Integrated Formulas of the Fine-structure Constant and Feigenbaum Constants (viXra:2021.0162v3)

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper is a subsequent paper to the previous paper "Formulas of Feigenbaum Constants and Their Physical Meanings" (viXra:2101.0187). In the previous paper, some formulas of Feigenbaum constants in fractional number format were given and the physical meanings of the factors in the formulas were exhibited, especially their relationships with nuclides, the fine-structure constant and 2π. In the previous paper, some integrated formulas of the fine-structure constant, Feigenbaum constants and 2π were also given, briefly denoted as $\alpha_1\delta^2(2\pi)\approx1$, and their relationships with nuclides were illustrated. In this paper, some formulas for $\alpha_1\delta^2(2\pi)\approx1$ are supplemented, some formulas for $\alpha_2(\delta\alpha)^2\approx1$, $[\alpha_1(2\pi)]/(\alpha_2\alpha^2)\approx1$ and $(2\pi)/\alpha^2\approx1$ are given, some formulas of the fine-structure constant (α_1 and α_2) based on the key number 103 instead of 112, 173, 137, 83 and 29 are supplemented. In the end, by introducing correction factors γ_1, γ_2 and γ, accurate formulas $\alpha_1(\delta/\gamma_1)^2(2\pi)=1$, $\alpha_2(\delta\alpha/\gamma_2)^2=1$ and $2\pi/(\alpha\gamma)^2=1$ are gained.

Keywords: Formulas; the fine-structure constant; Feigenbaum constants; 2π.

1. Introduction

In our previous papers[1,2,3,4,5], we gave or exhibited the following formulas.

$$(2\pi)_{Chen-k}=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}};\quad (2\pi)_{Wallis-k}=4\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{2k}{2k+1}\frac{2k+2}{2k+1})$$

$$(2\pi)_{GL-k}=8\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+(-1)^{k+1}\frac{1}{2k+1})\quad (GL\text{ means Gregory-Leibniz})$$

$$(2\pi)_{NC-k}=6+\sum_{n=1}^{k}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}}\quad (NC\text{ means Nilakantha-Chen})$$

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7\cdot(2\pi)_{Chen-112}} \frac{1}{112+\frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13\cdot(2\pi)_{Chen-278}}{100} \frac{1}{112-\frac{1}{64\cdot 3\cdot 29}} = 1/137.035999111818$$

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}} = \sqrt{112\times(168-\frac{1}{3}+\frac{1}{12\cdot 47}-\frac{1}{14\cdot 112\cdot(2\cdot 173+1)})}$$
$$= 137.035999074626$$

$$1/\alpha_1 = 56+81+\frac{1}{28-\frac{13\cdot(2\cdot 56\cdot 11-1)}{3\cdot 5\cdot(2\cdot 56\cdot 43+1)}} = 137.035999037435$$

$$1/\alpha_2 = 56+81+\frac{1}{28-\frac{2\cdot(16\cdot 27-1)}{3\cdot(16\cdot 81+1)}} = 137.035999111818$$

$$c_{au} = \frac{1}{\alpha_c} = 56+81+\frac{1}{28-\frac{5\cdot(4\cdot 3\cdot 7\cdot 17-1)}{2\cdot 5\cdot(4\cdot 5\cdot 7\cdot 23+1)+1}} = 137.035999074626$$

Note: c_{au} refers to the speed of light in vacuum in atomic units

Feigenbaum Constants: $\delta = 4.66920160910299$

$\alpha = 2.50290787509589$

$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326$$
$$= \frac{1}{4}-\frac{1}{27}+\frac{1}{4\cdot 9\cdot 23}-\frac{1}{2\cdot 3\cdot 7\cdot 23\cdot(2\cdot 3\cdot(4\cdot 3\cdot 11-1)+1)+\frac{2\cdot 23}{3\cdot 19}}$$

$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$
$$= \frac{1}{2}-\frac{1}{9}+\frac{1}{3\cdot 31}-\frac{1}{23\cdot(8\cdot 3\cdot 17+1)}+\frac{1}{17\cdot 23\cdot(8\cdot 3\cdot 11^4-1)}$$

Note: 136=8·17, 138=6·23

$\alpha_1\delta^2(2\pi) \approx 1$

On Feb. 8, 2021, we also noticed that Hieb uploaded a paper[6] in viXra in April of 2017, and gave an approximate formula of the fine-structure constant and Feigenbaum constant as follows, but without any explanations to its physical meanings.

$$\delta' = (1/(2\pi\alpha))^{1/2} = 4.670114 \approx \delta = 4.669201609$$
$$\delta' - \delta = 0.000912$$

α: the fine-structure constant, $\alpha \approx 1/137.036$

2. Integrated Formulas of α_1, δ and 2π

A Concise Deduction

The Fine-structure Constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7\cdot(2\pi)_{Chen-112}}\frac{1}{112+\frac{1}{75^2}} = 1/137.035999037435$$

Feigenbaum Constants: $\delta = 4.66920160910299$

$$\alpha_1 = \frac{36}{7\cdot(2\pi)_{Chen-112}}\frac{1}{112+\frac{1}{75^2}} \approx \frac{36}{7\cdot(2\pi)}\frac{1}{112} = (\frac{3}{14})^2\frac{1}{2\pi} \approx \frac{1}{\delta^2(2\pi)}$$

$$= \frac{1}{4.66920160910299^2\cdot(2\cdot3.14159265358979)} \approx \frac{1}{136.982}$$

So it should be resonable to assume the following approximate formulas:

$$\alpha_1\delta^2(2\pi) \approx 1 \text{ or } \frac{1}{\alpha_1\delta^2(2\pi)} \approx 1$$

Numerically: $\alpha_1\delta^2(2\pi) = \frac{4\cdot6692^2\times6.2832}{137.036} = 0.99961 \approx 1$

2021/2/1-3

The above approximate formula $\alpha_1\delta^2(2\pi)\approx1$ is assumed to be the brief form of integrated formulas of α_1, δ and 2π. There should be some corresponding accurate forms of integrated formulas of α_1, δ and 2π as follows.

$$\alpha_1\delta^2(2\pi)_{Chen-25\cdot17} = \frac{4.66920160910299^2\cdot(e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot3\cdot71}{25\cdot17})^{23\cdot37}})}{137.035999037435}$$

$$= \frac{4.66920160910299^2\cdot6.28564399787948}{137.035999037435}$$

$$= 1+\frac{1}{128\cdot9\cdot(2\cdot3\cdot173+1)+\frac{7}{10}} = 1.0000008354717 \approx 1$$

${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{64,66,67,68}_{30}Zn_{34,36,37,38}$ ${}^{80}_{34}Se_{46}$ ${}^{79,81}_{35}Br_{44,46}$

${}^{82,83,84,86}_{36}Kr_{46,47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{105,110}_{46}Pd_{59,64}$ ${}^{111,112}_{48}Cd_{63,64}$ ${}^{112,114,115-120,122,124}_{50}Sn_{62,64,65-70,72,74}$

${}^{121,123}_{51}Sb_{70,72}$ ${}^{126}_{52}Te_{74}$ ${}^{128}_{54}Xe_{74}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{157}_{64}Gd_{93}$ ${}^{168,170}_{68}Er_{100,102}$ ${}^{171,173}_{70}Yb_{101,103}$

${}^{169}_{69}Tm_{100}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{261,262}_{103}Lr^{*}_{158,159}$

${}^{257}_{100}Fm^{*}_{157}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{4\cdot71,286}_{113}Nh^{ie}_{171,173}$ ${}^{326}_{128}Ch^{ie}_{18\cdot11}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{6\cdot71}_{169}Ch^{ie}_{257}$ ${}^{434,435}_{173}Ch^{ie}_{261,262}$

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$$\frac{1}{\alpha_1\delta^2(2\pi)_{Chen-25\cdot17}}=\frac{137.035999037435}{4.66920160910299^2\cdot(e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot3\cdot71}{25\cdot17})^{23\cdot37}})}$$

$$=\frac{137.035999037435}{4.66920160910299^2\cdot6.28564399787948}$$

$$=1-\frac{1}{2\cdot5\cdot7\cdot(2\cdot83\cdot103+1)-\frac{9}{29}}=0.999999164529037\approx1$$

