss the methods and results.

2.1. Modeling regarding objects and their properties

We develop bases for modeling objects and their properties. We show a means for cataloging some properties of objects.

2.1.1. Bases for modeling objects and properties

Extant modeling models photons via two harmonic oscillators. For modeling a photon, one chooses two spatial axes. Each axis is perpendicular to the direction in which the photon moves. The two axes are perpendicular to each other. Extant modeling might label the two axes with, respectively, the symbols x and y. Each harmonic oscillator models a number of excitations that people attribute to the photon mode that people pair with the relevant axis. Equations (1), (2), and (3) show a number - n - of excitations, the raising operator, and the lowering operator. Equation (4) shows the extant modeling range for the integer n.

$$|n> \tag{1}$$

$$a^+|n>= (1+n)^{1/2}|n+1> \tag{2}$$

$$a^-|n>= n^{1/2}|n-1> \tag{3}$$

$$n \geq 0 \tag{4}$$

Extant modeling has bases in notions of three spatial dimensions. Proposed modeling suggests considering, regarding photons, a third harmonic oscillator. Considering this third oscillator provides a step toward proposed modeling. This essay de-emphasizes the notion of adding the third oscillator to extant modeling.

The third oscillator associates with the direction of motion. Modeling might label the axis associating with the direction of motion with the symbol z. Extant modeling states that photons have zero mass. Extant modeling states that longitudinal polarization does not pertain for photons. Proposed modeling suggests extending each of equations (1), (2), and (3) to pertain for the domain that equation (5) shows. Regarding the z oscillator, equation (6) shows that this extension is compatible with zero longitudinal polarization. Longitudinal polarization does not excite.

$$n \geq -1 \tag{5}$$

$$a^+|-1>= (1+(-1))^{1/2}|0>= 0|0> \tag{6}$$

Proposed modeling uses the construct $@_k$ to denote a value k that does not change. For example, equation (7) pertains.

$$@_0 = 0 \tag{7}$$

Equation (8) pertains regarding our conceptual extension - of extant modeling for photons - to include three spatial harmonic oscillators. The notation $\{\cdots\}$ denotes a set. The expression $KSAj$ parses as follows. The symbol K denotes kinematics modeling. (Elsewhere, we discuss notions of other modeling. See, for example, table 1.) The symbol S stands for the word spatial. (Elsewhere, we discuss notions of T and temporal. See, for example, discussion related to equation (12).) The symbol A stands for the word aspects. For example, one can read SA as denoting the two-word phrase spatial aspects. The symbol j varies over the range of applicable oscillators. Equation (9) pertains for mode x. Equation (10) pertains for mode y.

$$\{KSAj\} = \{KSAz, KSAx, KSAy\} \tag{8}$$

$$n_{KSAz} = -1,\ n_{KSAx} = n,\ n_{KSAy} = @_0 \tag{9}$$

$$n_{KSAz} = -1,\ n_{KSAx} = @_0,\ n_{KSAy} = n \tag{10}$$

For each of the two modes, equation (11) pertains. The symbol $\equiv$ denotes the notion of definition. The leftmost equality defines the symbol A_{KSA}.

$$A_{KSA} \equiv \sum_{\{KSAj\}} (n_{KSAj} + (1/2)) = n_{KSAz} + n_{KSAx} + n_{KSAy} + (3/2) = n + (1/2) \tag{11}$$

Extant modeling has bases in notions of one temporal dimension. Proposed modeling suggests including an oscillator that associates with the temporal dimension. Proposed modeling suggests that, for each of the two modes, equations (12), (13), and (14) pertain. Here, the symbol T stands for the word temporal. The symbol t denotes the one temporal coordinate.

$$\{KTAj\} = \{KTAt\} \tag{12}$$

$$n_{KTAt} = n \tag{13}$$

$$A_{KTA} \equiv \sum_{\{KTAj\}} (n_{KTAj} + (1/2)) = n_{KTAt} + (1/2) = n + (1/2) \tag{14}$$

Equation (15) pertains for each photon mode.

$$A_{KTA} - A_{KSA} = 0 \tag{15}$$

We use the two-element term double-entry bookkeeping to describe the equality that equation (16) shows. Adding a unit to one of A_{KTA} and A_{KSA} requires adding a unit to the other quantity.

$$A_{KA} \equiv A_{KTA} - A_{KSA} = 0 \tag{16}$$

Extant modeling includes two-mode photon models for which one mode features left circular polarization and the other mode features right circular polarization. Extant modeling circular polarization models are invariant with respect to choices of transverse axes. Compared to linear polarization models, circular polarization models are more invariant with respect to choice of observer. For models for a photon in a vacuum, all observers would agree on the number of excitations for left circular polarization and on the number of excitations for right circular polarization.

We convert kinematics notions above to pertain for circular polarization modes. From a perspective of equations underlying models, we use the substitutions that equation (17) shows. An expression of the form $a \leftarrow b$ denotes the six-element phrase b takes the place of a. The oscillator $KSA0$ associates with longitudinal polarization. We adopt the convention that an oscillator KSA(odd number) features left circular polarization. Oscillator $KSA1$ features left circular polarization. Oscillator $KSA2$ features right circular polarization.

$$KSAz \leftarrow KSA0,\ KSAx \leftarrow KSA1,\ KSAy \leftarrow KSA2 \tag{17}$$

We use the abbreviation KIN (for the word kinematics) and the two-word term kinematics modeling to characterize work and discussion leading to equation (17). Extant modeling KIN modeling features aspects regarding motions of and changes to objects. KIN modeling does not necessarily fully address the question of characterizing the objects.

We anticipate developing modeling that outputs representations that match or suggest elementary particles. We use the abbreviation ENT (for the word entity) and the two-word term entity modeling to contrast with - respectively - KIN and the two-word term kinematics modeling.

We show aspects of ENT modeling for the photon. Equations (18), (19), (20), (21), (22) and (23) pertain. Symbols of the form $ETAj$ denote oscillators that pair with the two-word term temporal aspects. However, space-time coordinates do not underlie ENT modeling. Symbols of the form $ESAj$ denote oscillators that pair with the two-word term spatial aspects. $ESA1$ pairs with left circular polarization. $ESA2$ pairs with right circular polarization. The two-word term longitudinal polarization pairs with $ESA0$. Equation (23) exemplifies double-entry bookkeeping.

$$\{ETAj\} = \{ETA0\} \tag{18}$$

$$n_{ETA0} = n \tag{19}$$

$$\{ESAj\} = \{ESA0, ESA1, ESA2\} \tag{20}$$

$$n_{ESA0} = -1,\ n_{ESA1} = n,\ n_{ESA2} = @_0 \tag{21}$$

$$n_{ESA0} = -1,\ n_{ESA1} = @_0,\ n_{ESA2} = n \tag{22}$$

$$A_{EA} \equiv A_{ETA} - A_{ESA} = 0 \tag{23}$$

ENT modeling for the photon has similarities to KIN modeling for photons. (Compare equation (17) and discussion related to equation (23).) We anticipate ENT modeling for the Higgs boson. Longitudinal polarization pertains. Circular polarization does not pertain. For the Higgs boson, $\{ESAj\} = \{ESA0\}$ pertains. For each of the photon and the set of weak interaction bosons, $\{ESAj\} = \{ESA0, ESA1, ESA2\}$ pertains.

Equation (24) defines the symbol Σ. Here, S is the spin - in the sense of the extant physics KIN modeling expression $S(S+1)\hbar^2$ that relates to (the square of) angular momentum. Σ is a nonnegative integer.

$$\Sigma \equiv 2S \tag{24}$$

For some elementary particles, the number of ENT modeling spatial oscillators does not equal three. For the elementary particles discussed just above, equation (25) pertains. The symbol $|$ denotes the two-word phrase such that. (Elsewhere, we show that equation (25) does not pertain for ENT modeling for some elementary particles. See discussion - that follows equation (49) - regarding elementary fermions.)

$$\Sigma = 2S = \max(j | n_{ESAj} = 0) \tag{25}$$

We anticipate that - in ENT modeling and for integer $j \geq 1$ - the oscillator $ESA(2j-1)$ associates with $\Sigma = 2j$ left circular polarization. The oscillator $ESA(2j)$ associates with $\Sigma = 2j$ right circular polarization. For example, $ESA3$ and $ESA4$ associate with $\Sigma = 4$, $S = 2$, and the would-be graviton.

For ENT modeling and other proposed modeling non-KIN modeling, double-entry bookkeeping continues to pertain.

For ENT modeling and other proposed modeling non-KIN modeling, we continue to use the words temporal and spatial, even though the modeling does not necessarily directly associate with space-time coordinates. For some ENT modeling, no continuous variables pertain. Some ENT modeling features essentially only integers. The integers are numbers of oscillators and the values of various n_{abAj}. (This essay de-emphasizes discussing the extent to which people might consider that a mathematical space that

Table 1: Some types of modeling

Modeling	a	Notes
KIN	K	KIN denotes the word kinematics. Modeling features motions of and changes to objects. Modeling might not yet suggest elementary particles that people have yet to find. Modeling has roots in the principle of stationary action and in mathematics of continuous functions. KIN modeling underlies much extant modeling. Proposed modeling suggests some re-interpretations of and some limits on extant modeling KIN modeling.
ENT	E	ENT denotes the word entity. Modeling matches all known elementary particles and suggests specific elementary particles - and properties of those particles - that people have yet to find. The set of elementary particles might suffice to explain much data that extant modeling seems not to explain. Modeling has roots in some aspects of KIN modeling, in symmetries that pair with physics conservation laws, and in discrete mathematics. ENT modeling is a subset of proposed modeling.
GFC	G	GFC denotes the two-element phrase G-family components. Modeling suggests associations between long-range forces and properties of objects. (Each of the notion of the G-family of elementary particles and the notion of long-range forces associates with the photon, the would-be graviton, and possibly other elementary particles.) Modeling has roots in ENT modeling, in symmetries that pair with physics conservation laws, and in discrete mathematics. GFC modeling is a subset of proposed modeling. GFC modeling echoes aspects of extant modeling.
UNI	U	UNI denotes the word united. Modeling produces a catalog of properties of objects and of relationships between properties. The notion of object includes both elementary particles and objects that include more than one elementary particle. Modeling unites aspects of KIN modeling, ENT modeling, and GFC modeling. UNI modeling unites aspects of extant modeling and aspects of proposed modeling.
Double-entry bookkeeping		Double-entry bookkeeping pertains for the following. Some proposed modeling re-interpretations of extant modeling KIN modeling. Proposed modeling ENT modeling. Proposed modeling GFC modeling. Some aspects of proposed modeling UNI modeling.
Quantum excitations		The notion of quantum excitations pertains for some KIN modeling and for ENT modeling. The notion of quantum excitations does not necessarily directly pertain to GFC modeling.

has bases in a combination of ENT modeling and notions of an energy-momentum space might serve as a tangent space to a mathematical space associated with KIN modeling.)

Equations (26) and (27) pertain throughout much of proposed modeling. For equation (26), the symbol a can be any one of K, E, G, and U. (See table 1.) The symbol b can be any one of T (for temporal) and S (for spatial). Equation (27) exemplifies double-entry bookkeeping.

$$A_{abA} \equiv \sum_{\{abAj\}} (n_{abAj} + (1/2)) \tag{26}$$

$$A_{aA} \equiv A_{aTA} - A_{aSA} = 0 \tag{27}$$

Table 1 discusses some types of modeling.

We note aspects of ENT modeling that pertain for more than just the photon.

Equation (23) exhibits an invariance with respect to a choice between KIN modeling that is quadratic in energy and KIN modeling that is linear in energy. Regarding a photon, the KIN expression $0 = E^2 - (pc)^2$ is quadratic in energy. The symbol E denotes energy. The symbol p denotes the magnitude of momentum. The symbol c denotes the speed of light. One can consider that an ENT raising operator associates with adding one unit of each of the two relevant items - E^2 and $(pc)^2$ - that have the dimensions of the square of energy. For an object with mass m and modeling based on the equation $E^2 = (mc^2)^2 + (pc)^2$ from special relativity, one can consider that an ENT raising operator associates with adding one unit of each of the three relevant items - E^2, $(mc^2)^2$, and p^2c^2. The Klein-Gordon equation provides an

Table 2: An ENT representation for photon ground states

$ETA4$	$ETA3$	$ETA2$	$ETA1$	$ETA0$	$ESA0$	$ESA1$	$ESA2$	$ESA3$	$ESA4$	$\Sigma\Phi$
				0	-1	0	0			2G

Table 3: A basis for ENT representations for G-family ground states (with LCP denoting left circular polarization; and with RCP denoting right circular polarization)

$ETA\cdots$	$ETA0$	$ESA0$	$ESA1$	$ESA2$	$ESA3$	$ESA4$	$ESA\cdots$
$\cdots$	0	-1	$\Sigma = 2$:LCP	$\Sigma = 2$:RCP	$\Sigma = 4$:LCP	$\Sigma = 4$:RCP	$\cdots$

example of KIN modeling - for other than just photons - that can be quadratic in energy. Regarding a photon, the KIN expression $0 = E - pc$ is linear in energy. One can consider that an ENT raising operator associates with adding one unit of each of the two relevant items - E and pc - that have the dimensions of energy. Each of the Dirac equation and the Schrodinger equation provides an example of KIN modeling - for other than just photons - that is linear in energy.

Either one of A_{ETA} and A_{ESA} can pair with the extant modeling KIN modeling notion of a photon ground state energy that associates with the expression $0 + (1/2)$ and with the number one-half. (See, for example, equation (14).) People interpret extant modeling KIN models as exhibiting notions of nonzero quantum energy of the vacuum. Proposed modeling suggests - via equations such as equation (16) - modeling that might obviate needs to consider nonzero quantum energy of the vacuum. Proposed modeling suggests a notion for which this essay uses the two-word term freeable energy. (See, for example, the use in table 13b of the three-word term freeable total energy.) For a proposed modeling model and a choice of object, the ground state of the object models as having zero freeable energy. (The following example features the topic of choice of model. A model for transitions between energy levels in an atom does not necessarily need to consider the rest energies of the relevant electrons and atomic nucleus as associating with freeable energy. Such a model can feature a ground state that associates with the ground state of the atom.)

We discuss ENT modeling for elementary particles that are not the photon.

This essay uses the notation Φ to denote so-called families of elementary particles. This essay uses the notation $\Sigma\Phi$ to denote so-called subfamilies of elementary particles. The two-element term G family includes the photon and the would-be graviton. Here, Φ=G.

Regarding ENT modeling, this essay tends to emphasize ground states and de-emphasize excited states. Such work in this essay tends to feature harmonic oscillator states that pair with the numbers 0 and -1. Such work tends not necessarily to state explicitly distinctions between $@_k$ and k.

Table 2 shows an ENT representation for photon ground states.

We assume that table 3 pertains for G-family ground states.

We explore aspects regarding G-family forces and regarding so-called components of G-family forces.

In extant modeling KIN modeling, an excitation of a photon carries information through which people infer aspects of an event that includes the excitation. For example, people measure the energy of a photon and might use that information to infer information about an atomic transition that excited the photon.

In proposed modeling ENT modeling, excitations of a photon carry similar information. We anticipate that GFC modeling points to encoded information to which extant modeling KIN modeling does not point. The additional encoded information features the isomer or isomers that participated in the creation of the photon. (See table 8 and table 9c.)

We consider the left circular polarization mode of 2G.

We consider an excitation that models conceptually as combining an excitation of the left circular mode of 4G and the right circular mode of 2G. (This essay de-emphasizes the possible relevance of an actual object that combines a graviton and a photon.) The combination yields a left circular polarization $\Sigma = 2$ (or spin one) excitation. The combination associates with 2G.

Equation (28) provides notation that we use for such combinations. The symbol ΣG denotes a subfamily of the G-family. The symbol Γ denotes a set of even positive integers. We use the symbol λ to denote an element of Γ. Each value of λ associates with the oscillator pair $GSA(\lambda-1)$-and-$GSA\lambda$. For the above example of subtracting spin one from spin two, the notation $\Gamma = 24$ pertains and equation (29) pertains.

$$\Sigma\mathrm{G}\Gamma \tag{28}$$

Table 4: A basis for GFC representations for G-family components (with LCP denoting left circular polarization; and with RCP denoting right circular polarization)

$GTA\cdots$	$GTA0$	$GSA0$	$GSA1$	$GSA2$	$GSA3$	$GSA4$	$GSA\cdots$
$\cdots$	0	-1	$\lambda = 2$:LCP	$\lambda = 2$:RCP	$\lambda = 4$:LCP	$\lambda = 4$:RCP	$\cdots$

Table 5: G-family solutions that may be relevant and for which $\lambda \leq 8$

Other	GFC monopole	GFC dipole	GFC quadrupole	GFC octupole
0G∅	2G2	ΣG24	ΣG246	ΣG2468
	4G4	ΣG26	ΣG248	
	6G6	ΣG28	ΣG268	
	8G8	ΣG46	ΣG468	
		ΣG48		
		ΣG68		

$$\Sigma = |-2+4| = 2 \tag{29}$$

Table 4 echoes table 3. Table 3 pertains for ENT modeling. Table 4 pertains for GFC modeling.

Table 5 points to possibly relevant solutions for which the limit $\lambda \leq 8$ pertains. (The word solution pertains regarding harmonic oscillator mathematics and double-entry bookkeeping. Here, a solution solves - or, satisfies - the equation $A_{GA} \equiv A_{GTA} - A_{GSA} = 0$. We anticipate that some solutions have relevance to models regarding G-family physics. We use the word component - as in component of a ΣG field or of a G-family force - regarding physics applications of solutions that are relevant to G-family physics. We anticipate that some solutions have relevance regarding modeling for aspects of physics other than G-family aspects.) The labels GFC monopole through GFC octupole pertain regarding GFC modeling. The label GFC monopole pairs with the existence of one mathematical solution for each item in the column labeled GFC monopole. The label GFC dipole pairs with the existence of two mathematical solutions for each item in the column labeled GFC dipole. For example, for $\Gamma = 24$, each one of the solutions 2G24 and 6G24 pertains. The symbol 6G24 pairs with the expression $\Sigma = |+2+4| = 6$. The label GFC quadrupole pairs with the existence of four mathematical solutions for each item in the column labeled GFC quadrupole. G-family physics does not include phenomena that might associate with the symbol 0G. For each of two GFC quadrupole items, the one 0GΓ mathematical solution is not relevant to G-family physics. For example, the solution 0G246, which pairs with $|-2-4+6|$, is not relevant to G-family physics. The label GFC octupole pairs with the existence of eight mathematical solutions for the one item in the column labeled GFC octupole. The solution 0G2468 is not relevant to G-family physics. The table notes a conceptually possible 0G∅ solution. The symbol ∅ denotes the empty set.

We use the symbol $\Sigma\gamma$ to refer to the set of G-family solutions ΣGΓ for which Σ appears in the list Γ. (See equation (30).) Here, the notation $\{a|b\}$ denotes the ten-element phrase the set of all a such that conditions b pertain. The symbol $\in$ denotes the four-word phrase is a member of (or, the four-word phrase is an element of). We use the symbol $\gamma\lambda$ to refer to the set of G-family solutions ΣGΓ for which λ appears in the list Γ and Σ does not appear in the list Γ. (See equation (31).) The symbol $\notin$ denotes the five-word phrase is not a member of.

$$\Sigma\gamma = \{\Sigma\mathrm{G}\Gamma | \Sigma \in \Gamma\} \tag{30}$$

$$\gamma\lambda = \{\Sigma\mathrm{G}\Gamma | \lambda \in \Gamma, \Sigma \notin \Gamma\} \tag{31}$$

Table 6 lists G-family solutions ΣGΓ for which both $\Sigma \leq 8$ and, for each $\lambda \in \Gamma$, $\lambda \leq 8$. The expressions $|-2+4-6+8|$ and $|-2-4-6+8|$ show that two solutions comport with the notion of 4G2468. We use the letters a and b to distinguish between the two solutions. We use each of the letters x and y to refer to either one of the solutions or to both solutions. The expressions $|+2+4-6+8|$ and $|-2-4+6+8|$ show that two solutions comport with the notion of 8G2468.

Work leading to table 5 does not depend on choosing a kinematics model. Examples of kinematics models include Newtonian physics and general relativity.

We posit that the words monopole through octupole pair, for extant modeling KIN Newtonian modeling, with force laws. RSDF abbreviates the five-word term radial spatial dependence of force. The notion of RSDF pertains regarding KIN modeling. (The notion of RSDF does not directly pertain regarding

Table 6: $\Sigma\gamma$ solutions for which both $\Sigma \leq 8$ and, for each $\lambda \in \Gamma$, $\lambda \leq 8$

Σ	GFC monopole	GFC dipole	GFC quadrupole	GFC octupole
2	2G2	2G24	2G248	
4	4G4	4G48	4G246	4G2468a, 4G2468b
6	6G6		6G468	
8	8G8			8G2468a, 8G2468b

Table 7: KIN modeling interpretations pairing with $\Sigma\gamma$ force components for which $\Sigma \leq 4$ and, for each $\lambda \in \Gamma$, $\lambda \leq 8$

(a) Interactions

Components	Property of an object (assuming that modeling pertains for zero translational motion)
2G2	Charge.
2G24	Magnetic dipole moment.
2G248	Magnetic dipole moment for which the direction of the axis (pairing with the dipole moment) changes over time. (Adjustment regarding 2G24. KIN spatial dipole. KIN RSDF r^{-3}.)
4G4	Mass.
4G48	Adjustment regarding 4G, to the extent that the object rotates. KIN spatial dipole. KIN RSDF r^{-3}.
4G246	Adjustment regarding 4G, to the extent that the object has a quadrupole moment of mass. KIN spatial quadruple. KIN RSDF r^{-4}.
4G2468a, 4G2468b	Adjustments regarding 4G, to the extents that quadrupole moments of mass rotate. KIN spatial octupole. KIN RSDF r^{-5}.

(b) An interpretation of $8 \in \Gamma$ and a preview of an interpretation of $[\![16]\!] \in \Gamma$ (with the notion that, for $\lambda \geq 10$, this essay uses $[\![\lambda]\!]$ to denote elements of Γ)

Aspect	Interpretation
$8 \in \Gamma$	Rotation
$[\![16]\!] \in \Gamma$	Change (other than rotation) over time

GFC modeling.) Extant modeling pairs the word monopole with a potential energy that varies as r^{-1} and with the RSDF of r^{-2}. Here, r denotes an extant modeling KIN radial coordinate and the distance from the center of the one relevant object. Here, we de-emphasize angular aspects of forces. A series that starts with monopole continues. For example, extant modeling pairs the word dipole with a potential energy that varies as r^{-2} and with the RSDF of r^{-3}. (Perhaps, see table 7.)

Table 7 notes some aspects related to table 6. Table 7a discusses measurable properties for an object that measures as not moving. In table 7, we use the notion that - for 2γ - $8 \in \Gamma$ does not add a factor - to RSDF - of r^{-1}. In table 7, we posit that - for 4γ - $8 \in \Gamma$ adds a factor - to RSDF - of r^{-1}. (Regarding the case of 4γ, see discussion immediately below.)

We discuss aspects of table 7. Elsewhere, we further discuss the adjustments - regarding 4G - to which table 7a alludes. (See table 16.) Regarding non-4G $\Sigma\gamma$ G-family solutions for which $8 \in \Gamma$, $[\![16]\!] \notin \Gamma$, and at least one of two, four, and six is a member of Γ, one can consider that the presence of $\lambda = 8$ pairs with a KIN factor of $(ct)^{-1}$ and not with a KIN factor of r^{-1}. (For $\lambda \geq 10$, this essay uses $[\![\lambda]\!]$ to denote elements of Γ.) Here, t denotes an extant modeling KIN temporal coordinate and c denotes the speed of light. (Perhaps, consider the notion that - at least regarding propagation in a vacuum - $r^{-1} = (ct)^{-1}$.) Regarding non-4G $\Sigma\gamma$ G-family solutions for which $8 \in \Gamma$, $[\![16]\!] \notin \Gamma$, and at least one of two, four, and six is a member of Γ, the GFC (or ENT) notion of quadrupole pairs with the KIN notion of $r^{-3}t^{-1}$ and with the KIN notion of spatial dipole. Regarding KIN modeling, 2G248 associates with an adjustment - that varies with time - to 2G24 and magnetic dipole moment. (See the 2G248 row in table 7a. Perhaps, consider the following example. For the planet earth, the axis of rotation does not match the axis for the magnetic dipole moment.) Similarly, the GFC notion of $[\![16]\!] \in \Gamma$ might associate - for non-4G G-family solutions that are relevant to G-family physics - with a KIN factor of $(ct)^{-1}$ and not with a KIN factor of r^{-1}. (Note table 7b. For the earth and 2G, $[\![16]\!] \in \Gamma$ might associate with changes regarding the relative alignment of the axis of rotation and the axis of magnetic moment.) Such an association with a KIN

Table 8: PRι_IISP modeling and isomers of simple particles

Note
• The two-word phrase simple particles denotes all elementary particles except G-family elementary particles and J-family elementary particles. The set $\{\Sigma\Phi \vert \Phi \neq \text{G and } \Phi \neq \text{J}\}$ of subfamilies associates with all simple particles.
• Proposed modeling includes so-called PRι_IISP modeling, with ι_I being one of the integers one, six, and 36. The models address aspects of astrophysics and aspects of cosmology. The two letters PR denote the term physics-relevant. The three letters ISP denote the four-word term isomers of simple particles (or, the five-word term isomers of simple elementary particles). The integer ι_I denotes a number of so-called isomers of the set of all simple particles.
• In this respect, PR1ISP modeling associates with extant modeling.
• Proposed modeling suggests that PR6ISP models explain more astrophysics data and more cosmology data than do PR1ISP models. For example, PR6ISP modeling explains some observed ratios of dark matter to ordinary matter.
• PR36ISP models might explain more data than do PR6ISP models. In particular, PR36ISP models offer a new explanation for the dark energy density of the universe.

factor of $(ct)^{-1}$ might pertain only to the extent that six is a member of Γ. Discussion related to table 10 suggests that there might not be any adequately relevant G-family physics relevant non-4G $\Sigma\gamma$ G-family solutions for which $[\![16]\!] \in \Gamma$.

