

Integrated Formulas of the Fine-structure Constant and Feigenbaum Constants (viXra:2021.0162v3)

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper is a subsequent paper to the previous paper “Formulas of Feigenbaum Constants and Their Physical Meanings” (viXra:2101.0187). In the previous paper, some formulas of Feigenbaum constants in fractional number format were given and the physical meanings of the factors in the formulas were exhibited, especially their relationships with nuclides, the fine-structure constant and 2π . In the previous paper, some integrated formulas of the fine-structure constant, Feigenbaum constants and 2π were also given, briefly denoted as $\alpha_1\delta^2(2\pi)\approx 1$, and their relationships with nuclides were illustrated. In this paper, some formulas for $\alpha_1\delta^2(2\pi)\approx 1$ are supplemented, some formulas for $\alpha_2(\delta\alpha)^2\approx 1$, $[\alpha_1(2\pi)]/(\alpha_2\alpha^2)\approx 1$ and $(2\pi)/\alpha^2\approx 1$ are given, some formulas of the fine-structure constant (α_1 and α_2) based on the key number 103 instead of 112, 173, 137, 83 and 29 are supplemented. In the end, by introducing correction factors γ_1 , γ_2 and γ , accurate formulas $\alpha_1(\delta/\gamma_1)^2(2\pi)=1$, $\alpha_2(\delta\alpha/\gamma_2)^2=1$ and $2\pi/(\alpha\gamma)^2=1$ are gained.

Keywords: Formulas; the fine-structure constant; Feigenbaum constants; 2π .

1. Introduction

In our previous papers^{1,2,3,4,5}, we gave or exhibited the following formulas.

$$(2\pi)_{Chen-k} = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{k+1}{k}\right)^{2k+1}}; \quad (2\pi)_{Wallis-k} = 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \cdots \frac{2k}{2k+1} \frac{2k+2}{2k+1}\right)$$

$$(2\pi)_{GL-k} = 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + (-1)^{k+1} \frac{1}{2k+1}\right) \quad (GL \text{ means Gregory-Leibniz})$$

$$(2\pi)_{NC-k} = 6 + \sum_{n=1}^k \frac{(-1)^{n+1}}{n(n+1/2)(n+1)} \quad (NC \text{ means Nilakantha-Chen})$$

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7 \cdot (2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13 \cdot (2\pi)_{Chen-278}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818$$

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = \sqrt{112 \times (168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{14 \cdot 112 \cdot (2 \cdot 173 + 1)})}$$

$$= 137.035999074626$$

$$1/\alpha_1 = 56 + 81 + \frac{1}{28 - \frac{13 \cdot (2 \cdot 56 \cdot 11 - 1)}{3 \cdot 5 \cdot (2 \cdot 56 \cdot 43 + 1)}} = 137.035999037435$$

$$1/\alpha_2 = 56 + 81 + \frac{1}{28 - \frac{2 \cdot (16 \cdot 27 - 1)}{3 \cdot (16 \cdot 81 + 1)}} = 137.035999111818$$

$$c_{au} = \frac{1}{\alpha_c} = 56 + 81 + \frac{1}{28 - \frac{5 \cdot (4 \cdot 3 \cdot 7 \cdot 17 - 1)}{2 \cdot 5 \cdot (4 \cdot 5 \cdot 7 \cdot 23 + 1) + 1}} = 137.035999074626$$

Note: c_{au} refers to the speed of light in vacuum in atomic units

Feigenbaum Constants: $\delta = 4.66920160910299$

$\alpha = 2.50290787509589$

$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326$$

$$= \frac{1}{4} - \frac{1}{27} + \frac{1}{4 \cdot 9 \cdot 23} - \frac{1}{2 \cdot 3 \cdot 7 \cdot 23 \cdot (2 \cdot 3 \cdot (4 \cdot 3 \cdot 11 - 1) + 1) + \frac{2 \cdot 23}{3 \cdot 19}}$$

$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

$$= \frac{1}{2} - \frac{1}{9} + \frac{1}{3 \cdot 31} - \frac{1}{23 \cdot (8 \cdot 3 \cdot 17 + 1)} + \frac{1}{17 \cdot 23 \cdot (8 \cdot 3 \cdot 11^4 - 1)}$$

Note: $136=8 \cdot 17$, $138=6 \cdot 23$

$$\alpha_1 \delta^2 (2\pi) \approx 1$$

On Feb. 8, 2021, we also noticed that Hieb uploaded a paper⁶ in viXra in April of 2017, and gave an approximate formula of the fine-structure constant and Feigenbaum constant as follows, but without any explanations to its physical meanings.

$$\delta' = (1/(2\pi\alpha))^{1/2} = 4.670114 \approx \delta = 4.669201609$$

$$\delta' - \delta = 0.000912$$

α : the fine-structure constant, $\alpha \approx 1/137.036$

2. Integrated Formulas of α_1 , δ and 2π

A Concise Deduction

The Fine-structure Constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7 \cdot (2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

Feigenbaum Constants: $\delta = 4.66920160910299$

$$\begin{aligned} \alpha_1 &= \frac{36}{7 \cdot (2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} \approx \frac{36}{7 \cdot (2\pi)} \frac{1}{112} = \left(\frac{3}{14}\right)^2 \frac{1}{2\pi} \approx \frac{1}{\delta^2(2\pi)} \\ &= \frac{1}{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)} \approx \frac{1}{136.982} \end{aligned}$$

So it should be reasonable to assume the following approximate formulas:

$$\alpha_1 \delta^2(2\pi) \approx 1 \text{ or } \frac{1}{\alpha_1 \delta^2(2\pi)} \approx 1$$

$$\text{Numerically: } \alpha_1 \delta^2(2\pi) = \frac{4 \cdot 6692^2 \times 6.2832}{137.036} = 0.99961 \approx 1$$

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The above approximate formula $\alpha_1 \delta^2(2\pi) \approx 1$ is assumed to be the brief form of integrated formulas of α_1 , δ and 2π . There should be some corresponding accurate forms of integrated formulas of α_1 , δ and 2π as follows.

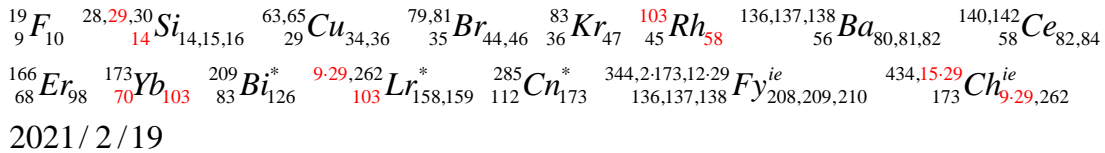
$$\begin{aligned} \alpha_1 \delta^2(2\pi)_{Chen-25-17} &= \frac{4.66920160910299^2 \cdot \left(\frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}} \right)}{137.035999037435} \\ &= \frac{4.66920160910299^2 \cdot 6.28564399787948}{137.035999037435} \\ &= 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717 \approx 1 \end{aligned}$$