$^{19}_{9}F_{10}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{63,65}_{29}Cu_{34,36}$ $^{79,81}_{35}Br_{44,46}$ $^{83}_{36}Kr_{47}$ $^{103}_{45}Rh_{58}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{140,142}_{58}Ce_{82,84}$
$^{166}_{68}Er_{98}$ $^{173}_{70}Yb_{103}$ $^{209}_{83}Bi^*_{126}$ $^{9\cdot29,262}_{103}Lr^*_{158,159}$ $^{285}_{112}Cn^*_{173}$ $^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{434,15\cdot29}_{173}Ch^{ie}_{9\cdot29,262}$

2021/2/19

$$\alpha_1\delta^2(2\pi)_{Walllis-9\cdot71}=\frac{4.66920160910299^2\cdot\cdot4\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{1278}{1279}\frac{1280}{2\cdot9\cdot71+1})}{137.035999037435}$$

$$=\frac{4.66920160910299^2\cdot6.28564015562186}{137.035999037435}$$

$$=1+\frac{1}{2\cdot9\cdot(8\cdot3\cdot25\cdot7\cdot59-1)-\frac{5}{17}}=1.00000022419606\approx1$$

$^{28,29,30}_{14}Si_{14,15,16}$ $^{35,37}_{17}Cl_{18,20}$ $^{55}_{25}Mn_{30}$ $^{59}_{27}Co_{32}$ $^{64,66,70}_{30}Zn_{34,36,40}$ $^{69,71}_{31}Ga_{38,40}$ $^{105}_{46}Pd_{59}$
$^{82,83,84,86}_{36}Kr_{46,47,48,50}$ $^{85,87}_{37}Rb_{48,50}$ $^{112}_{48}Cd_{64}$ $^{118,120,122}_{50}Sn_{68,70,72}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{3\cdot47}_{59}\mathrm{Pr}_{82}$
$^{144,145,150}_{60}Nd_{84,85,90}$ $^{173}_{70}Yb_{103}$ $^{175,176}_{71}Lu_{104,105}$ $^{3\cdot59}_{72}Hf_{105}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{208}_{82}Pb_{126}$ $^{209}_{83}Bi^*_{126}$
$^{209}_{84}Po^*_{125}$ $^{210}_{85}At^*_{125}$ $^{285}_{112}Cn^*_{173}$ $^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

2021/2/1

$$\frac{1}{\alpha_1\delta^2(2\pi)_{Wallis-9\cdot71}}=\frac{137.035999037435}{4.66920160910299^2\cdot4\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{1278}{1279}\frac{1280}{2\cdot9\cdot71+1})}$$

$$=\frac{137.035999037435}{4.66920160910299^2\cdot6.28564015562186}$$

$$=1-\frac{1}{2\cdot9\cdot(8\cdot3\cdot25\cdot7\cdot59-1)+\frac{3\cdot4}{17}}=0.999999775803991\approx1$$

$^{28,29,30}_{14}Si_{14,15,16}$ $^{35,37}_{17}Cl_{18,20}$ $^{55}_{25}Mn_{30}$ $^{59}_{27}Co_{32}$ $^{64,66,68,70}_{30}Zn_{34,36,38,40}$ $^{69,71}_{31}Ga_{38,40}$ $^{76,80,82}_{34}Se_{42,46,48}$
$^{79,81}_{35}Br_{44,46}$ $^{82,83,84,86}_{36}Kr_{46,47,48,50}$ $^{85,87}_{37}Rb_{48,50}$ $^{100}_{44}Ru_{56}$ $^{105}_{46}Pd_{59}$ $^{112}_{48}Cd_{64}$ $^{118,120,122}_{50}Sn_{68,70,72}$
$^{136,137,138}_{56}Ba_{80,81,82}$ $^{3\cdot47}_{59}\mathrm{Pr}_{82}$ $^{168}_{68}Er_{100}$ $^{173}_{70}Yb_{103}$ $^{175,176}_{71}Lu_{104,105}$ $^{3\cdot59}_{72}Hf_{105}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$
$^{208}_{82}Pb_{126}$ $^{209}_{83}Bi^*_{126}$ $^{209}_{84}Po^*_{125}$ $^{210}_{85}At^*_{125}$ $^{285}_{112}Cn^*_{173}$ $^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

2021/2/20

$$\alpha_1\delta^2(2\pi)_{GL-22\cdot37}=\frac{4.66920160910299^2\cdot8\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot11\cdot37+1})}{137.035999037435}$$

$$=\frac{4.66920160910299^2\cdot6.28563929398602}{137.035999037435}$$

$$=1+\frac{1}{163\cdot(6\cdot11^2\cdot97+1)+\frac{9}{10}}=1.00000008711598\approx1$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{48,50}_{22}Ti_{26,28}$ ${}^{66,67}_{30}Zn_{36,37}$ ${}^{75}_{33}As_{42}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{97}_{42}Mo_{55}$ ${}^{97}_{43}Tc^{*}_{54}$ ${}^{163}_{66}Dy_{97}$

${}^{165}_{67}Ho_{98}$ ${}^{3\cdot67}_{80}Hg_{121}$ ${}^{6\cdot37}_{86}Rn^{*}_{136}$ ${}^{247}_{97}Bk^{*}_{150}$ ${}^{4\cdot67}_{105}Db^{*}_{163}$ ${}^{269,271}_{106}Sg^{*}_{163,165}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{2\cdot163}_{128}Ch^{*}_{3\cdot66}$ ${}^{410}_{163}Ch^{ie}_{247}$

2021/2/1

$$\frac{1}{\alpha_1\delta^2(2\pi)_{GL-22\cdot37}}=\frac{137.035999037435}{4.66920160910299^2\cdot8\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot11\cdot37+1})}$$

$$=\frac{137.035999037435}{4.66920160910299^2\cdot6.28563929398602}$$

$$=1-\frac{1}{9\cdot11\cdot47\cdot(2\cdot9\cdot137+1)-\frac{2}{25}}=0.999999912884025\approx1$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{47,50}_{22}Ti_{25,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{75}_{33}As_{42}$ ${}^{83}_{36}Kr_{47}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{98,99,100}_{44}Ru_{54,55,56}$

${}^{107,109}_{47}Ag_{60,62}$ ${}^{116,118,119}_{50}Sn_{66,68,69}$ ${}^{128,129,132}_{54}Xe_{74,75,78}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$

${}^{5\cdot37,16\cdot11}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{6\cdot37}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{208,209,210}$

2021/2/19

$$\alpha_1\delta^2(2\pi)_{NC-3}=\frac{4.66920160910299^2\cdot(6+\sum_{n=1}^{3}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}})}{137.035999037435}$$

$$=\frac{4.66920160910299^2\cdot6.29047619047619}{137.035999037435}$$

$$=1+\frac{1}{3\cdot(2\cdot7\cdot31-1)}-\frac{1}{13\cdot89\cdot(2\cdot37\cdot53+1)-\frac{2\cdot9}{25}}=1.00076960262352\approx1$$

${}^{25}_{12}Mg_{13}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{50,52,53}_{24}Cr_{26,28,29}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{74}_{32}Ge_{42}$

${}^{89}_{39}Y_{50}$ ${}^{95}_{42}Mo_{53}$ ${}^{127}_{53}I_{74}$ ${}^{150,152}_{62}Sm_{88,90}$ ${}^{178}_{72}Hf_{106}$ ${}^{180,186}_{74}W_{106,112}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{3\cdot53}$ ${}^{14\cdot31,435}_{173}Ch^{ie}_{261,262}$

2021/2/1,3

$$\frac{1}{\alpha_1\delta^2(2\pi)_{NC-3}}=\frac{137.035999037435}{4.66920160910299^2\cdot(6+\sum_{n=1}^{3}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}})}$$

$$=\frac{137.035999037435}{4.66920160910299^2\cdot6.29047619047619}$$

$$=1-\frac{1}{4\cdot 25\cdot 13}+\frac{1}{4\cdot 9\cdot 25\cdot(2\cdot 25\cdot(4\cdot 25+1)+1)+\frac{2}{7}}=0.999230989209198\approx 1$$

${}^{12,13}_{6}C_{6,7}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{24,25,26}_{12}Mn_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{47,48,50}_{22}Ti_{25,26,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{84,86}_{36}Kr_{48,50}$