Table 8 defines the two-word term simple particles and notes some aspects regarding the proposed modeling notion of isomers of simple particles. (This proposed modeling notion of isomers does not necessarily parallel the nuclear physics notion - same numbers of protons and neutrons, but different energy states - of isomers. This proposed modeling notion of isomers does not necessarily parallel the chemistry notion - same numbers of various atoms, but different spatial arrangements - of molecular isomers.)

This essay generally de-emphasizes possible applications of PR36ISP modeling, except in regard to a discussion of dark energy density of the universe. (Regarding dark energy density of the universe and PR36ISP, see discussion related to equation (132).)

Before continuing our discussion of GFC modeling, we discuss notions related to group theory and to harmonic oscillator mathematics.

We note a relationship between $SU(j)$ groups and the group $U(1)$.

Equation (32) echoes mathematics and some extant modeling. Here, each of the positive integers j_1 and j_2 is at least two. The symbol $\supset$ denotes the notion that each group to the right of the symbol is a subgroup of the group to the left of the symbol.

$$SU(j_1 + j_2) \supset SU(j_1) \times SU(j_2) \times U(1) \tag{32}$$

We use a symbol of the form g_{group} to denote the number of generators for a group. Equation (33) pertains.

$$g_{SU(j)} = j^2 - 1 \tag{33}$$

For $U(1)$, $g_{U(1)} = 2$. One of the two generators of the group $U(1)$ pairs with the raising operator that equation (2) shows. The other of the two generators of the group $U(1)$ pairs with the lowering operator that equation (3) shows.

We posit that equations (34) and (35) have relevance for the domain $-1 \leq n \leq 0$. We use the symbol $U(1)_b$ to denote a construct that associates with this pair of one raising operator and one lowering operator. We posit that applications of equation (32) pertain for which one replaces the $U(1)$ (in equation (32)) with $U(1)_b$.

$$b^+|n>= n^{1/2}|n+1> \tag{34}$$

$$b^-|n>= (1+n)^{1/2}|n-1> \tag{35}$$

Extant modeling includes the notion of the Poincare group. Equation (36) pertains. The construct for which this essay uses the symbol $S1g$ associates with conservation of energy and with a group with one generator. One instance of $SU(2)$ pairs with conservation of angular momentum. One instance of $SU(2)$ pairs with conservation of momentum. One instance of $SU(2)$ pairs with boost symmetry.

$$S1g \times SU(2) \times SU(2) \times SU(2) \tag{36}$$

We posit that applications of equation (32) pertain for which one envisions, for one of $k = 1$ and $k = 2$, that j_k equals one and that one replaces the would-be $SU(1)$ with $S1g$.

We posit that - for GTA aspects of GFC modeling - the substitutions (in either of the two directions) that equation (37) suggests can be appropriate when $S1g$ associates with the $GTA0$ oscillator.

$$SU(j) \ \leftrightarrow \ SU(j-1) \times S1g \tag{37}$$

We discuss relationships between the numbers of generators for some $SU(j)$ groups.

In equation (38), g_j denotes the number of generators of the group $SU(j)$. The symbol $|$ denotes the word divides (or, the two-word phrase divides evenly). The symbol $\nmid$ denotes the four-word phrase does not divide evenly. For some aspects of proposed modeling, equation (38) associates with ending the series $SU(3)$, $SU(5)$, $\cdots$ at the item $SU(7)$. (See discussion related to equation (42).) For some aspects of proposed modeling, the series $SU(3)$, $SU(5)$, $SU(7)$, and $SU(17)$ might pertain.

$$g_3|g_5,\ g_3|g_7,\ g_5|g_7 \quad g_5\nmid g_9,\ g_7\nmid g_9,\ g_7\nmid g_{11} \quad g_3|g_{17},\ g_5|g_{17},\ g_7|g_{17} \tag{38}$$

We continue discussion regarding GFC modeling.

Table 9 shows GFC representations for the G-family solutions for which - for each $\lambda \in \Gamma$ - $\lambda \leq 8$. The solutions associate with symmetries pertaining to ENT modeling and ground states. In table 9, the rightmost seven columns comport with double-entry bookkeeping. (See table 9b. Regarding table 9b and the notion of $S1g$ symmetry, see discussion related to equation (37).) Table 9c discusses the notion of span. (Regarding information in the column - in table 9a - regarding span, see discussion regarding equation (39) and equation (40).) Information about GTA symmetries has two roles. One role pertains to the number of relevant isomers. (See tables 8 and 9c.) One role pertains to the extent to which solutions associate with interactions with individual elementary particles. (See discussion related to equation (42).) The word component - as in component of a G-family force - pertains for (at least) each $\Sigma\text{G}\Gamma$ in table 9a for which $2 \leq \Sigma \leq 8$. (Perhaps, compare with table 7.) Some components can interact with multicomponent objects and not with individual elementary particles. Elsewhere, this essay discusses using PR6ISP modeling and the notion of six isomers to explain the observed ratio - of five-plus to one - of dark matter density of the universe to ordinary matter density of the universe. (See discussion related to table 28.)

Table 10 points to some G-family solutions that one might extrapolate from aspects that underlie table 9.

We preview notions regarding some aspects of table 10. We associate the 4G2468⟦16⟧ solution with an attractive component - of 4G - that might dominate early in the evolution of the universe. (See table 16. See discussion related to equation (115).) Paralleling the notion that some instances of $\lambda = 8$ associate with rotation, some instances of $\lambda = 16$ might associate with changes over time. (See table 7b.) The 4G246⟦16⟧ solution might associate with an attractive KIN octupole component of 4G. The corresponding force might participate regarding ending the inflationary epoch. (See discussion related to equation (118).) This essay de-emphasizes the possible physics relevance of some possible extrapolations. Solution 10G⟦10⟧ provides an example. Per equation (100), a strength factor of four pertains regarding 2G2 and a strength factor of three pertains regarding 4G4. We assume that a strength factor of two pertains regarding 6G6. We assume that a strength factor of one pertains regarding 8G8. We assume that a strength factor of zero pertains regarding 10G⟦10⟧. We pair some 0GΓ solutions with some elementary bosons. (See table 23.) The following notions provide an example - that is not specific to elementary particles - regarding the 2G248⟦16⟧ row in table 10. For the earth, 2G24 pairs with nominal magnetic dipole moment. 2G248 pairs with non-alignment of the axis of planetary spin and the axis of the nominal magnetic dipole moment. Speculatively, 2G248⟦16⟧ might associate with changes over time regarding the difference between the axis of planetary spin and the axis of the nominal magnetic dipole moment. However, proposed modeling suggests that changes over time might associate with freeable energy and a need to have $6 \in \Gamma$. (See discussion related to table 7b.)

We discuss spans for components of G-family forces. We develop the second column - Span (for $\iota_I > 1$) - in table 9a.

For any one value of ι_I (as in PRι_IISP), equation (39) pertains for each simple particle, for each component of G-family force, for the jay (or, 2J) boson, and for each hadron-like particle. For example,

Table 9: GFC information regarding G-family solutions for which, for each $\lambda \in \Gamma$, $\lambda \leq 8$

(a) $\Sigma\Phi\Gamma$, GTA symmetries, and other aspects (with NR denoting not relevant)

$\Sigma\Phi\Gamma$	Span (for $\iota_I > 1$)	GTA $SU(\_)$ symmetry	$GTA0$	$GSA0$	$GSA1$ and $GSA2$	$GSA3$ and $GSA4$	$GSA5$ and $GSA6$	$GSA7$ and $GSA8$
0G∅	NR	NR	-1	-1				
2G2	1	None	0	-1	$\pi_{0,@_0}$			
4G4	6	$SU(3)$	0	-1	A0+	$\pi_{0,@_0}$		
ΣG24	1	None	0	-2	$\pi_{0,@_0}$	$\pi_{0,@_0}$		
6G6	2	$SU(5)$	0	-1	A0+	A0+	$\pi_{0,@_0}$	
ΣG26	6	$SU(3)$	0	-2	$\pi_{0,@_0}$	A0+	$\pi_{0,@_0}$	
ΣG46	6	$SU(3)$	0	-2	A0+	$\pi_{0,@_0}$	$\pi_{0,@_0}$	
ΣG246	1	None	0	-3	$\pi_{0,@_0}$	$\pi_{0,@_0}$	$\pi_{0,@_0}$	
8G8	1	$SU(7)$	0	-1	A0+	A0+	A0+	$\pi_{0,@_0}$
ΣG28	2	$SU(5)$	0	-2	$\pi_{0,@_0}$	A0+	A0+	$\pi_{0,@_0}$
ΣG48	2	$SU(5)$	0	-2	A0+	$\pi_{0,@_0}$	A0+	$\pi_{0,@_0}$
ΣG68	2	$SU(5)$	0	-2	A0+	A0+	$\pi_{0,@_0}$	$\pi_{0,@_0}$
ΣG248	6	$SU(3)$	0	-3	$\pi_{0,@_0}$	$\pi_{0,@_0}$	A0+	$\pi_{0,@_0}$
ΣG268	6	$SU(3)$	0	-3	$\pi_{0,@_0}$	A0+	$\pi_{0,@_0}$	$\pi_{0,@_0}$
ΣG468	6	$SU(3)$	0	-3	A0+	$\pi_{0,@_0}$	$\pi_{0,@_0}$	$\pi_{0,@_0}$
ΣG2468	1	None	0	-4	$\pi_{0,@_0}$	$\pi_{0,@_0}$	$\pi_{0,@_0}$	$\pi_{0,@_0}$

(b) Notes regarding notation that table 9a uses and regarding GTA symmetries

Note
• The symbol A0+ pertains for an oscillator pair for which, for each of the two oscillators, the symbol $@_0$ pertains.
• The symbol $\pi_{0,@_0}$ associates with the notion that either $n_{GSA(odd)} = 0$ and $n_{GSA(even)} = @_0$ pertains or $n_{GSA(odd)} = @_0$ and $n_{GSA(even)} = 0$ pertains. For example, equation (29) and 2G24 associate with $n_{GSA1} = @_0$ and $n_{GSA2} = 0$ and $n_{GSA3} = 0$ and $n_{GSA4} = @_0$. Here, the two values of zero anti-align with respect to odd and even. In contrast, 6G24 associates with $n_{GSA1} = 0$ and $n_{GSA2} = @_0$ and $n_{GSA3} = 0$ and $n_{GSA4} = @_0$. Here, the two values of zero align with respect to odd and even.
• For each row for which table 9a shows a GTA $SU(\_)$ symmetry of none, oscillator $GTA0$ suffices regarding double-entry bookkeeping.
• For the case of GTA $SU(\_)$ symmetry of none, the symmetry $S1g$ pertains.
• For each row for which table 9a shows a GTA symmetry of $SU(j)$, double-entry bookkeeping suggests adding $j-1$ GTA oscillators. For each added $GTAk$ oscillator, the value of n_{GTAk} is zero. The result satisfies double-entry bookkeeping. The $SU(j)$ symmetry pairs with mathematics for an isotropic harmonic oscillator that features j component harmonic oscillators. Here, the set of component oscillators includes $GTA0$.

(c) Notes regarding G-family excitations, regarding information that associates with specific $\Sigma G\Gamma$, and regarding the notion of span

Note
• An excitation of a ΣG field does not (directly) encode information about a relevant $\Sigma G\Gamma$.
• For PRι_IISP modeling for which $\iota_I > 1$, the word span denotes the isomers among which a specific instance of a specific $\Sigma G\Gamma$ intermediates interactions.
• For PRι_IISP modeling for which $\iota_I > 1$, this essay tends (when not discussing specific isomers of simple particles) to use the word span to denote the number of isomers among which a specific instance of a specific $\Sigma G\Gamma$ intermediates interactions. (See, for example, table 9a.)
• For PRι_IISP modeling for which $\iota_I > 1$, an excitation of a ΣG field encodes information that specifies relevant isomers of particles. The number of relevant isomers associates with the Γ of the relevant $\Sigma G\Gamma$. The word span denotes that number of relevant isomers.
• For PRι_IISP modeling for which $\iota_I > 1$, a de-excitation of a ΣG field must associate with an isomer in the list of isomers that associates with the relevant excitation.
• For PR1ISP modeling, there is one isomer of simple particles and the span is always one.

Table 10: Some G-family solutions that one might extrapolate from aspects that underlie table 9

Solutions that associate with table 9 and with the limits $\Gamma \neq \emptyset$ and $\lambda \leq 8$	Other solution, assuming the limits $\Gamma \neq \emptyset$ and $\lambda \leq 16$	Possibilities, regarding the other solution
4G4, 4G48, 4G246, 4G2468x	4G2468⟦16⟧	Might associate with the dominant force component for an era two eras before inflation.
4G4, 4G246	4G246⟦16⟧	Might associate with a significant force component around the time of inflation.
2G2, 4G4, 6G6, 8G8	10G⟦10⟧	Seemingly not relevant. The strength of 10G⟦10⟧ would be zero.
0G246, 0G2468	0G2468⟦16⟧	Might associate with the 0I elementary boson.
0G268	0G268⟦16⟧	Might associate with the 2U elementary bosons.
2G2, 2G24, 2G248	2G248⟦16⟧	Seemingly not necessarily relevant. $6 \notin \Gamma$.
4G4, 4G48	4G48⟦16⟧	Seemingly not necessarily relevant. $6 \notin \Gamma$.
8G8	8G8⟦16⟧	Seemingly not necessarily relevant. $6 \notin \Gamma$.

for PR6ISP modeling, for the electron, the number of isomers is six and the span of each isomer is one. (The electron does not pair directly with a GFC solution.) For PR6ISP modeling, for the 4G4 component of 4G, the number of isomers is one and the span of each isomer is six. (Gravity intermediates interactions between the six isomers of simple particles.)

$$\text{(number of isomers)} \times \text{(span of one isomer)} = \iota_I \tag{39}$$

We start from the span of six that we posit for 4G4. We consider GTA symmetries for G-family solutions. (See table 9a.) We aim to develop numbers that belong in the table 9a column that has the label span (for $\iota_I \geq 6$). The number of generators of each of $SU(3)$, $SU(5)$, and $SU(7)$ divides evenly the integer 48, which is the number of generators of $SU(7)$. Regarding 4G4, we posit that the expression $6 = g_{SU(7)}/g_{SU(3)}$ provides the span. We generalize. We assert that, for each G-family solution for which a GTA symmetry of $SU(j)$ pertains, equation (40) provides the span. We assume that we can generalize from the assumption that the span of 2G2 is one. (Ordinary matter photons do not interact - or, at least, do not interact much - with dark matter.) For each G-family solution with no GTA $SU(\_)$ symmetry, the span is one. (Here, we consider that the 0G$\emptyset$ solution is not relevant.) We anticipate that some G-family solutions - for which some λ exceed eight - have relevance and that equation (40) does not pertain. (See discussion related to equation (114).)

$$g_{SU(7)}/g_{SU(j)} \tag{40}$$

Equation (41) shows notation for denoting the span, s, for an elementary particle or for a component of a long-range force.

$$\Sigma(\mathrm{s})\Phi \quad \text{or} \quad \Sigma(\mathrm{s})\Phi\Gamma \tag{41}$$

We explore the extents to which components of G-family forces interact with simple particles.

Elsewhere, we pair an $SU(4)$ symmetry with the notion of additivity - across systems or objects - of energy that modeling associates with ground state total energy of the systems or objects. (See the row - in table 13 - that discusses minimal total energy or ground state total energy.) We deploy equation (37). Here, we assume that an $SU(5)$ symmetry pertains. The $SU(5)$ symmetry associates with UTA UNI modeling and with ETA ENT modeling. The symmetry pertains - in ENT modeling - for each G-family force ΣG.

We posit that aspects of the UTA UNI modeling $SU(5)$ symmetry and the GTA $SU(\_)$ symmetry column in table 9 combine. For example, for 8G8, a GTA $SU(11)$ symmetry would pertain. (In table 9, seven GTA oscillators pertain. For the symmetry pairing with UTA UNI modeling, five GTA oscillators pertain. The two aspects that combine share their respective $n_{aTA0} = 0$ values. Seven plus five minus one is 11.) For such work, equation (42) pertains. For example, for 4G4, a GTA $SU(7)$ symmetry would pertain. For example, for 2G2 or 2G24, a GTA $SU(5)$ symmetry would pertain. We posit a limit that associates with aspects of equation (38). We posit that each component that appears in table 9 and has a

Table 11: Possibilities regarding conservation of some properties of an elementary fermion, from before to after an interaction with an elementary boson

Symbol	Note
CEFGE	The symbol denotes conservation of elementary fermion generation. CEFGE pertains (at least) regarding isolated interactions between weak interaction bosons and elementary fermions. For example, for an incoming electron (which is a generation one charged lepton) and an incoming W^+ boson, the outgoing neutrino has the same generation - one - as does the incoming charged lepton.
SCEFGE	The symbol denotes somewhat conservation of elementary fermion generation, which pertains regarding (at least) interactions (in hadrons) between W bosons and quarks. Extant modeling associates this lack of CEFGE with notions of CP violation. The notion of CP-symmetry pairs with the four-word phrase charge conjugation parity symmetry.
CEFMA	The symbol denotes conservation of elementary fermion (rest) mass. The after-interaction fermion has the same mass as has the before-interaction fermion. (Compare with CEFGE and with SCEFGE.)
CEFCC	The symbol denotes conservation of elementary fermion color charge. Interactions between gluons and quarks do not necessarily exhibit CEFCC.

GTA symmetry of none or $SU(3)$ can interact with simple particles. (Here, combining the GTA symmetry that table 9 shows with the additivity - across objects - of energy symmetry produces, respectively, $SU(5)$ or $SU(7)$.) We posit that components that appear in table 9 and have a GTA symmetry of none or $SU(3)$ can interact with multicomponent objects. We posit that each component that appears in table 9 and has a GTA symmetry of $SU(5)$ or $SU(7)$ does not interact with simple particles. (Here, combining the GTA symmetry that table 9 shows with the UTA UNI modeling symmetry produces, respectively, $SU(9)$ or $SU(11)$.) We posit that a combined symmetry of either $SU(9)$ or $SU(11)$ associates with possible interactions with multicomponent objects.

$$SU(j_1) \text{ combines with } SU(j_2) \text{ to yield } SU(j_1 + j_2 - 1) \tag{42}$$

For example, 2G68 can interact with an atom but not with an isolated electron. (Table 9 shows, regarding 2G68, a GTA $SU(5)$ symmetry.) We associate 2G68 with at least the 21-centimeter hyperfine interaction with hydrogen atoms. (See discussion related to equation (133).) Generally, $6 \in \lambda$ can associate with interactions regarding freeable energies of objects. (See table 12b and table 20.) Generally, $8 \in \lambda$ can associate with interactions regarding rotations of objects or spins of objects. (See table 7b and table 20.)

We posit conservation laws that might pertain regarding interactions between an elementary fermion and an elementary boson.

Table 11 defines symbols for some possibilities regarding conservation of some properties of an elementary fermion, from before to after an interaction with an elementary boson. This essay uses the symbols to describe aspects that pertain to elementary bosons.

2.1.2. Objects and their properties

We consider the possibility that table 7 points toward useful modeling regarding objects and properties of objects. Table 7 links aspects of GFC modeling (and, hence, aspects of proposed modeling ENT modeling) with properties that associate with extant modeling KIN models.

We consider the topic of how modeling might characterize an object.

We start from a perspective of extant modeling KIN modeling for classical physics.

Extant modeling considers attributes - of objects - that people measure or infer. Attributes can include energy, charge, mass (or, rest energy), angular momentum, and momentum.

We consider one object. People deploy notions that measured attributes might change without the object losing its identity. For example, a force might produce a change of momentum but not produce a change of object.

Extant modeling includes a notion of ground state. The ground state associates with a least observable or inferable energy for the object. The notion of least observable or inferable energy depends on assumptions and modeling. For example, a model for an atom might focus on energy states for the electrons in the atom. The model might not need to consider the rest energies of electrons as being freeable. (For example, the model might not need to take into account possible changes having bases in the annihilation - via an incoming positron - of an electron in the atom.)

We posit a generalization. Modeling for each property - such as energy or angular momentum - might embrace three values. The values are an actual value (which people can measure or infer), a minimal value (which depends on the choice of model), and a so-called freeable value. Equation (43) pertains. Equation (44) shows proposed modeling uses of equation (43).

$$\text{Actual} = \text{Minimal} + \text{Freeable} \tag{43}$$

$$\text{Minimal} = \text{Actual} - \text{Freeable} \tag{44}$$

In extant modeling classical physics models, for each of angular momentum and momentum, equation (44) pertains three times, based on the relevance of three spatial axes. For each of angular momentum and momentum, additivity - across objects - of actual pertains for each of the three spatial axes. In a system, a total actual property is the sum of actual properties for each object that is part of the system. This notion of additivity pairs with a conservation law. For example, the objects can change their individual z-axis components of momentum, as long as the total - for all objects in the system - z-axis momentum is constant. Extant modeling pairs - via the Poincare group - each one of conservation of angular momentum and conservation of momentum with an $SU(2)$ symmetry. (See discussion related to equation (36).) Proposed modeling pairs each of the three generators of $SU(2)$ with one of the three relevant (orthogonal) spatial axes. Notions, similar to notions relevant to actual, pertain regarding freeable. Based on equation (44), we anticipate the relevance - for each of angular momentum and momentum - of two instances of $SU(2)$ symmetry. One of the two instances associates with actual. One of the two instances associates with freeable. Proposed modeling uses models that associate each instance of $SU(2)$ with two harmonic oscillators. Proposed modeling can include modeling that does not consider or assume specific minimal values. Proposed modeling can include modeling for which the minimal value for one property depends on the choices of minimal values for other properties. Here, such choices need to be compatible with aspects associating with a minimal (or, ground state) energy. (This essay does not explore possible relationships between notions of entropy and the notion of numbers of sums that add to a minimal energy.)

For each of angular momentum and momentum, we posit that the pair of $SU(2)$ symmetries associates - via equation (32) and $j_1 = j_2 = 2$ - with one instance of $SU(4)$ and with one instance of $U(1)$. We posit that the $U(1)$ symmetry pairs with additivity regarding the actual value. We posit that the $SU(4)$ symmetry pairs with the notion of minimal value.

Extant modeling KIN classical physics can treat each of actual angular momentum and actual momentum as a three-vector. We now admit into our discussion aspects that associate with extant modeling KIN quantum physics.

For angular momentum, the freeable $SU(2)$ pairs with $\hbar$, which is a minimum unit of exchange for angular momentum. Here, the three generators of $SU(2)$ associate with $D = 3$ in equation (45). Equation (45) associates with the extant modeling expression $S(S+1)\hbar^2$ regarding angular momentum for objects with spin S. Equation (45) associates with an aspect of solutions involving Laplacian operators associating - in KIN models - with $D = 3$ spatial dimensions. The $SU(2)$ symmetry pairs with an extending - from extant modeling before quantum modeling - extant modeling to include quantized spin and to include the notion of a minimal unit, $\hbar$, of angular momentum that pertains to exchanges - between objects - of angular momentum.

$$S(S + D - 2) = S(S + 1) \tag{45}$$

Regarding momentum, the freeable $SU(2)$ pairs with the $D = 3$ dimensions that are relevant to extant modeling KIN models for special relativity. The $SU(2)$ symmetry pairs with an extending - from extant modeling before special relativity modeling - extant modeling to include special relativity. People pair the two-word term boost symmetry with this $SU(2)$ symmetry.

The notion of three-vector does not pertain for each one of energy, charge, and mass. Nevertheless, proposed modeling UNI modeling regarding each of (total) energy, charge, and mass includes (based on, in effect, equation (44) and parallels to other notions above) symmetries that parallel the symmetries (two $SU(2)$, one $U(1)$, and one $SU(4)$) that pair with either one of angular momentum and momentum.

We postpone discussing the case of total energy until we complete discussion leading to table 12.

Regarding charge, an $SU(2)$ symmetry pairs with extending extant modeling before quantized charge modeling to include a notion of a minimal unit, $|q_e|$, of charge that pertains to exchanges - between objects - of charge. The symbol q_e denotes the charge of the electron. Here, $D = 3$ (again pairs with three dimensions and) pairs with three spatial-like dimensions for a charge-current four-vector.

Similar methods seem not to apply regarding freeable passive gravitational energy. For elementary bosons, some aspects regarding additivity associate - not with masses but - with squares of masses. (See discussion related to table 23.) For elementary fermions, some aspects of additivity loosely associate - not with masses but - with logarithms of mass and with either or both of generation and charge. (See discussion related to equation (62).)

Regarding actual passive gravitational energy, the $U(1) \times SU(2)$ symmetry associates with six isomers. (For $U(1) \times SU(2)$, the number of generators is two times three - or, is six.)

Regarding freeable passive gravitational energy, the $SU(2)$ symmetry associates with aspects that depend on the relevant object and the relevant model. (See table 12b.)

We posit that - in accord with table 7 - the following notions pertain. Actual charge associates with $\lambda = 2$. Actual rest energy associates with $\lambda = 4$. Actual angular momentum associates with $\lambda = 8$.

We define aspects of UNI modeling.