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$$\frac{1}{\alpha_1 \delta^2 (2\pi)_{Chen-25-17}} = \frac{137.035999037435}{4.66920160910299^2 \cdot \left(e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}} \right)}$$

$$= \frac{137.035999037435}{4.66920160910299^2 \cdot 6.28564399787948}$$

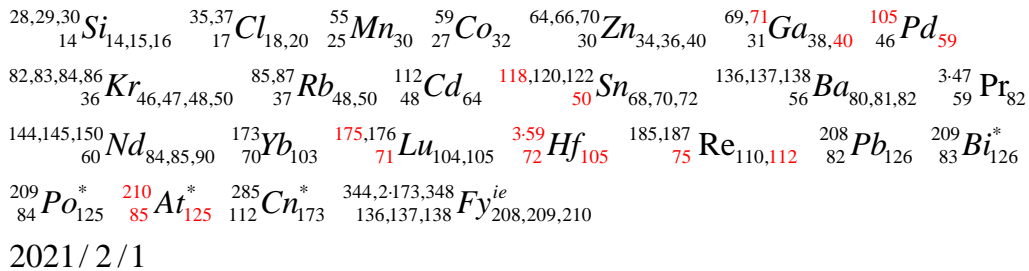
$$= 1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \approx 1$$



$$\alpha_1 \delta^2 (2\pi)_{Wallis-9-71} = \frac{4.66920160910299^2 \cdot 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{1278}{1279} \frac{1280}{2 \cdot 9 \cdot 71 + 1} \right)}{137.035999037435}$$

$$= \frac{4.66920160910299^2 \cdot 6.28564015562186}{137.035999037435}$$

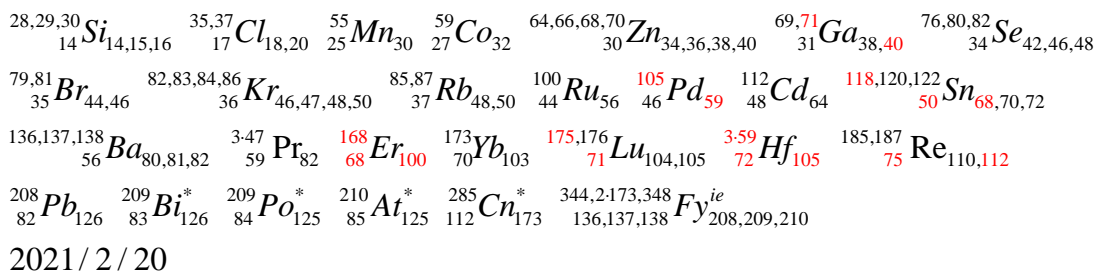
$$= 1 + \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) - \frac{5}{17}} = 1.00000022419606 \approx 1$$



$$\frac{1}{\alpha_1 \delta^2 (2\pi)_{Wallis-9-71}} = \frac{137.035999037435}{4.66920160910299^2 \cdot 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{1278}{1279} \frac{1280}{2 \cdot 9 \cdot 71 + 1} \right)}$$

$$= \frac{137.035999037435}{4.66920160910299^2 \cdot 6.28564015562186}$$

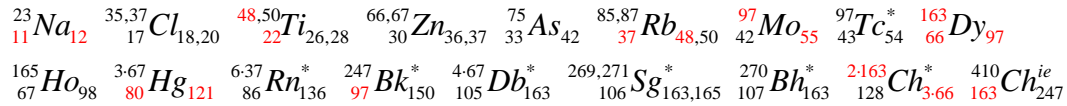
$$= 1 - \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) + \frac{3 \cdot 4}{17}} = 0.999999775803991 \approx 1$$



$$\alpha_1 \delta^2 (2\pi)_{GL-22,37} = \frac{4.66920160910299^2 \cdot 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1}\right)}{137.035999037435}$$

$$= \frac{4.66920160910299^2 \cdot 6.28563929398602}{137.035999037435}$$

$$= 1 + \frac{1}{163 \cdot (6 \cdot 11^2 \cdot 97 + 1) + \frac{9}{10}} = 1.00000008711598 \approx 1$$

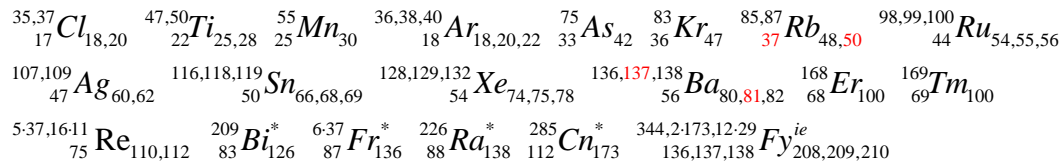


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$$\alpha_1 \delta^2 (2\pi)_{GL-22,37} = \frac{1}{4.66920160910299^2 \cdot 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1}\right)}$$

$$= \frac{137.035999037435}{4.66920160910299^2 \cdot 6.28563929398602}$$

$$= 1 - \frac{1}{9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) - \frac{2}{25}} = 0.999999912884025 \approx 1$$

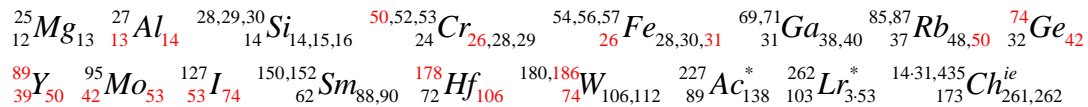


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$$\alpha_1 \delta^2 (2\pi)_{NC-3} = \frac{4.66920160910299^2 \cdot \left(6 + \sum_{n=1}^3 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)}\right)}{137.035999037435}$$

$$= \frac{4.66920160910299^2 \cdot 6.29047619047619}{137.035999037435}$$

$$= 1 + \frac{1}{3 \cdot (2 \cdot 7 \cdot 31 - 1)} - \frac{1}{13 \cdot 89 \cdot (2 \cdot 37 \cdot 53 + 1) - \frac{2 \cdot 9}{25}} = 1.00076960262352 \approx 1$$

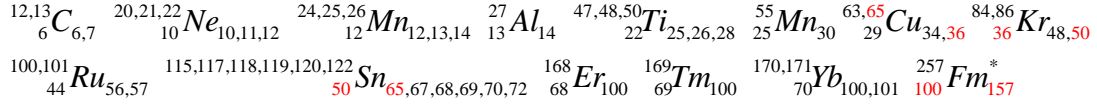


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$$\alpha_1 \delta^2 (2\pi)_{NC-3} = \frac{1}{4.66920160910299^2 \cdot \left(6 + \sum_{n=1}^3 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)}\right)}$$

$$= \frac{137.035999037435}{4.66920160910299^2 \cdot 6.29047619047619}$$

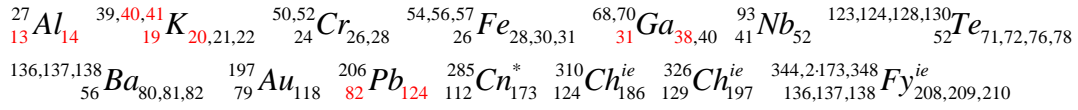
$$= 1 - \frac{1}{4 \cdot 25 \cdot 13} + \frac{1}{4 \cdot 9 \cdot 25 \cdot (2 \cdot 25 \cdot (4 \cdot 25 + 1) + 1) + \frac{2}{7}} = 0.999230989209198 \approx 1$$



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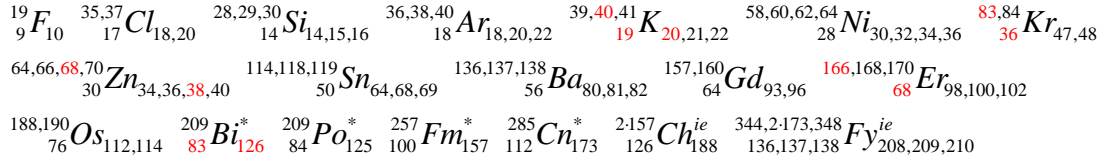
$$\alpha_1 \delta^2 (2\pi) = \frac{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)}{137.035999037435}$$

$$= 1 - \frac{1}{13 \cdot 197} + \frac{1}{2 \cdot 7 \cdot 41 \cdot (4 \cdot 5 \cdot 19 \cdot 31 - 1)} = 0.99960967543223 \approx 1$$



$$\frac{1}{\alpha_1 \delta^2 (2\pi)} = \frac{137.035999037435}{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)}$$

$$= 1 + \frac{1}{512 \cdot 5} - \frac{1}{4 \cdot 9 \cdot 7 \cdot 17 \cdot 19 \cdot 83} = 1.00039047698053 \approx 1$$



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3. Integrated Formulas of α_2 , δ and α

The Fine-structure Constant:

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13 \cdot (2\pi)_{\text{Chen-278}}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818$$