${}^{100,101}_{44}Ru_{56,57}$ ${}^{115,117,118,119,120,122}_{50}Sn_{65,67,68,69,70,72}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{170,171}_{70}Yb_{100,101}$ ${}^{257}_{100}Fm^{*}_{157}$

2021/2/20

$$\alpha_1\delta^2(2\pi)=\frac{4.66920160910299^2\cdot(2\cdot 3.14159265358979)}{137.035999037435}$$

$$=1-\frac{1}{13\cdot 197}+\frac{1}{2\cdot 7\cdot 41\cdot(4\cdot 5\cdot 19\cdot 31-1)}=0.99960967543223\approx 1$$

${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,52}_{24}Cr_{26,28}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{68,70}_{31}Ga_{38,40}$ ${}^{93}_{41}Nb_{52}$ ${}^{123,124,128,130}_{52}Te_{71,72,76,78}$

${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{197}_{79}Au_{118}$ ${}^{206}_{82}Pb_{124}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{310}_{124}Ch^{ie}_{186}$ ${}^{326}_{129}Ch^{ie}_{197}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\frac{1}{\alpha_1\delta^2(2\pi)}=\frac{137.035999037435}{4.66920160910299^2\cdot(2\cdot 3.14159265358979)}$$

$$=1+\frac{1}{512\cdot 5}-\frac{1}{4\cdot 9\cdot 7\cdot 17\cdot 19\cdot 83}=1.00039047698053\approx 1$$

${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{58,60,62,64}_{28}Ni_{30,32,34,36}$ ${}^{83,84}_{36}Kr_{47,48}$

${}^{64,66,68,70}_{30}Zn_{34,36,38,40}$ ${}^{114,118,119}_{50}Sn_{64,68,69}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{157,160}_{64}Gd_{93,96}$ ${}^{166,168,170}_{68}Er_{98,100,102}$

${}^{188,190}_{76}Os_{112,114}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot 157}_{126}Ch^{ie}_{188}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

2021/2/8

3. Integrated Formulas of α_2, δ and α

The Fine-structure Constant:

$$\alpha_2=\frac{2\pi r_e}{\lambda_e}=\frac{13\cdot(2\pi)_{Chen-278}}{100}\frac{1}{112-\frac{1}{64\cdot 3\cdot 29}}=1/137.035999111818$$

Feigenbaum Constants: $\delta=4.66920160910299$

$\alpha=2.50290787509589$

$$\alpha_2=\frac{13\cdot(2\pi)_{Chen-278}}{100}\frac{1}{112-\frac{1}{64\cdot 3\cdot 29}}\approx\frac{13\cdot(2\pi)}{100}\frac{1}{112}\approx\frac{(2\pi)}{(\delta\alpha)^2(2\pi)}$$

$$\approx\frac{1}{(\delta\alpha)^2}=\frac{1}{(4.66920160910299\cdot 2.50290787509589)^2}\approx 1/136.575$$

So it should be resonable to assume the following approximate formulas:

$\alpha_2(\delta\alpha)^2\approx 1$ or $\frac{1}{\alpha_2(\delta\alpha)^2}\approx 1$

Numerically: $\alpha_2(\delta\alpha)^2=\frac{(4\cdot 6692\times 2.5029)^2}{137.036}=0.99664\approx 1$

2021/2/7

The above approximate formula $\alpha_2(\delta\alpha)^2 \approx 1$ is assumed to be the brief form of integrated formulas of α_2, δ and α. There should be some corresponding accurate forms of integrated formulas of α_2, δ and α as follows.

$$\alpha_2(\delta\alpha)^2 = \frac{(4.66920160910299 \cdot 2.50290787509589)^2}{137.035999111818}$$

$$= 1 - \frac{1}{2 \cdot 149} + \frac{1}{29 \cdot 31 \cdot (2 \cdot 3 \cdot 49 \cdot 13 + 1) - \frac{16}{19}} = 0.996644586263908 \approx 1$$

$^{12,13}_{6}C_{6,7}$ $^{14,15}_{7}N_{7,8}$ $^{19}_{9}F_{10}$ $^{24,25,26}_{12}Mg_{12,13,14}$ $^{27}_{13}Al_{14}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{31}_{15}P_{16}$ $^{39,40,41}_{19}K_{20,21,22}$

$^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{50,52,53}_{24}Cr_{26,28,29}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{58,60,62,64}_{28}Ni_{30,32,34,36}$

$^{63,65}_{29}Cu_{34,36}$ $^{69,71}_{31}Ga_{38,40}$ $^{70,74,76}_{32}Ge_{38,42,44}$ $^{76,78}_{34}Se_{42,44}$ $^{85,87}_{37}Rb_{48,50}$ $^{89}_{39}Y_{50}$ $^{93}_{41}Nb_{52}$

$^{94,95,96,98,100}_{42}Mo_{52,53,54,56,58}$ $^{112,113}_{48}Cd_{64,65}$ $^{113,115}_{49}In_{64,66}$ $^{7\cdot 19}_{54}Xe_{76,78}$ $^{7\cdot 19}_{55}Cs_{78}$ $^{149}_{62}Sm_{87}$

$^{134,136,137,138}_{56}Ba_{78,80,81,82}$ $^{155,156,157,160}_{64}Gd_{91,92,93,96}$ $^{186,187,189,190,192}_{76}Os_{110,112,113,114,116}$

$^{190,192,194,195,196}_{78}Pt_{112,114,116,117,118}$ $^{223}_{87}Fr^{*}_{136}$ $^{227}_{89}Ac^{*}_{138}$ $^{21\cdot 11}_{91}Pa^{*}_{140}$ $^{237}_{93}Np^{*}_{144}$ $^{15\cdot 19}_{112}Cn^{*}_{173}$

$^{22\cdot 13}_{113}Nh^{ie}_{173}$ $^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{8\cdot 47}_{149}Ch^{ie}_{227}$ $^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 19,262}$

2021/2/7

$$\frac{1}{\alpha_2(\delta\alpha)^2} = \frac{137.035999111818}{(4.66920160910299 \cdot 2.50290787509589)^2}$$

$$= 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (4 \cdot 11 \cdot 47 + 1) - \frac{2}{5}}$$

$$= 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (2 \cdot 9 \cdot 5 \cdot 23 - 1) - \frac{2}{5}} = 1.00336671044256 \approx 1$$

$^{10,11}_{5}Be_{5,6}$ $^{23}_{11}Na_{12}$ $^{24,25}_{12}Mg_{12,13}$ $^{46,47,49,50}_{22}Ti_{24,25,27,28}$ $^{50,51}_{23}V_{27,28}$ $^{55}_{25}Mn_{30}$ $^{75}_{33}As_{42}$

$^{80,82,83,86}_{36}Kr_{44,46,47,50}$ $^{98,99,100,104}_{44}Ru_{54,55,56,60}$ $^{107,109}_{47}Ag_{60,62}$ $^{115,116,119,120}_{50}Sn_{65,66,69,70}$

$^{129,131,132}_{54}Xe_{75,77,78}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{169}_{69}Tm_{100}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{226}_{88}Ra^{*}_{138}$

$^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

2021/2/9

4. Integrated Formulas of α_1, α_2, α and 2π

$$\alpha_1\delta^2(2\pi) = 0.99961 \approx 1$$

$$\alpha_2(\delta\alpha)^2 = 0.99664 \approx 1$$

$$\frac{\alpha_1\delta^2(2\pi)}{\alpha_2(\delta\alpha)^2} = \frac{\alpha_1(2\pi)}{\alpha_2\alpha^2} \approx \frac{2\pi}{\alpha^2} = 1.002975 \approx 1$$

2021/2/11

$$\frac{\alpha_1(2\pi)}{\alpha_2\alpha^2}=\frac{137.035999111818\cdot(2\cdot 3.14159265358979)}{137.035999037435\cdot 2.50290787509589^2}$$

$$=1+\frac{1}{16\cdot 3\cdot 7}-\frac{1}{2\cdot 3\cdot 7\cdot(4\cdot 17\cdot(2\cdot 157-1)-1)-\frac{1}{4}}=1.00297507176499\approx 1$$