Table 12 shows USA aspects of UNI modeling. Table 12a shows modeling regarding a system that includes at least one object. The modeling associates with extant modeling classical physics. The word additivity refers to the notion that modeling associates with an ability to add, across more than one system, the respective system property. The column with the label USA defines associations with oscillators that underlie the modeling. (UNI modeling does not necessarily directly reflect mathematics associating with excitations of harmonic oscillators.) The assignments comport with aspects of table 6, table 7a, table 8, and table 9. Each instance of $U(1)$ pairs with additivity. (Additivity does not necessarily pertain regarding $U(1)_b$.)

We discuss notions regarding table 12. The column - in table 12a - labeled symmetry is compatible with applying - starting with $SU(17)$ - equation (32) four times. (Here, we assume that equation (32) pertains once with $j_1 = 1$.) Table 12b pertains to an object that is part of the system. The column labeled with the two-word phrase object property differentiates cases. For example, the $USA0$ row differentiates between elementary fermions and other objects, including elementary bosons. Relative to table 12a, table 12b has bases in four applications of equation (32). For each application, $j_1 = j_2 = 2$.

We discuss notions of passive gravitational mass, active gravitational mass, and not necessarily gravitational mass. Regarding table 12b, for an object, the passive gravitational energy equals the sum of the minimal passive gravitational energy and the freeable passive gravitational energy. For this essay, the three-word term passive gravitational energy is synonymous with the four-word term passive gravitational rest energy. The three-word term passive gravitational mass denotes the mass that modeling attributes to the object when modeling the gravitational field that the object - in effect - produces. The passive gravitational mass equals c^{-2} times the passive gravitational energy. In this context, each of the three-word term active gravitational mass and the two-word term inertial mass contrasts with passive gravitational mass. Active gravitational mass associates with the notion of interaction between an object and the gravitational field that other objects - in effect - produce. Inertial mass interrelates notions of accelerations and forces (in general). Inertial mass can refer to a ratio of the force (which does not necessarily associate with gravity) that acts on the object to the acceleration that the object exhibits (because of the force). We use the four-word phrase not necessarily gravitational mass to denote notions of mass that do not necessarily associate with passive gravitational mass or with active gravitational mass. Inertial mass provides an example of not necessarily gravitational mass. Equation (112) might point to an example of not necessarily gravitational mass that differs from both passive gravitational mass and active gravitational mass.

We engage with the topic of total energy. Total energy associates with UTA aspects of UNI modeling. UTA modeling for total energy exhibits parallels to USA modeling for passive gravitational energy.

Table 13 shows UTA aspects of UNI modeling. Table 13 is a UTA analog to the USA centric table 12. For table 13b, the column labeled symmetry is compatible with applying - starting with $SU(7)$ - equation (32) twice. The symbol k_B denotes the Boltzmann constant. The symbol T denotes temperature.

We discuss notions regarding table 12 and table 13.

Table 14 brings together aspects regarding oscillators $zTA0$ and $zSA0$, for ENT modeling and for UNI modeling.

For each of table 12 and table 13, the appearances (in a row) of the word additivity and of the symmetry $U(1)$ pair with a conservation law. Conservation of energy is an example of such a conservation law.

Modeling - for an object - that associates with a change in minimal (or, ground state) total energy associates with a change of object or with a change of model.

Table 15 places, in one framework, various physics constants. The constants include masses (for

Table 12: USA symmetries

(a) Some system properties that associate with classical physics

System property	Trio	USA	Note	Symmetry
-	-	0	Not necessarily applicable, unless the system consists of just one object	$U(1)_b \times S1g$
Charge	3 signs	1-2, 15-16	Additivity pertains for each sign	$U(1) \times SU(4)$
Minimal passive gravitational energy	-	3-6	Scalar quantity	$SU(4)$
Minimal angular momentum	3 axes	7-10	Additivity pertains for each axis	$U(1) \times SU(4)$
Minimal momentum	3 axes	11-14	Additivity pertains for each axis	$U(1) \times SU(4)$

(b) Some object properties that associate with proposed modeling

Object property	Trio	USA	Note	Symmetry
Nonzero / zero property choice (charge for elementary fermions, mass otherwise)	-	0	Associates with a binary choice	$U(1)_b$
Charge	3 signs	1-2	Additivity pertains for each sign	$U(1) \times SU(2)$
Passive gravitational energy	-	3-4	Associates with a scalar quantity and with six isomers of PR1ISP (and with PR6ISP models)	$U(1) \times SU(2)$
Generation (for elementary fermions)	one, two, three	5-6	Three values of freeable energy	$SU(2)$
Freeable passive gravitational energy (any object)	3 spatial dimensions	5-6	Associates with a scalar quantity (The symmetry might not be relevant.)	$SU(2)$
Angular momentum (classical physics)	3 axes	7-8	Additivity pertains for each axis	$U(1) \times SU(2)$
Freeable angular momentum (classical physics)	3 axes	9-10	Three axes of freeable angular momentum	$SU(2)$
Quantized unit of angular momentum exchange (some models)	3 spatial dimensions	9-10	Associates with a scalar quantity (The symmetry might not be relevant.)	$SU(2)$
Momentum	3 axes	11-12	Additivity pertains for each axis	$U(1) \times SU(2)$
Boost symmetry (special relativity)	3 axes	13-14	Specific to special relativity	$SU(2)$
Non-quantized charge (some models)	-	15-16	Associates with a scalar quantity (The symmetry might not be relevant.)	$SU(2)$
Quantized unit of charge exchange (some models)	3 spatial dimensions	15-16	Associates with a scalar quantity (The symmetry might not be relevant.)	$SU(2)$
Magnitude of one unit of nonzero charge divided by the magnitude of the charge of an electron	1, 2/3, 1/3	15-16	Allows for quarks	$SU(2)$
Change (other than rotation) over time (some models)	-	15-16	Associates with a scalar quantity (The symmetry might not be relevant.)	$SU(2)$

Table 13: UTA symmetries

(a) Some system properties that associate with classical physics

System property	Trio	UTA	Note	Symmetry
-	-	0	Not necessarily applicable, unless the system consists of just one object	$U(1)_b \times S1g$
-	-	1-2	Not applicable, unless the system consists of just one elementary fermion that interacts with color charge	$SU(2)$
Minimal (or, ground state) total energy	-	3-6	Additivity pertains regarding the scalar quantity	$U(1) \times SU(4)$

(b) Some object properties that associate with proposed modeling

Object property	Trio	UTA	Note	Symmetry
Property choice (whether an elementary particle or other object can model as not entangled)	-	0	Associates with a binary choice	$U(1)_b$
Color charge	red, blue, green	1-2	Associates with a three-fold choice	$SU(2)$
Total energy (any object)	-	3-4	Associates with a scalar quantity and with six isomers of PR6ISP (and with PR36ISP models)	$U(1) \times SU(2)$
Temperature (thermodynamics, when applicable)	3 DoF	5-6	$(1/2)k_BT$ per DoF (or, degree of freedom)	$SU(2)$
Freeable total energy (any object)	3 spatial dimensions	5-6	Associates with a scalar quantity (The symmetry might not be relevant.)	$SU(2)$

Table 14: Aspects associating with oscillators $zTA0$ and $zSA0$, for ENT modeling and for UNI modeling

Object	Parameters ($z = E$ or U)	Note
Any	$n_{zTA0} = 0$	The model associates with some notions of no entanglement.
Any	$n_{zTA0} = -1$	The model associates with entanglement.
Elementary particle	$n_{zTA0} = 0$, $n_{zSA0} = -1$	In a vacuum, the object travels at the speed of light. (The minimal passive gravitational energy equals zero.)
Elementary fermion	$n_{zSA0} = -1$	The object has zero charge.
Elementary boson	$n_{zSA0} = -1$	The object has zero mass.

Table 15: Modeling that catalogs four types of physics constants - masses, $\hbar$, c, and q_e

(a) Catalog that includes four types of physics constants - masses, $\hbar$, c, and q_e

M_EAotO	Basic	λ	Subtlety	λ	Example re subtlety (freeable, plus ...)
passive gravitational	USA:3-4	4	USA:5-6	6	Δ of mc^2- e.g., regarding Δ of generation for an elementary fermion
angular momentum	USA:7-8	8	USA:9-10	10	$\hbar$ - quantum of exchange of angular momentum
momentum	USA:11-12	12	USA:13-14	14	c - speed of light and speed of gravity
momentum	USA:11-12	12	USA:13-14	14	$p \leq E/c$ for each object
electromagnetic	USA:1-2	2	USA:15-16	16	$\lvert q_e \rvert$ - quantum of exchange of charge

(b) Notes

Note
• The construct M_EAotO abbreviates the seven-element phrase minimal _ energy aspect of the object.
• A symbol λ associates with the column immediately preceding the column in which the symbol appears.
• The symbol Δ denotes the word change.

example, of elementary particles), $\hbar$, c, and q_e.

We mention some aspects - that this essay de-emphasizes discussing further - of proposed modeling UNI modeling. The aspects seem to link to aspects of extant modeling KIN models. The aspects might pertain also for proposed modeling ENT modeling.

Double-entry bookkeeping and the possibilities that oscillators $zTA16$, $zTA15$, ..., and $zTA7$ have relevance might point to KIN modeling involving stress-energy tensors. Here, z can be U and possibly can be E. The ten otherwise seemingly unused oscillators might associate with the ten independent aspects of 16-element (or, four-by-four) stress-energy tensors. (General relativity features 16-element stress-energy tensors with ten independent aspects.) Cases might feature the following. Models have bases in non-isotropic harmonic oscillators. Non-isotropic pertains regarding (at least) oscillators $zTAj$ for which $j = 0$ or $7 \leq j \leq 16$. The parameter n_{zTA0} satisfies $-1 < n_{zTA0} < 0$.

Similarly, one might consider modeling based on $-1 < n_{zSA0} < 0$ and oscillators $zSA13$ and $zSA14$. Modeling could link to, for example, KIN modeling regarding photon modes in the cavity of a laser. Modeling could link, also, to KIN modeling regarding photons in situations for which the index of refraction is not one. For each example, boost symmetry would not pertain. (Compare with table 12b.)

2.1.3. Gravitational properties

We discuss gravitational properties of objects. For example, we explore aspects related to components of 4G. (See, for example, 4G4, 4G48, and so forth in table 7.)

We explore PR1ISP modeling.

We discuss adjustments - to the strength of 4G4 - to which table 7a alludes. Data about the rate of expansion of the universe seems to support some of the adjustments. (See table 35.) Modeling regarding the masses of some elementary bosons might echo some of the adjustments. (See discussion regarding equation (53).)

Table 16 discusses some aspects regarding the strength of gravitation and some components of 4γ plus 2γ.

Proposed modeling suggests that the results that table 16 shows pertain for KIN Newtonian modeling. We posit that these results are compatible with extant modeling KIN general relativity modeling.

We explore PR6ISP modeling.

We consider some related thought experiments.

We consider three cases regarding a non-rotating, spherically symmetric ordinary matter star. Each case involves the idealization of a small, non-rotating, spherically symmetric planet. In the first case, the planet includes only ordinary matter. In the second case, the planet includes only the isomer (other than isomer zero) for which 4(2)G48 intermediates repulsion regarding ordinary matter. In the third case, the planet includes only one of the other four isomers. In each case, the planet starts at the same point (relative to the star) and with the same velocity. The orbits of the three planets are identical. (One might say the following. With respect to 4(6)G4, the planets behave identically.)

Table 16: Aspects regarding the strength of gravitation and some components of 4γ plus 2γ

Component and aspect
• 4G48: We consider a thought experiment in which a first object has a spherically symmetric distribution of matter and has no angular momentum. A second object has the same spherically symmetric distribution of the same matter and has some angular momentum. The second object uses more (than does the first object) freeable energy to maintain its shape. (Without use of that energy, the second object would bulge near its equator and flatten near its poles.) A lesser amount of freeable energy associates with a lesser amount of passive gravitational energy. (See discussion regarding table 12b. Also, perhaps, note a parallel to equation (53).) The first object does not exhibit a 4G48 component of passive gravitational rest energy. The second object exhibits a 4G48 component of passive gravitational rest energy. 4G48 associates with a repulsive component that detracts from attraction that associates with 4G.
• 4G246: We consider a thought experiment in which a first object has a spherically symmetric distribution of matter and has no angular momentum. A second object has a non-spherically symmetric distribution of the same matter and has no angular momentum. The second object has more (than does the first object) freeable energy. (The second object would - during a transition to having the shape of the first object - lose freeable energy. A greater amount of freeable energy associates with a greater amount of passive gravitational energy. See discussion regarding table 12b.) The first object does not exhibit a 4G246 component of passive gravitational rest energy. The second object exhibits a 4G246 component of passive gravitational rest energy. 4G246 associates with an attractive component that augments attraction that associates with 4G.
• 4G246⟦16⟧: We consider a thought experiment in which a first object has a distribution of matter and does not exhibit changes over time. A second object has the same distribution of the same matter and exhibits changes over time. The second object has more (compared to the first object) freeable energy. (The second object would - during a transition to having the characteristics of the first object - lose freeable energy. A greater amount of freeable energy associates with a greater amount of passive gravitational energy. See discussion regarding table 12b.) The first object does not exhibit a 4G246⟦16⟧ component of passive gravitational rest energy. The second object exhibits a 4G246⟦16⟧ component of passive gravitational rest energy. 4G246⟦16⟧ associates with an attractive component that augments attraction that associates with 4G.
• 4G2468a and 4G2468b: We consider a thought experiment in which a first object has a non-spherically symmetric distribution of matter and has no angular momentum. A second object has the same non-spherically symmetric distribution of the same matter and has some angular momentum. The second object uses more (than does the first object) freeable energy to maintain its shape. A lesser amount of freeable energy associates with a lesser amount of passive gravitational energy. (See discussion regarding table 12b.) 4G2468a and 4G2468b associate with repulsive components that detract from attraction that associates with 4G.
• 4G2468⟦16⟧: We consider a thought experiment in which a first object has a distribution of matter, perhaps has some angular momentum, and does not change over time. A second object has the same distribution of the same matter, has the same angular momentum, and exhibits changes over time. The second object has more (compared to the first object) freeable energy. (The second object would - during a transition to having the characteristics of the first object - lose freeable energy. A greater amount of freeable energy associates with a greater amount of passive gravitational energy. See discussion regarding table 12b.) The first object does not exhibit a 4G2468⟦16⟧ component of passive gravitational rest energy. The second object exhibits a 4G2468⟦16⟧ component of passive gravitational rest energy. 4G2468⟦16⟧ associates with an attractive component that augments attraction that associates with 4G.
• 2G2: We consider a thought experiment in which a first object has a spherically symmetric distribution of matter and has no charge. A second object has the same spherically symmetric distribution of the same matter and has some net charge. The second object uses more (than does the first object) freeable energy to maintain its net charge. (Without use of that energy, the charge would repel itself and the object would bulge outward.) A lesser amount of freeable energy associates with a lesser amount of passive gravitational energy. (Perhaps, note a parallel to equation (53). Perhaps, also, consider solutions - to the Einstein field equations - regarding a spherically symmetric non-rotating charged object.) Net charge associates with a repulsive component that detracts from attraction that associates with 4G.

We vary the three original cases. We assume that the star rotates. Based on 4(2)G48, the following notions pertain. For each of case one and case two, the orbit of the planet changes. Across case one and case two, the orbits are identical. For case three, the orbit matches the orbit pertaining to the cases in which the star does not rotate.

We vary the three original cases. We assume that the star is not spherically symmetric. We assume that the star does not rotate. Based on 4(1)G246, the following notions pertain, relative to the cases in which the star is spherically symmetric and does not rotate. For case one, the orbit of the planet changes. For each of cases two and three, the orbit of the planet does not change.

The thought experiments do not mention the passive gravitational masses of the planets.

The thought experiments suggest an analogy between electromagnetism and gravity. In extant modeling, the effect - on an object - of the (externally generated or active) electromagnetic field depends on properties (such as charge) of the object. In proposed modeling, the effect - on an object (in the cases of the thought experiments, the planets) - of the (externally generated or active) gravitational field depends on properties (such as isomer or isomers) of the object.

We suggest that such dependence on isomer does not disturb results of core extant modeling. Possibly, so far, direct tests regarding general relativity have yet to explore adequately the motions of objects other than isomer zero objects. For example, the bending, by the sun, of paths of light involves only isomer zero phenomena. For another example, the bending of light by a galaxy cluster involves only isomer zero photons. Here, the source of gravity that causes the bending might have roughly equal populations of stuff correlating with each of the six isomers.

We suggest that such dependence on isomer points to opportunities to look again at aspects of unverified extant modeling. Attempts to explain the recent (few billion years of) increases in the rate of expansion of the universe fall in the category of unverified extant modeling. (Proposed physics suggests relevance of 4(2)G48. The span of an instance of 4(2)G48 is two, not one or six.)

We explore PR36ISP modeling. One instance of 4(6)G intermediates interactions between ordinary matter and (itself plus) five dark matter isomers. Ordinary matter does not interact via 4G with 30 isomers.

We discuss terminology.

Perhaps, people will find useful the following notions. People can associate the three-word term passive gravitational energy with just the 4G4 component of 4G. (Doing so would have similarities to associating the word charge with 2G2 and associating other terms, such as magnetic moment, with components - other than 2G2 - of 2G.) People can develop other terms (or use notation of the form 4GΓ) for other aspects of passive gravitational effects. Then, people can re-explore uses for terms such as the three-word term active gravitational energy.

2.1.4. Perspective about modeling, objects, and properties

This essay uses notions that the essay discusses regarding modeling, objects, and properties. Seemingly, those notions suffice for the purposes of this essay. Such purposes include explaining and predicting some data about elementary particles, dark matter, some aspects of cosmology, and some aspects of astrophysics.

We think that the notions have more applications than the applications for which this essay uses the notions.

We suggest that, ultimately, aspects of the notions, proposed modeling in general, and extant modeling point to useful opportunities to rethink physics notions regarding modeling, objects, properties, entanglement, and related topics. For example, from this perspective, the notion of entanglement might apply broadly and the notion of distinct objects might decrease in usefulness. People might want to rethink notions of passive gravitational mass and active gravitational mass. For another example, given aspects such as components of - and spans of components of - G-family elementary particles, people might rethink the notion of elementary particle.

2.2. Elementary particles and dark matter

Table 17 previews elementary particles that proposed modeling suggests. Table 17 alludes to all known elementary particles and to elementary particles that proposed modeling suggests. Elsewhere, we depict some aspects regarding subfamilies. (See figure 3.)

Table 18 explores the following analogy. Elementary particle is to subfamily as atom is to chemical element.

Discussion related to table 28 provides details about proposed modeling regarding dark matter. Table 29 alludes to data - related to dark matter - that proposed modeling seems to explain. (For more details,

Table 17: Known and proposed elementary particles

(a) Known and proposed elementary particles (with SM denoting known or Standard Model; with PM denoting proposed or proposed modeling; with (Di) denoting the seven-word phrase if the particles model as Dirac fermions; with (Ma) denoting the seven-word phrase if the particles model as Majorana fermions; and with TBD denoting the three-word phrase to be determined)

Description	Sub-family	Spin	Models as free or entangled	Mass	Number of zero-charge particles (includes anti-particles)	Number of charged particles (includes anti-particles)	Number of modes	Status
Higgs boson	0H	0	Free	>0	1	0	-	SM
Aye	0I	0	Entangled	=0	1	0	-	PM
Charged leptons	1C	1/2	Free	>0	0	6	-	SM
Neutrinos	1N	1/2	Free	>0	6(Di) or 3(Ma)	0	-	SM
Quarks	1Q	1/2	Entangled	>0	0	12	-	SM
Arcs	1R	1/2	Entangled	>0	6(Di) or 3(Ma)	0	-	PM
Weak interaction bosons	2W	1	Free	>0	1	2	-	SM
Gluons	2U	1	Entangled	=0	8	0	-	SM
Jay	2J	1	Entangled	=0	1	0	-	PM
Photon	2G	1	Free	=0	-	-	2	SM
Graviton	4G	2	Free	=0	-	-	2	PM
TBD	6G	3	Free	=0	-	-	2	PM
TBD	8G	4	Free	=0	-	-	2	PM

(b) Notes regarding items designated as PM in table 17a

Item	Note
0I	Aye (or, inflaton) - would be a zero-mass analog to the Higgs boson; might have a role during the inflationary epoch
1R	Arcs - would be zero-charge fermions; would be analogs to quarks; might be components of (dark matter) hadron-like particles
2J	Jay - would be a zero-mass spin-one boson; might have a role before inflation; might associate with the Pauli exclusion force
4G	Graviton - would be a zero-mass spin-two boson; might associate with extant modeling notions regarding quantum gravity
6G	Name to be determined - would be a zero-mass spin-three boson; might associate with observations which people interpret as implying that there are at least two distinct rest energies for neutrinos
8G	Name to be determined - would be a zero-mass spin-four boson

Table 18: An analogy regarding modeling for elementary particles and modeling for atoms (with PM denoting proposed modeling)

An elementary particle models as ...	An atom models as ... (with ((...)) denoting a PM suggestion regarding extant modeling)
• Associating with a subfamily	• Associating with a chemical element
• Associating with a specific PM isomer of simple particles	• ((Associating with a specific PM isomer of simple particles))
• Being - or not being - entangled	• Being - or not being - part of a molecule or other structure
• Having a specific charge	• Having a specific charge
• Having a specific mass	• Having a specific mass
• Having a specific spin state	• Having a specific spin state
-	• Associating with a specific nuclear isotope
-	• Associating with a specific (nuclear) isomer of the isotope
• (If it is a fermion,) having a specific generation	-

see table 36.) Elsewhere, we depict some aspects regarding dark matter and ordinary matter. (See figure 5.)

2.2.1. Elementary particles

We show a method for matching known elementary particles and suggesting new elementary particles. We use the method. We suggest elementary particles that people have yet to find.

Table 19 previews aspects of our work to match and suggest elementary particles. In table 19a, the leftmost five columns show representations for subfamilies. Each representation satisfies double-entry bookkeeping. The column with label $\Sigma\Phi$ shows the subfamily that pertains. (Regarding 1Q - or, quarks - the table devotes one row to each of the two magnitudes of charge.) Table 19b defines acronyms that table 19a uses. Some of the acronyms echo aspects of table 11. Table 11 also defines the symbol SCEFGE (or, the five-word term somewhat conservation of fermion generation). In table 19a in the column labeled EFCC, 0,0 denotes the $SU(2)$ symmetry that associates with the number of color charges being three. (See table 13b.) The column labeled with the three-element phrase Σ_T relates to provides information about the two oscillators to which the column labeled with the one-element symbol Σ_T alludes. (See table 19b.) For each subfamily $\Sigma\Phi$ for which $\Sigma \geq 1$, the oscillator pair relevant for the column labeled Σ_S is SA($\Sigma - 1$)-and-SAΣ. The column with the one-word label measures points - for interactions in which one elementary fermion enters, one elementary fermion leaves, and one elementary boson either (but not both) enters or leaves - to a property of the elementary fermion that the interaction measures. We assume that the notion of measures implies the notion of conserved. The column with the one-word label breaks points to conservation laws with which the interaction does not necessarily conform. Discussion below develops the information - including the information in the column with the one-word label note - that table 19a shows. (Extant modeling might consider some of the items in that column to be internal symmetries.)

We define four symbols. (See table 19a.) The symbol $ETAe$ denotes the even-numbered oscillator that associates with Σ_T. The symbol $ETAo$ denotes the odd-numbered oscillator that associates with Σ_T. The relationship $o = e - 1$ pertains. Similarly, the following pertain. The symbol $ESAo$ denotes the odd-numbered oscillator that associates with Σ_S. The symbol $ESAe$ denotes the even-numbered oscillator that associates with Σ_S. The relationship $o = e - 1$ pertains.

We review proposed modeling ENT models for the photon. We note an association between proposed modeling ENT models and the extant modeling elementary particle Standard Model.

Table 2 pertains. Aspects related to oscillator $ETA0$ associate with the extant modeling Standard Model notion that a $U(1)$ symmetry pertains regarding the photon.

We discuss proposed modeling ENT models for the weak interaction bosons.

Each of the Z and W bosons has nonzero mass. Three spin states can pertain. Regarding KIN modeling, equation (46) pertains. The ENT equation (47) pertains. We extend work regarding 2G. We associate $ESAo$ with left circular polarization. We associate $ESAe$ with right circular polarization. We associate $ESA0$ with longitudinal polarization.