Feigenbaum Constants: $\delta = 4.66920160910299$

$$\alpha = 2.50290787509589$$

$$\alpha_2 = \frac{13 \cdot (2\pi)_{\text{Chen-278}}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} \approx \frac{13 \cdot (2\pi)}{100} \frac{1}{112} \approx \frac{(2\pi)}{(\delta\alpha)^2 (2\pi)}$$

$$\approx \frac{1}{(\delta\alpha)^2} = \frac{1}{(4.66920160910299 \cdot 2.50290787509589)^2} \approx 1/136.575$$

So it should be reasonable to assume the following approximate formulas:

$$\alpha_2 (\delta\alpha)^2 \approx 1 \text{ or } \frac{1}{\alpha_2 (\delta\alpha)^2} \approx 1$$

$$\text{Numerically: } \alpha_2 (\delta\alpha)^2 = \frac{(4 \cdot 6692 \times 2.5029)^2}{137.036} = 0.99664 \approx 1$$

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The above approximate formula $\alpha_2(\delta\alpha)^2 \approx 1$ is assumed to be the brief form of integrated formulas of α_2 , δ and α . There should be some corresponding accurate forms of integrated formulas of α_2 , δ and α as follows.

$$\alpha_2(\delta\alpha)^2 = \frac{(4.66920160910299 \cdot 2.50290787509589)^2}{137.035999111818}$$

$$= 1 - \frac{1}{2 \cdot 149} + \frac{1}{29 \cdot 31 \cdot (2 \cdot 3 \cdot 49 \cdot 13 + 1) - \frac{16}{19}} = 0.996644586263908 \approx 1$$

$^{12,13}_6C_{6,7}$ $^{14,15}_7N_{7,8}$ $^{19}_9F_{10}$ $^{24,25,26}_{12}Mg_{12,13,14}$ $^{27}_{13}Al_{14}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{31}_{15}P_{16}$ $^{39,40,41}_{19}K_{20,21,22}$
 $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{50,52,53}_{24}Cr_{26,28,29}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{58,60,62,64}_{28}Ni_{30,32,34,36}$
 $^{63,65}_{29}Cu_{34,36}$ $^{69,71}_{31}Ga_{38,40}$ $^{70,74,76}_{32}Ge_{38,42,44}$ $^{76,78}_{34}Se_{42,44}$ $^{85,87}_{37}Rb_{48,50}$ $^{89}_{39}Y_{50}$ $^{93}_{41}Nb_{52}$
 $^{94,95,96,98,100}_{42}Mo_{52,53,54,56,58}$ $^{112,113}_{48}Cd_{64,65}$ $^{113,115}_{49}In_{64,66}$ $^{7-19}_{54}Xe_{76,78}$ $^{7-19}_{55}Cs_{78}$ $^{149}_{62}Sm_{87}$
 $^{134,136,137,138}_{56}Ba_{78,80,81,82}$ $^{155,156,157,160}_{64}Gd_{91,92,93,96}$ $^{186,187,189,190,192}_{76}Os_{110,112,113,114,116}$
 $^{190,192,194,195,196}_{78}Pt_{112,114,116,117,118}$ $^{223}_{87}Fr_{136}$ $^{227}_{89}Ac_{138}$ $^{21-11}_{91}Pa_{140}$ $^{237}_{93}Np_{144}$ $^{15-19}_{112}Cn_{173}$
 $^{22-13}_{113}Nh_{173}^{ie}$ $^{344,2,173,12-29}_{136,137,138}Fy_{208,209,210}^{ie}$ $^{8-47}_{149}Ch_{227}^{ie}$ $^{14-31,15-29}_{173}Ch_{9-19,262}^{ie}$

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$$\frac{1}{\alpha_2(\delta\alpha)^2} = \frac{137.035999111818}{(4.66920160910299 \cdot 2.50290787509589)^2}$$

$$= 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (4 \cdot 11 \cdot 47 + 1) - \frac{2}{5}}$$

$$= 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (2 \cdot 9 \cdot 5 \cdot 23 - 1) - \frac{2}{5}} = 1.00336671044256 \approx 1$$

$^{10,11}_5Be_{5,6}$ $^{23}_{11}Na_{12}$ $^{24,25}_{12}Mg_{12,13}$ $^{46,47,49,50}_{22}Ti_{24,25,27,28}$ $^{50,51}_{23}V_{27,28}$ $^{55}_{25}Mn_{30}$ $^{75}_{33}As_{42}$
 $^{80,82,83,86}_{36}Kr_{44,46,47,50}$ $^{98,99,100,104}_{44}Ru_{54,55,56,60}$ $^{107,109}_{47}Ag_{60,62}$ $^{115,116,119,120}_{50}Sn_{65,66,69,70}$
 $^{129,131,132}_{54}Xe_{75,77,78}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{169}_{69}Tm_{100}$ $^{185,187}_{75}Re_{110,112}$ $^{226}_{88}Ra_{138}^*$
 $^{344,346,348}_{136,137,138}Fy_{208,209,210}^{ie}$

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4. Integrated Formulas of α_1 , α_2 , α and 2π

$$\alpha_1\delta^2(2\pi) = 0.99961 \approx 1$$

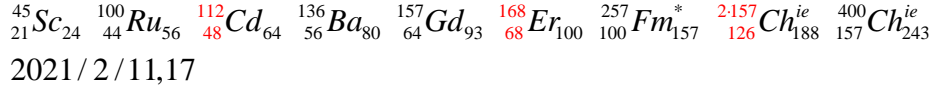
$$\alpha_2(\delta\alpha)^2 = 0.99664 \approx 1$$

$$\frac{\alpha_1\delta^2(2\pi)}{\alpha_2(\delta\alpha)^2} = \frac{\alpha_1(2\pi)}{\alpha_2\alpha^2} \approx \frac{2\pi}{\alpha^2} = 1.002975 \approx 1$$

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$$\frac{\alpha_1(2\pi)}{\alpha_2\alpha^2} = \frac{137.035999111818 \cdot (2 \cdot 3 \cdot 14159265358979)}{137.035999037435 \cdot 2.50290787509589^2}$$

$$= 1 + \frac{1}{16 \cdot 3 \cdot 7} - \frac{1}{2 \cdot 3 \cdot 7 \cdot (4 \cdot 17 \cdot (2 \cdot 157 - 1) - 1) - \frac{1}{4}} = 1.00297507176499 \approx 1$$

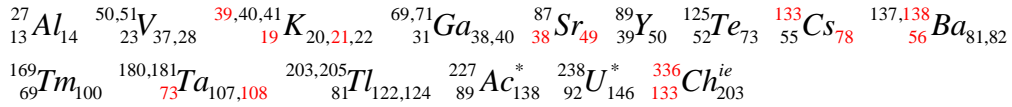


$$\frac{\alpha_2\alpha^2}{\alpha_1(2\pi)} = \frac{137.035999037435 \cdot 2.50290787509589^2}{137.035999111818 \cdot (2 \cdot 3 \cdot 14159265358979)}$$

$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{7 \cdot 19 \cdot (2 \cdot 3 \cdot 7^2 \cdot 23 - 1) + \frac{16}{3 \cdot 13}}$$

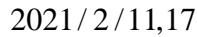
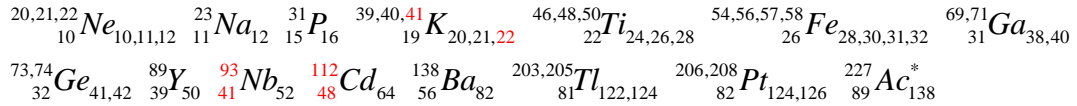
$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{7 \cdot 19 \cdot (8 \cdot 5 \cdot 13^2 + 1) + \frac{16}{3 \cdot 13}}$$

$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{2 \cdot 3 \cdot 73 \cdot (4 \cdot 27 \cdot 19 + 1) - \frac{23}{3 \cdot 13}} = 0.997033753032614 \approx 1$$