${}^{45}_{21}Sc_{24}$ ${}^{100}_{44}Ru_{56}$ ${}^{112}_{48}Cd_{64}$ ${}^{136}_{56}Ba_{80}$ ${}^{157}_{64}Gd_{93}$ ${}^{168}_{68}Er_{100}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{2\cdot 157}_{126}Ch^{ie}_{188}$ ${}^{400}_{157}Ch^{ie}_{243}$

2021/2/11,17

$$\frac{\alpha_2\alpha^2}{\alpha_1(2\pi)}=\frac{137.035999037435\cdot 2.50290787509589^2}{137.035999111818\cdot(2\cdot 3.14159265358979)}$$

$$=1-\frac{1}{16\cdot 3\cdot 7+1}+\frac{1}{7\cdot 19\cdot(2\cdot 3\cdot 7^2\cdot 23-1)+\frac{16}{3\cdot 13}}$$

$$=1-\frac{1}{16\cdot 3\cdot 7+1}+\frac{1}{7\cdot 19\cdot(8\cdot 5\cdot 13^2+1)+\frac{16}{3\cdot 13}}$$

$$=1-\frac{1}{16\cdot 3\cdot 7+1}+\frac{1}{2\cdot 3\cdot 73\cdot(4\cdot 27\cdot 19+1)-\frac{23}{3\cdot 13}}=0.997033753032614\approx 1$$

${}^{27}_{13}Al_{14}$ ${}^{50,51}_{23}V_{37,28}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{87}_{38}Sr_{49}$ ${}^{89}_{39}Y_{50}$ ${}^{125}_{52}Te_{73}$ ${}^{133}_{55}Cs_{78}$ ${}^{137,138}_{56}Ba_{81,82}$
${}^{169}_{69}Tm_{100}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{203,205}_{81}Tl_{122,124}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{238}_{92}U^{*}_{146}$ ${}^{336}_{133}Ch^{ie}_{203}$

2021/2/21

$$\frac{2\pi}{\alpha^2}=\frac{2\cdot 3.14159265358979}{2.50290787509589^2}=1+\frac{1}{16\cdot 3\cdot 7}-\frac{1}{3\cdot 13\cdot 31\cdot(2\cdot 9\cdot 41+1)-\frac{1}{22}}$$

$$=1.00297507122057\approx 1$$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,48,50}_{22}Ti_{24,26,28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{69,71}_{31}Ga_{38,40}$
${}^{73,74}_{32}Ge_{41,42}$ ${}^{89}_{39}Y_{50}$ ${}^{93}_{41}Nb_{52}$ ${}^{112}_{48}Cd_{64}$ ${}^{138}_{56}Ba_{82}$ ${}^{203,205}_{81}Tl_{122,124}$ ${}^{206,208}_{82}Pt_{124,126}$ ${}^{227}_{89}Ac^{*}_{138}$

2021/2/11,17

$$\frac{\alpha^2}{2\pi}=\frac{2.50290787509589^2}{2\cdot 3.14159265358979}=1-\frac{1}{16\cdot 3\cdot 7+1}+\frac{1}{8\cdot 81\cdot 19\cdot 73+\frac{1}{3\cdot 13}}$$

$$=0.997033753573803\approx 1$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{73}_{32}Ge_{41}$ ${}^{125}_{52}Te_{73}$ ${}^{137}_{56}Ba_{81}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{203,205}_{81}Tl_{122,124}$ ${}^{336}_{133}Ch^{ie}_{203}$

2021/2/21

5. Marvelous Coincidences

There are some marvelous coincidences of factors with nuclides in the above formulas. One typical example of these coincidences is listed as follows, which indicates the methodology and the formulas in this paper should be correct.

$$\alpha_1\delta^2(2\pi)_{Chen-25\cdot 17} = 1 + \frac{1}{128\cdot 9\cdot(2\cdot 3\cdot 173+1)+\frac{7}{10}} = 1.0000008354717 \approx 1$$

$$\frac{1}{\alpha_1\delta^2(2\pi)_{Chen-25\cdot 17}} = 1 - \frac{1}{2\cdot 5\cdot 7\cdot(2\cdot 83\cdot 103+1)-\frac{9}{29}} = 0.999999164529037 \approx 1$$

$$(2\pi)_{Chen-25\cdot 17} = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots \frac{e^2}{(\frac{2\cdot 3\cdot 71}{25\cdot 17})^{23\cdot 37}}$$

$^{28,29,30}_{14}Si_{14,15,16}$ $^{35,37}_{17}Cl_{18,20}$ $^{50,51}_{23}V_{27,28}$ $^{63,65}_{29}Cu_{34,36}$ $^{80}_{34}Se_{46}$ $^{79,81}_{35}Br_{44,46}$ $^{82,83,84,86}_{36}Kr_{46,47,48,50}$
$^{85,87}_{37}Rb_{48,50}$ $^{103}_{45}Rh_{58}$ $^{112}_{48}Cd_{64}$ $^{128}_{54}Xe_{74}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{140,142}_{58}Ce_{82,84}$ $^{173}_{70}Yb_{103}$ $^{209}_{83}Bi^*_{126}$
$^{209}_{84}Po^*_{125}$ $^{210}_{85}At^*_{125}$ $^{9\cdot 29,262}_{103}Lr^*_{158,159}$ $^{285}_{112}Cn^*_{173}$ $^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{434,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$

6. Formulas of the Fine-structure Constant based on 103

In our previous paper[1,2,4], many formulas of the fine-structure constant based on the key numbers 112, 173, 137, 83 and 29 were given. As shown in the above two formulas in **Section 5**, it seems 103 is another key number comparable to the above stated key numbers, so some formulas of the fine-structure constant based on the key number 103 instead of them are constructed as follows.

$$\alpha_1 = \frac{137}{29\cdot e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots \frac{e^2}{(\frac{11\cdot 47}{4\cdot 3\cdot 43})^{1033}}} \frac{1}{103+\frac{1}{32\cdot(32\cdot 29+1)-\frac{3}{2\cdot 17}}}$$

$$= \frac{137}{29\cdot e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots \frac{e^2}{(\frac{11\cdot 47}{4\cdot 3\cdot 43})^{1033}}} \frac{1}{103+\frac{1}{81\cdot(2\cdot 3\cdot 61+1)+\frac{31}{2\cdot 17}}}$$

$$= \frac{137}{29\cdot e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots \frac{e^2}{(\frac{11\cdot 47}{4\cdot 3\cdot 43})^{1033}}} \frac{1}{103+\frac{1}{81\cdot(16\cdot 23-1)+\frac{31}{2\cdot 17}}}$$

$$= 1/137.035999037435$$

$^{23}_{11}Na_{12}$ $^{35,37}_{17}Cl_{18,20}$ $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{50,51}_{23}V_{27,28}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{63,65}_{29}Cu_{34,36}$
$^{69,71}_{31}Ga_{38,40}$ $^{70,72,76}_{32}Ge_{38,40,44}$ $^{77,78}_{34}Se_{43,44}$ $^{79,81}_{35}Br_{44,46}$ $^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ $^{97,98,99}_{43}Tc^*_{54,55,56}$
$^{103}_{45}Rh_{58}$ $^{107,109}_{47}Ag_{60,62}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{5\cdot 29,146,147}_{61}Pm^*_{84,85,86}$ $^{173}_{70}Yb_{103}$ $^{7\cdot 29,205}_{81}Tl_{122,124}$
$^{9\cdot 29,262}_{103}Lr^*_{158,159}$ $^{285}_{112}Cn^*_{173}$ $^{8\cdot 43,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$

2021/2/25

$$\alpha_1 = \frac{137}{29\cdot 4\cdot (2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{1548}{1549}\frac{2\cdot 25\cdot 31}{2\cdot 2\cdot 9\cdot 43+1})}\frac{1}{103+\frac{1}{2\cdot 3\cdot 5\cdot (2\cdot 3\cdot 5\cdot 7\cdot 13-1)-\frac{3}{17}}}$$

$=1/137.035999037435$

${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{103}_{45}Rh_{58}$

${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{173}_{70}Yb_{103}$ ${}^{9\cdot 29,262}_{103}Lr^{*}_{158,159}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{8\cdot 43,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$

$$\alpha_1 = \frac{137}{29\cdot 8\cdot (1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 17\cdot 29+1})}\frac{1}{103+\frac{1}{7\cdot (4\cdot 7\cdot 199+1)+\frac{4}{7}}}$$