Table 19: Representations for elementary particle subfamilies

(a) Representations for subfamilies

EFCC	Σ_T	n_{aTA0}	n_{aSA0}	Σ_S	$\Sigma\Phi$	Σ_T relates to	Measures	Breaks	Note
-	-	0	-1	0,0	2G	-	EFCH	-	$U(1)$
-	0,0	0	0	0,0	2W	EFCH	-	CEFCH, CEFGE	$SU(2) \times U(1)$
-	-	0	0	-	0H	-	-	-	
-	-	-1	-1	-	0I	-	-	-	
-	$-1, -1$	-1	-1	$-1, -1$	2U	EFCC	-	CEFCC	$SU(3)$
-	$-1, -1$	-1	-1	$-1, -1$	2J	EFIS	EFIS	-	$SU(2) \times U(1)$
-	$\pi_{0,-1}$	0	0	$\pi_{0,-1}$	1C	EFCH	-	-	
-	$-1, -1$	0	-1	$\pi_{0,-1}$	1N	EFCH	-	-	
0,0	$\pi_{0,-1}$	-1	0	$\pi_{0,-1}$	$1\mathrm{Q}^{\lvert 2/3 \rvert}$	EFCH	-	-	
0,0	$\pi_{0,-1}$	-1	0	$\pi_{0,-1}$	$1\mathrm{Q}^{\lvert 1/3 \rvert}$	EFCH	-	-	
0,0	$-1, -1$	-1	-1	$\pi_{0,-1}$	1R	EFCH	-	-	
-	-	0	-1	0,0	4G	-	EFMA	CEFGE	
-	-	0	-1	0,0	6G	-	EFGE		
-	-	0	-1	0,0	8G	-	EFAM		

(b) Acronyms

Symbol	Explanation	Relevant λ or λ_T	Conservation law
EFCC	Elementary fermion color charge	TA2	CEFCC
EFCH	Elementary fermion charge	SA2 and SA16	CEFCH
EFIS	Elementary fermion isomer	SA4 and SA6	CEFIS
EFMA	Elementary fermion mass	SA4	CEFMA
EFGE	Elementary fermion generation	SA6	CEFGE
EFAM	Elementary fermion angular momentum	SA8 and SA10	CEFAM

(c) Notes regarding interactions for which one elementary fermion enters, one elementary fermion leaves, and one elementary boson either (but not both) enters or leaves

Conserved fermion property	Notes
Charge	CEFCH implies CEFIS
Mass	CEFMA can conflict with CEFGE
Generation	CEFGE can conflict with CEFMA, but not for 6G

$$n_{KSA0} = 0,\ n_{KSA1} = 0,\ n_{KSA2} = 0 \tag{46}$$

$$n_{ESA0} = 0,\ n_{ESAo} = 0,\ n_{ESAe} = 0 \tag{47}$$

Double-entry bookkeeping suggests that equation (48) pertains. We associate n_{ETAe} with the W$^+$ boson and with positive charge. We associate n_{ETAo} with the W$^-$ boson and with negative charge. We associate n_{ETA0} with the Z boson and with zero charge. Equation (49) pertains for ground states.

$$\{ETAj\} = \{ETAe, ETAo, ETA0\} \tag{48}$$

$$n_{ETA0} = 0,\ n_{ETAo} = 0,\ n_{ETAe} = 0 \tag{49}$$

We discuss a thought experiment that associates with the extant modeling notion of an excitation of one W$^-$ boson during an isolated interaction that converts an electron into a neutrino. Proposed modeling suggests modeling in which - for the W$^-$ boson - the $ETAo$ oscillator excites by one unit and one of the three $ESAj$ oscillators excites by one unit. The four other oscillators do not excite. These four oscillators associate with two $SU(2)$ symmetries.

We discuss a thought experiment that associates with extant modeling notions of CP violation within a hadron. Extant modeling considers the production of two virtual W bosons. Exciting once each of a W$^+$ and a W$^-$ associates - regarding proposed modeling ENT models - with an ETA factor of one (or, $(1+0)^{1/2} \cdot (1+0)^{1/2}$). (See equation (2).) Raising one $ESAj$ by two units would produce a factor of $2^{1/2}$ (or $(1+0)^{1/2} \cdot (1+1)^{1/2}$). The mismatch between one and $2^{1/2}$ violates double-entry bookkeeping. Double-entry bookkeeping suggests that the ESA result should feature - for some $j \neq k$ - $n_{ESAj} = 1$ and $n_{ESAk} = 1$. We let l denote the one relevant integer that satisfies $l \neq j$ and $l \neq k$. Only $ETA0$ and $ESAl$ remain relevant regarding relevant symmetries. One $SU(2)$ symmetry pertains. (Compare with the two $SU(2)$ symmetries for isolated interactions involving charged leptons.) CEFGE does not pertain.

Overall, for interactions involving W bosons, SCEFGE pertains. (For non-isolated interactions involving leptons, SCEFGE pertains. CEFGE does not necessarily pertain.)

Aspects related to oscillators $ETAe$, $ETAo$, and $ETA0$ associate with the extant modeling Standard Model notion that an $SU(2) \times U(1)$ symmetry pertains regarding the weak interaction bosons. From the ground state and for any j such that $j \in \{e, o, 0\}$, an excitement of n_{ETAj} associates a $U(1)$ symmetry with oscillator $ETAj$. An $SU(2)$ symmetry associates with the other two $ETAk$ oscillators.

We discuss proposed modeling ENT models for the Higgs boson.

Proposed modeling interpretation of extant modeling for the Higgs boson associates with the set $\{KSAj\}$ having one member - $KSA0$. Longitudinal polarization and nonzero mass pertain. Circular polarization does not pertain.

Proposed modeling ENT models use the notion that excitation associates with the oscillator pair $ETA0$-and-$ESA0$. For a ground state, $n_{ETA0} = n_{ESA0} = 0$. Unlike for the 2W bosons, for the Higgs boson, there is only one relevant particle and only one relevant polarization. CEFGE pertains.

We discuss proposed modeling ENT models for the aye boson.

ENT modeling for the aye boson reflects ENT modeling for the Higgs boson. For the aye boson, $n_{ETA0} = -1$ and $n_{ESA0} = -1$ pertain for the ground state. Excitation associating with n_{ETA0} can occur in entangled environments. The CEFGE aspects that pertain for the Higgs boson pertain for the aye boson.

We discuss proposed modeling ENT models for gluons.

The following notions associate with modeling for the ground state of gluons. The expression $n_{ESA0} = -1$ associates with zero mass. The expressions $n_{ESAo} = -1$ and $n_{ESAe} = -1$ pertain. We invoke double-entry bookkeeping. The expressions $n_{ETAe} = -1$, $n_{ETAo} = -1$, and $n_{ETA0} = -1$ pertain. For each j, $ETAj$ associates with a color charge.

Based on the notion of entangled environment, oscillators $ESAo$ (left circular polarization) and $ESAe$ (right circular polarization) can excite. Modeling for each possible excitement preserves $n_{ESA0} = -1$. We invoke double-entry bookkeeping. We consider phenomena pertaining to one interaction vertex. Modeling regarding absorption (by a gluon) of a unit of color charge and depositing (by the gluon) of a unit of (possibly different) color charge preserves one $n_{ETAj} = -1$. A corresponding $ETAj$-and-$ESA0$ $SU(2)$ symmetry pertains. The lack of a second $SU(2)$ symmetry associates with the notion that CEFCC does not pertain.

Aspects related to oscillators $ETAe$, $ETAo$, and $ETA0$ associate with the extant modeling Standard Model notion that an $SU(3)$ symmetry pertains regarding gluons.

We discuss proposed modeling ENT models for the jay boson.

The following notions associate with modeling for the ground state of the jay boson. The expression $n_{ESA0} = -1$ associates with zero mass. The expressions $n_{ESAo} = -1$ and $n_{ESAe} = -1$ pertain. We invoke double-entry bookkeeping. The expressions $n_{ETAe} = -1$, $n_{ETAo} = -1$, and $n_{ETA0} = -1$ pertain.

Aspects related to oscillators $ETAe$, $ETAo$, and $ETA0$ parallel similar aspects regarding the weak interaction bosons. An $SU(2) \times U(1)$ symmetry pertains regarding the jay boson. (Perhaps note, regarding table 12b and table 19a, that $U(1) \times SU(2)$ equals $SU(2) \times U(1)$.)

We explore the topic of the properties with which the jay boson interacts. We suggest that the symmetry of $SU(2) \times SU(1)$ pertains and associates - for PR6ISP modeling - with six isomers. We suggest that one property with which the jay boson interacts is isomer. The span of 2G2 is one. (See table 9a.) Each isomer of simple particles associates, in effect, with its own isomer of charge. We suggest that - regarding properties with which the jay boson interacts - the jay boson differentiates between isomers of charge. We suggest that - regarding PR36ISP modeling - the jay boson differentiates between six isomers of charge and between six isomers of mass.

Discussion just above suggests the possibility of one jay boson with two modes. (Compare with the representation, in table 2, for the photon.) For this case, oscillator $ESA0$ does not excite. Discussion regarding the 0I and 2U bosons might suggest that modeling for the jay boson might embrace the notion that oscillator $ESA0$ can excite. For this case, there would be one particle with three spin states. (This essay does not make a selection among these two - and possibly other - cases.) Here and elsewhere - we use wording that assumes that there is just one jay boson.

The symbol $2\mathrm{J}_o$ associates with left circular polarization. The symbol $2\mathrm{J}_e$ associates with right circular polarization. The symbol $2\mathrm{J}_0$ associates with the possibility of nonzero longitudinal polarization. This essay continues to discuss the notion of $2\mathrm{J}_0$. Seemingly, proposed modeling results that this essay shows do not depend on $2\mathrm{J}_0$ being physics relevant.

We discuss proposed modeling ENT models for charged leptons.

ENT modeling for charged leptons reflects ENT modeling for weak interaction bosons. An electron has negative charge. Modeling uses $n_{ETAe} = -1$ and $n_{ETAo} = 0$. Regarding one of the two possible spin states, $n_{ESAo} = 0$, and $n_{ESAe} = -1$. The $ETAe$-and-$ESAe$ oscillator pair associates with an $SU(2)$ symmetry. The three generators of $SU(2)$ associate with three generations. (For this spin state, equation (25) might seem to pertain explicitly.) Regarding the other one of the two possible spin states, $n_{ESAo} = -1$, and $n_{ESAe} = 0$. The $ETAe$-and-$ESAo$ oscillator pair associates with an $SU(2)$ symmetry. The three generators of $SU(2)$ associate with three generations. (For this spin state, a notion similar to equation (25) might seem to pertain implicitly.) This modeling associates with the electron, muon, and tauon. A swap featuring $n_{ETAe} \leftrightarrow n_{ETAo}$ leads to modeling for the three respective antiparticles.

We discuss proposed modeling ENT models for neutrinos.

ENT modeling for neutrinos reflects ENT modeling for charged leptons. Neutrinos have zero charge. The expression $n_{ETAe} = n_{ETAo} = n_{ESA0} = -1$ associates with zero-charge. Double-entry bookkeeping suggests that one of $n_{ESAo} = -1$ and $n_{ESAe} = -1$ pertains. The choice of $n_{ESAo} = 0$ and $n_{ESAe} = -1$ comports with observations that suggest that (ordinary matter) neutrinos are left-handed. This essay does not recommend extents to which neutrinos model as Dirac fermions and as Majorana fermions. The case of Dirac fermions associates with six neutrinos. The case of Majorana fermions associates with three neutrinos, with each neutrino being its own antiparticle.

We discuss proposed modeling ENT models for quarks.

Compared to modeling for charged leptons, modeling for quarks changes n_{ETA0} from zero (which associates with the notion that a charged lepton can model as not entangled) to minus one (which associates with the notion that quarks model as entangled). Based on double-entry bookkeeping, we add (compared to models for charged leptons) an oscillator pair. (See the EFCC column in table 19a.) We set each of the corresponding two new n_{ETAj} to zero. The new oscillator pair associates with an $SU(2)$ symmetry and three generators. The three generators associate with three color charges. These notions associate with quarks for which the magnitude of charge is two-thirds of the charge of a positron. The same notions associate with quarks for which the magnitude of charge is one-third of the charge of a positron. For each magnitude of charge, swapping n_{ETAo} and n_{ETAe} associates with changing the sign of charge.

We discuss proposed modeling ENT models for arcs.

ENT models for arcs reflect ENT models for quarks. Arcs have zero charge. The expression $n_{ESA0} = -1$ associates with zero charge. The expression $n_{ETAe} = n_{ETAo} = -1$ associates with zero charge. The result satisfies double-entry bookkeeping. This essay does not recommend extents to which arcs model as Dirac fermions and as Majorana fermions. The case of Dirac fermions associates with six arcs. The

Table 20: Some associations between G-family elementary particles and some properties of objects

ΣG	Property of the object
2G	Charge.
4G	Passive gravitational energy (or, equivalently, passive gravitational mass).
6G	Generation (for elementary fermions). Also, freeable passive gravitational energy.
8G	Angular momentum.

case of Majorana fermions associates with three arcs, with each arc being its own antiparticle.

We discuss proposed modeling ENT models for G-family elementary particles.

Table 20 associates G-family elementary particles with some properties of objects. An interaction between a G-family elementary particle and an object might - in effect - measure the property of the object. For an interaction that does not change the object, the interaction does not change the property of the object. (Regarding an interaction that ionizes an atom, modeling generally associates with not leaving the atom intact.) Table 20 associates with and extends aspects of table 12. (The notion of Σ in ΣG in table 20 differs from notions of λ, such as in $USA\lambda$ in table 12.)

Proposed modeling suggests that 2G associates with extant modeling classical physics notions of electromagnetism. Proposed modeling suggests that 2G associates with extant modeling quantum physics notions of the photon. We are not aware of any evidence that photons associate with other than CEFMA and CEFGE. (See table 19b.)

Regarding 4G, 6G, and 8G, we associate Σ_S with SA($\Sigma - 1$)-and-SAΣ. (See discussion regarding table 19.) 4G associates with CEFMA. (See table 20.) 4G does not necessarily associate with CEFGE (or, conservation of fermion generation). 4G catalyzes neutrino oscillations. 6G associates with CEFGE. (See table 20.)

We discuss the possible completeness of the list of elementary particles to which table 19a alludes. Discussion related to table 23 suggests that each one of some non-G-family elementary bosons might associate with a ΣG solution for which $\Sigma = 0$. To the extent that each non-G-family elementary boson associates with a ΣG solution for which $\Sigma = 0$, the list of non-G-family elementary bosons to which table 19a alludes might be complete. (Mathematically, for $\Sigma > 8$, $\Sigma = 14$ is the least value of Σ for which 0G solutions exist.) The list of elementary fermions to which table 19a alludes might also be complete. Proposed modeling points to ΣG solutions for which $\Sigma \geq 10$. (See table 9a.) These solutions seem not to associate directly with properties that associate with $\lambda \geq 10$. (See table 12b.) This essay de-emphasizes - but does not entirely dismiss - the notion that people might want to associate some ΣG solutions for which $\Sigma \geq 10$ pertains with the notion of elementary particles.

Table 21 summarizes information regarding spans for simple particles, for hadron-like particles, and for some components of long-range forces. The table summarizes information regarding types of objects with which boson simple particles and some long-range force components interact. The table separates, based on a proposed modeling view, elementary particle Standard Model aspects from aspects that the elementary particle Standard Model does not embrace. The symbol 1Q⊗2U associates with known and possible hadrons. (See discussion regarding equation (108).) The symbol 1R⊗2U associates with possible hadron-like particles. (See discussion regarding equation (109).) Regarding the PR6ISP case, the pairings of isomers that isomers of 4G48 span might not equal the pairings of isomers that isomers of 2G68 span. The symbols †4G and †2G associate with this possible mismatch regarding pairings. Table 21 shows the extent to which each of the simple bosons and some of the long-range force components interacts directly with each of at least some simple fermions and with each of at least some multicomponent objects. The word yes denotes that interactions occur. The word no denotes that interactions do not occur. Proposed modeling suggests the possibility that neither the 0H boson nor the 0I boson interacts directly with multicomponent objects. Table 21c summarizes some concepts relevant to tables 21a and 21b.

The following proposed modeling notions seem to suggest that - for PR6ISP modeling - the isomer pairings 0-and-3, 1-and-4, and 2-and-5 pertain for each 4(2)GΓ solution and for each 2(2)GΓ solution. (See table 21c.) Isomer three - and not the other four dark matter isomers - echoes isomer zero relationships between masses of charged leptons and generation numbers for charged leptons. (See discussion related to table 32.) Of isomers one through five, possibly only isomer three has enough hydrogen atom like entities to explain data about some depletion of CMB (or, cosmic microwave background radiation). (See discussion related to table 34 and see discussion related to equation (133).) The possibility that case A - not case B - pertains regarding galaxy evolution. (See discussion related to table 40.) Nevertheless, this essay does not ignore other possibilities regarding pairings of isomers.

Table 22 shows the span for each component of G-family forces for which λ does not exceed eight

Table 21: Particles and solutions that associate with one isomer and particles and solutions that might associate with more than one isomer; plus, the extents to which simple bosons and some long-range force components interact with simple fermions and with multicomponent objects (with the symbol 1f+1b↔1f denoting interactions for which one elementary fermion enters, one elementary fermion exits, and one elementary boson either enters or exits; and with the symbol MCO denoting multicomponent objects)

(a) Particles

Standard Model entities	Possible entities	PRι_IISP span	1b interactions: 1f+1b↔1f	1b interactions with MCO
0H	0I	1	Yes	No
1C	-	1	-	-
1N	-	1	-	-
1Q	1R	1	-	-
2U	-	1	Yes	No
2W	-	1	Yes	No
-	2J	ι_I	Yes	Yes
1Q$\otimes$2U	1R$\otimes$2U	1	-	-
2G	-	(See table 21b.)	Yes	Yes
-	4G	(See table 21b.)	Yes	Yes
-	6G	(See table 21b.)	Yes	Yes
-	8G	(See table 21b.)	Yes	Yes

(b) Selected G-family components (with symbols of the form (†_) denoting aspects that table 21c discusses)

G-family component	PR1ISP span	PR6ISP span	PR36ISP span (†36)	1b interactions: 1f+1b↔1f	1b interactions with MCO
2G2	1	1	1	Yes	Yes
2G24	1	1	1	Yes	Yes
2G248	1	6	6	Yes	Yes
2G68	1	2 (†2G)	2	No	Yes
4G4	1	6	6	Yes	Yes
4G48	1	2 (†4G)	2	Yes	Yes
4G246	1	1	1	Yes	Yes
4G246⟦16⟧	1	? (†G4)	? (†G4)	Yes	Yes
4G2468a	1	1	1	Yes	Yes
4G2468b	1	1	1	Yes	Yes
4G2468⟦16⟧	1	6 (†G5)	36 (†G5)	Yes	Yes
6G6	1	2	2	No	Yes
6G468	1	6	6	Yes	Yes
8G8	1	1	1	No	Yes
8G2468a	1	1	1	Yes	Yes
8G2468b	1	1	1	Yes	Yes

(c) Notes regarding spans

Note
• (†36): For the case of PR36ISP modeling, $2(>1)\text{G}\Gamma$, and $4(>1)\text{G}\Gamma$, the span of $2(>1)\text{G}\Gamma$ is orthogonal to the span of $4(>1)\text{G}\Gamma$.
• (†2G) and (†4G): For PR6ISP modeling, the following notions pertain. For one of $4\text{G}\Gamma_4$ with a span of two and $2\text{G}\Gamma_2$ with a span of two (and for a numbering system that numbers isomers using the integers zero through five), the pairings 0-and-3, 1-and-4, and 2-and-5 might pertain. For the other one of the two ($4\text{G}\Gamma_4$ and $2\text{G}\Gamma_2$), different pairings might pertain.
• (†G4): For PR6ISP modeling, the span might be one or six. For PR36ISP modeling, the span might be one, might be six, or might be 36. (Perhaps note that six equals 288/48, which equals $g_{SU(17)}/g_{SU(7)}$. Perhaps, see discussion related to equation (118).) Possibly, $[\![16]\!]\in\Gamma$ implies that the span is ι_I. (This possibility might associate with the following notions. $\lambda = 16$ associates with $USA16$. $USA16$ associates with charge. Each isomer associates with its own isomer of 2G2 and, therefore, with its own isomer of charge.)
• (†G5): See discussion related to equation (115).

Table 22: A catalog of components of G-family forces for which $\Sigma \leq 8$ and λ does not exceed eight

(a) G-family force components for which $\Sigma \in \Gamma$, $\Sigma \leq 8$, and λ does not exceed eight (with r^{-k} associating with KIN modeling RSDF)

$\Sigma \in \Gamma$	S	GFC monopole	GFC dipole	GFC quadrupole	GFC octupole
Yes	1	2(1)G2 (r^{-2})	2(1)G24 (r^{-3})	2(6)G248 (r^{-3})	
Yes	2	4(6)G4 (r^{-2})	4(2)G48 (r^{-3})	4(1)G246 (r^{-4})	4(1)G2468a (r^{-5})
Yes	2				4(1)G2468b (r^{-5})
Yes	3	6(2)G6 (r^{-2})		6(6)G468 (r^{-3})	
Yes	4	8(1)G8 (r^{-2})			8(1)G2468a (r^{-4})
Yes	4				8(1)G2468b (r^{-4})

(b) G-family force components for which $\Sigma \notin \Gamma$, $\Sigma \leq 8$, and λ does not exceed eight

$\Sigma \in \Gamma$	S	GFC monopole	GFC dipole	GFC quadrupole	GFC octupole
No	1		2(6)G46	2(6)G468	
No	1		2(2)G68		
No	2		4(6)G26	4(6)G268	
No	3		6(1)G24	6(6)G248	
No	3		6(2)G28		
No	4		8(6)G26	8(1)G246	

and Σ does not exceed eight. (This essay de-emphasizes discussing the possible relevance - to G-family physics - of ΣG for which $\Sigma \geq 10$.) The table pertains for PR6ISP modeling and for PR36ISP modeling. Rows in table 22a list $\Sigma\gamma$ components. Table 22a lists 2(6)G248 and does not list 2(1)G248. Rows in table 22b list G-family force components that do not associate with $\Sigma\gamma$.

We discuss concepts regarding the 2(2)G68 solution and regarding interactions between dark matter and ordinary matter. Here, we assume that PR6ISP modeling comports with nature.

Elsewhere, we posit that 2(2)G68 associates with some electromagnetic (or, $\Sigma = 2$) interactions with atoms and other objects. (See discussion regarding table 9.) We posit that those interactions include hyperfine interactions.

Each of 2(1)G2 and 2(1)G24 associates with some electromagnetic (or, $\Sigma = 2$) interactions with atoms and other objects that include both baryons and leptons.

Unlike for the cases of electromagnetic interactions that associate with 2(1)G2 and 2(1)G24, 2G produced by ordinary matter objects interacts with non-ordinary-matter dark matter objects (for the case in which PR6ISP pertains to nature) via 2(2)G68. Unlike for the cases of electromagnetic interactions that associate with 2(1)G2 and 2(1)G24, 2G produced by some dark matter objects (for the case in which PR6ISP pertains to nature) interacts with ordinary matter via 2(2)G68.

We discuss other aspects that associate with table 7a and table 9.

Table 9 does not point to a G-family solution that would associate with an interaction with nonzero magnetic monopole moment. To the extent that proposed modeling adequately comports with nature, proposed modeling ENT modeling seems to suggest that nature does not exhibit magnetic monopole elementary particles.

Table 9 does not point to a G-family solution that would associate with a nonzero electric dipole moment for an object that does not feature - within the object - non-uniformity of charge. To the extent that an elementary particle models - with respect to KIN modeling - as having zero size, proposed modeling ENT modeling seems to suggest that the particle has zero electric dipole moment.

2.2.2. Properties of elementary bosons

We discuss the masses of elementary bosons.

We suggest that equation (50) comports with data. (For data, see reference [1].) The most accurately known of the masses is the mass of the Z boson. We use the nominal mass of the Z boson as a base for calculations. Regarding the Higgs and W bosons, the larger deviation from equation (50) associates with the 9 : 7 ratio. Equation (50) suggests a W boson mass that is about 3.4 standard deviations high with respect to the measured mass of the W boson.

$$(m_{\text{Higgs boson}})^2 : (m_{\text{Z}})^2 : (m_{\text{W}})^2 \ :: \ 17 : 9 : 7 \tag{50}$$

Table 23: Some relationships among all elementary bosons to which table 21a alludes

(a) Relationships between non-G-family elementary bosons and GFC items for which $\Sigma = 0$

$0\mathrm{G}\Gamma$	j_λ (for $[\![16]\!] \notin \Gamma$)	j_λ (for $[\![16]\!] \in \Gamma$)	Z_{S4}	Z_{T4}	Z_{S2}	Z_{S6}	Z_{S8}	Bosons	n_{aTA0}
0G2468	4	-	17	17	0	0	0	0H (or, Higgs)	$+1$
0G246 or 0G268	3	-	9	10	0	0	1	2W: Z	$+1$
0G268 or 0G246	3	-	7	10	2	0	1	2W: W	$+1$
$0\mathrm{G}\emptyset$	0	-	0	1	0	0	1	2J	-1
$0\mathrm{G}2468[\![16]\!]$	-	i	0	0	0	0	0	0I	-1
$0\mathrm{G}268[\![16]\!]$	-	0	0	1	0	0	1	2U	-1

(b) Notes regarding table 23a

Note
• In table 23a, i denotes a square root of minus one.
• For $[\![16]\!] \notin \Gamma$, the integer j_λ denotes the number of integers λ that appear in the Γ that associates with $0\mathrm{G}\Gamma$.
• Except regarding the column with the label Z_{S4}, each integer in the columns labeled with an expression of the form $Z...$ satisfies - for some k in the set $\{i$, 0, 1, 2, 3, or $4\}$ - the expression $k^2 + 1$.
• This essay does not fully address the topic of which of 0G246 and 0G268 associates with the Z boson. (The other of 0G246 and 0G268 associates with the W boson.)
• Regarding table 23a and table 12, the notion of Z_{S10} blends into the notion of Z_{S8} in a manner parallel to the blending of Z_{S16} into Z_{S2}.
• Regarding table 23a and table 12, a term (such as Z_{S12} or Z_{S14}) associating with momentum (or with motion) is not appropriate for the purposes of table 23a.

Discussion regarding table 5 alludes to $0\mathrm{G}\Gamma$ solutions. Within the constraints of $\Gamma \neq \emptyset$ and $\lambda \leq 8$, there are three $0\mathrm{G}\Gamma$ solutions - 0G2468, 0G246, and 0G268. Removing the constraint of $\Gamma \neq \emptyset$ admits the $0\mathrm{G}\emptyset$ solution. For each of the four solutions, we define j_λ to be the number of λ elements in Γ.