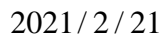
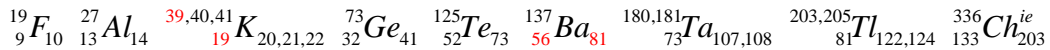
$$\frac{2\pi}{\alpha^2} = \frac{2 \cdot 3 \cdot 14159265358979}{2.50290787509589^2} = 1 + \frac{1}{16 \cdot 3 \cdot 7} - \frac{1}{3 \cdot 13 \cdot 31 \cdot (2 \cdot 9 \cdot 41 + 1) - \frac{1}{22}}$$

$$= 1.00297507122057 \approx 1$$



$$\frac{\alpha^2}{2\pi} = \frac{2.50290787509589^2}{2 \cdot 3 \cdot 14159265358979} = 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{8 \cdot 81 \cdot 19 \cdot 73 + \frac{1}{3 \cdot 13}}$$

$$= 0.997033753573803 \approx 1$$



5. Marvelous Coincidences

There are some marvelous coincidences of factors with nuclides in the above formulas. One typical example of these coincidences is listed as follows, which indicates the methodology and the formulas in this paper should be correct.

$$\alpha_1 \delta^2 (2\pi)_{Chen-25:17} = 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717 \approx 1$$

$$\frac{1}{\alpha_1 \delta^2 (2\pi)_{Chen-25:17}} = 1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \approx 1$$

$$(2\pi)_{Chen-25:17} = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}}$$

$^{28,29,30}_{14}Si_{14,15,16}$ $^{35,37}_{17}Cl_{18,20}$ $^{50,51}_{23}V_{27,28}$ $^{63,65}_{29}Cu_{34,36}$ $^{80}_{34}Se_{46}$ $^{79,81}_{35}Br_{44,46}$ $^{82,83,84,86}_{36}Kr_{46,47,48,50}$
 $^{85,87}_{37}Rb_{48,50}$ $^{103}_{45}Rh_{58}$ $^{112}_{48}Cd_{64}$ $^{128}_{54}Xe_{74}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{140,142}_{58}Ce_{82,84}$ $^{173}_{70}Yb_{103}$ $^{209}_{83}Bi^*_{126}$
 $^{209}_{84}Po^*_{125}$ $^{210}_{85}At^*_{125}$ $^{9-29,262}_{103}Lr^*_{158,159}$ $^{285}_{112}Cn^*_{173}$ $^{344,2-173,12-29}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{434,15-29}_{173}Ch^{ie}_{9-29,262}$

6. Formulas of the Fine-structure Constant based on 103

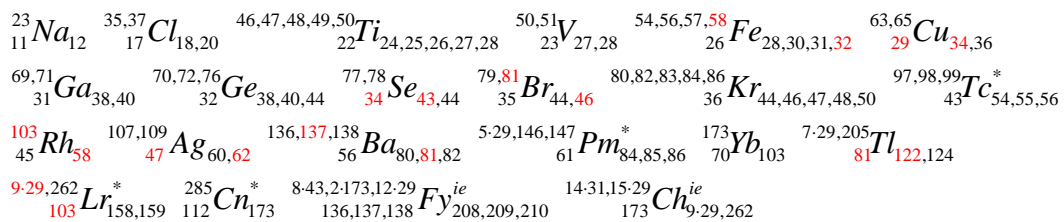
In our previous paper^{1,2,4}, many formulas of the fine-structure constant based on the key numbers 112, 173, 137, 83 and 29 were given. As shown in the above two formulas in **Section 5**, it seems 103 is another key number comparable to the above stated key numbers, so some formulas of the fine-structure constant based on the key number 103 instead of them are constructed as follows.

$$\alpha_1 = \frac{137}{29 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{11 \cdot 47}{4 \cdot 3 \cdot 43}\right)^{1033}}} \frac{1}{103 + \frac{1}{32 \cdot (32 \cdot 29 + 1) - \frac{3}{2 \cdot 17}}}$$

$$= \frac{137}{29 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{11 \cdot 47}{4 \cdot 3 \cdot 43}\right)^{1033}}} \frac{1}{103 + \frac{1}{81 \cdot (2 \cdot 3 \cdot 61 + 1) + \frac{31}{2 \cdot 17}}}$$

$$= \frac{137}{29 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{11 \cdot 47}{4 \cdot 3 \cdot 43}\right)^{1033}}} \frac{1}{103 + \frac{1}{81 \cdot (16 \cdot 23 - 1) + \frac{31}{2 \cdot 17}}}$$

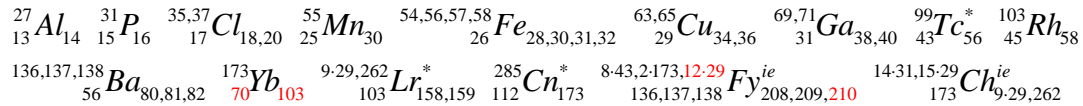
$$= 1/137.035999037435$$



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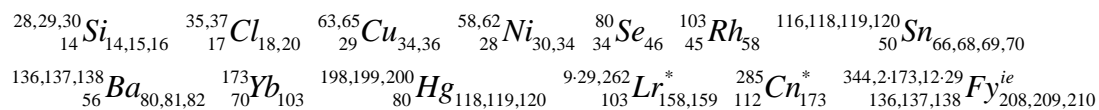
$$\alpha_1 = \frac{137}{29 \cdot 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{1548}{1549} \frac{2 \cdot 25 \cdot 31}{2 \cdot 2 \cdot 9 \cdot 43 + 1}\right)} \frac{1}{103 + \frac{1}{2 \cdot 3 \cdot 5 \cdot (2 \cdot 3 \cdot 5 \cdot 7 \cdot 13 - 1) - \frac{3}{17}}}$$

$$= 1/137.035999037435$$



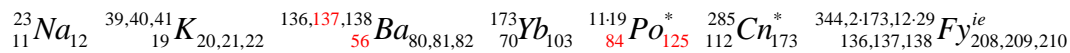
$$\alpha_1 = \frac{137}{29 \cdot 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 17 \cdot 29 + 1}\right)} \frac{1}{103 + \frac{1}{7 \cdot (4 \cdot 7 \cdot 199 + 1) + \frac{4}{7}}}$$

$$= 1/137.035999037435$$



$$\alpha_1 = \frac{137}{29 \cdot \left(6 + \sum_{n=1}^7 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)}\right)} \frac{1}{103 + \frac{1}{3 \cdot 19} - \frac{1}{125 \cdot (8 \cdot 7 \cdot 11 + 1) + \frac{1}{4}}}$$

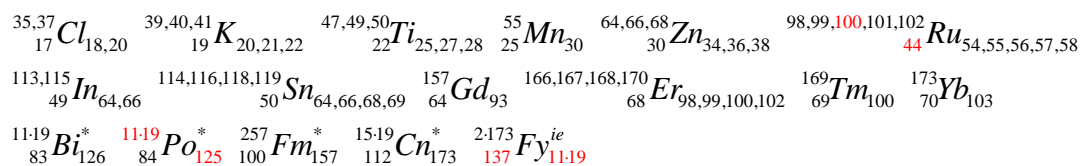
$$= 1/137.035999037435$$



2021/2/26

$$\alpha_2 = \frac{25 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2500}{3 \cdot 49 \cdot 17}\right)^{4999}}}{11 \cdot 19} \frac{1}{103 - \frac{1}{32 \cdot (512 \cdot 25 - 1) + \frac{3}{10}}}$$