$=1/137.035999037435$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{58,62}_{28}Ni_{30,34}$ ${}^{80}_{34}Se_{46}$ ${}^{103}_{45}Rh_{58}$ ${}^{116,118,119,120}_{50}Sn_{66,68,69,70}$

${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{173}_{70}Yb_{103}$ ${}^{198,199,200}_{80}Hg_{118,119,120}$ ${}^{9\cdot 29,262}_{103}Lr^{*}_{158,159}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_1 = \frac{137}{29\cdot (6+\sum_{n=1}^{7}\frac{(-1)^{n+1}}{n(n+1/2)(n+1)})}\frac{1}{103+\frac{1}{3\cdot 19}-\frac{1}{125\cdot (8\cdot 7\cdot 11+1)+\frac{1}{4}}}$$

$=1/137.035999037435$

${}^{23}_{11}Na_{12}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{173}_{70}Yb_{103}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$

2021/2/26

$$\alpha_2 = \frac{25\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2500}{3\cdot 49\cdot 17})^{4999}}}{11\cdot 19}\frac{1}{103-\frac{1}{32\cdot (512\cdot 25-1)+\frac{3}{10}}}$$

$=1/137.035999111818$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{47,49,50}_{22}Ti_{25,27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{64,66,68}_{30}Zn_{34,36,38}$ ${}^{98,99,100,101,102}_{44}Ru_{54,55,56,57,58}$

${}^{113,115}_{49}In_{64,66}$ ${}^{114,116,118,119}_{50}Sn_{64,66,68,69}$ ${}^{157}_{64}Gd_{93}$ ${}^{166,167,168,170}_{68}Er_{98,99,100,102}$ ${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$

${}^{11\cdot 19}_{83}Bi^{*}_{126}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{11\cdot 19}$

2021/2/25

$$\alpha_2 = \frac{25\cdot 4\cdot (2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{7496}{7497}\frac{2\cdot 23\cdot 163}{9\cdot 49\cdot 17})}{11\cdot 19}\frac{1}{103-\frac{1}{7\cdot (16\cdot 3\cdot 19\cdot 257-1)}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{98,99,100,101}_{44}Ru_{54,55,56,57}$

${}^{112}_{48}Cd_{64}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{118,119}_{50}Sn_{68,69}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{157}_{64}Gd_{93}$ ${}^{163}_{66}Dy_{97}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$

${}^{173}_{70}Yb_{103}$ ${}^{11\cdot 19}_{83}Bi^{*}_{126}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{8\cdot 43,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_2=\frac{25\cdot 8\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{5\cdot 23\cdot 83})}{11\cdot 19}\frac{1}{103-\frac{1}{2\cdot 3\cdot 5\cdot 13\cdot 23\cdot(2\cdot 11\cdot 17-1)}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{11\cdot 19}_{83}Bi^{*}_{126}$

$$\alpha_2=\frac{25\cdot(6+\sum_{n=1}^{11}\frac{(-1)^{n+1}}{n(n+1/2)(n+1)})}{11\cdot 19}\frac{1}{103-\frac{1}{4\cdot 5\cdot 23}+\frac{1}{2\cdot(2\cdot 3\cdot 5\cdot 7+1)\cdot(2\cdot 9\cdot 17+1)+\frac{2}{3}}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{103}_{45}Rh_{58}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{173}_{70}Yb_{103}$
${}^{11\cdot 19}_{83}Bi^{*}_{126}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{238}_{92}U^{*}_{143}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

2021/2/26

7. Integrated Formulas of α_1, δ, 2π and γ_1

By introducing a correction factor γ_1, some integrated formulas of α_1, δ, 2π and γ_1 in the format of $\alpha_1(\delta\gamma_1)^2(2\pi)=1$ could be obtained as follows.

$\alpha_1(\delta/\gamma_1)^2(2\pi)=1$

$$\gamma_1=\sqrt{2\pi\alpha_1}\delta=\sqrt{\frac{6.28564399787948}{137.035999037435}}4.66920160910299$$

$$=(1-\frac{1}{47\cdot 109}+\frac{1}{27\cdot 7\cdot(3\cdot 8\cdot(3\cdot 8\cdot(4\cdot 137-1)-1)-1)})^2=0.999804818668238$$

${}^{45}_{21}V_{24}$ ${}^{59}_{27}Co_{32}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{158}_{64}Gd_{94}$ ${}^{183}_{74}W_{109}$ ${}^{4\cdot 47}_{76}Os_{112}$ ${}^{137}_{56}Ba_{81}$ ${}^{209}_{83}Bi^{*}_{126}$
${}^{209}_{84}Po^{*}_{125}$ ${}^{278}_{109}Mt^{*}_{169}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$

$$\frac{1}{\gamma_1}=\frac{1}{\sqrt{2\pi\alpha_1}\delta}=\sqrt{\frac{137.035999037435}{2\cdot 3.14159265358979}}\frac{1}{4.66920160910299}$$

$$=(1+\frac{1}{2\cdot 13\cdot 197}-\frac{16\cdot 7\cdot 17\cdot(16\cdot 3\cdot 23-1)}{125\cdot 10^{12}})^2=1.00019521943495\approx 1$$

${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{46,48,50}_{22}Ti_{24,26,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$
${}^{58,60,62,64}_{28}Ni_{30,32,34,36}$ ${}^{74,76,78,80,82}_{34}Se_{40,42,44,46,48}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{90,91,92,96}_{40}Zr_{50,51,52,56}$ ${}^{112,116}_{48}Cd_{64,68}$
${}^{102,104,105,106,110}_{46}Pd_{56,58,59,60,64}$ ${}^{114,115,118,119,120}_{50}Sn_{64,65,68,69,70}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{156,160}_{64}Gd_{92,96}$
${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{197}_{79}Au_{118}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$
${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{238}_{92}U^{*}_{146}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{24\cdot 13}_{125}Ch^{ie}_{187}$ ${}^{326}_{129}Ch^{ie}_{197}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{2\cdot 197}_{156}Ch^{ie}_{238}$

2021/2/28

$$\alpha_1(\delta/\gamma_{1-Chen-25\cdot17})^2(2\pi)_{Chen-25\cdot17}=1$$

$$\gamma_{1-Chen-25\cdot17}=\sqrt{(2\pi)_{Chen-25\cdot17}\alpha_1\delta}$$

$$=\frac{\sqrt{e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot3\cdot71}{25\cdot17})^{23\cdot37}}}\cdot4.66920160910299}{\sqrt{137.035999037435}}$$

$$=\frac{\sqrt{6.28564399787948\cdot4.66920160910299}}{\sqrt{137.035999037435}}$$

$$=1+\frac{1}{2\cdot79\cdot109\cdot139-\frac{2}{17}or\frac{3}{25}}=1.00000041773574$$

${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{85,87}_{37}Rb_{48,50}$

${}^{100}_{44}Ru_{56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{118,119}_{50}Sn_{68,69}$ ${}^{157,2\cdot79}_{64}Gd_{93,94}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{183}_{74}W_{109}$ ${}^{197}_{79}Au_{118}$ ${}^{257}_{100}Fm^{*}_{157}$

${}^{2\cdot139}_{109}Mt^{*}_{169}$ ${}^{6\cdot71}_{169}Ch^{ie}_{257}$

$$\frac{1}{\gamma_{1-Chen-25\cdot17}}=\frac{1}{\sqrt{(2\pi)_{Chen-25\cdot17}\alpha_1\delta}}$$

$$=1-\frac{1}{2\cdot79\cdot109\cdot139+\frac{15}{17}or\frac{22}{25}}=0.999999582264432$$

${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{85,87}_{37}Rb_{48,50}$

${}^{100}_{44}Ru_{56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{118,119}_{50}Sn_{68,69}$ ${}^{157,2\cdot79}_{64}Gd_{93,94}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{183}_{74}W_{109}$ ${}^{197}_{79}Au_{118}$ ${}^{257}_{100}Fm^{*}_{157}$