We use the notation and the expression that equation (51) shows. (This essay does not explore the extent to which Z_{T4} associates with $UTA4$.)

$$Z_{T4} = (j_\lambda)^2 + 1 \tag{51}$$

We use - for each of the values of λ of two, four, six, eight, and 16 - the notation $Z_{S\lambda}$. Passive gravitational energy associates with Z_{S4}. Freeable energy associates with Z_{S6}. Spin associates with Z_{S8}. Charge associates with one of Z_{S2} and Z_{S16}. In this essay, we assume (seemingly without inducing problems) that charge associates with Z_{S2} and that we can de-emphasize Z_{S16}. We assume that Z_{S2} is zero for zero-charge elementary bosons and is two for nonzero charge elementary bosons. We assume that Z_{S6} is zero for all elementary bosons. We assume that Z_{S8} is zero for zero-spin elementary bosons and is one for spin-one elementary bosons. We posit that equation (52) pertains for the 0H, 2W, and 2J bosons. We explore the notion that equation (53) shows. (The rightmost relationship follows from equation (52).)

$$Z_{T4} \approx Z_{S2} + Z_{S4} + Z_{S6} + Z_{S8} \tag{52}$$

$$m^2 \propto Z_{S4} \approx Z_{T4} - Z_{S2} - Z_{S6} - Z_{S8} \tag{53}$$

Table 23 shows modeling that interrelates all elementary bosons to which table 21a alludes. The first three rows of table 23a associate with equation (50) and equation (53). The first four rows of table 23a use equation (53). Each G-family boson has representation in (table 23a) via a corresponding $Z_{S\Sigma}$. The ordering of the columns (in table 23a) associating with $USA\Sigma$ aspects associates with the ordering of terms in equation (53). The one 0I boson represents a zero-mass association to the one 0H boson. (Compare with table 10.) The eight 2U bosons represent zero-mass associations to the two weak interaction bosons.

Table 24: Approximate rest energies (in MeV) for quarks and charged leptons (with the symbol q denoting charge)

M''	Legend	M'=3, $q = -1 \cdot \lvert q_e \rvert$	M'=2, $q = +(2/3) \cdot \lvert q_e \rvert$	M'=1, $q = -(1/3) \cdot \lvert q_e \rvert$
0	name	electron	up	down
0	data	$(0.511 \text{ to } 0.511) \times 10^0$	$(1.8 \text{ to } 2.7) \times 10^0$	$(4.4 \text{ to } 5.2) \times 10^0$
0	calculation	$m_e c^2 \approx 0.511 \times 10^0$	$m_u c^2 \approx 2.2 \times 10^0$	$m_d c^2 \approx 4.8 \times 10^0$
1	name		charm	strange
1	data		$(1.24 \text{ to } 1.30) \times 10^3$	$(0.92 \text{ to } 1.04) \times 10^2$
1	calculation		$m_c c^2 \approx 1.263 \times 10^3$	$m_s c^2 \approx 0.938 \times 10^2$
2	name	muon	top	bottom
2	data	$(1.06 \text{ to } 1.06) \times 10^2$	$(1.56 \text{ to } 1.74) \times 10^5$	$(4.15 \text{ to } 4.22) \times 10^3$
2	calculation	$m_\mu c^2 \approx 1.06 \times 10^2$	$m_t c^2 \approx 1.72 \times 10^5$	$m_b c^2 \approx 4.18 \times 10^3$
3	name	tauon		
3	data	$(1.777 \text{ to } 1.777) \times 10^3$		
3	calculation	$m_\tau c^2 \approx 1.777 \times 10^3$		

Table 23 associates with a notion that G-family solutions might point to all elementary bosons and, thus perhaps, to the notion that table 17 points to all elementary particles. (Note discussion - following on from equation (48) - that seemingly relates all elementary fermions to weak interaction bosons.)

2.2.3. Properties of elementary fermions

We discuss formulas that - based on the accuracy of measured quantities - predict a tauon mass that is consistent with and would be more accurate than the measured tauon mass.

Equation (54) shows an experimental result for the tauon mass, m_τ. (See reference [1].)

$$m_{\tau,\ \text{experimental}} \approx 1776.86 \pm 0.12 \text{ MeV}/c^2 \tag{54}$$

Equation (55) defines the symbol β'. Equation (56) defines β. Here, m denotes mass, e denotes electron, q denotes charge, ε_0 denotes the vacuum permittivity, and G_N denotes the gravitational constant. Equation (57) possibly pertains. Equation (57) predicts a tauon mass, which equation (58) shows. (For relevant data, see reference [1].) Eight standard deviations fit within one experimental standard deviation of the nominal experimental result. Equation (59) shows an approximate value of β that we calculate, using data that reference [1] shows, via equation (56).

$$\beta' = m_\tau / m_e \tag{55}$$

$$(4/3) \times \beta^{12} = ((q_e)^2/(4\pi\varepsilon_0))/(G_N (m_e)^2) \tag{56}$$

$$\beta' = \beta \tag{57}$$

$$m_{\tau,\ \text{calculated}} \approx 1776.8400 \pm 0.0115 \text{ MeV}/c^2 \tag{58}$$

$$\beta \approx 3477.1891 \pm 0.0226 \tag{59}$$

We discuss formulas that - based on the accuracy of measured quantities - fit the masses of the six quarks and three charged leptons.

Table 24 shows, regarding the rest energies of quarks and charged leptons, data that people report and numbers that we calculate via equation (62). Below, we discuss the table and the data before we discuss the equation and the calculations. Equation (62) results from fitting data. (Equation (62) provides - for elementary fermions - a somewhat analog to equation (53) for elementary bosons. For elementary fermions, a notion of $\log(m/m_{ref})$ - and not a notion of m^2 - pertains. The choice of a positive value of m_{ref} can be arbitrary. Equation (62) associates with $m_{ref} = m_e$. This essay does not show modeling that would generate equation (62).)

The data in table 24 reflect information from reference [1]. For each particle other than the top quark, reference [1] provides one estimate. For the top quark, reference [1] provides estimates associating with each of three bases. For each quark, table 24 shows a data range that runs from one standard deviation below the minimum nominal value that reference [1] shows to one standard deviation above the maximum nominal value that reference [1] shows. Each standard deviation associates with the reported standard deviation that associates with the nominal value. For charged leptons (that is, for $M' = 3$), the table does not completely specify accuracy regarding ranges.

The following concepts pertain regarding developing equation (62). Use of modular arithmetic in equation (64) anticipates uses of equation (62) that pertain to neutrino masses and that pertain regarding inferences about dark matter. The notion of $M'' = 3/2$ associates with modeling. (No elementary particle associates with $M'' = 3/2$.) Regarding equations (66) and (67), uses of $M' = 0$ anticipate uses of equation (62) that pertain to arc masses. Equation (60) produces a meaningful value for $m(3,1)$. (No elementary particle associates with $M'' = 3$ and $M' = 1$.) For each $0 \leq M'' \leq 2$, equation (61) produces a meaningful value of $m(M'', 3/2)$. (No elementary particle associates with $M' = 3/2$. The notion of $M' = 3/2$ associates with the average of $M' = 2$ and $M' = 1$ and associates with equation (61). Aspects of equations (62), (66), and (67) associate with the concept that $m(M'', 3/2)$ values have meaning. The concepts of $M' = 3/2$ and $m(M'', 3/2)$ are useful mathematically, though not necessarily directly relevant to physics.) Within each cluster of rows - in table 24 - for which $M'' \neq 3$, the fine-structure constant plays a role regarding linking the masses that pertain for that cluster of rows. (Aspects of equation (62) comport with this role.) Regarding equations (68), (69), and (70), we choose values that fit data. Regarding each charged lepton, our calculations fit data to more significant figures than the numbers in table 24 show.

$$m(3,1)m(3,2) = m(3,0)m(3,3) \tag{60}$$

$$(m(M'', 3/2))^2 = m(M'', 2)m(M'', 1) \tag{61}$$

The following concepts pertain regarding developing and using equation (62). We use equation (56) to calculate β. Equation (62) calculates the same value of m_τ that equation (58) calculates.

Equation (62) shows a formula that approximately fits the masses of the six quarks and three charged leptons. The formula includes two integer variables and seven parameters. One integer variable, M'', associates somewhat with generation. For the electron and each of the six quarks, the generation equals $M''+1$. For each of the muon and the tauon, the generation equals M''. The other integer variable, M', associates with magnitude of charge. The seven parameters can be m_e, m_μ (or, the mass of a muon), β, α, $d'(0)$, $d'(1)$, and $d'(2)$. The symbol α denotes the fine-structure constant. (See equation (63).) Here, $d'(k)$ pertains regarding generation-$(k+1)$ quarks. For each generation, the number $d'(k)$ associates with the extent to which the two relevant quark masses do not equal the geometric mean of the two quark masses. (See equation (61).) Regarding charged leptons, $M' = 3$, the term $g(M')$ is zero, and the factor - in equation (62) - that includes the fine-structure constant is one. (See equation (66).)

$$m(M'', M') = m_e \times (\beta^{1/3})^{M'' + (j''_{M''})d''} \times (\alpha^{-1/4})^{g(M')\cdot(1+M'') + j'_{M'}d'(M''))} \tag{62}$$

$$\alpha = ((q_e)^2/(4\pi\varepsilon_0))/(\hbar c) \tag{63}$$

$$j''_{M''} = 0, +1, 0, -1 \text{ for, respectively, } M'' \bmod 3 = 0, 1, 3/2, 2 \tag{64}$$

$$d'' = (2 - (\log(m_\mu/m_e)/\log(\beta^{1/3}))) \approx 3.840679 \times 10^{-2} \tag{65}$$

$$g(M') = 0, 3/2, 3/2, 3/2, 3/2, \text{ for, respectively, } M' = 3, 2, 3/2, 1, 0 \tag{66}$$

$$j'_{M'} = 0, -1, 0, +1, +3 \text{ for, respectively, } M' = 3, 2, 3/2, 1, 0 \tag{67}$$

$$d'(0) \sim 0.318 \tag{68}$$

$$d'(1) \sim -1.057 \tag{69}$$

$$d'(2) \sim -1.5091 \tag{70}$$

$$m(1,3) \approx 8.59341\mathrm{MeV}/c^2 \tag{71}$$

We explore possibly useful variations and extensions regarding uses of equation (62).

Equations (72), (73), and (74) characterize a possible approach to re-estimating rest energies for the six quarks.

$$d'(0) \approx 0.264835 \tag{72}$$

$$d'(1) = -1 \tag{73}$$

$$d'(2) = -3/2 \tag{74}$$

The calculations yield new calculated rest energies for the six quarks. (See table 25.) Of the six quarks, the rest energies that one calculates via equation (62) differ from measured values (that reference [1] provides) by more than 1.2 units of estimated error for, at most, $m(1,2)$ or the charm quark and $m(2,2)$ or the top quark. (Our calculations use the estimated errors - regarding experimental data - that reference [1] provides.) For the charm quark, the calculated number differs from the experimental number by about 4.6 units of estimated error. For the top quark, the largest (of the three differences associating with the three experimental interpretations) difference would be about 4.0 units of estimated error and one other difference would be about 0.6 units of estimated error.

To the extent that table 25 comports with nature, various straightforward equations interrelate the masses of elementary fermions. Equation (75) provides an example.

$$(m_s)^2 m_\mu = m_e m_\tau m_c \tag{75}$$

Equation (76) points to possibilities for estimating rest energies for arcs and neutrinos. Equations (77) and (78) would pertain.

$$m(M'',0) = m(M'',1) \cdot (m(M'',1)/m(M'',2)) \tag{76}$$

$$m(0,0) \approx m(1,0) = m(1,3) \tag{77}$$

$$m(2,0) = m(2,3) \tag{78}$$

To the extent that $m(0,0)$, $m(1,0)$, and $m(2,0)$ associate with masses of arc particles, approximate rest energies (in MeV/c^2) for arcs are 8.593 for generation one, 8.593 for generation two, and 105.66 for generation three.

We consider the possible extension that has bases in equations (79) and (80).

$$m(-1,3) = (\beta')^{-1} m(2,3) \tag{79}$$

$$d'(-1) = 0 \tag{80}$$

Equation (81) pertains.

$$m(-1,M')c^2 \approx 3.0386 \times 10^{-2}\ \mathrm{MeV},\ \text{for}\ M' = 3, 2, 3/2, 1,\ \text{and}\ 0 \tag{81}$$

We discuss possible rest energies for neutrinos.

Equation (82) provides extant modeling limits for the sum, across three generations, of neutrino masses. (The limits have bases in interpretations of astrophysics data. See reference [1].) The integer j associates with an index for counting neutrinos.

$$0.06\mathrm{eV}/c^2 \lesssim \sum_{j=1}^{3} m_j \lesssim 0.12\mathrm{eV}/c^2 \tag{82}$$

Table 25: Suggested rest energies for some elementary fermions

Particles	Approximate rest energy	Note
Tauon	1776.8400 ± 0.0115 MeV	The error reflects the measured error re G_N
Up quark	2.335 MeV	
Down quark	4.479 MeV	
Charm quark	1.178×10^3 MeV	
Strange quark	1.006×10^2 MeV	
Top quark	1.695×10^5 MeV	
Bottom quark	4.232×10^3 MeV	
Arcs - generation one	8.593 MeV	
Arcs - generation two	8.593 MeV	
Arcs - generation three	1.0566×10^2 MeV	Equals the muon rest energy
Neutrinos (each of at least two mass eigenstates)	3.4475×10^{-2} eV	
Neutrinos (no more than one mass eigenstate)	4.1629×10^{-6} eV	Might instead equal 4.4305×10^{-4} eV

Extending work that produces equation (81) produces equations (83), (84), and (85). (Here, equation (80) extends to the notion that $d'(-4) = 0$ pertains. Here, $m(-4,3) = (\beta')^{-1} m(-1,3)$ pertains. Compare with equation (79). We assume that $d'(-5) = 0$ and $d'(-6) = 0$ pertain.)

$$m(-6,0)c^2 = m(-6,3/2)c^2 \approx 4.1629 \times 10^{-6} \text{ eV} \tag{83}$$

$$m(-5,0)c^2 = m(-5,3/2)c^2 \approx 4.4305 \times 10^{-4} \text{ eV} \tag{84}$$

$$m(-4,0)c^2 = m(-4,3/2)c^2 \approx 3.4475 \times 10^{-2} \text{ eV} \tag{85}$$

We posit that equation (85) provides the rest energies for either just two neutrinos or for all three neutrinos. Either case can comport with equation (82).

The case for which the rest energies of just two neutrinos associate with equation (85) might comport with the extant modeling notion that at least two neutrino masses are unequal. (Extant modeling suggests that indirect observations imply at least two neutrino masses differ from each other. See, for example, reference [1]) Either of equations (83) and (84) might pertain for the other neutrino. (Perhaps, note that no other lepton associates with $1 \equiv M''$ (mod 3). Here, the symbol $\equiv$ denotes the three-word phrase is congruent with. The lack of such a congruence for other leptons might suggest that equation (84) does not yield a neutrino mass.)

The case for which the rest energies of all three neutrinos associate with equation (85) might comport with data. CEFGE symmetry does not pertain regarding 4G interactions with neutrinos. Gravity catalyzes neutrino oscillations. (See discussion related to table 20. CEFMA symmetry pertains regarding 4G interactions with neutrinos.) Extant modeling interpretations of data suggest that the squares of masses of neutrinos might differ from each other. Proposed modeling suggests that such inferred differences regarding squares of masses might associate with effects of neutrino interactions with at least one of 6G468, 8G2468a, and 8G2468b. 6G associates with CEFGE. The differences - regarding CEFMA and CEFGE - between 4G and 6G might echo extant modeling KIN notions that, for neutrinos, mass eigenstates differ from generation eigenstates.

Table 25 lists approximate rest energies that proposed modeling suggests for some elementary fermions. (Some results regarding quarks differ from those that table 24 shows. Equations (72), (73), and (74) lead to results that table 25 shows for quarks.)

We discuss the possibility that proposed modeling can produce useful results regarding the topic of anomalous magnetic dipole moments for charged leptons. (This essay de-emphasizes discussing the extent to which the 2G248 solution might associate with anomalous magnetic dipole moments for elementary particles. Perhaps, note table 7a.)

Equations (86), (87), and (88) show extant modeling KIN interpretations of results of experiments regarding anomalous magnetic dipole moments. (See reference [1].) The subscripts e, μ, and τ denote, respectively, electron, muon, and tauon. The symbol a associates with anomalous magnetic dipole moment.)

$$a_e \approx 0.00115965218091 \tag{86}$$

$$a_\mu \approx 0.0011659209 \tag{87}$$

$$-0.052 < a_\tau < +0.013 \tag{88}$$

Extant modeling provides means, associating with Feynman diagrams, to calculate an anomalous magnetic dipole moment for each of, at least, the electron and the muon. The extant modeling Standard Model suggests computations whereby the anomalous magnetic dipole moment for a charged lepton is a sum of terms. The first term is $\alpha/(2\pi)$. The second term is proportional to α^2. The third term is proportional to α^3. The exponent associated with α associates with a number of virtual photons.

Regarding the tauon, equation (89) shows a result associating with a first-order Standard Model (or, extant modeling) calculation. (See reference [2].)

$$a_{\tau,\mathrm{SM}} \approx +1.177 \times 10^{-3} \tag{89}$$

Proposed modeling suggests that notions of anomalous electromagnetic moments associate with $\gamma 2$ solutions. Electromagnetic dipole solutions associate with $\gamma 2$ solutions for which RSDF is r^{-3}. The following remarks pertain for other than the 2G24 solution, which associates with the extant modeling nominal magnetic moment result of $g \approx 2$. (2G24 associates with 2γ and not with $\gamma 2$.) Relevant G-family solutions (for which $\lambda \leq 8$) might be 4G26, 6G24, 6G28, 8G26, and (if we allow $\Sigma \geq 10$) 10G28. However, 6G28 and 10G28 do not interact with individual simple fermions. (Each of 6G28 and 10G28 associates with a *GTA* $SU(5)$ symmetry. See table 9a. Perhaps, note table 21. Perhaps, also note that - technically - $8 \in \Gamma$ might associate with $(ct)^{-1}$ and might not necessarily associate with r^{-1}.) Solutions 6G28 and 10G28 might associate with, for example, the Lamb shift. Regarding anomalous electromagnetic dipole moments, we assume that 4G26, 6G24, and 8G26 pertain.

For each of solutions 4G26 and 8G26, $4 \notin \Gamma$. Solutions 4G26 and 8G26 might associate with results that do not vary with charged lepton rest mass. For solution 6G24, $4 \in \Gamma$. Solution 6G24 might associate with a result that varies with charged lepton rest mass.

We explore modeling for which equation (90) pertains. Here, the subscript cl can be any one of e, μ, and τ. The symbol $a_{4\mathrm{G}26^*}$ associates with the notion of combining effects of 4G26 and 8G26. We explore the notions that t_{cl} might be one of $(\log(m_{\mathrm{cl}}/m_e))^2$, $(M'')^2$, and $(\text{generation})^2$. For each of the three possibilities regarding t_{cl}, $(a_\tau - a_{\tau,\mathrm{SM}})/a_{\tau,\mathrm{SM}}$ is more than -0.003 and less than -0.0006. For t_{cl} being $(\log(m_{\mathrm{cl}}/m_e))^2$, $(a_\tau - a_{\tau,\mathrm{SM}})/a_{\tau,\mathrm{SM}}$ is approximately -0.00228.

$$a_{\mathrm{cl}} \approx a_{4\mathrm{G}26^*} + a_{6\mathrm{G}24} t_{\mathrm{cl}} \tag{90}$$

Proposed modeling might provide modeling relevant to anomalous magnetic dipole moments for charged leptons.

2.2.4. Strengths of long-range forces

We explore concepts that might associate with the extant modeling notion that the strength of gravity is much less than the strength of electromagnetism.

We explore modeling for interactions that involve a charged simple fermion, such as an electron, that models as not entangled.

We assume that we can work within aspects of proposed modeling that de-emphasize translational motion and multicomponent objects. We assume that conservation of angular momentum pertains.

We associate the symbol 1F with that fermion. We explore interactions that model as if the number of incoming elementary bosons equals the number of outgoing elementary bosons. Equation (91) shows an interaction in which the fermion absorbs a photon. Conservation of angular momentum pertains. The spin of the fermion flips. Trying to replace, in equation (91), 2G with 4G does not work. The angular momentum associated with the fermion can change by no more than one unit. The interaction would not conserve angular momentum. Equation (92) can pertain. One can consider that the 2J particle in equation (92) associates with $2\mathrm{J}_o$ or $2\mathrm{J}_e$. (See table 26.)

$$1\mathrm{F} + 2\mathrm{G} \rightarrow 1\mathrm{F} + 0\mathrm{I} \tag{91}$$

$$1\mathrm{F} + 4\mathrm{G} \rightarrow 1\mathrm{F} + 2\mathrm{J} \tag{92}$$

The notion that 1F + 4G → 1F + 0I does not pertain might associate with extant modeling notions that the strength of gravity is much less than the strength of electromagnetism.

We explore the strengths - for the monopole components of interactions between pairs of identical charged leptons - of electromagnetism and gravity. We use KIN Newtonian modeling.

For each of the three charged leptons, equation (93) characterizes the strength of the 2G2 component of electromagnetism. Here, r denotes the distance between the two particles. Here, F denotes the strength of the force. The equation associates with a magnitude of the force. The interaction is repulsive. Equation (94) shows notation regarding the masses of charged leptons. (See discussion related to table 24.) Here, the three in $m(M'',3)$ associates with charged leptons. (Compare with equation (62), which pertains to the masses of quarks and charged leptons.) Equation (95) repeats equation (55). Equation (96) shows results that reflect data. (We used data that reference [1] shows.) Equation (97) provides a 4G4 analog to the 2G2 equation (93). The symbol G_N denotes the gravitational constant. The equation associates with a magnitude of the force. Here, the interaction is attractive.

$$r^2F = (q_e)^2/(4\pi\varepsilon_0) \tag{93}$$

$$m(M'',3) = m_x, \text{ for the pairs } M''=0, x=e;\ M''=2, x=\mu; \text{ and } M''=3, x=\tau \tag{94}$$

$$\beta' = m_\tau/m_e \tag{95}$$

$$m(M'',3) = y_{M''}(\beta')^{M''/3}m_e, \text{with } y_0 = y_3 = 1 \text{ and } y_2 \approx 0.9009 \tag{96}$$

$$r^2F = G_N(m(M'',3))^2 \tag{97}$$

We pursue the concept that a value of M'' can point to a relationship between the strength of electromagnetism and the strength of gravity. Based on the definitions just above, equation (98) pertains within experimental errors regarding relevant data. (Reference [1] provides the data.) Here, in essence, the equation $y_{18} = y_0 = 1$ pertains. Equation (98) echoes equation (56).

$$((q_e)^2/(4\pi\varepsilon_0))/4 = (G_N(m(18,3))^2)/3, \text{ with } m(18,3) = (\beta')^6m_e \tag{98}$$

The following notes pertain. Equation (98) links the ratio of the masses of two simple particles to a ratio of the strengths of two G-family force components. Equation (98) links the strength of 2G2 interactions to the strength of 4G4 interactions. Equation (99) associates the fine-structure constant, α, with a function of the tauon mass and the electron mass. (Regarding the fine-structure constant, see equation (63).) Equation (100) recasts equation (56) to feature, in effect, the magnitudes of three interactions, with each one of the interactions involving two similar particles. (For example, $G_N(m_\tau)^2$ associates with a gravitational interaction between two tauons.) Equation (101) shows a ratio that pertains for interactions between two electrons.

$$\alpha = ((q_e)^2/(4\pi\varepsilon_0\hbar c)) = (4/3) \times (m_\tau/m_e)^{12}G_N(m_e)^2/(\hbar c) \tag{99}$$

$$(4/3)((G_N(m_\tau)^2)/(G_N(m_e)^2))^6 = ((q_e)^2/(4\pi\varepsilon_0))/(G_N(m_e)^2) \tag{100}$$

$$(((q_e)^2/(4\pi\varepsilon_0))/4)\ /\ ((G_N(m_e)^2)/3) \approx 3.124 \times 10^{42} \tag{101}$$

We explore a possible relationship between the strength of electromagnetism associating with G-family monopole interactions with charge and the strength of electromagnetism associating with G-family dipole interactions with nominal magnetic dipole moment.

Equation (102) provides one definition of the fine-structure constant. (Compare with equation (63), which provides a more common definition.) In equation (102), $(q_e)^2/(4\pi\varepsilon_0 c)$ associates with the strength of 2G2.

$$\alpha = ((q_e/\hbar)^2/(4\pi\varepsilon_0 c)) \cdot \hbar \tag{102}$$

Equation (102) provides a link between the strength of 2G2 and the strength of 2G24. The equation includes the term $(q_e/\hbar)^2$. The Josephson constant K_J equals $2q_e/h$ (or, $q_e/(2\pi\hbar)$). Extant modeling considers that magnetic flux is always an integer multiple of $h/(2q_e)$. (We note the existence of an analog

- to equation (102) - for which $\alpha = (\cdots) \cdot K_{\mathrm{J}}$. Elsewhere, this essay links spin to aspects pertaining to the squares of masses of elementary bosons. See, for example, discussion related to equation (50). Elsewhere, this essay mentions the notion that aspects pertaining to squares of masses of elementary bosons might link with nominal magnetic dipole moment. See discussion related to equation (50). Possibly, the $\alpha = (\cdots) \cdot K_{\mathrm{J}}$ analog to equation (102) has relevance to aspects pertaining to squares of masses of elementary bosons. This essay does not further discuss possible relevance of the $\alpha = (\cdots) \cdot K_{\mathrm{J}}$ analog to equation (102).)