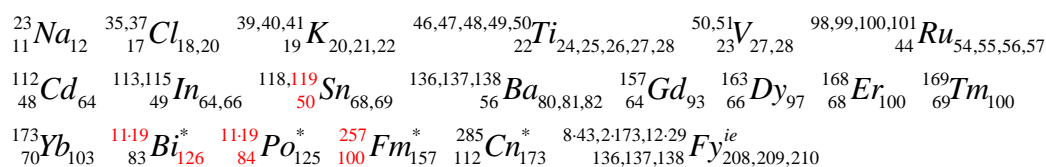
$$= 1/137.035999111818$$



2021/2/25

$$\alpha_2 = \frac{25 \cdot 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{7496}{7497} \frac{2 \cdot 23 \cdot 163}{9 \cdot 49 \cdot 17}\right)}{11 \cdot 19} \frac{1}{103 - \frac{1}{7 \cdot (16 \cdot 3 \cdot 19 \cdot 257 - 1)}}$$

$$= 1/137.035999111818$$



$$\alpha_2 = \frac{25 \cdot 8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{5 \cdot 23 \cdot 83})}{11 \cdot 19} \frac{1}{103 - \frac{1}{2 \cdot 3 \cdot 5 \cdot 13 \cdot 23 \cdot (2 \cdot 11 \cdot 17 - 1)}}$$

$$= 1/137.035999111818$$

²³Na₁₂ ²⁷Al₁₄ ^{35,37}Cl_{18,20} ^{39,40,41}K_{20,21,22} ^{50,51}V_{27,28} ⁵⁵Mn₃₀ ¹⁶⁸Er₁₀₀ ¹⁶⁹Tm₁₀₀ ¹¹⁹Bi*₁₂₆

$$\alpha_2 = \frac{25 \cdot (6 + \sum_{n=1}^{11} \frac{(-1)^{n+1}}{n(n+1/2)(n+1)})}{11 \cdot 19} \frac{1}{103 - \frac{1}{4 \cdot 5 \cdot 23} + \frac{1}{2 \cdot (2 \cdot 3 \cdot 5 \cdot 7 + 1) \cdot (2 \cdot 9 \cdot 17 + 1) + \frac{2}{3}}}$$

$$= 1/137.035999111818$$

²³Na₁₂ ^{35,37}Cl_{18,20} ^{39,40,41}K_{20,21,22} ^{50,51}V_{27,28} ⁵⁵Mn₃₀ ¹⁰³Rh₅₈ ^{136,137,138}Ba_{80,81,82} ¹⁷³Yb₁₀₃
¹¹⁹Bi*₁₂₆ ¹¹⁹Po*₁₂₅ ²¹⁰At*₁₂₅ ²³⁸U*₁₄₃ ¹⁵¹⁹Cn*₁₇₃ ^{344,2173,348}Fy^{ie}_{208,209,210}

2021/2/26

7. Integrated Formulas of α_1 , δ , 2π and γ_1

By introducing a correction factor γ_1 , some integrated formulas of α_1 , δ , 2π and γ_1 in the format of $\alpha_1(\delta/\gamma_1)^2(2\pi)=1$ could be obtained as follows.

$$\alpha_1(\delta/\gamma_1)^2(2\pi) = 1$$

$$\gamma_1 = \sqrt{2\pi\alpha_1\delta} = \sqrt{\frac{6.28564399787948}{137.035999037435}} 4.66920160910299$$

$$= (1 - \frac{1}{47 \cdot 109} + \frac{1}{27 \cdot 7 \cdot (3 \cdot 8 \cdot (3 \cdot 8 \cdot (4 \cdot 137 - 1) - 1) - 1)})^2 = 0.999804818668238$$

⁴⁵V₂₄ ⁵⁹Co₃₂ ^{83,84}Kr_{47,48} ^{107,109}Ag_{60,62} ³⁴⁷Pr₈₂ ¹⁵⁸Gd₉₄ ¹⁸³W₁₀₉ ⁴⁴⁷Os₁₁₂ ¹³⁷Ba₈₁ ²⁰⁹Bi*₁₂₆
²⁰⁹Po*₁₂₅ ²⁷⁸Mt*₁₆₉ ²⁸⁵Cn*₁₇₃ ²¹⁷³Fy^{ie}₂₀₉

$$\frac{1}{\gamma_1} = \frac{1}{\sqrt{2\pi\alpha_1\delta}} = \sqrt{\frac{137.035999037435}{2 \cdot 3.14159265358979}} \frac{1}{4.66920160910299}$$

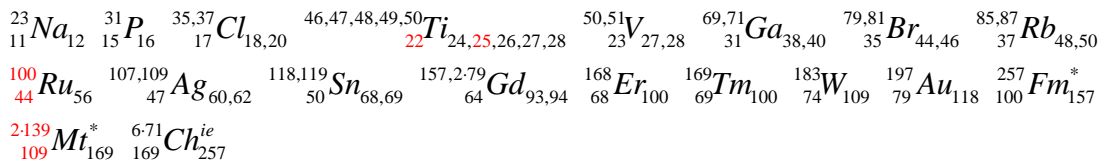
$$= (1 + \frac{1}{2 \cdot 13 \cdot 197} - \frac{16 \cdot 7 \cdot 17 \cdot (16 \cdot 3 \cdot 23 - 1)}{125 \cdot 10^{12}})^2 = 1.00019521943495 \approx 1$$

^{24,25,26}Mg_{12,13,14} ²⁷Al₁₄ ^{28,29,30}Si_{14,15,16} ^{35,37}Cl_{18,20} ^{46,48,50}Ti_{24,26,28} ^{50,51}V_{23,27,28} ^{54,56,58}Fe_{28,30,32}
^{58,60,62,64}Ni_{28,30,32,34,36} ^{74,76,78,80,82}Se_{34,40,42,44,46,48} ^{79,81}Br_{35,44,46} ^{90,91,92,96}Zr_{40,50,51,52,56} ^{112,116}Cd_{48,64,68}
^{102,104,105,106,110}Pd_{46,56,58,59,60,64} ^{114,115,118,119,120}Sn_{50,64,65,68,69,70} ^{135,136,137,138}Ba_{56,79,80,81,82} ^{156,160}Gd_{64,92,96}
¹⁶⁸Er_{68,100} ¹⁶⁹Tm_{69,100} ¹⁷³Yb_{70,103} ^{185,187}Re_{75,110,112} ¹⁹⁷Au_{79,118} ²⁰⁹Po*_{84,125} ²¹⁰At*_{85,125} ²²²Rn*_{86,136} ²²³Fr*_{87,136}
²²⁶Ra*_{88,138} ²²⁷Ac*_{89,138} ²³⁸U*_{92,146} ²⁸⁵Cn*_{112,173} ²⁴¹³Ch^{ie}_{125,187} ³²⁶Ch^{ie}_{129,197} ^{344,2173,348}Fy^{ie}_{136,137,138,208,209,210} ²¹⁹⁷Ch^{ie}_{156,238}

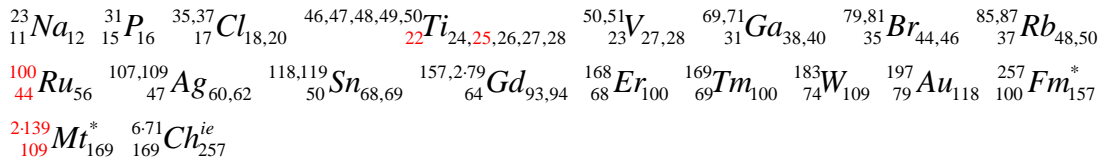
2021/2/28

$$\alpha_1(\delta / \gamma_{1-\text{Chen}-25.17})^2 (2\pi)_{\text{Chen}-25.17} = 1$$

$$\begin{aligned} \gamma_{1-\text{Chen}-25.17} &= \sqrt{(2\pi)_{\text{Chen}-25.17} \alpha_1 \delta} \\ &= \frac{\sqrt{e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}} \cdot 4.66920160910299}}{\sqrt{137.035999037435}} \\ &= \frac{\sqrt{6.28564399787948 \cdot 4.66920160910299}}{\sqrt{137.035999037435}} \\ &= 1 + \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 - \frac{2}{17} \text{ or } \frac{3}{25}} = 1.00000041773574 \end{aligned}$$