${}^{2\cdot139}_{109}Mt^{*}_{169}$ ${}^{6\cdot71}_{169}Ch^{ie}_{257}$

2021/3/1

$$\alpha_1(\delta^2/\gamma_{1-Wallis-9\cdot71})(2\pi)_{Wallis-9\cdot71}$$

$$\gamma_{1-Wallis-9\cdot71}=\sqrt{(2\pi)_{Wallis-9\cdot71}\alpha_1\delta}$$

$$=\frac{\sqrt{4\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{1278}{1279}\frac{1280}{2\cdot9\cdot71+1})}\cdot4.66920160910299}{\sqrt{137.035999037435}}$$

$$=\frac{\sqrt{6.28564015562186\cdot4.66920160910299}}{\sqrt{137.035999037435}}$$

$$=1+\frac{1}{4\cdot9\cdot(8\cdot3\cdot25\cdot7\cdot59-1)}=1.00000011209802$$

${}^{27}_{13}Al_{14}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{82,83,84,86}_{36}Kr_{46,47,48,50}$ ${}^{105}_{46}Pd_{59}$ ${}^{112}_{48}Cd_{64}$ ${}^{121,123}_{51}Sb_{70,72}$

${}^{112,114,118,120,122}_{50}Sn_{62,64,68,70,72}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{173}_{70}Yb_{103}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{175}_{75}\mathrm{Re}_{112}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$

${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

2021/3/2

$$\frac{1}{\gamma_{1-Wallis-9\cdot 71}}=\frac{1}{\sqrt{(2\pi)_{Wallis-9\cdot 71}\alpha_1\delta}}$$

$$=1-\frac{1}{5\cdot 7\cdot(32\cdot 27\cdot 5\cdot 59-1)}=0.999999887901990$$

$^{27}_{13}Al_{14}$ $^{50,51}_{23}V_{27,28}$ $^{59}_{27}Co_{32}$ $^{69,71}_{31}Ga_{38,40}$ $^{82,83,84,86}_{36}Kr_{46,47,48,50}$ $^{105}_{46}Pd_{59}$ $^{112}_{48}Cd_{64}$ $^{121,123}_{51}Sb_{70,72}$ $^{112,114,118,120,122}_{50}Sn_{62,64,68,70,72}$ $^{141}_{59}\mathrm{Pr}_{82}$ $^{173}_{70}Yb_{103}$ $^{175,176}_{71}Lu_{104,105}$ $^{175}_{75}\mathrm{Re}_{112}$ $^{208}_{82}Pb_{126}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{210}_{85}At^{*}_{125}$ $^{285}_{112}Cn^{*}_{173}$ $^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

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$$\alpha_1(\delta/\gamma_{1-GL-22\cdot 37})^2(2\pi)_{GL-22\cdot 37}=1$$

$$\gamma_{1-GL-22\cdot 37}=\sqrt{(2\pi)_{GL-22\cdot 37}\alpha_1\delta}$$

$$=\frac{\sqrt{8\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 11\cdot 37+1})\cdot 4.66920160910299}}{\sqrt{137.035999037435}}$$

$$=\frac{\sqrt{6.28563929398602\cdot 4.66920160910299}}{\sqrt{137.035999037435}}$$

$$=1+\frac{1}{4\cdot 25\cdot 7\cdot(2\cdot 23^2\cdot 31-1)+\frac{6}{11}}=1.00000004355799$$

$^{23}_{11}Na_{12}$ $^{47}_{22}Ti_{25}$ $^{50,51}_{23}V_{27,28}$ $^{55}_{25}Mn_{30}$ $^{69,71}_{31}Ga_{38,40}$ $^{75}_{33}As_{42}$ $^{85,87}_{37}Rb_{48,50}$ $^{99,100}_{44}Ru_{55,56}$ $^{108}_{46}Pd_{62}$ $^{112,115,116,119,120,124}_{50}Sn_{62,65,66,69,70,74}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{169}_{69}Tm_{100}$ $^{184}_{74}W_{110}$ $^{257}_{100}Fm^{*}_{157}$

$$\frac{1}{\gamma_{1-GL-22\cdot 37}}=\frac{1}{\sqrt{(2\pi)_{GL-22\cdot 37}\alpha_1\delta}}$$

$$=1-\frac{1}{2\cdot 9\cdot 11\cdot 47\cdot(2\cdot 9\cdot 137+1)-\frac{5}{11}}=0.999999956442012$$

$^{23}_{11}Na_{12}$ $^{47}_{22}Ti_{25}$ $^{55}_{25}Mn_{30}$ $^{83,84}_{36}Kr_{47,48}$ $^{85,87}_{37}Rb_{48,50}$ $^{99,100}_{44}Ru_{55,56}$ $^{107,109}_{47}Ag_{60,62}$ $^{137}_{56}Ba_{81}$ $^{160,162}_{66}Dy_{94,96}$ $^{182,184}_{74}W_{108,110}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{285}_{112}Cn^{*}_{173}$ $^{2\cdot 173}_{137}Fy^{*}_{209}$

2021/3/2

$$\alpha_1(\delta/\gamma_{1-NC-3})^2(2\pi)_{NC-3}=1$$

$$\gamma_{1-NC-3}=\sqrt{(2\pi)_{NC-3}\alpha_1\delta}$$

$$=\frac{\sqrt{(6+\sum_{n=1}^{3}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}})\cdot 4.66920160910299}}{\sqrt{137.035999037435}}$$

$$=\frac{\sqrt{6.29047619047619\cdot 4.66920160910299}}{\sqrt{137.035999037435}}$$

$$=1+\frac{1}{23\cdot 113}-\frac{1}{2\cdot 3\cdot 257\cdot(4\cdot 5\cdot 29\cdot 31+1)}=1.00038472730421$$

$^{50,51}_{23}V_{27,28}$ $^{63,65}_{29}Cu_{34,36}$ $^{69,71}_{31}Ga_{38,40}$ $^{104,108}_{46}Pd_{58,62}$ $^{113,115}_{49}In_{64,66}$ $^{157}_{64}Gd_{93}$ $^{169}_{69}Tm_{100}$ $^{189}_{76}Os_{113}$

$^{226}_{88}Ra^{*}_{138}$ $^{257}_{100}Fm^{*}_{157}$ $^{284,286}_{113}Nh^{ie}_{171,173}$ $^{426}_{169}Ch^{ie}_{257}$

2021/3/3

$$\frac{1}{\gamma_{1-NC-3}}=\frac{1}{\sqrt{(2\pi)_{NC-3}\alpha_1}\delta}$$

$$=1-\frac{1}{8\cdot 25\cdot 13}+\frac{1}{179\cdot(8\cdot 9\cdot(2\cdot 3\cdot(2\cdot 179+1)-1)+1)-\frac{3}{7}}=0.999615420653963$$

$^{56}_{26}Fe_{30}$ $^{100}_{44}Ru_{56}$ $^{115,119}_{50}Sn_{65,69}$ $^{124}_{52}Te_{72}$ $^{169}_{69}Tm_{100}$ $^{179}_{72}Hf_{107}$ $^{257}_{100}Fm^{*}_{157}$ $^{298}_{119}Ch^{ie}_{179}$

2021/3/2

8. Integrated Formulas of α_2, α, δ and γ_2

By introducing a correction factor γ_2, some integrated formulas of α_2, α, δ and γ_2 in the format of $\alpha_2(\delta\alpha\gamma_2)^2=1$ could be obtained as follows.

$$\alpha_2(\delta\alpha/\gamma_2)^2=1$$

$$\gamma_2=\sqrt{\alpha_2(\alpha\delta)^2}=\frac{4.66920160910299\cdot 2.50290787509589}{\sqrt{137.035999111818}}$$

$$=1-\frac{1}{5\cdot 7\cdot 17}+\frac{1}{4\cdot 3\cdot 17\cdot 23\cdot 137-\frac{11}{59}}=0.998320883415699$$

$^{23}_{11}Na_{12}$ $^{35,37}_{17}Cl_{18,20}$ $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{50,51}_{23}V_{27,28}$ $^{59}_{27}Co_{32}$ $^{78,80}_{34}Se_{44,46}$ $^{79,81}_{35}Br_{44,46}$

$^{82,83,84,86}_{36}Kr_{46,47,48,50}$ $^{99,100}_{44}Ru_{55,56}$ $^{105}_{46}Pd_{59}$ $^{118,7\cdot 17,120}_{50}Sn_{68,69,70}$ $^{121,123}_{51}Sb_{70,72}$ $^{136,137,138}_{56}Ba_{80,81,82}$