We explore a concept regarding extant modeling notions that associate with relationships between the strengths of the electromagnetic, weak, and strong interactions.

We use the symbol ΣB to denote an elementary boson having a spin of Σ/2. The expression 1F+2B→1F+0B can pertain for each of the following cases - 2B associates with 2G, 2B associates with 2W, and 2B associates with 2U. This notion might associate with extant modeling notions that associate with relationships between the strengths of the electromagnetic, weak, and strong interactions.

We explore the relative strengths of interactions regarding G-family bosons with spins of at least two.

Equations (103) and (104) parallel equation (92). Compared to equation (92), equation (103) requires dissipation (from the incoming G-family boson) of one more unit (of magnitude $\hbar$) of spin. Compared to equation (103), equation (104) requires dissipation (from the incoming G-family boson) of one more unit (of magnitude $\hbar$) of spin.

$$1\mathrm{F} + 6\mathrm{G} + 0\mathrm{I} \rightarrow 1\mathrm{F} + 2\mathrm{J} + 2\mathrm{J} \tag{103}$$

$$1\mathrm{F} + 8\mathrm{G} + 0\mathrm{I} + 0\mathrm{I} \rightarrow 1\mathrm{F} + 2\mathrm{J} + 2\mathrm{J} + 2\mathrm{J} \tag{104}$$

Each of 4G4, 6G6, and 8G8 interacts with a different property of objects. In effect, 4G4 interacts with (at least some) elementary fermions, while neither one of 6G6 and 8G8 interacts with elementary fermions. (See table 21b.)

We explore the notion that a strength scaling relationship might pertain regarding G-family components ΣGΓ that share a value of Γ. For two such ΣGΓ, Σ_1GΓ and Σ_2GΓ, equation (105) pertains.

$$|\Sigma_2 - \Sigma_1|/4 \text{ is an integer} \tag{105}$$

We interpret equation (102) as suggesting that a factor of α might pertain regarding modeling the absorbing of a unit of spin. For a step from equation (92) to equation (104), two factors of α would pertain.

2.2.5. Interactions involving the jay boson

We note one observational result that might associate with effects associating with the jay boson.

Reference [3] reports a possible discrepancy between the observed energy associating with one type of fine-structure transition in positronium and a prediction based on core extant modeling. (Perhaps, see also reference [4].) Equation (106) states a transition frequency. The observed value of transition frequency associates with the energy that associates with the transition. Equation (107) associates with extant modeling. The observed energy might exceed the predicted energy. Reference [3] characterizes the transition via the expression $2^3S_1 \rightarrow 2^3P_0$.

$$18501.02 \pm 0.61 \text{ MHz} \tag{106}$$

$$18498.25 \pm 0.08 \text{ MHz} \tag{107}$$

We explore the topic of interactions and effects associating with the jay boson.

Table 26 discusses aspects regarding physics, interactions, and modeling involving the jay (or, 2J) boson.

Table 27 shows some possible reactions involving pairs of jay bosons. The leftmost column describes the pair of incoming jay bosons. We discuss, as an example, the case of incoming $2\mathrm{J}_o$+$2\mathrm{J}_e$. The incoming particles associate with units of spin that have opposite circular polarizations. In effect, the circular polarizations sum to zero circular polarization. The outgoing pair 0I+0I is possible. The outgoing pair 2G+0I is not possible. The outgoing circular polarizations would sum to plus one or minus one.

Table 26: Aspects regarding the 2J boson

(a) Aspects - associating with observations and modeling - that might associate with the 2J boson

Aspect
• Interactions - between identical fermions - that associate with extant modeling notions of a Pauli exclusion force. (A pair of such identical fermions can be, for example, two hadrons in an atomic nucleus or two elementary particles. In extant modeling, the notion of identical can involve rest energy, charge, generation, and - for example, in an atom - spin orientation and orbital state. Aspects such as spin orientation and orbital state associate with extant modeling KIN aspects. Proposed modeling would suggest - regarding the notion of identical - including a number that associates with isomer. This inclusion would add to the list that associates with extant modeling.)
• Forces associating with some energy levels of positronium atoms. (See discussion related to equation (106).)
• Some interaction vertices that involve an incoming spin-one-half elementary fermion, an incoming or outgoing ΣG for which $\Sigma \geq 4$, and an outgoing spin-one-half elementary fermion. (See discussion related to equation (92). For this example, a 2J boson absorbs, in effect, one unit of spin that associates originally with an incoming fermion. The unit associates with $\hbar$. Equation (92) associates with - for example - neutrino oscillations.)
• Some interaction vertices that involve no fermions. (See discussion related to equation (117). For this example, two incoming 2J bosons associate with, in effect, two units of spin that associate with an outgoing component of a graviton. Each unit of spin associates with $\hbar$.)

(b) Suggested aspects regarding the 2J boson

Aspect
• The Pauli exclusion force (in extant modeling) associates with (in proposed modeling) a repulsive force based on 2J_o and 2J_e. The proposed modeling 2J force, in effect, tries to flip the spin of a fermion.
• The positronium energy shift involves the notion that the two fermions - an electron and a positron - have identical properties (including the spin orientations), except for the signs of the charges. We posit that an energy level shift (regarding at least one of the two positronium states) associates with, in effect, aspects of 2J_o and 2J_e. Here, at least with respect to extant modeling based on the Dirac equation, a notion associating with charge exchange (between the electron and positron) might be appropriate.
• We posit that the 2J boson associates with some interaction vertices that involve an incoming spin-one fermion, an incoming or outgoing ΣG for which $\Sigma \geq 4$, and an outgoing spin-one fermion. (See, for example, equation (92).)
• We posit that the 2J boson can associate with some interaction vertices that involve no fermions. (See, for example, discussion related to equation (117).)

Table 27: Some possible reactions involving pairs of jay bosons

Incoming particles	Allowed outgoing particles	Precluded outgoing particles
$2\text{J}_o+2\text{J}_o$ or $2\text{J}_e+2\text{J}_e$	4G+0I	2G+0I
$2\text{J}_o+2\text{J}_e$	0I+0I	2G+0I
$2\text{J}_0+2\text{J}_0$	0I+0I	2G+0I

Table 28: Perspective regarding PR6ISP modeling

PR6ISP modeling ...
• Explains observed dark matter to ordinary matter ratios of five-plus to one, four to one, one to one, zero-plus to one, and one to zero-plus.
• Associates with a $U(1) \times SU(2)$ symmetry to which table 12b alludes.
• Echoes the notion that ENT modeling intertwines 2G-related aspects and 4G-related aspects in ways that extant modeling does not. (See, for example, equation (62).)
• Echoes the exponent of six that equation (98) discusses.
• Echoes the six ranges that equation (110) and table 32 feature.

2.2.6. Dark matter particles

We discuss one type of dark matter.

We discuss the symbols that equations (108) and (109) show. The symbol 1Q⊗2U denotes a particle that includes (regarding non-virtual particles) just quarks and gluons. The word hadron pertains for the particle. The one-element term hadron-like pertains for the particle. Examples of 1Q⊗2U particles include protons, neutrons, and pions. The symbol 1R⊗2U denotes a particle that includes just arcs and gluons. The one-element term hadron-like pertains for the particle. The particle does not include (non-virtual) quarks.

$$1\text{Q} \otimes 2\text{U} \tag{108}$$

$$1\text{R} \otimes 2\text{U} \tag{109}$$

A 1R⊗2U hadron-like particle contains no (non-virtual) charged simple particles. The 1R⊗2U hadron-like particles do not interact with 2γ. The 1R⊗2U hadron-like particles measure as being dark matter.

If we associate notions above with PR1ISP modeling, the existence of 1R⊗2U hadron-like particles seems insufficient to explain observed ratios of dark matter effects to ordinary matter effects (for example) of five-plus to one for densities of the universe.

We explore the notion that some five-plus to one ratios reflect something fundamental in nature. We associate some results from this exploration with PR6ISP modeling. (See table 9c.)

The notion of isomer associates with a $U(1) \times SU(2)$ symmetry . (See table 12b.)

GFC modeling interrelates interactions with charge and the 2G2 component of the 2G force. We posit that nature includes six isomers of charge. GFC modeling interrelates interactions with nominal magnetic dipole moment and the 2G24 component of the 2G force. We posit that each isomer of charge associates with one isomer of nominal magnetic dipole moment. We posit that each of six pairings of one isomer of charge and one isomer of nominal magnetic moment associates with its own isomer of all simple particles. One isomer of charge, nominal magnetic dipole moment, and related simple particles measures mostly as ordinary matter. (Regarding PR6ISP modeling, the one isomer of charge, nominal magnetic dipole moment, and simple particles associates with 1R ⊗ 2U hadron-like particles that measure as dark matter. Hence, we used the word mostly.) We label that isomer as isomer zero. We posit that each of the other five isomers of charge, nominal magnetic dipole moment, and related simple particles measures as dark matter. (PR1ISP modeling does not include these five isomers.) We label those isomers as isomer one, isomer two, . . ., and isomer five. Each of the six isomers associates with its own 2U particles (or, gluons). We posit that one isomer of 4G4 interacts with each one of the one (mostly) ordinary matter isomer and five dark matter isomers.

We posit that the next two sentences pertain. The six-isomer notion explains the five that pertains regarding five-plus to one ratios of amounts of dark matter to ordinary matter. The existence of isomer zero 1R⊗2U hadron-like particles explains the plus that pertains regarding five-plus to one ratios of amounts of dark matter to ordinary matter. Such five-plus to one ratios pertain regarding densities of the universe and regarding the compositions of some (perhaps, most) galaxy clusters.

Table 28 provides perspective regarding PR6ISP modeling.

Regarding each one of the six isomers that associate with PR6ISP models, we suggest that each combination - that table 24 shows - of magnitude of charge and magnitude of mass pertains to a simple fermion that associates with the isomer. For example, each isomer includes a charged lepton for which the magnitude of charge equals the magnitude of the charge of the ordinary matter electron and for which the rest energy equals the rest energy of the electron. However, regarding charged leptons, the combination of mass and generation number does not necessarily match across isomers. (See table 32.)

Table 29: Cumulative features of various types of modeling (with NR denoting not relevant)

Modeling	ι_I	New descriptions and new explanations	New subtleties
Extant modeling	NR	• (Baseline)	• Ranges of applicability of some kinematics models
PR1ISP	1	• New elementary particles • One type of dark matter • Possible eras early in the development of the universe	• Internal symmetries • Known eras regarding the rate of expansion of the universe
PR6ISP	6	• More types of dark matter • Ratios of dark matter effects to ordinary matter effects • Objects, smaller than galaxies, that feature dark matter	• Galaxy formation and evolution • Eras regarding the rate of expansion of the universe • Spans
PR36ISP	36	• Possible dark energy stuff	• Dark energy density of the universe • Spans

Table 30: Relationships regarding PR1ISP, PR6ISP, and G-family forces

Aspect
• Absent the notion that the jay boson has a span of more than one and the notion that some components of G-family forces have spans of more than one, PR6ISP would associate with six non-interacting sub-universes.
• In PR6ISP models, each sub-universe consists of an isomer of PR1ISP. The six isomers of PR1ISP might exhibit differing matches between generation of charged lepton and mass of charged lepton. (See discussion related to table 32.)
• In PR6ISP models, the main interactions between PR1ISP-like isomers associate - except before the era of inflation - with the monopole component (or, 4G4) of gravity (or, 4G). Some other interactions between PR1ISP-like isomers associate with a KIN dipole (or, 4G48) component of gravity (or, 4G). Some other interactions between PR1ISP-like isomers associate with a KIN dipole component (or, 2G248 - which associates with the notion of GFC quadrupole) of electromagnetism (or, 2G).

For example, for isomer one, the generation three charged lepton may have the same mass as the ordinary matter electron. (See table 24.) The ordinary matter electron has a generation number of one.

We preview features of each of PR1ISP modeling, PR6ISP modeling, and PR36ISP modeling.

Table 29 discusses cumulative features of various types of modeling. Generally, each row augments the rows above that row. (Table 9c discusses the symbol ι_I.) Regarding extant modeling, the symbol NR denotes the concept that the notion of isomers is not relevant. We think that PR6ISP provides useful insight about nature. We think that PR36ISP might provide a new description for phenomena that measure as dark energy density of the universe.

Table 30 shows relationships regarding PR1ISP, PR6ISP, and G-family forces.

2.2.7. Isomers of quarks and charged leptons

We consider PR6ISP modeling.

Table 31 lists aspects that seem to associate with each other regarding the one isomer that associates with ordinary matter (and some dark matter) and the five isomers that associate with (most) dark matter.

We explore modeling that associates each of the six relevant isomers with a range of M''. (Regarding M'', perhaps see discussion related to equation (62).) In equation (110), the integer n numbers the isomers. The ordinary matter isomer associates with $n = 0$.

$$\text{isomer } n \leftrightarrow 3n \leq M'' \leq 3n + 3, \text{ for } 0 \leq n \leq 5 \tag{110}$$

Table 32 shows, for each value of n, relationships between quark generation and charged lepton aspects. For each n, the order for quarks is generation one, generation two, and then generation three. We de-emphasize the following notions. Dark matter lepton passive gravitational masses might associate with $m(M'', 3)$ and $M'' > 3$. Results that associate with $M'' < 0$ might be useful for estimating magnitudes of ordinary matter 2G interactions with dark matter analogs to ordinary matter charged leptons.

Table 31: Aspects that seem to associate with each other regarding the one isomer that associates with ordinary matter (and some dark matter) and the five isomers that associate with (most) dark matter

Aspect
• The exponent of six in equation (98) associates with the notion of six isomers, one of which associates with ordinary matter and five of which associate with (most) dark matter.
• The number, six, of isomers associates with the number, six, of generators of a $U(1) \times SU(2)$ symmetry. (See table 12b.)
• The $U(1) \times SU(2)$ symmetry breaks - across the six isomers - based on aspects that associate with relationships between - for charged leptons - passive gravitational mass and generation.

Table 32: Relationships between quark generation and charged lepton aspects

M''	n	Quark n	Quark generation	Lepton n (for n even)	Lepton aspect (for even n)	Lepton n (for n odd)	Lepton aspect (for odd n)
0	0	0	1	0	1	-	-
1	0	0	2	-	-	-	-
2	0	0	3	0	2	-	-
3	0 or 1	1	1	0	3	1	3
4	1	1	2	-	-	-	-
5	1	1	3	-	-	1	1
6	1 or 2	2	1	2	2	1	2
7	2	2	2	-	-	-	-
8	2	2	3	2	3	-	-
9	2 or 3	3	1	2	1	3	1
10	3	3	2	-	-	-	-
11	3	3	3	-	-	3	2
12	3 or 4	4	1	4	3	3	3
13	4	4	2	-	-	-	-
14	4	4	3	4	1	-	-
15	4 or 5	5	1	4	2	5	2
16	5	5	2	-	-	-	-
17	5	5	3	-	-	5	3
18	5	-	-	-	-	5	1

Table 32 has roots in models that associate with the relative strengths of 2G2 and 4G4. We posit that, for each item (in table 32) that associates with a particle, equation (111) provides the passive gravitational mass. Here, the notions of $n = 0$ and $m_{grav}(M'', M')$ associate with work that associates with isomer zero and equation (62). For example, for the dark matter lepton for which $n = 1$ and M''=3, the generation is three and the passive gravitational mass equals the passive gravitational mass of the ordinary matter electron.

$$m_{grav}(M'' + 3n, M') = m_{grav}(M'', M'), \text{ for } 0 \leq n \leq 5 \tag{111}$$

We speculate regarding the extent to which aspects of table 32 associate with origins for baryon asymmetry.

Aspects of extant modeling consider that early in the universe baryon symmetry likely pertained. Unverified extant modeling posits mechanisms that might have led to asymmetry. Some conjectured mechanisms would suggest asymmetries between matter simple fermions and antimatter simple fermions. One set of such simple fermions might feature the neutrinos. (See reference [5].)

Observed baryon asymmetry associates with isomer zero (or, ordinary matter).

We think that some aspects of proposed modeling might shed light on baryon asymmetry. For example, a modeling centric notion of baryon symmetry might pertain regarding the combination of isomer zero and isomer three.

We consider a thought experiment. We consider that modeling for isomer three quarks parallels modeling for isomer zero quarks. Per table 32, modeling for isomer three leptons can differ from modeling for isomer zero leptons. One difference might associate with handedness, for example regarding (let us use the word interactive) neutrinos. Such differences might associate with the two-fold symmetry that associates with the $U(1)$ component of the $U(1) \times SU(2)$ symmetry that table 12b shows regarding the oscillator pair $USA1$-and-$USA2$.

2.2.8. Right-handed W bosons and neutrinos

Reference [6] notes that the (extant modeling) Standard Model predicts that the fraction f_+ of W bosons - produced by decays of top quarks - that are right-handed is $f_+ = 3.6 \times 10^{-4}$. Reference [1] suggests that, with a confidence level of 90 percent, the rest energy of a W_R (or, right-handed W) would exceed 715 GeV. (Perhaps, note also, reference [7].)

Proposed modeling suggests that each of isomers one through five includes its own isomer of W bosons. The suggested passive gravitational mass for dark matter W bosons is the same as the passive gravitational mass for the ordinary matter W boson.

We suggest that leptons associating with isomers zero, two, and four might associate with left-handedness and that leptons associating with isomers one, three, and five might associate with right-handedness. (Note the pattern that table 32 exhibits regarding charged leptons.) We suggest that W bosons associating with isomers zero, two, and four might associate with left-handedness and that W bosons associating with isomers one, three, and five might associate with right-handedness. Table 31 and equation (98) suggest that equation (112) pertains regarding measurements that feature aspects centric to ordinary matter and interactions intermediated by span-six aspects of 2G. (Note, for example, 2(6)G248 in table 22a.) We know of no measurements that associate with interactions intermediated by 4G. To the extent that equation (112) has relevance to nature, one might use the four-word phrase not necessarily gravitational mass to describe $m_{W_R(\text{isomer one}), \text{ inferred not via 4G}}$.

$$m_{W_R(\text{isomer one}), \text{ inferred not via 4G}} c^2 = \beta m_W c^2 \approx 2.8 \times 10^5 \text{ GeV} \tag{112}$$

We consider a thought experiment. We consider a possibly relevant notion that would have bases in statistics related to inferable not necessarily gravitational masses. Perhaps equation (113) approximates fractions of non-longitudinal polarization W bosons observed via ordinary matter non-4G interactions. (For isomers not numbered as zero or one, the $m_{W_R(\text{isomer \_}), \text{ inferred} \ldots} c^2$ would be larger than $m_{W_R(\text{isomer one}), \text{ inferred} \ldots} c^2$. Effects based on the existence of isomer three W bosons and isomer five W bosons would be small compared to effects associating with each of isomer zero W bosons and isomer one W bosons.)

$$f_+/f_- \sim e^{(\beta^{-1})} \approx \beta^{-1} \approx 2.9 \times 10^{-4} \tag{113}$$

Equation (113) is not necessarily incompatible with the estimate - $f_+ = 3.6 \times 10^{-4}$ - based on the Standard Model.

Table 33: Opportunities for advances regarding cosmology

Opportunity
• Describe aspects of the universe that occurred before inflation.
• Identify - within a context that is broader than inflation - the inflaton elementary particle that extant modeling hypothesizes.
• Describe mechanisms underlying three eras in the rate of expansion of the universe.
• Explain the magnitude of the current increase in the rate of expansion of the universe.
• Describe bases leading to the ratio of dark matter density of the universe to ordinary matter density of the universe.

Regarding neutrinos, similar notions might pertain. Proposed modeling suggests that neutrinos do not interact with 2G. Direct inferences of the presence of right-handed neutrinos might associate with isomer one neutrinos and with interactions - mediated by 4G - with isomer zero. This essay de-emphasizes discussing the question of when people might have observations that would point to right-handed neutrinos.

2.3. Cosmology

Table 33 lists opportunities for advances regarding cosmology. Proposed modeling suggests advances regarding each opportunity.

2.3.1. An earlier of two eras that might occur before inflation

We discuss possibilities regarding times before the inflationary epoch.

We explore possibilities pertaining to an era before a later (but also before inflation) era that proposed modeling associates with prominence for the jay boson and the 4G2468x components of 4γ. (Regarding the later of the two eras before inflation, see discussion related to equation (117). Regarding the symbol 4G2468x, see discussion related to table 6.)

We assume that modeling associating with G-family solutions for which the RSDF is r^{-6} pertains. No solutions of the form ΣG2468⟦10⟧ comport with $\Sigma = 4$. One solution of the form ΣG2468⟦16⟧ comports with $\Sigma = 4$. (Here, $|-2-4-6-8+16|$ equals four. Perhaps, see table 10.) Regarding KIN Newtonian modeling, the RSDF (or, radial spatial dependence of force) would be r^{-6}. Table 16 notes that attraction (not repulsion) pertains. (Perhaps, also note that extrapolation based on aspects of table 35 might point to attraction.)

We consider interactions between two similar, neighboring, non-overlapping objects (or clumps of energy). Equation (114) suggests scaling for a 4G2468⟦16⟧ component of G-family force. Here, υ is a non-dimensional scaling factor that associates with linear size (or, a length) pertaining to each object and that associates with the distance between the centers of the objects, ρ is the relevant object property for the case for which $\upsilon = 1$, and r is the distance (for the case of $\upsilon = 1$) between the centers of the objects. The factor υ^3 provides for scaling for an object that has three spatial dimensions. The force would be independent of υ. That independence might suggest, from a standpoint of physics, that a 4G2468⟦16⟧ component of 4G would associate with concentrating matter or energy before the suggested era in which much of the matter in the universe consists of jay bosons.

$$(\upsilon^3\rho)^2/(\upsilon r)^6 \tag{114}$$

The method that we use to calculate spans for other components of G-family forces would not pertain for 4G2468⟦16⟧. (See discussion regarding equation (40).) We assume that the span ι_I - as in PRι_IISP - pertains for 4G2468⟦16⟧. The notation that equation (115) shows pertains.

$$4(\iota_I)\text{G2468}\,[\![16]\!] \tag{115}$$

We assume that 4G provides the dominant phenomena that pertain early in this era. (For later eras, we identify a combination of stuff - or non-G-family phenomena - and dominant components of G-family forces.)

We assume that interactions of the form that equation (116) shows pertain. Here, we assume that the net circular polarization for before the interaction is zero.

$$4(\iota_I)\text{G2468}[\![16]\!]+4(\iota_I)\text{G2468}[\![16]\!] \rightarrow 2(\iota_I)\text{J}_o+2(\iota_I)\text{J}_e \tag{116}$$

2.3.2. The later of two eras that might occur before inflation

We explore the notion that, just before the inflationary epoch, the main component of the universe might have consisted of jay bosons.

Extant modeling seems to suggest that nature creates photons (or, 2G) primarily after the inflationary epoch. Regarding times just before inflation, we assume that the allowed reactions that table 27 shows pertain.

We assume that the particle density is sufficiently large that modeling can associate the production of 4G with the 4G2468x components of 4G.

Equation (117) describes a possible interaction. For PRι_IISP models for which ι_I exceeds one, we posit that modeling suggests roughly equal creation of ι_I isomers for each of 4G2468x and 0I.

$$2(\iota_I)\mathrm{J}_o + 2(\iota_I)\mathrm{J}_o \rightarrow 4(1)\mathrm{G2468x} + 0(1)\mathrm{I} \tag{117}$$

4G4 has a span of six. To the extent that ι_I exceeds one, isomers interact with each other during and after this period.

Table 27 suggests that interactions between pairs of jay bosons do not create photons. A lack of photons is compatible with extant modeling that suggests that significant presence of photons starts after inflation.

2.3.3. Inflation

We discuss possibilities regarding the inflationary epoch.

Extant modeling suggests that an inflationary epoch might have occurred. Extant modeling suggests that the epoch started around 10^{-36} seconds after the Big Bang. Extant modeling suggests that the epoch ended around 10^{-33} seconds to 10^{-32} seconds after the Big Bang. We are not certain as to the extent to which data confirms the occurrence of an inflationary epoch.

Extant modeling includes models that people claim would support notions of inflation. The models point to states of the universe, at and somewhat after the inflationary epoch, that would provide bases for evolution that would be consistent with observations about later phenomena and would be consistent with aspects of extant modeling. (Reference [8] summarizes aspects related to inflation, points to references regarding extant modeling, and discusses some extant modeling work.)

Reference [9] suggests the possibility that a repulsive aspect of gravity drove phenomena associated with the inflationary epoch. The reference suggests that the composition of the universe was nearly uniform spatially. The reference suggests the importance of a so-called inflaton field.

Proposed modeling suggests the possibility that, during the inflationary epoch, aye particles (or, 0I particles) provided a major non-long-range-force component of the universe. The aye particle matches extant modeling notions of a boson with zero spin. (See reference [8].) Extant modeling uses the word inflaton to name that boson. Proposed modeling suggests the possibility that the octupole components of 4γ provided the repulsive aspect of gravity. (Components 4G4268x associate with GFC octupole and with KIN octupole.) Those components interact with individual simple particles and are repulsive. Equation (118) shows such an interaction. Here, x and y might be either of a and b.

$$0(1)\mathrm{I} + 4(1)\mathrm{G2468x} \rightarrow 0(1)\mathrm{I} + 4(1)\mathrm{G2468y} \tag{118}$$

Around the time of the inflationary epoch, octupole attraction associating with 4G246⟦16⟧ might play a role. (Perhaps, see table 16.)