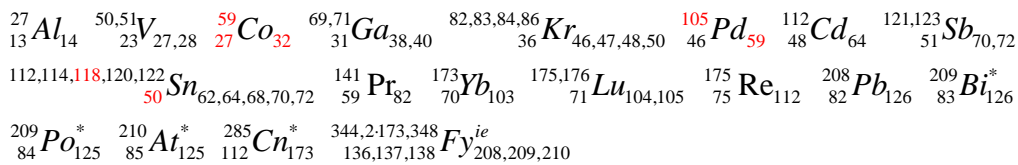
$$\begin{aligned} \gamma_{1-\text{Chen}-25.17} &= \frac{1}{\sqrt{(2\pi)_{\text{Chen}-25.17} \alpha_1 \delta}} \\ &= 1 - \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 + \frac{15}{17} \text{ or } \frac{22}{25}} = 0.999999582264432 \end{aligned}$$



2021/3/1

$$\alpha_1(\delta^2 / \gamma_{1-\text{Wallis}-9.71})(2\pi)_{\text{Wallis}-9.71}$$

$$\begin{aligned} \gamma_{1-\text{Wallis}-9.71} &= \sqrt{(2\pi)_{\text{Wallis}-9.71} \alpha_1 \delta} \\ &= \frac{\sqrt{4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \cdots \frac{1278}{1279} \frac{1280}{2 \cdot 9 \cdot 71 + 1}\right) \cdot 4.66920160910299}}{\sqrt{137.035999037435}} \\ &= \frac{\sqrt{6.28564015562186 \cdot 4.66920160910299}}{\sqrt{137.035999037435}} \\ &= 1 + \frac{1}{4 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1)} = 1.00000011209802 \end{aligned}$$



2021/3/2

$$\frac{1}{\gamma_{1-\text{Wallis}-9.71}} = \frac{1}{\sqrt{(2\pi)_{\text{Wallis}-9.71} \alpha_1 \delta}}$$

$$= 1 - \frac{1}{5 \cdot 7 \cdot (32 \cdot 27 \cdot 5 \cdot 59 - 1)} = 0.999999887901990$$

²⁷₁₃Al₁₄ ^{50,51}₂₃V_{27,28} ⁵⁹₂₇Co₃₂ ^{69,71}₃₁Ga_{38,40} ^{82,83,84,86}₃₆Kr_{46,47,48,50} ¹⁰⁵₄₆Pd₅₉ ¹¹²₄₈Cd₆₄ ^{121,123}₅₁Sb_{70,72}
^{112,114,118,120,122}₅₀Sn_{62,64,68,70,72} ¹⁴¹₅₉Pr₈₂ ¹⁷³₇₀Yb₁₀₃ ^{175,176}₇₁Lu_{104,105} ¹⁷⁵₇₅Re₁₁₂ ²⁰⁸₈₂Pb₁₂₆ ²⁰⁹₈₃Bi₁₂₆^{*}
²⁰⁹₈₄Po₁₂₅^{*} ²¹⁰₈₅At₁₂₅^{*} ²⁸⁵₁₁₂Cn₁₇₃^{*} ^{344,2-173,348}_{136,137,138}Fy_{208,209,210}^{ie}

2021/3/2

$$\alpha_1(\delta / \gamma_{1-\text{GL}-22.37})^2 (2\pi)_{\text{GL}-22.37} = 1$$

$$\gamma_{1-\text{GL}-22.37} = \sqrt{(2\pi)_{\text{GL}-22.37} \alpha_1 \delta}$$

$$= \frac{\sqrt{8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1})} \cdot 4.66920160910299}{\sqrt{137.035999037435}}$$

$$= \frac{\sqrt{6.28563929398602} \cdot 4.66920160910299}{\sqrt{137.035999037435}}$$

$$= 1 + \frac{1}{4 \cdot 25 \cdot 7 \cdot (2 \cdot 23^2 \cdot 31 - 1) + \frac{6}{11}} = 1.00000004355799$$

²³₁₁Na₁₂ ⁴⁷₂₂Ti₂₅ ^{50,51}₂₃V_{27,28} ⁵⁵₂₅Mn₃₀ ^{69,71}₃₁Ga_{38,40} ⁷⁵₃₃As₄₂ ^{85,87}₃₇Rb_{48,50} ^{99,100}₄₄Ru_{55,56} ¹⁰⁸₄₆Pd₆₂
^{112,115,116,119,120,124}₅₀Sn_{62,65,66,69,70,74} ^{136,137,138}₅₆Ba_{80,81,82} ¹⁶⁹₆₉Tm₁₀₀ ¹⁸⁴₇₄W₁₁₀ ²⁵⁷₁₀₀Fm₁₅₇^{*}

$$\frac{1}{\gamma_{1-\text{GL}-22.37}} = \frac{1}{\sqrt{(2\pi)_{\text{GL}-22.37} \alpha_1 \delta}}$$

$$= 1 - \frac{1}{2 \cdot 9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) - \frac{5}{11}} = 0.999999956442012$$

²³₁₁Na₁₂ ⁴⁷₂₂Ti₂₅ ⁵⁵₂₅Mn₃₀ ^{83,84}₃₆Kr_{47,48} ^{85,87}₃₇Rb_{48,50} ^{99,100}₄₄Ru_{55,56} ^{107,109}₄₇Ag_{60,62} ¹³⁷₅₆Ba₈₁
^{160,162}₆₆Dy_{94,96} ^{182,184}₇₄W_{108,110} ²⁰⁹₈₃Bi₁₂₆^{*} ²⁰⁹₈₄Po₁₂₅^{*} ²⁸⁵₁₁₂Cn₁₇₃^{*} ²⁻¹⁷³₁₃₇Fy₂₀₉^{*}

2021/3/2

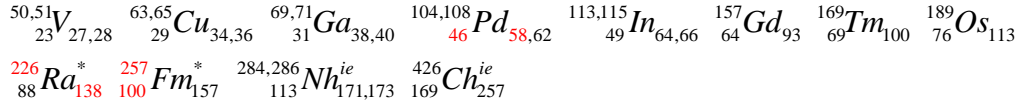
$$\alpha_1(\delta / \gamma_{1-\text{NC}-3})^2 (2\pi)_{\text{NC}-3} = 1$$

$$\gamma_{1-\text{NC}-3} = \sqrt{(2\pi)_{\text{NC}-3} \alpha_1 \delta}$$

$$= \frac{\sqrt{(6 + \sum_{n=1}^3 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)})} \cdot 4.66920160910299}{\sqrt{137.035999037435}}$$

$$= \frac{\sqrt{6.29047619047619} \cdot 4.66920160910299}{\sqrt{137.035999037435}}$$

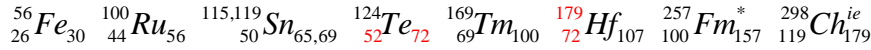
$$= 1 + \frac{1}{23 \cdot 113} - \frac{1}{2 \cdot 3 \cdot 257 \cdot (4 \cdot 5 \cdot 29 \cdot 31 + 1)} = 1.00038472730421$$



2021/3/3

$$\frac{1}{\gamma_{1-NC-3}} = \frac{1}{\sqrt{(2\pi)_{NC-3} \alpha_1 \delta}}$$

$$= 1 - \frac{1}{8 \cdot 25 \cdot 13} + \frac{1}{179 \cdot (8 \cdot 9 \cdot (2 \cdot 3 \cdot (2 \cdot 179 + 1) - 1) + 1) - \frac{3}{7}} = 0.999615420653963$$



2021/3/2

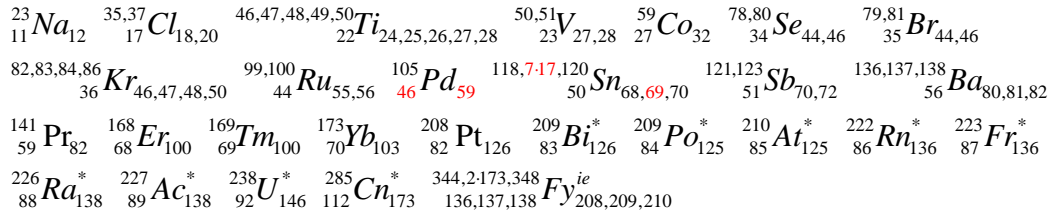
8. Integrated Formulas of α_2 , α , δ and γ_2