$^{141}_{59}\mathrm{Pr}_{82}$ $^{168}_{68}Er_{100}$ $^{169}_{69}Tm_{100}$ $^{173}_{70}Yb_{103}$ $^{208}_{82}\mathrm{Pt}_{126}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{210}_{85}At^{*}_{125}$ $^{222}_{86}Rn^{*}_{136}$ $^{223}_{87}Fr^{*}_{136}$

$^{226}_{88}Ra^{*}_{138}$ $^{227}_{89}Ac^{*}_{138}$ $^{238}_{92}U^{*}_{146}$ $^{285}_{112}Cn^{*}_{173}$ $^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

2021/2/28

$$\frac{1}{\gamma_2}=\frac{1}{\sqrt{\alpha_2}\delta\alpha}=\frac{\sqrt{137.035999111818}}{4.66920160910299\cdot 2.50290787509589}$$

$$=1+\frac{1}{2\cdot 27\cdot 11}-\frac{1}{5\cdot 17\cdot(16\cdot 3\cdot 157+1)+\frac{16}{17}}=1.00168194075892$$

$^{19}_{9}F_{10}$ $^{23}_{11}Na_{12}$ $^{35,37}_{17}Cl_{18,20}$ $^{36,38,40}_{18}Ar_{18,20,22}$ $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{59}_{27}Co_{32}$ $^{74,78,80,82}_{34}Se_{40,44,46,48}$

$^{82,83,84,86}_{36}Kr_{46,47,48,50}$ $^{85,87}_{37}Rb_{48,50}$ $^{99,100}_{44}Ru_{55,56}$ $^{112}_{48}Cd_{64}$ $^{118,120}_{50}Sn_{68,70}$ $^{136,137,138}_{56}Ba_{80,81,82}$

$^{143,144,145}_{60}Nd_{83,84,85}$ $^{157}_{64}Gd_{93}$ $^{167,168}_{68}Er_{99,100}$ $^{200}_{80}Hg_{120}$ $^{208}_{82}\mathrm{Pt}_{126}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{210}_{85}At^{*}_{125}$

$^{257}_{100}Fm^{*}_{157}$ $^{2\cdot 157}_{126}Ch^{ie}_{188}$ $^{400}_{157}Ch^{ie}_{243}$

2021/2/27

9. Integrated Formulas of α_1, α_2, α, 2π, γ_1 and γ_2

$$\frac{\alpha_1(2\pi)}{\alpha_2(\alpha\gamma_1/\gamma_2)^2}=1$$

$$\frac{\gamma_1}{\gamma_2}=\frac{\sqrt{137.035999111818\cdot(2\cdot 3.14159265358979)}}{\sqrt{137.035999037435}\cdot 2.50290787509589}$$

$$=1+\frac{1}{32\cdot 3\cdot 7}-\frac{1}{32\cdot 89\cdot(4\cdot 53-1)-\frac{25}{2\cdot 23}}=1.00148643114372$$

$^{50,51}_{23}V_{27,28}$ $^{53}_{24}Cr_{29}$ $^{89}_{39}Y_{50}$ $^{95}_{42}Mo_{53}$ $^{2\cdot 53}_{46}Pd_{60}$ $^{112}_{48}Cd_{64}$ $^{119}_{50}Sn_{69}$ $^{127}_{53}I_{74}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{151,153}_{63}Eu_{88,90}$
$^{169}_{69}Tm_{100}$ $^{2\cdot 89}_{72}Hf_{2\cdot 53}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{227}_{89}Ac^{*}_{138}$ $^{285}_{112}Cn^{*}_{173}$ $^{6\cdot 53-320}_{127}Ch^{ie}_{191-193}$ $^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\frac{\gamma_2}{\gamma_1}=1-\frac{1}{32\cdot 3\cdot 7+1}+\frac{1}{4\cdot 53\cdot(2\cdot 49\cdot 29+1)+\frac{2\cdot 3}{23}}=0.99851577505446$$

$^{28,29,30}_{14}Si_{14,15,16}$ $^{50,51}_{23}V_{27,28}$ $^{53}_{24}Cr_{29}$ $^{63,65}_{29}Cu_{34,36}$ $^{95}_{42}Mo_{53}$ $^{2\cdot 53}_{46}Pd_{60}$ $^{112}_{48}Cd_{64}$ $^{127}_{53}I_{74}$ $^{136,137,138}_{56}Ba_{80,81,82}$
$^{285}_{112}Cn^{*}_{173}$ $^{336}_{133}Ch^{ie}_{7\cdot 29}$ $^{6\cdot 53,11\cdot 29,320}_{127}Ch^{ie}_{191,192,193}$ $^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$

2021/3/3

10. Integrated Formulas α, 2π and γ

By introducing a correction factor γ, some integrated formulas of 2π, α and γ in the format of $2\pi/(\alpha\gamma)^2=1$ could be obtained as follows.

$$\frac{2\pi}{(\alpha\gamma)^2}=1$$

$$\gamma=\frac{\sqrt{(2\cdot 3.14159265358979)}}{2.50290787509589^2}$$

$$=1+\frac{1}{32\cdot 3\cdot 7}-\frac{1}{23\cdot 151\cdot 173+\frac{9}{4\cdot 7}}=1.00148643087192$$

$^{23}_{11}Na_{12}$ $^{50,51}_{23}V_{27,28}$ $^{82,83,84}_{36}Kr_{46,47,48}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{151,153}_{63}Eu_{88,90}$ $^{208}_{82}Pd_{126}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$
$^{247}_{96}Cm^{*}_{151}$ $^{285}_{112}Cn^{*}_{173}$ $^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\frac{1}{\gamma}=1-\frac{1}{32\cdot 3\cdot 7+1}+\frac{1}{2\cdot 41\cdot(2\cdot 3\cdot 25\cdot 49-1)-\frac{1}{6}}=0.99851577532546$$

$^{55}_{25}Mn_{30}$ $^{73}_{32}Ge_{41}$ $^{82,83,84,86}_{36}Kr_{46,47,48,50}$ $^{93}_{41}Nb_{52}$ $^{140,142}_{58}Ce_{82,84}$ $^{208}_{82}Pd_{126}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{210}_{85}At^{*}_{125}$

2021/3/3

11. Summary

The above integrated formulas of the fine-structure constant and Feigenbaum constants are summarized as follows.

The Fine-structure Constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7\cdot(2\pi)_{Chen-112}}\frac{1}{112+\frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13\cdot(2\pi)_{Chen-278}}{100}\frac{1}{112-\frac{1}{64\cdot3\cdot29}} = 1/137.035999111818$$

Feigenbaum Constants: $\delta = 4.66920160910299$

$\alpha = 2.50290787509589$

$2\pi = 2\cdot3.14159265358979$

$$\alpha_1(\delta/\gamma_1)^2(2\pi) = 1$$

$$\gamma_1 = (1-\frac{1}{47\cdot109}+\frac{1}{27\cdot7\cdot(3\cdot8\cdot(3\cdot8\cdot(4\cdot137-1)-1)-1)})^2 = 0.999804818668238$$

$$\frac{1}{\gamma_1} = (1+\frac{1}{2\cdot13\cdot197}-\frac{16\cdot7\cdot17\cdot(16\cdot3\cdot23-1)}{125\cdot10^{12}})^2 = 1.00019521943495$$

$$\gamma_1^{\ 2} = 1-\frac{1}{13\cdot197}+\frac{1}{2\cdot7\cdot41\cdot(4\cdot5\cdot19\cdot31-1)} = 0.99960967543223$$

$$\frac{1}{\gamma_1^{\ 2}} = 1+\frac{1}{512\cdot5}-\frac{1}{4\cdot9\cdot7\cdot17\cdot19\cdot83} = 1.00039047698053$$

$$\alpha_1(\delta/\gamma_{1-Chen-25\cdot17})^2(2\pi)_{Chen-25\cdot17} = 1$$

$$(2\pi)_{Chen-25\cdot17} = e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot3\cdot71}{25\cdot17})^{23\cdot37}}6.28564399787948$$