2.3.4. Just after inflation

The end of the inflationary epoch might associate with a change, regarding effects of 4γ, from octupole repulsion being dominant to quadrupole attraction being dominant. The end of the inflationary epoch might also associate with a growth of spatial inhomogeneities regarding (at least) aye particles. The quadrupole component of 4γ might help catalyze some of the spatial inhomogeneities. The quadrupole component of 4γ might amplify some of the spatial inhomogeneities.

Proposed modeling suggests the possibility that, for some time just after the inflationary epoch, the aye particle might have been a dominant non-long-range-force component of the universe. Interactions between aye particles would produce components of 2G forces. (See equation (119).) Interactions of 2G with itself produce matter-and-antimatter pairs of simple fermions. Proposed modeling suggests the possibility that attraction based on the (quadrupole) 4G246 component of 4γ contributed to clumping.

$$0\mathrm{I} + 0\mathrm{I} \rightarrow 2\mathrm{G} + 2\mathrm{G} \tag{119}$$

Table 34: Ordinary matter, four cold dark matter isomers, and the one other dark matter isomer

Isomers (n)	Aspect - regarding each isomer
0	Includes ordinary matter.
3	Evolves similarly to isomer zero.
1, 2, 4, and 5	Evolves into cold dark matter.

2.3.5. Dissimilarities between isomers

We consider a thought experiment regarding isomer zero (or, the isomer that includes ordinary matter) and a so-called isomer alt zero. Here, alt zero is one of one, two, four, and five.

The stuff that associates with isomer alt zero and the stuff that associates with isomer zero exhibit similarities with respect to phenomena involving quarks, gluons, and W-family bosons.

We consider a time at which the densities of stuff are high and the compositions of stuff associating with the isomers are essentially similar. Similar evolution would occur to the extent that one considers just quarks, gluons, and W-family bosons.

We consider three-quark baryons (real or virtual) that consist of generation three quarks. The charged baryons are more massive than the neutral (or, charge-neutral) baryons. (Consider the masses - per table 25 - of the constituent quarks.)

For the alt zero isomer, generation three leptons are less massive than the tauon that associates with isomer zero generation three. Interactions that produce generation three leptons (and produce or consume W bosons) facilitate - in the alt zero isomer compared to isomer zero - more transitions from all-generation-three charged baryons to all-generation-three neutral baryons.

Over time, in both isomers, generation three quarks and generation two quarks evolve, via interactions that entangle multiple W bosons, into generation one quarks.

We consider a time when the transitions to all-generation-one quarks have just completed. Densities of stuff have dropped. We consider all-generation-one baryons. Compared to isomer zero, the alt isomer contains more alt neutrons than isomer zero contains neutrons. The mass of the alt isomer generation one charged lepton exceeds the mass of the isomer zero generation one charged lepton (or, the mass of the electron). The (already more abundant, compared to isomer zero) alt one neutrons have difficulties (compared to isomer zero neutrons) decaying into charged baryons.

From then on, the alt isomer has, compared to isomer zero, more neutrons and fewer protons. The alt isomer has, compared to isomer zero, fewer charged leptons. The alt isomer has, compared to isomer zero, fewer charged leptons with masses equal to the mass of the isomer zero electron.

Even to the extent that stuff associating with isomer alt zero forms some stars, isomer alt zero becomes cold dark matter consisting mainly of alt neutrons and alt hydrogen atoms. Also, the collection of - mostly old - alt isomer photons cools.

We consider isomer zero and isomer three.

Presumably, similar evolution pertains regarding isomer three and isomer zero. For example, isomer three stuff forms stars in numbers similar to isomer zero numbers.

Table 34 pertains.

2.3.6. Filaments and baryon acoustic oscillations

Proposed modeling is compatible with the extant modeling notion that ordinary matter baryon acoustic oscillations contributed to the formation of filaments.

Regarding models for which ι_I (as in PRι_IISP) exceeds one, each of the five dark matter isomers has its own baryon-like particles and its own 2(1)G physics. Proposed modeling suggests, for models for which ι_I exceeds one, that dark matter baryon-like acoustic oscillations occurred in the early universe. Proposed modeling suggests that dark matter baryon-like acoustic oscillations contributed (along with ordinary matter baryon acoustic oscillations) to the formation of filaments.

2.3.7. The rate of expansion of the universe

Table 35 posits concepts regarding three eras in the rate of expansion of the universe. (Regarding observations that associate with the eras that associate with decrease and recent increase, see references [10], [11], [12], and [13].) We know of no observations that pertain directly to the era of inflation. Extant modeling suggests the existence of an era of inflation.

Table 35 suggests associations between repulsion and 4G2468a, 4G2468b, and 4G48. Table 35 suggests associations between attraction and 4G246. We suggest these associations, based on data.

Table 35: Aspects regarding three eras associating with the expansion of the universe

Aspect	Era: Inflation	Era: Next billions of years	Era: Most recent billions of years
Observed changes in the rate	?	Decrease	Increase
Extant modeling KIN model-based changes in the rate	Increase	Decrease	Increase
Proposed modeling ENT model-based changes in the rate	Increase	Decrease	Increase
Drivers, as suggested by ENT modeling and GFC modeling (4G components that dominate between largest objects)	4G2468a, 4G2468b	4G246	4G48
KIN RSDF for the 4G components	r^{-5}	r^{-4}	r^{-3}
Proposed modeling interpretation of KIN modeling for the net force associating with the components	Repulsive	Attractive	Repulsive

Work elsewhere in this essay reinforces the notions that 4G246 associates with attraction and that 4G2468a, 4G2468b, and 4G48 associate with repulsion. (See table 16.)

Two thought experiments provide notions that lead to table 35.

We consider one thought experiment. We consider two similar neighboring clumps of stuff. We assume that the clumps are moving away from each other. We assume that the clumps will continue to move away from each other. We assume that, initially, interactions associating with RSDF $r^{-(n+1)}$ dominate regarding interactions between the two clumps. We assume that the two clumps interact via interactions associating with RSDF r^{-n}. We assume that no other forces have adequate relevance. We assume that the distance between the objects increases adequately. Eventually, the RSDF r^{-n} force dominates the RSDF $r^{-(n+1)}$ force.

We consider a similar thought experiment. We consider two similar neighboring clumps. We assume that these clumps are less interactive (for example, less massive) than the two clumps in the first thought experiment. Generally, dominance of the RSDF r^{-n} force over the RSDF $r^{-(n+1)}$ force occurs sooner for the two clumps in the second thought experiment than it does for the two clumps in the first thought experiment.

Interactions between galaxy-like clumps transit to 4G4 RSDF r^{-2} dominance quickly compared to the current age of the universe. Mutual attraction occurs. Interactions between adequately larger clumps can still exhibit 4G48 RSDF r^{-3} dominance. Mutual repulsion occurs.

We discuss modeling regarding recent increases in the rate of expansion.

People suggest that extant modeling underestimates recent increases in the rate of expansion. (See, for example, reference [14], reference [15], reference [16], and reference [17]. However, some people note possible objections to some notions of underestimates. See, for example, references [18] and [19].) People suggest phenomenological remedies regarding the modeling. (See, for example, reference [20].)

Proposed modeling suggests a basis for such underestimates.

We consider a thought experiment.

Here, we assume that people use models that associate with data about the rate of expansion during the era of decreases in that rate. We assume that the models have bases in equations of state and in general relativity.

Proposed modeling associates dominant effects - for the era of decreasing rate - with the span of one that associates with 4G246. Proposed modeling associates dominant effects for the recent era with the span of two that associates with 4G48.

Applying decreasing-rate era equations of state and general relativity to current era phenomena associates with underestimating a key factor - 4G48 repulsion - by, conceptually, a factor of two.

2.3.8. Dark matter density of the universe

Extant modeling discusses five partial densities of the universe. The symbol Ω_{c} denotes dark matter (or, cold dark matter) density of the universe. The symbol Ω_{b} denotes ordinary matter (or, baryonic matter) density of the universe. The symbol Ω_{ν} denotes neutrino density of the universe. The symbol Ω_{γ} denotes photon density of the universe. The symbol Ω_{Λ} denotes dark energy density of the universe.

Each of the five densities associates with data. Equation (120) pertains regarding the total density of the universe, Ω.

$$\Omega = \Omega_{\mathrm{c}} + \Omega_{\mathrm{b}} + \Omega_{\nu} + \Omega_{\gamma} + \Omega_{\Lambda} \tag{120}$$

Reference [1] provides the data that equations (121), (122), (123), and (124) show.

$$\Omega_{\mathrm{c}} \approx 0.265 \pm 0.007 \tag{121}$$

$$\Omega_{\mathrm{b}} \approx 0.0493 \pm 0.0006 \tag{122}$$

$$\Omega_{\nu} \leq 0.003, \text{ also } \Omega_{\nu} \geq 0.0012 \tag{123}$$

$$\Omega_{\gamma} \approx 0.0000538 \pm 0.0000015 \tag{124}$$

In extant modeling, the symbol Ω_{c} associates with all dark matter. To the extent that proposed modeling PR6ISP modeling comports with nature, the symbol Ω_{c} associates with all of the three aspects - isomer zero 1R⊗2U hadron-like particles, the four dark matter isomers that we associate above with the word cold, and the one dark matter isomer that we do not necessarily associate above with the word cold - that proposed modeling associates with the term dark matter.

Proposed modeling suggests considering - for each isomer j, with $0 \leq j \leq 5$ - equation (125). (Technically, the isomers share a fraction of Ω_{γ}, but the total Ω_{γ} is small.) The symbol $\Omega_{\mathrm{1R2U,j}}$ denotes the density of the universe that associates with the 1R⊗2U hadron-like particles that associate with isomer j. From here on, we de-emphasize the densities of neutrinos and the densities of photons. Equation (126) pertains. Even though isomers evolve differently with respect to quark-based hadrons, we assume that there is adequate similarity in evolution so that equation (127) pertains. Equations (128) and (129) pertain.

$$\Omega_{j} = \Omega_{\mathrm{b},j} + \Omega_{\mathrm{1R2U},j} + \Omega_{\nu,j} + \Omega_{\gamma,j} \tag{125}$$

$$\Omega_{\mathrm{b}} + \Omega_{\mathrm{c}} \approx \sum_{j=0}^{5} \Omega_{j} \tag{126}$$

$$\Omega_{\mathrm{1R2U},j} \approx \Omega_{\mathrm{1R2U,0}}, \text{ for } 0 \leq j \leq 5 \tag{127}$$

$$\Omega_{\mathrm{b}} + \Omega_{\mathrm{c}} \approx \Omega_{\mathrm{b}} + \Omega_{\mathrm{1R2U,0}} + 5(\Omega_{\mathrm{1R2U,0}} + \Omega_{\mathrm{b}}) \tag{128}$$

$$\Omega_{\mathrm{1R2U,0}} \approx (\Omega_{\mathrm{c}} - 5\Omega_{\mathrm{b}})/6 \tag{129}$$

Equation (130) estimates $\Omega_{\mathrm{1R2U,0}}$ for the current state of the universe.

$$\Omega_{\mathrm{1R2U,0}} \approx 0.0031 \tag{130}$$

Except possibly regarding dark energy density (or, Ω_{Λ}), proposed modeling suggests that ratios of the actual values of the various $\Omega\_$ in equation (120) remain constant for essentially the entire history of the universe. (This essay does not speculate - regarding this topic - regarding the very earliest times after the Big Bang. Regarding Ω_{Λ}, see discussion related to equation (132).) PR6ISP proposes no significant mechanisms for transferring stuff between ordinary matter and dark matter. (We assume that net transfers based on components - for which the spans are greater than one - of 2G are negligible.)

We discuss measurements via which people infer densities - of dark matter and ordinary matter - of the universe.

People use data from observations of CMB (or, cosmic microwave background radiation) to infer ratios - of dark matter density of the universe to ordinary matter density of the universe - to which equations (121), (122), (123), and (124) point. A ratio of five-plus to one might pertain for billions of years.

Regarding data based on CMB, measured ratios of dark matter density of the universe to ordinary matter density of the universe would not much change regarding times for which equation (131) pertains. That time range starts somewhat after 380,000 years after the Big Bang and continues through now.

$$\Omega_{\gamma} \ll \Omega_{\mathrm{b}} \text{ and } \Omega_{\nu} \ll \Omega_{\mathrm{b}} \tag{131}$$

2.3.9. Dark energy density of the universe

We explore possible explanations for nonzero dark energy density of the universe.

Equation (132) shows a ratio of presently inferred density of the universe of dark energy to presently inferred density of the universe of dark matter plus ordinary matter plus (ordinary matter) neutrinos plus (ordinary matter) photons. (Reference [1] provides the five items of data.) Inferences that reference [21] discusses might suggest that inferred dark energy density increases with time. Reference [22] suggests that an inferred dark energy density of essentially zero associates with times around 380,000 years after the Big Bang. We know of no inferences that would not comport with a somewhat steady increase - regarding the inferred ratio associating with equation (132) - from approximately zero over time since somewhat after the Big Bang.

$$\Omega_\Lambda/(\Omega_\mathrm{c}+\Omega_\mathrm{b}+\Omega_\nu+\Omega_\gamma)\approx 2.18 \tag{132}$$

Some aspects of extant modeling associate inferred dark energy densities of the universe with phenomena for which people use terms such as vacuum energy, vacuum fluctuations, or quintessence. Proposed modeling is not necessarily incompatible with such extant modeling. Nevertheless, we discuss possibilities for proposed modeling that might explain nonzero dark energy density.

For any one of PR1ISP modeling, PR6ISP modeling, and PR36ISP modeling, aspects related to the aye (or, 0I) boson or the jay (or, 2J) boson might lead to phenomena similar to effects that extant modeling associates with vacuum energy, vacuum fluctuations, or quintessence. (See discussion related to equations (91) and (92). Perhaps, also note discussion related to equation (114).)

For PR6ISP modeling, proposed modeling includes the notion of 2(6)G248, whereas extant modeling associates with the notion of 2(1)G248. The difference, in proposed modeling, between 2(6)G248 and 2(1)G248 might associate with nature's indirectly producing effects, regarding CMB, that people associate (via extant modeling) with some nonzero dark energy density. The difference associates with interactions between ordinary matter and dark matter.

PR36ISP modeling offers another possibility. (This possibility associates with a six-fold symmetry that associates with the instance of $U(1)\times SU(2)$ that table 13b shows.) We assume that the spans of 4(6)G4 and the other 4(>1)GΓ components are orthogonal to the spans of 2(6)G248 and the other 2(>1)GΓ components. The PR36ISP universe associates with six isomers of a PR6ISP sub-universe. Each PR6ISP sub-universe includes its own isomer of 4(6)G4. We continue to associate ordinary matter with isomer zero and most dark matter with isomers one through five. We use the numbers six, 12, 18, 24, and 30 to number the five isomers for which 2(6)GΓ components intermediate interactions with isomer zero. We use the three-word term doubly dark matter to associate with isomers six through 35. Doubly dark matter isomers do not interact with ordinary matter via 4G. Dark matter isomers do not interact with ordinary matter via 2G. Differences between 2(>1)GΓ and 2(1)GΓ associate with interactions between ordinary matter plus dark matter and doubly dark matter. All interactions - mediated by 2G - that PR6ISP modeling would associate with interactions between ordinary matter and dark matter isomers become - for PR36ISP modeling - interactions between ordinary matter and doubly dark matter. Dark energy density might associate with stuff associating with the 30 doubly dark matter isomers. Modeling suggests an upper bound of approximately five regarding a possible future value for the ratio that associates with equation (132).

2.4. Astrophysics

Table 36 lists opportunities for advances regarding astrophysics. Proposed modeling suggests advances regarding each opportunity.

We discuss ratios that proposed modeling PR6ISP models might predict or explain.

Table 37 lists some approximate ratios of dark matter effects to ordinary matter effects that PR6ISP modeling might explain. We designed PR6ISP modeling to explain the five-plus to one ratio that people observe regarding densities of the universe. Here, the five associates with dark matter isomers of simple elementary particles (that is, of elementary particles other than the jay boson and G-family elementary particles) and the plus associates with (ordinary matter isomer) hadron-like particles that do not interact with 2γ force components. Galaxy clusters seem to be sufficiently large to comport with similar ratios. (However, galaxy clusters that are remnants of collisions of galaxy clusters might be exceptions. See discussion related to table 38.) Discussion regarding 2(2)G68 associates with the approximately one to one ratio. (See discussion related to equation (42) and discussion related to equation (133).) DMA:OMA ratios of zero-plus to one, four to one, and one to zero-plus comport with roles of non-monopole components of gravity in scenarios regarding galaxy formation. (See discussion related to table 40.) DMA:OMA

Table 36: Opportunities for advances regarding astrophysics

Opportunity
• Describe mechanisms leading to an observed amount of depletion - some of which has bases in hyperfine interactions with hydrogen atoms - of cosmic microwave background radiation.
• Hone scenarios associating with the formation of galaxies.
• Explain data - that extant modeling seems not to explain - about the following.
○ Large clumps of ordinary matter gas and of dark matter.
○ Ratios of dark matter to ordinary matter in galaxy clusters.
○ Amounts of stuff that does and does not pass through - with mainly just gravitational interactions - collisions of galaxy clusters.
○ Some aspects of interactions between galaxies.
○ Ratios - within galaxies - of dark matter to ordinary matter.
○ Dark matter effects within the Milky Way galaxy.

Table 37: Approximate ratios of dark matter effects to ordinary matter effects (with DM denoting dark matter; with OM denoting ordinary matter; with A denoting amount; and with OM CMB denoting cosmic microwave background radiation)

Approximate DMA:OMA	Amounts
5^+:1	Density of the universe
5^+:1	Amount of stuff in some galaxy clusters
1:1 or 1^+:1	Amount of absorption of OM CMB via some interactions with DM atoms or OM atoms.
0^+:1	Amount of stuff in some early galaxies
≈4:1	Amount of stuff in some early galaxies
1:0^+	Amount of stuff in some early galaxies
0^+:1	Amount of stuff in some later galaxies
≈4:1	Amount of stuff in some later galaxies
1:0^+	Amount of stuff in some later galaxies

ratios of zero-plus to one, four to one, and one to zero-plus comport with scenarios regarding some galaxies for which observations associate with times well after galaxy formation. (See other discussion related to table 40.)

2.4.1. CMB depletion via hyperfine interactions

People measure specific depletion of CMB and attribute some of that depletion to hyperfine interactions with (ordinary matter) hydrogen atoms. (See reference [23].) The amount of depletion is twice or somewhat more than twice the amount that people expected. At least one person speculates that the amount above expectations associates with effects of dark matter. (See reference [24].)

Proposed modeling suggests the following explanation. Solution 2(2)G68 (or, 2G68) might associate with hyperfine interactions. (See discussion related to equation (42). Perhaps, also note equation (133).) Solution 2G68 has a span of two. (See table 21b.) Solution 2G68 does not associate with interactions with individual simple fermions. (See table 21b.) Half or somewhat less than half of the observed absorption associates with the ordinary matter isomer of hydrogen atoms. An approximately equal amount of the observed effect associates with hydrogen-atom isomers that associate with one dark matter isomer.

$$2\mathrm{G}68 \notin 2\gamma,\ \ 2\mathrm{G}68 \notin \gamma 2 \tag{133}$$

To the extent that the absorption by ordinary matter is less than half of the total absorption, the following explanations might pertain regarding the difference between less than half and equal to half. One explanation associates with the notion that the evolution of the relevant non-ordinary-matter isomer might differ from the evolution of the ordinary matter isomer. The non-ordinary-matter isomer might have more hydrogen-atom-like objects than does the ordinary matter isomer. One explanation associates with 2GΓ solutions with spans of at least two. Each one of solutions 2(6)G46 and 2(6)G468 might pertain. For each one, the solution is not a member of 2γ and is not a member of $\gamma 2$. The number six appears in both the Γ for 2(6)G46 and the Γ for 2(6)G468. Solution 2(6)G46 associates with a KIN spatial dipole effect. Solution 2(6)G468 associates with a KIN spatial dipole effect (and with the notion of GFC quadrupole solution).

Proposed modeling might contribute to credibility for assumptions and calculations that led to the prediction for the amount of depletion that associates with ordinary matter hydrogen atoms. (Regarding the assumptions and calculations, see reference [25].)

2.4.2. Large clumps of ordinary matter gas and of dark matter

Reference [26] discusses observations that point to the notion that - on a large scale - clumping of matter - ordinary matter gas and dark matter - might be less than extant modeling models suggest. Observed phenomena have bases in gravitational lensing of light. The article alludes to a dozen observational studies and points to at least two papers - reference [27] and reference [28]. Clumps would be - to use wording from reference [26] - too thin. (Reference [26] suggests a result of too thin by about ten percent. This essay does not explore the topic of quantifying such thinness.) A distribution of galaxies would be - to use wording from reference [17] - too smooth. Reference [17] suggests a notion of ten percent more evenly spread than extant modeling predicts.

Proposed modeling suggests that such effects might associate with the notion that 4(2)G48 repels more stuff than would 4(1)G48. (See table 22a and table 16.) Early formation of clumps associates with 4(1)G246 attraction. Early clumps associate with single isomers. Effects of 4(2)G48 repulsion would dilute matter around early clumps more than would effects that extant modeling might associate with, in effect, 4(1)G48 repulsion. Also, effects of dilution might carry into the times for which 4(6)G4 attraction dominates.

2.4.3. Galaxy clusters - ratios of dark matter to ordinary matter

Regarding some galaxy clusters, people report inferred ratios of dark matter amounts to ordinary matter amounts.

References [29] and [30] report ratios of five-plus to one. The observations have bases in gravitational lensing. Reference [31] reports, for so-called massive galaxy clusters, a ratio of roughly 5.7 to one. (Perhaps, note reference [32].) The observations have bases in X-ray emissions.

Proposed modeling PR6ISP modeling is not incompatible with these galaxy cluster centric ratios.

Reference [33] suggests a formula that associates - across 64 galaxy clusters - dark matter mass, hot gas baryonic mass (or, essentially, ordinary matter mass), and two radii from the centers of each galaxy cluster. The reference suggests that the formula supports the notion of a relationship between dark matter and baryons. This essay de-emphasizes discussing the extent to which proposed modeling comports with this formula. Proposed modeling might suggest a relationship, based on proposed similarities between dark matter and ordinary matter.

2.4.4. Galaxy clusters - collisions

People use the two-word term Bullet Cluster to refer, specifically, to one of two galaxy clusters that collided and, generally, to the pair of galaxy clusters. The clusters are now moving away from each other. Extant modeling makes the following interpretations based on observations. For each of the two clusters, dark matter continues to move along trajectories generally consistent with just gravitational interactions during the collision. For each of the two clusters, stars move along trajectories generally consistent with just gravitational interactions during the collision. For each of the two clusters, (ordinary matter) gas somewhat generally moves along with the cluster, but generally lags behind the other two components (dark matter and stars). Regarding such gas, people use the acronym IGM and the two-word term intergalactic medium. Extant modeling suggests that the IGM component of each original cluster interacted electromagnetically with the IGM component of the other original cluster. Electromagnetic interactions led to slowing the motion of the gas.

If each of the six dark matter or ordinary matter isomers evolved similarly, there might be problems regarding explaining aspects of the Bullet Cluster. One might expect that, in each galaxy cluster, more (than the observed amount of) dark matter would lag. The lag would occur because of one-isomer 2G-mediated interactions within each of the five dark matter isomers. Possibly, for each dark matter isomer, there would not be enough star-related stuff to explain the amount of dark matter that is not lagging. Possibly, across the six (five dark matter and one ordinary matter) isomers, there would not be enough $1R\otimes 2U$ dark matter to significantly help regarding explaining the amount of dark matter that is not lagging.

We assume that four dark matter isomers associate with proposed modeling notions of cold dark matter and that one dark matter isomer exhibits behavior similar to behavior that ordinary matter exhibits. (See discussion related to table 32 and see table 34.)

Table 38: Aspects regarding a collision between two galaxy clusters (with the assumption that each of the two galaxy clusters has not undergone earlier collisions)

Aspect
• Up to essentially nearly all ordinary matter IGM (in each galaxy cluster) interacts with ordinary matter IGM (in the other galaxy cluster) and slows down. (The notion of up to essentially all associates with equally sized colliding galaxy clusters and with a head-on collision.)
• Much of the stuff associating with ordinary matter stars passes through with just gravitational interactions having significance.
• No more than somewhat less than 20 percent of dark matter significantly interacts non-gravitationally with dark matter and, based on non-gravitational interactions, slows down. (For each galaxy cluster, this dark matter associates with the IGM associating with isomer three.)
• At least 80 percent of dark matter passes through with just gravitational interactions having significance.
• Essentially all of the incoming 1R⊗2U passes through the collision with just gravitational interactions having significance.

Proposed modeling suggests that, for each of the two galaxy clusters, essentially all the stuff associating with isomers one, two, four, and five would pass through the collision with just gravitational interactions having significance. For isomer three, incoming 1R⊗2U would pass through. For isomer zero, incoming 1R⊗2U (which measures as dark matter) would pass through. Thus, at least 80 percent of the incoming dark matter would pass through the collision with just gravitational interactions having significance.

Table 38 lists aspects regarding a collision between two galaxy clusters. Here, we assume that each of the two galaxy clusters has not undergone earlier collisions.

We suggest that these proposed modeling notions might comport with various possible findings about IGM after a collision such as the Bullet Cluster collision. The findings might point to variations regarding the fractions of IGM that, in effect, stay with (the cores of) outgoing galaxy clusters and the fractions of IGM that, in effect, (at least somewhat) detach from (the cores of) outgoing galaxy clusters.

We discuss possible aspects regarding an outgoing galaxy cluster.