By introducing a correction factor γ_2 , some integrated formulas of α_2 , α , δ and γ_2 in the format of $\alpha_2(\delta\alpha\gamma_2)^2=1$ could be obtained as follows.

$$\alpha_2(\delta\alpha/\gamma_2)^2 = 1$$

$$\gamma_2 = \sqrt{\alpha_2(\alpha\delta)^2} = \frac{4.66920160910299 \cdot 2.50290787509589}{\sqrt{137.035999111818}}$$

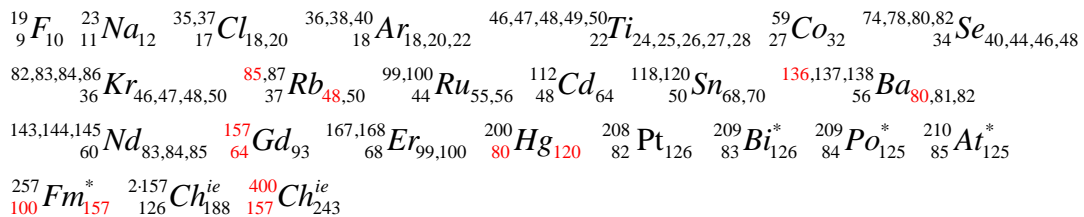
$$= 1 - \frac{1}{5 \cdot 7 \cdot 17} + \frac{1}{4 \cdot 3 \cdot 17 \cdot 23 \cdot 137 - \frac{11}{59}} = 0.998320883415699$$



2021/2/28

$$\frac{1}{\gamma_2} = \frac{1}{\sqrt{\alpha_2 \delta \alpha}} = \frac{\sqrt{137.035999111818}}{4.66920160910299 \cdot 2.50290787509589}$$

$$= 1 + \frac{1}{2 \cdot 27 \cdot 11} - \frac{1}{5 \cdot 17 \cdot (16 \cdot 3 \cdot 157 + 1) + \frac{16}{17}} = 1.00168194075892$$



2021/2/27

9. Integrated Formulas of α_1 , α_2 , α , 2π , γ_1 and γ_2

$$\frac{\alpha_1(2\pi)}{\alpha_2(\alpha\gamma_1/\gamma_2)^2} = 1$$

$$\frac{\gamma_1}{\gamma_2} = \frac{\sqrt{137.035999111818 \cdot (2 \cdot 3.14159265358979)}}{\sqrt{137.035999037435 \cdot 2.50290787509589}}$$

$$= 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{32 \cdot 89 \cdot (4 \cdot 53 - 1) - \frac{25}{2 \cdot 23}} = 1.00148643114372$$

^{50,51}V_{27,28} ⁵³Cr₂₉ ⁸⁹Y₃₉ ⁹⁵Mo₄₂ ²⁻⁵³Pd₄₆ ¹¹²Cd₄₈ ¹¹⁹Sn₆₀ ¹²⁷I₅₃ ^{136,137,138}Ba₅₆_{80,81,82} ^{151,153}Eu₆₃_{88,90}
¹⁶⁹Tm₆₉ ²⁻⁸⁹Hf₇₂ ²⁻⁵³Re₇₅ ^{185,187}Re_{110,112} ²²⁷Ac₈₉^{*} ²⁸⁵Cn₁₁₂^{*} ⁶⁻⁵³⁻³²⁰Ch₁₂₇^{ie}₁₉₁₋₁₉₃ ^{344,2-173,348}Fy_{136,137,138}^{ie}_{208,209,210}

$$\frac{\gamma_2}{\gamma_1} = 1 - \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{4 \cdot 53 \cdot (2 \cdot 49 \cdot 29 + 1) + \frac{2 \cdot 3}{23}} = 0.99851577505446$$

^{28,29,30}Si₁₄^{ie}_{14,15,16} ^{50,51}V₂₃_{27,28} ⁵³Cr₂₄₂₉ ^{63,65}Cu₂₉_{34,36} ⁹⁵Mo₄₂₅₃ ²⁻⁵³Pd₄₆ ¹¹²Cd₄₈₆₄ ¹²⁷I₅₃₇₄ ^{136,137,138}Ba₅₆_{80,81,82}
²⁸⁵Cn₁₁₂^{*} ³³⁶Ch₁₁₃^{ie}₇₋₂₉ ^{6-53,11-29,320}Ch₁₂₇^{ie}_{191,192,193} ^{344,2-173,12-29}Fy_{136,137,138}^{ie}_{208,209,210}

2021/3/3

10. Integrated Formulas α , 2π and γ

By introducing a correction factor γ , some integrated formulas of 2π , α and γ in the format of $2\pi/(\alpha\gamma)^2=1$ could be obtained as follows.

$$\frac{2\pi}{(\alpha\gamma)^2} = 1$$

$$\gamma = \frac{\sqrt{(2 \cdot 3.14159265358979)}}{2.50290787509589^2}$$

$$= 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{23 \cdot 151 \cdot 173 + \frac{9}{4 \cdot 7}} = 1.00148643087192$$

²³Na₁₁₁₂ ^{50,51}V₂₃_{27,28} ^{82,83,84}Kr₃₆_{46,47,48} ^{136,137,138}Ba₅₆_{80,81,82} ^{151,153}Eu₆₃_{88,90} ²⁰⁸Pd₈₂₁₂₆ ²⁰⁹Bi₈₃^{*}₁₂₆ ²⁰⁹Po₈₄^{*}₁₂₅
²⁴⁷Cm₉₆^{*}₁₅₁ ²⁸⁵Cn₁₁₂^{*}₁₇₃ ^{344,2-173,348}Fy_{136,137,138}^{ie}_{208,209,210}

$$\frac{1}{\gamma} = 1 - \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{2 \cdot 41 \cdot (2 \cdot 3 \cdot 25 \cdot 49 - 1) - \frac{1}{6}} = 0.99851577532546$$

⁵⁵Mn₂₅₃₀ ⁷³Ge₃₂₄₁ ^{82,83,84,86}Kr₃₆_{46,47,48,50} ⁹³Nb₄₁₅₂ ^{140,142}Ce₅₈_{82,84} ²⁰⁸Pd₈₂₁₂₆ ²⁰⁹Bi₈₃^{*}₁₂₆ ²⁰⁹Po₈₄^{*}₁₂₅ ²¹⁰At₈₅^{*}₁₂₅

2021/3/3

11. Summary

The above integrated formulas of the fine-structure constant and Feigenbaum constants are summarized as follows.

The Fine-structure Constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7 \cdot (2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13 \cdot (2\pi)_{Chen-278}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818$$

Feigenbaum Constants: $\delta = 4.66920160910299$

$$\alpha = 2.50290787509589$$

$$2\pi = 2 \cdot 3.14159265358979$$

$$\alpha_1 (\delta / \gamma_1)^2 (2\pi) = 1$$

$$\gamma_1 = \left(1 - \frac{1}{47 \cdot 109} + \frac{1}{27 \cdot 7 \cdot (3 \cdot 8 \cdot (3 \cdot 8 \cdot (4 \cdot 137 - 1) - 1) - 1)}\right)^2 = 0.999804818668238$$

$$\frac{1}{\gamma_1} = \left(1 + \frac{1}{2 \cdot 13 \cdot 197} - \frac{16 \cdot 7 \cdot 17 \cdot (16 \cdot 3 \cdot 23 - 1)}{125 \cdot 10^{12}}\right)^2 = 1.00019521943495$$

$$\gamma_1^2 = 1 - \frac{1}{13 \cdot 197} + \frac{1}{2 \cdot 7 \cdot 41 \cdot (4 \cdot 5 \cdot 19 \cdot 31 - 1)} = 0.99960967543223$$

$$\frac{1}{\gamma_1^2} = 1 + \frac{1}{512 \cdot 5} - \frac{1}{4 \cdot 9 \cdot 7 \cdot 17 \cdot 19 \cdot 83} = 1.00039047698053$$

$$\alpha_1 (\delta / \gamma_{1-Chen-25 \cdot 17})^2 (2\pi)_{Chen-25 \cdot 17} = 1$$