$$\gamma_{1-Chen-25\cdot17} = 1+\frac{1}{2\cdot79\cdot109\cdot139-\frac{3}{25}} = 1.00000041773574$$

$$\frac{1}{\gamma_{1-Chen-25\cdot17}} = 1-\frac{1}{2\cdot79\cdot109\cdot139+\frac{22}{25}} = 0.999999582264432$$

$$\gamma_{1-Chen-25\cdot17}^{\ 2} = 1+\frac{1}{128\cdot9\cdot(2\cdot3\cdot173+1)+\frac{7}{10}} = 1.0000008354717$$

$$\frac{1}{\gamma_{1-Chen-25\cdot17}^{\ 2}} = 1-\frac{1}{2\cdot5\cdot7\cdot(2\cdot83\cdot103+1)-\frac{9}{29}} = 0.999999164529037 \approx 1$$

$$\alpha_1(\delta/\gamma_{1-Wallis-9\cdot71})^2(2\pi)_{Wallis-9\cdot71}=1$$

$$(2\pi)_{Wallis-9\cdot71}=4\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{1278}{1279}\frac{1280}{2\cdot9\cdot71+1})=6.28564015562186$$

$$\gamma_{1-Wallis-9\cdot71}=1+\frac{1}{4\cdot9\cdot(8\cdot3\cdot25\cdot7\cdot59-1)}=1.00000011209802$$

$$\frac{1}{\gamma_{1-Wallis-9\cdot71}}=1-\frac{1}{5\cdot7\cdot(32\cdot27\cdot5\cdot59-1)}=0.999999887901990$$

$$\gamma_{1-Wallis-9\cdot71}{}^2=1+\frac{1}{2\cdot9\cdot(8\cdot3\cdot25\cdot7\cdot59-1)-\frac{5}{17}}=1.00000022419606$$

$$\frac{1}{\gamma_{1-Wallis-9\cdot71}{}^2}=1-\frac{1}{2\cdot9\cdot(8\cdot3\cdot25\cdot7\cdot59-1)+\frac{3\cdot4}{17}}=0.99999977580399$$

$$\alpha_1\delta^2(2\pi/\gamma_{1-GL-22\cdot37})_{GL-22\cdot37}=1$$

$$(2\pi)_{GL-22\cdot37}=8\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot11\cdot37+1})=6.28563929398602$$

$$\gamma_{1-GL-22\cdot37}=1+\frac{1}{4\cdot25\cdot7\cdot(2\cdot23^2\cdot31-1)+\frac{6}{11}}=1.00000004355799$$

$$\frac{1}{\gamma_{1-GL-22\cdot37}}=1-\frac{1}{2\cdot9\cdot11\cdot47\cdot(2\cdot9\cdot137+1)-\frac{5}{11}}=0.999999956442012$$

$$\gamma_{1-GL-22\cdot37}{}^2=1+\frac{1}{163\cdot(6\cdot11^2\cdot97+1)+\frac{9}{10}}=1.00000008711598$$

$$\frac{1}{\gamma_{1-GL-22\cdot37}{}^2}=1-\frac{1}{9\cdot11\cdot47\cdot(2\cdot9\cdot137+1)-\frac{2}{25}}=0.999999912884025$$

$$\alpha_1(\delta/\gamma_{1-NC-3})^2(2\pi)_{NC-3}=1$$

$$(2\pi)_{NC-3}=(6+\sum_{n=1}^{3}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}})=6.29047619047619$$

$$\gamma_{1-NC-3}=1+\frac{1}{23\cdot113}-\frac{1}{2\cdot3\cdot257\cdot(4\cdot5\cdot29\cdot31+1)}=1.00038472730421$$

$$\frac{1}{\gamma_{1-NC-3}}=1-\frac{1}{8\cdot25\cdot13}+\frac{1}{179\cdot(8\cdot9\cdot(2\cdot3\cdot(2\cdot179+1)-1)+1)-\frac{3}{7}}$$

$$=0.999615420653963$$

$$\gamma_{1-NC-3}{}^{2} = 1 + \frac{1}{3\cdot(2\cdot7\cdot31-1)} - \frac{1}{13\cdot89\cdot(2\cdot37\cdot53+1) - \frac{2\cdot9}{25}} = 1.00076960262352$$

$$\frac{1}{\gamma_{1-NC-3}{}^{2}} = 1 - \frac{1}{4\cdot25\cdot13} + \frac{1}{4\cdot9\cdot25\cdot(2\cdot25\cdot(4\cdot25+1)+1) + \frac{2}{7}} = 0.999230989209198$$

$$\alpha_2(\delta\alpha/\gamma_2)^2 = 1$$

$$\gamma_2 = 1 - \frac{1}{5\cdot7\cdot17} + \frac{1}{4\cdot3\cdot17\cdot23\cdot137 - \frac{11}{59}} = 0.998320883415699$$

$$\frac{1}{\gamma_2} = 1 + \frac{1}{2\cdot27\cdot11} - \frac{1}{5\cdot17\cdot(16\cdot3\cdot157+1) + \frac{16}{17}} = 1.00168194075892$$

$$\gamma_2{}^{2} = 1 - \frac{1}{2\cdot149} + \frac{1}{29\cdot31\cdot(2\cdot3\cdot49\cdot13+1) - \frac{16}{19}} = 0.996644586263908$$

$$\frac{1}{\gamma_2{}^{2}} = 1 + \frac{1}{27\cdot11} - \frac{1}{2\cdot3\cdot25\cdot11\cdot(4\cdot11\cdot47+1) - \frac{2}{5}} = 1.00336671044256$$

$$\frac{\alpha_1(2\pi)}{\alpha_2(\alpha\gamma_1/\gamma_2)^2} = 1$$

$$\frac{\gamma_1}{\gamma_2} = 1 + \frac{1}{32\cdot3\cdot7} - \frac{1}{32\cdot89\cdot(4\cdot53-1) - \frac{25}{2\cdot23}} = 1.00148643114372$$

$$\frac{\gamma_2}{\gamma_1} = 1 - \frac{1}{32\cdot3\cdot7+1} + \frac{1}{4\cdot53\cdot(2\cdot49\cdot29+1) + \frac{2\cdot3}{23}} = 0.99851577505446$$

$$(\frac{\gamma_1}{\gamma_2})^2 = 1 + \frac{1}{16\cdot3\cdot7} - \frac{1}{2\cdot3\cdot7\cdot(4\cdot17\cdot(2\cdot157-1)-1) - \frac{1}{4}} = 1.00297507176499$$

$$(\frac{\gamma_2}{\gamma_1})^2 = 1 - \frac{1}{16\cdot3\cdot7+1} + \frac{1}{7\cdot19\cdot(2\cdot3\cdot7^2\cdot23-1) + \frac{16}{3\cdot13}}$$

$$= 1 - \frac{1}{16\cdot3\cdot7+1} + \frac{1}{7\cdot19\cdot(8\cdot5\cdot13^2+1) + \frac{16}{3\cdot13}}$$

$$= 1 - \frac{1}{16\cdot3\cdot7+1} + \frac{1}{2\cdot3\cdot73\cdot(4\cdot27\cdot19+1) - \frac{23}{3\cdot13}} = 0.997033753032614$$

$$\frac{2\pi}{(\alpha\gamma)^2}=1$$

$$\gamma=1+\frac{1}{32\cdot 3\cdot 7}-\frac{1}{23\cdot 151\cdot 173+\frac{9}{4\cdot 7}}=1.00148643087192$$

$$\frac{1}{\gamma}=1-\frac{1}{32\cdot 3\cdot 7+1}+\frac{1}{2\cdot 41\cdot(2\cdot 3\cdot 25\cdot 49-1)-\frac{1}{6}}=0.99851577532546$$

$$\gamma^2=1+\frac{1}{16\cdot 3\cdot 7}-\frac{1}{3\cdot 13\cdot 31\cdot(2\cdot 9\cdot 41+1)-\frac{1}{22}}=1.0029750712205$$

$$\frac{1}{\gamma^2}=1-\frac{1}{16\cdot 3\cdot 7+1}+\frac{1}{8\cdot 81\cdot 19\cdot 73+\frac{1}{3\cdot 13}}=0.997033753573803$$

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Appendix I: Research History

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10	15	2021/3/3	Chengdu
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Note: Time was recorded according to Beihing Time.