Suppose that, before a collision, ordinary matter IGM comprised much of the ordinary matter in the galaxy cluster. Suppose that, because of the collision, the galaxy cluster has a significant net loss of ordinary matter IGM. After the collision, the galaxy cluster could have a (perhaps somewhat arbitrarily) large ratio of amount of dark matter to amount of ordinary matter.

To the extent that IGM detaches from galaxy clusters after the galaxy clusters collide, the detached IGM might form one or more objects. Some such objects might have roughly equal amounts of dark matter and ordinary matter. The dark matter would associate with isomer three.

2.4.5. Interactions between galaxies

Reference [34] reports measurements pertaining to external gravitational effects on components of individual galaxies. The article suggests that - compared to expected results based on notions that associate with the strong equivalence principle and with general relativity - observations point to unexpected effects regarding galaxy rotation curves. The article suggests the possibility of associating the unexpected effects with the notion of an external field effect and possibly with aspects of MOND (or, Milgromian dynamics or modified Newtonian dynamics).

Proposed modeling provides the possibility that the unexpected results associate with differences in spans between 4G4 (for which the span is six) and (perhaps just) 4G48 (for which the span is two) and (maybe also) other components of 4γ (for which the spans are one).

2.4.6. Galaxies - formation

We discuss scenarios regarding galaxy formation and evolution. We anticipate that such galaxy formation and evolution scenarios will explain galaxy centric data that table 37 shows.

Models for galaxy formation and evolution might take into account the following factors - one-isomer repulsion (which associates with the 4G2468a and 4G2468b solutions), one-isomer attraction (which associates with 4G246), two-isomer repulsion (which associates with 4G48), six-isomer attraction (which associates with 4G4), dissimilarities between isomers, the compositions of filaments and galaxy clusters, statistical variations in densities of stuff, and collisions between galaxies. Modeling might feature a notion of a multicomponent fluid with varying concentrations of gas-like or dust-like components and of objects

Table 39: A scenario for the formation and evolution of a galaxy for which the original clump contains essentially just one isomer (with the two-word phrase featured isomer associating with that one isomer)

Step
• Early on, stuff associating with each one of the six isomers expands, essentially independently from the stuff associating with other isomers, based on repulsion associating with 4(1)G2468a and 4(1)G2468b.
• Then, each isomer starts to clump, essentially independently from the other isomers, based on attraction associating with 4(1)G246.
• With respect to clumps associating with any one isomer, 4(2)G48 repels one other isomer and repels some stuff associating with the first-mentioned isomer.
• A galaxy forms based on a clump that contains mostly the featured isomer.
• The galaxy attracts and accrues, via 4(6)G4 attraction, stuff associating with the four isomers that the featured isomer does not repel. The galaxy can contain small amounts of stuff associating with the isomer that the featured isomer repels.

(such as stars, black holes, galaxies, and galaxy clusters) for which formation associates significantly with six-isomer (or 4G4) attraction.

We focus on early-stage galaxy formation and evolution. For purposes of this discussion, we assume that we can de-emphasize collisions between galaxies. We suggest the two-word term untouched galaxy for a galaxy that does not collide, before and during the time relevant to observations, with other galaxies. We emphasize formation scenarios and evolution scenarios for untouched galaxies. (Reference [35] and reference [36] discuss data that pertains regarding a time range from about one billion years after the Big Bang to about 1.5 billion years after the Big Bang. Observations suggest that, out of a sample of more than 100 galaxies or galaxy-like rotating disks of material, about 15 percent of the objects might have been untouched.)

We assume that differences - in early evolution - regarding the various isomers do not lead, for the present discussion, to adequately significant differences - regarding 4G interactions and galaxy formation - between isomers. (We think that this assumption can be adequately useful, even given our discussion regarding cold dark matter and our discussion regarding the Bullet Cluster. Regarding cold dark matter, see discussion related to table 34. Regarding the Bullet Cluster, see discussion related to table 38.)

We organize this discussion based on the isomer or isomers that originally clump based, respectively, on 4G246 attraction or on 4G246 attraction and 4G4 attraction. Each one of some galaxies associates with an original clump that associates with just one isomer. Multi-isomer original clumps are possible. Because of 4G48 repulsion, an upper limit on the number of isomers that an original clump features might be three.

Table 39 discusses a scenario for the formation and evolution of a galaxy for which the original clump contains essentially just one isomer. Regarding this isomer, we use the word featured. We assume that stuff that will become the galaxy is always in somewhat proximity with itself. We assume that no collisions between would-be galaxies or between galaxies occur.

2.4.7. Galaxies - ratios of dark matter to ordinary matter

We continue to explore the realm of one-isomer clumps.

One of two cases pertains. For so-called case A, one isomer of 4(2)G48 spans (or connects) isomers zero and three. (Regarding numbering for isomers, see n in table 34.) For so-called case B, one isomer of 4(2)G48 spans isomer zero and one isomer out of isomers one, two, four, and five. The existence of many spiral galaxies might point to the notion that case A pertains. (Compare the rightmost column in table 40a and the rightmost column in table 40b.) However, we consider the possibility that people might not know of data or current modeling that would adequately point to the one of case A and case B that pertains. We discuss both cases.

Table 40 pertains. (See table 37.) The following sentences illustrate the notion that some statements in table 40 are at least somewhat conceptual. We assume that local densities for the isomers are somewhat the same. We assume that the galaxy remains adequately untouched. For each row in the table, OM stars can form (and become visible) over time, whether or not significant OM star formation occurs early on. The notation DMA:OMA=1:0^+ denotes the notion that the ratio of OMA to DMA might be arbitrarily small. (Table 37 defines the three-letter terms DMA and OMA.) The notion of three or four DM isomers in a halo refers to the notion that one or zero (respectively) of the DM isomers in the halo is the featured isomer. We de-emphasize some aspects regarding 1R⊗2U hadron-like particles.

Table 40: Aspects regarding untouched galaxies that associate with original one-isomer clumps (with just one of cases A and B pertaining to all galaxies)

(a) Case A

Label	Featured isomer (n)	Early aspects regarding the galaxy	Possible later aspects regarding the galaxy
A0	0	Forms some ordinary matter stars early on. Starts at DMA:OMA=0^+:1.	Attracts cold dark matter over time. Can get to DMA:OMA≈4:1, with most DM in a halo. Might be a spiral galaxy.
A3	3	Forms some dark matter stars early on. Starts at DMA:OMA=1:0^+.	Attracts the four other DM isomers over time. Some OM stars can form over time. Can settle at DMA:OMA=1:0^+. The three-word term dark matter galaxy pertains.
AX	Any one of 1, 2, 4, and 5	Might form dark matter stars early on. Starts at DMA:OMA=1:0^+.	Attracts the OM isomer and three other isomers over time. OM stars can form over time. Can get to DMA:OMA≈4:1, with three or four DM isomers in a halo. Might become an elliptical galaxy.

(b) Case B

Label	Featured isomer (n)	Early aspects regarding the galaxy	Possible later aspects regarding the galaxy
B0	0	Forms some ordinary matter stars early on. Starts at DMA:OMA=0^+:1.	Attracts isomer three and three cold dark matter isomers over time. Can get to DMA:OMA≈4:1, with three DM isomers in a halo. Might appear to be an elliptical galaxy.
BP	The DM isomer that 4(2)G48 connects to the OM isomer	Might form dark matter stars early on. Starts at DMA:OMA=1:0^+.	Attracts the other DM isomers over time. OM stars can form over time. Can settle at DMA:OMA=1:0^+. The three-word term dark matter galaxy pertains.
B3	3	Forms some dark matter stars early on. Starts at DMA:OMA=1:0^+.	Attracts the OM isomer and three other DM isomers over time. OM stars can form over time. Can get to DMA:OMA≈4:1, with three DM isomers in a halo. Might appear to be an elliptical galaxy.
BY	Any one of the other three DM isomers	Might form dark matter stars early on. Starts at DMA:OMA=1:0^+.	Attracts the OM isomer and three other DM isomers over time. OM stars can form over time. Can get to DMA:OMA≈4:1, with three or four DM isomers in a halo. Might appear to be an elliptical galaxy.

Table 40 reflects at least two assumptions. Each core clump features one isomer. Each galaxy does not collide with other galaxies. Yet, data of which we know and discussion below seem to indicate that ratios that table 40 features might pertain somewhat broadly. We think that galaxies that have core clumps that feature more than one isomer are more likely to appear as elliptical galaxies (and not as spiral galaxies) than are galaxies that have core clumps that feature only one isomer. Such likelihood can associate with starting as being elliptical. Such likelihood can associate with earlier transitions - via collisions - from spiral to elliptical.

We explore the extent to which the galaxy formation scenarios comport with observations.

Observations regarding stars and galaxies tend to have bases in ordinary matter isomer 2G phenomena (or, readily observable electromagnetism). (The previous sentence de-emphasizes some observations - regarding collisions between black holes or neutron stars - that have bases in 4G phenomena.) People report ratios of amounts of dark matter to amounts of ordinary matter.

We discuss observations associating with early in the era of galaxy formation. Table 37 comports with these results. We suggest that visible early galaxies associate with generalization of label-A0 or with generalization of label-B0. (See table 40.) Label-A3 or label-B3 evolves similarly to label-A0 or label-B0, but is not necessarily adequately visible early on.

- Reference [37] provides data about early-stage galaxies. (See, for example, figure 7 in reference [37]. The figure provides two graphs. Key concepts include redshift, stellar mass, peak halo mass, and a stellar - peak halo mass ratio.) Data associated with redshifts of at least seven suggests that some galaxies accrue, over time, dark matter, with the original fractions of dark matter being small. Use of reference [38] suggests that redshifts of at least seven pertain to times ending about 770 million years after the Big Bang.
- Reference [39] reports zero-plus to one ratios. The observations have bases in the velocities of stars within galaxies and associate with the three-word term galaxy rotation curves. Proposed modeling suggests that the above galaxy evolution scenario comports with this data.

We discuss observations associating with later times. Table 37 comports with these results.

- Reference [40] discusses some MED09 spiral - or, disk - galaxies. A redshift of approximately $z = 1.57$ pertains. (See reference [41].) The redshift associates with a time of 4.12 billion years after the Big Bang. (We used reference [38] to calculate the time.) Reference [40] reports ratios of amount of dark matter to amount of ordinary matter of approximately four to one. The observations have bases in gravitational lensing. We suggest that each label - other than label-A3 or label-BP - that table 40 shows might pertain. (We note, without further comment, that this example might associate with the notion that case A pertains to nature and that case B does not pertain to nature. This example features spiral galaxies. Label-A0 suggests an association with spiral galaxies. Each other label - pertaining to case A or to case B - either associates with dark matter galaxies or might suggest an association with - at least statistically - evolution into elliptical galaxies. See table 40.)
  - To the extent that such an MED09 galaxy models as being nearly untouched, proposed modeling offers the following possibility. The galaxy began based on a one isomer clump. The clump might have featured the ordinary matter isomer. The clump might have featured a dark matter isomer that does not repel ordinary matter. Over time, the galaxy accrued stuff associating with the isomers that the original clump did not repel. Accrual led to a DMA:OMA ratio of approximately four to one.
  - To the extent that such an MED09 galaxy models as not being untouched, proposed modeling offers the following possibility. One type of collision merges colliding galaxies. One type of collision features galaxies that separate after exchanging material. For either type of collision, incoming galaxies having approximately four times as much dark matter as ordinary matter might produce outgoing galaxies having approximately four times as much dark matter as ordinary matter.
- Reference [42] discusses the Dragonfly 44 galaxy. A redshift of $z = 0.023$ pertains. The redshift associates with a time of 13.45 billion years after the Big Bang. (We used reference [38] to calculate the time.) People discuss the notion that ordinary matter accounts for perhaps as little as one part in 10 thousand of the matter in the galaxy. (See reference [43].) The observations have bases in light emitted by visible stars. This case associates with the three-word term dark matter galaxy. We suggest that label-A3 or label-BP might pertain. (See table 40.)

The following notions pertain regarding other data of which we know. Here, the ratios are ratios of dark matter amounts to ordinary matter amounts. Table 37 seems to comport with these results. (See table 40.)

- Reference [44] discusses six baryon-dominated ultra-diffuse galaxies that seem to lack dark matter, at least to the radii studied (regarding gas kinematics) via observations of light with a wavelength of 21 centimeters. These observations seem not to be incompatible with the early stages of label-A0 or label-B0.

- Reference [45] discusses 19 dwarf galaxies that lack having much dark matter, from their centers to beyond radii for which extant modeling suggests that dark matter should dominate. These observations measure r-band light that the galaxies emitted. These observations seem not to be incompatible with the early stages of label-A0 or label-B0.

- People report two disparate results regarding the galaxy NGC1052-DF2. Proposed modeling seems to be able to explain either ratio. Proposed modeling might not necessarily explain ratios that would lie between the two reported ratios.

  - Reference [46] suggests a ratio of much less than one to one. The observation has bases in the velocities of stars - or, galaxy rotation curves. This observation seems not to be incompatible with the early stages of label-A0 or label-B0.
  - Reference [47] suggests that at least 75 percent of the stuff within the half mass radius is dark matter. This ratio seems similar to ratios that reference [40] discusses regarding some MED09 galaxies. (See discussion above regarding MED09 galaxies.) We suggest that each label - other than label-A3 or label-BP - that table 40 shows can pertain.

- The galaxy NGC1052-DF4 might associate with a ratio of much less than one to one. (See reference [48].) The observation has bases in the velocities of stars - or, galaxy rotation curves. This observation seems not to be incompatible with the early stages of label-A0 or label-B0.

- The compact elliptical galaxy Markarian 1216 has an unexpectedly large amount of dark matter in its core and may have stopped accumulating each of ordinary matter and dark matter approximately 4 billion years after the Big Bang. (See reference [49].) Observations feature the X-ray brightness and temperature of hot gas. This galaxy might associate with an original clump that features three isomers. One isomer would be the ordinary matter isomer. Around the time that the galaxy stopped accruing material, there might have been - near the galaxy - essentially nothing left for the galaxy to attract via 4(6)G4.

- The galaxy XMM-2599 stopped producing visible stars by approximately 1.8 billion years after the Big Bang. (See reference [50].) People speculate regarding a so-called quenching mechanism. Proposed modeling suggests that phenomena similar to phenomena that might pertain regarding Markarian 1216 might pertain regarding XMM-2599.

People report other data. Table 37 and table 40 seem not to be incompatible with these results. We are uncertain as to the extents to which proposed modeling provides insight that extant modeling does not provide.

- One example features a rotating disk galaxy, for which observations pertain to the state of the galaxy about 1.5 billion years after the Big Bang. (See reference [51].) People deduce that the galaxy originally featured dark matter and that the galaxy attracted ordinary matter.

- One example features so-called massive early-type strong gravitation lens galaxies. (See reference [52].) Results suggest, for matter within one so-called effective radius, a minimum ratio of dark matter to dark matter plus ordinary matter of about 0.38. Assuming, for example, that measurements associating with material within larger radii would yield larger ratios, these observational results might support the notion that the galaxies accumulated dark matter over time.

- One example pertains to early stages of galaxies that are not visible at visible light wavelengths. (See reference [53].) Observations feature sub-millimeter wavelength light. We might assume that proposed modeling galaxy formation scenarios comport with such galaxies. We are not certain about the extent to which proposed modeling might provide insight regarding subtleties, such as regarding star formation rates, associating with this example.

- We are uncertain as to the extent to which proposed modeling might provide insight regarding possible inconsistencies - regarding numbers of observed early-stage galaxies and numbers of later stage galaxies - that associate with various observations and models. (For a discussion of some possible inconsistencies, see reference [54].)
- We are uncertain as to the extent to which proposed modeling might provide insight regarding the existence of two types - born and tidal - of ultra-diffuse galaxies. (See reference [55].)

Observations that we discuss above indicate that some galaxies do not exhibit dark matter halos. Proposed modeling that we discuss above comports with the notion that some galaxies do not exhibit dark matter halos.

2.4.8. Some components of galaxies

We discuss effects, within galaxies, that might associate with dark matter.

Reference [56] reports, based on a study of 11 galaxy clusters, more instances of more gravitational lensing - likely associating with clumps of dark matter that associate with individual galaxies - than extant modeling simulations predict. Reference [57] suggests that the number of instances - 13 - compares with an expected number of about one. We suggest the possibility that the clumps might be dark matter galaxies. (See, for example, table 40.) Perhaps some of the dark matter galaxies are dwarf dark matter galaxies. We suggest the possibility that galaxies with significant amounts of ordinary matter gravitationally captured (or at least attracted) such dark matter clumps.

People study globular cluster systems within ultra-diffuse galaxies. Regarding 85 globular cluster systems in ultra-diffuse galaxies in the Coma cluster of galaxies, reference [58] suggests that 65 percent of the ultra-diffuse galaxies are more massive than people might expect based on extant modeling relationships, for so-called normal galaxies, between stellar mass and halo mass. We are uncertain as to the extent to which proposed modeling might explain this result. For example, proposed modeling might suggest that phenomena related to isomers might play a role. (See, for example, table 40.) Higher-mass galaxies might tend to feature more dark matter isomers (or tend to feature more material that associates with such isomers) than do lower-mass galaxies.

Discussion related to table 40 is not incompatible with the notion that visible stars do not include much dark matter.

Discussion related to table 40 is not incompatible with the notion that some black holes that form based on the collapse of stars might originally associate with single isomers. Discussion above is not incompatible with the notion that supermassive black holes might contain material associating with more than one isomer. (Perhaps, note references [59] and [60].)

We suggest that proposed modeling might provide insight about other aspects regarding black holes. People suggest gaps in understanding about the formation of intermediate-mass and large-mass black holes. (Perhaps, note reference [61].) Proposed modeling suggests the possibility that the 4G(1)246 attractive component of G-family forces plays key roles in the early formation of some intermediate-mass and large-mass black holes.

Regarding the coalescing of two black holes, proposed modeling suggests that people might be able to estimate the extent to which 4(2)G48 repulsion pertains. Effects of 4(2)G48 repulsion would vary based on the amounts of various isomers that each black hole in a pair of colliding black holes features.

2.4.9. Dark matter effects within the Milky Way galaxy

People look for possible effects, within the Milky Way galaxy, that might associate with dark matter.

For one example, data regarding the stellar stream GD-1 suggests effects of an object of 10^6 to 10^8 solar masses. (See reference [62].) Researchers tried to identify and did not identify an ordinary matter object that might have caused the effects. The object might be a clump of dark matter. (See reference [63].) Proposed modeling offers the possibility that the object is an originally dark matter centric clump of stuff.

For other examples, people report inhomogeneities regarding Milky Way dark matter. (See references [63] and [64].) Researchers note that simulations suggest that such dark matter may have velocities similar to velocities of nearby ordinary matter stars. We suggest that these notions are not incompatible with proposed modeling notions of the existence of dark matter stars that would be similar to ordinary matter stars.

3. Results

This unit summarizes results that proposed modeling produces.

3.1. Physics properties

Table 12 and table 13 show an organizing and a uniting of various properties of objects. Examples of extant modeling properties include charge, energy, angular momentum, and momentum. The property of isomer (of simple elementary particles) arises from proposed modeling. Figure 2 summarizes some aspects of table 12.

Principles for organizing and uniting the properties come from proposed modeling models that feature components of long-range forces. (See, for example, table 7, table 8, and table 9.)

3.2. Elementary particles

Table 17 alludes to all known elementary particles and to candidate elementary particles that proposed modeling suggests. Table 19 and table 21 provide further information. Figure 3 summarizes some information about elementary particles. Figure 4 shows suggested rest energies for all elementary fermions other than the electron and muon (for which people have determined masses rather accurately).

This essay suggests that particles associating with table 17 might suffice - from the standpoint of elementary particles - to explain data that extant modeling does not yet explain and to predict data that extant modeling does not necessarily predict. Some of that data associates with the field of cosmology. Some of that data associates with the field of astrophysics. Some of that data associates with the field of elementary particles.

Proposed modeling points to various associations among properties of elementary particles and strengths of interactions.

3.3. Cosmology

Proposed modeling suggests advances associating with the opportunities that table 33 lists. Figure 6 suggests eras - in the evolution of the universe - that might precede inflation. Figure 6 also suggests insight regarding mechanisms leading to eras regarding the rate of expansion of the universe. Figure 5 depicts information about the ratio of dark matter density of the universe to ordinary matter density of the universe.

3.4. Astrophysics

Proposed modeling suggests advances associating with the opportunities that table 36 lists and with data to which table 37 alludes. Figure 5 depicts information about the ratio of dark matter density of the universe to ordinary matter density of the universe. Figure 7 notes seemingly prevalent ratios of dark matter to ordinary matter. This essay discusses aspects of galaxy formation and other phenomena that seem to lead to the seemingly prevalent ratios.

3.5. Physics modeling

Proposed modeling suggests perspective about modeling and about notions associating with the word object. For example, table 12 and table 13 suggest perspective about relationships between models, modeling that purports to discuss distinguishable (or, generally non-entangled) objects, and properties that associate with objects.

Figure 8 suggests that proposed modeling provides a framework for cataloging, comparing, and uniting aspects of proposed modeling and aspects of extant modeling. Figure 8 uses and extends notions that table 12, table 13, and figure 2 show.

4. Discussion

This unit provides perspective about some physics topics and about proposed modeling.

4.1. Possible variations and extensions

We discuss the possibility for another subfamily of elementary fermions.

Nature might embrace another - in effect - sibling of 1N. (See table 19.) We use the symbol 1N′ to denote this possible sibling. This sibling might augment or supplant the notion of 1R (or, arc) elementary particles. The 1N′ elementary particles might associate with the rest energies that table 25 shows for arcs. Absent evidence for so-called sterile neutrinos, this essay de-emphasizes the notion of 1N′ elementary particles.

We discuss the possibility for another subfamily of elementary bosons.

For the set - of λ - $\{2, 4, 6, 8, 10, 12, 14\}$, four 0G solutions pertain. These solutions might associate with $Z_{T4} = 7^2 + 1 = 50$ and with elementary bosons with rest energies somewhat more than 200 GeV. (Compare with discussion related to table 23.) These solutions might associate - paralleling the 2W subfamily - with elementary bosons that have spin one. Two of the bosons might have no charge. One of the bosons might have a magnitude of charge equal to $|q_e|/3$. One of the bosons might have a magnitude of charge equal to $2|q_e|/3$. The bosons might have a role regarding catalyzing baryon asymmetry. Absent evidence for so-called leptoquarks, this essay de-emphasizes the notion of such elementary bosons.

We discuss a possibility for developing modeling that would pertain for ι_I equals six or 36, would parallel general relativity, and would pertain for some circumstances in which general relativity might not provide adequate accuracy.

Such modeling might have similarities to Maxwell's equations for electromagnetism. Presumably, there would be at least one field for each one of the RSDF that pertains regarding 4G. Presumably, for each such field for which the span is not ι_I, the field would have components that echo relevant spans.

This essay does not attempt to develop such modeling. This essay does not attempt to develop a gravitational analog to extant modeling electromagnetism models that have bases in vector potentials.

We discuss dynamics within black holes.

People might consider applying the notion of components of 4G to dynamics within black holes. For example, octupole repulsion might prevent conditions that extant modeling might associate with the notion of a singularity.

Aside from aspects regarding 4(2)G48 near the edges of black holes, this essay de-emphasizes discussing dynamics within black holes.

We discuss possible bases for so-called high-mass neutron stars.

Observations associate with most known neutron star pairs having masses in the range that equation (134) shows and one neutron star pair having a mass of about 3.4 solar masses. (See references [65] and [66].) Here, M denotes the mass of a pair. The symbol $M_\odot$ denotes the mass of the sun. The 3.4 number results from the second detection via gravitational waves of a merger of two neutron stars. People assign the name GW190425 to that detection.

$$2.5M_\odot \lesssim M \lesssim 2.9M_\odot \tag{134}$$

People speculate - based on, at least, the GW190425 result - about needs for new modeling regarding neutron stars. (See references [65] and [67].)

The span of 4G4 is six.

Some high-mass neutron stars might, in effect, result from mergers of neutron stars, with each merging neutron star associating with an isomer that differs from the isomer pertaining to each other neutron star that forms part of the merged object.

4.2. Proposed modeling

The following notions were essential to the development of proposed modeling.

There might be a straightforward explanation for three eras in the rate of expansion of the universe.

There might be a straightforward explanation for the ratio of dark matter density of the universe to ordinary matter density of the universe.

Solutions - that were seemingly previously essentially unknown and that extant modeling might consider to associate with the three-word term below ground state - regarding harmonic oscillator mathematics exist and might have use in physics modeling.

People might use observational data about dark matter and objects (especially, but not only just, galaxies) to evaluate the usefulness - regarding elementary particle physics, astrophysics, and cosmology - of proposed modeling.

The following notions might pertain regarding proposed modeling.

People might find that some aspects of proposed modeling are incomplete or are not compatible with data. We suggest that people might be able to adjust proposed modeling - to remedy such lacks of completeness or compatibility - without abandoning much of proposed modeling. Some incompleteness might feature the extents to which neutrinos and arcs model as being Dirac fermions or Majorana fermions.

5. Concluding remarks

Proposed modeling might provide impetus for people to tackle broad agendas that our work suggests. Proposed modeling might provide means to fulfill aspects of such agendas. Proposed modeling might fulfill aspects of such agendas.

Opportunities might exist to develop more sophisticated modeling than the modeling that we present. Such a new level of work might provide more insight than we provide.

Proposed modeling suggests applied mathematics techniques that might have uses other than uses that we make.

Proposed modeling might suggest - directly or indirectly - opportunities for observational research, experimental research, development of precision measuring techniques, development of data analysis techniques, numerical simulations, and theoretical research regarding elementary particle physics, astrophysics, and cosmology.

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