$$(2\pi)_{Chen-25 \cdot 17} = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}} = 6.28564399787948$$

$$\gamma_{1-Chen-25 \cdot 17} = 1 + \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 - \frac{3}{25}} = 1.00000041773574$$

$$\frac{1}{\gamma_{1-Chen-25 \cdot 17}} = 1 - \frac{1}{2 \cdot 79 \cdot 109 \cdot 139 + \frac{22}{25}} = 0.999999582264432$$

$$\gamma_{1-Chen-25 \cdot 17}^2 = 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717$$

$$\frac{1}{\gamma_{1-Chen-25 \cdot 17}^2} = 1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \approx 1$$

$$\alpha_1(\delta / \gamma_{1-\text{Wallis-9.71}})^2(2\pi)_{\text{Wallis-9.71}} = 1$$

$$(2\pi)_{\text{Wallis-9.71}} = 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \cdots \frac{1278}{1279} \frac{1280}{2 \cdot 9 \cdot 71 + 1}\right) = 6.28564015562186$$

$$\gamma_{1-\text{Wallis-9.71}} = 1 + \frac{1}{4 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1)} = 1.00000011209802$$

$$\frac{1}{\gamma_{1-\text{Wallis-9.71}}} = 1 - \frac{1}{5 \cdot 7 \cdot (32 \cdot 27 \cdot 5 \cdot 59 - 1)} = 0.999999887901990$$

$$\gamma_{1-\text{Wallis-9.71}}^2 = 1 + \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) - \frac{5}{17}} = 1.00000022419606$$

$$\frac{1}{\gamma_{1-\text{Wallis-9.71}}^2} = 1 - \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) + \frac{3 \cdot 4}{17}} = 0.99999977580399$$

$$\alpha_1 \delta^2 (2\pi / \gamma_{1-\text{GL-22.37}})_{\text{GL-22.37}} = 1$$

$$(2\pi)_{\text{GL-22.37}} = 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1}\right) = 6.28563929398602$$

$$\gamma_{1-\text{GL-22.37}} = 1 + \frac{1}{4 \cdot 25 \cdot 7 \cdot (2 \cdot 23^2 \cdot 31 - 1) + \frac{6}{11}} = 1.00000004355799$$

$$\frac{1}{\gamma_{1-\text{GL-22.37}}} = 1 - \frac{1}{2 \cdot 9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) - \frac{5}{11}} = 0.999999956442012$$

$$\gamma_{1-\text{GL-22.37}}^2 = 1 + \frac{1}{163 \cdot (6 \cdot 11^2 \cdot 97 + 1) + \frac{9}{10}} = 1.00000008711598$$

$$\frac{1}{\gamma_{1-\text{GL-22.37}}^2} = 1 - \frac{1}{9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) - \frac{2}{25}} = 0.999999912884025$$

$$\alpha_1(\delta / \gamma_{1-\text{NC-3}})^2(2\pi)_{\text{NC-3}} = 1$$

$$(2\pi)_{\text{NC-3}} = \left(6 + \sum_{n=1}^3 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)}\right) = 6.29047619047619$$

$$\gamma_{1-\text{NC-3}} = 1 + \frac{1}{23 \cdot 113} - \frac{1}{2 \cdot 3 \cdot 257 \cdot (4 \cdot 5 \cdot 29 \cdot 31 + 1)} = 1.00038472730421$$

$$\frac{1}{\gamma_{1-\text{NC-3}}} = 1 - \frac{1}{8 \cdot 25 \cdot 13} + \frac{1}{179 \cdot (8 \cdot 9 \cdot (2 \cdot 3 \cdot (2 \cdot 179 + 1) - 1) + 1) - \frac{3}{7}}$$

$$= 0.999615420653963$$

$$\gamma_{1-NC-3}^2 = 1 + \frac{1}{3 \cdot (2 \cdot 7 \cdot 31 - 1)} - \frac{1}{13 \cdot 89 \cdot (2 \cdot 37 \cdot 53 + 1) - \frac{2 \cdot 9}{25}} = 1.00076960262352$$

$$\frac{1}{\gamma_{1-NC-3}^2} = 1 - \frac{1}{4 \cdot 25 \cdot 13} + \frac{1}{4 \cdot 9 \cdot 25 \cdot (2 \cdot 25 \cdot (4 \cdot 25 + 1) + 1) + \frac{2}{7}} = 0.999230989209198$$

$$\alpha_2(\delta\alpha / \gamma_2)^2 = 1$$

$$\gamma_2 = 1 - \frac{1}{5 \cdot 7 \cdot 17} + \frac{1}{4 \cdot 3 \cdot 17 \cdot 23 \cdot 137 - \frac{11}{59}} = 0.998320883415699$$

$$\frac{1}{\gamma_2} = 1 + \frac{1}{2 \cdot 27 \cdot 11} - \frac{1}{5 \cdot 17 \cdot (16 \cdot 3 \cdot 157 + 1) + \frac{16}{17}} = 1.00168194075892$$

$$\gamma_2^2 = 1 - \frac{1}{2 \cdot 149} + \frac{1}{29 \cdot 31 \cdot (2 \cdot 3 \cdot 49 \cdot 13 + 1) - \frac{16}{19}} = 0.996644586263908$$

$$\frac{1}{\gamma_2^2} = 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (4 \cdot 11 \cdot 47 + 1) - \frac{2}{5}} = 1.00336671044256$$

$$\frac{\alpha_1(2\pi)}{\alpha_2(\alpha\gamma_1 / \gamma_2)^2} = 1$$

$$\frac{\gamma_1}{\gamma_2} = 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{32 \cdot 89 \cdot (4 \cdot 53 - 1) - \frac{25}{2 \cdot 23}} = 1.00148643114372$$

$$\frac{\gamma_2}{\gamma_1} = 1 - \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{4 \cdot 53 \cdot (2 \cdot 49 \cdot 29 + 1) + \frac{2 \cdot 3}{23}} = 0.99851577505446$$

$$\left(\frac{\gamma_1}{\gamma_2}\right)^2 = 1 + \frac{1}{16 \cdot 3 \cdot 7} - \frac{1}{2 \cdot 3 \cdot 7 \cdot (4 \cdot 17 \cdot (2 \cdot 157 - 1) - 1) - \frac{1}{4}} = 1.00297507176499$$

$$\left(\frac{\gamma_2}{\gamma_1}\right)^2 = 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{7 \cdot 19 \cdot (2 \cdot 3 \cdot 7^2 \cdot 23 - 1) + \frac{16}{3 \cdot 13}}$$

$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{7 \cdot 19 \cdot (8 \cdot 5 \cdot 13^2 + 1) + \frac{16}{3 \cdot 13}}$$

$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{2 \cdot 3 \cdot 73 \cdot (4 \cdot 27 \cdot 19 + 1) - \frac{23}{3 \cdot 13}} = 0.997033753032614$$

$$\frac{2\pi}{(\alpha\gamma)^2} = 1$$

$$\gamma = 1 + \frac{1}{32 \cdot 3 \cdot 7} - \frac{1}{23 \cdot 151 \cdot 173 + \frac{9}{4 \cdot 7}} = 1.00148643087192$$

$$\frac{1}{\gamma} = 1 - \frac{1}{32 \cdot 3 \cdot 7 + 1} + \frac{1}{2 \cdot 41 \cdot (2 \cdot 3 \cdot 25 \cdot 49 - 1) - \frac{1}{6}} = 0.99851577532546$$

$$\gamma^2 = 1 + \frac{1}{16 \cdot 3 \cdot 7} - \frac{1}{3 \cdot 13 \cdot 31 \cdot (2 \cdot 9 \cdot 41 + 1) - \frac{1}{22}} = 1.0029750712205$$

$$\frac{1}{\gamma^2} = 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{8 \cdot 81 \cdot 19 \cdot 73 + \frac{1}{3 \cdot 13}} = 0.997033753573803$$

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Appendix I: Research History

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Note: Time was recorded according to Beijing Time.