

Integrated Formulas of the Fine-structure Constant and Feigenbaum Constants

Gang Chen[†], Tianman Chen, Tianyi Chen

Guangzhou Huifu Research Institute Co., Ltd., Guangzhou, P. R. China

7-20-4, Greenwich Village, Wangjianglu 1, Chengdu, P. R. China

[†]Correspondence to: gang137.chen@connect.polyu.hk

Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper is a subsequent paper to the previous paper “Formulas of Feigenbaum Constants and Their Physical Meanings” (viXra:2101.0187). In the previous paper, some formulas of Feigenbaum constants in fractional number format were given and the physical meanings of the factors in the formulas were exhibited, especially their relationships with nuclides, the fine-structure constant and 2π . In the previous paper, some integrated formulas of the fine-structure constant, Feigenbaum constants and 2π were also given, briefly denoted as $\alpha_1\delta^2(2\pi)\approx 1$, and their relationships with nuclides were illustrated. In this paper, some formulas for $\alpha_1\delta^2(2\pi)\approx 1$ are supplemented, some formulas for $\alpha_2(\delta\alpha)^2\approx 1$, $[\alpha_1(2\pi)]/(\alpha_2\alpha^2)\approx 1$ and $(2\pi)/\alpha^2\approx 1$ are given, some formulas of the fine-structure constant (α_1 and α_2) based on the key number 103 instead of 112, 173, 137, 83 and 29 are given, and their relationships with nuclides are illustrated.

Keywords: Formulas; the fine-structure constant; Feigenbaum constants; 2π .

1. Introduction

In our previous papers^{1,2,3,4,5}, we gave or exhibited the following formulas.

$$(2\pi)_{Chen-k} = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{k+1}{k}\right)^{2k+1}}$$

$$(2\pi)_{Wallis-k} = 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \cdots \frac{2k}{2k+1} \frac{2k+2}{2k+1}\right)$$

$$(2\pi)_{GL-k} = 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + (-1)^{k+1} \frac{1}{2k+1}\right) \quad (GL \text{ means Gregory-Leibniz})$$

$$(2\pi)_{NC-k} = 6 + \sum_{n=1}^k \frac{(-1)^{n+1}}{n(n+1/2)(n+1)} \quad (NC \text{ means Nilakantha-Chen})$$

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7 \cdot (2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13 \cdot (2\pi)_{Chen-278}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818$$

$$c_{au} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \alpha_2}} = \sqrt{112 \times (168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{14 \cdot 112 \cdot (2 \cdot 173 + 1)})}$$

$$= 137.035999074626$$

$$1/\alpha_1 = 56 + 81 + \frac{1}{28 - \frac{13 \cdot (2 \cdot 56 \cdot 11 - 1)}{3 \cdot 5 \cdot (2 \cdot 56 \cdot 43 + 1)}} = 137.035999037435$$

$$1/\alpha_2 = 56 + 81 + \frac{1}{28 - \frac{2 \cdot (16 \cdot 27 - 1)}{3 \cdot (16 \cdot 81 + 1)}} = 137.035999111818$$

$$c_{au} = \frac{1}{\alpha_c} = 56 + 81 + \frac{1}{28 - \frac{5 \cdot (4 \cdot 3 \cdot 7 \cdot 17 - 1)}{2 \cdot 5 \cdot (4 \cdot 5 \cdot 7 \cdot 23 + 1) + 1}} = 137.035999074626$$

Note: c_{au} refers to the speed of light in vacuum in atomic units

Feigenbaum Constants: $\delta = 4.66920160910299$

$$\alpha = 2.50290787509589$$

$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326$$

$$= \frac{1}{4} - \frac{1}{27} + \frac{1}{4 \cdot 9 \cdot 23} - \frac{1}{2 \cdot 3 \cdot 7 \cdot 23 \cdot (2 \cdot 3 \cdot (4 \cdot 3 \cdot 11 - 1) + 1) + \frac{2 \cdot 23}{3 \cdot 19}}$$

$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

$$= \frac{1}{2} - \frac{1}{9} + \frac{1}{3 \cdot 31} - \frac{1}{23 \cdot (8 \cdot 3 \cdot 17 + 1)} + \frac{1}{17 \cdot 23 \cdot (8 \cdot 3 \cdot 11^4 - 1)}$$

Note: $136=8 \cdot 17$, $138=6 \cdot 23$

$$\alpha_1 \delta^2 (2\pi) \approx 1$$

On Feb. 8, 2021, we also noticed that Hieb uploaded a paper⁶ in viXra in April of 2017, and gave an approximate formula of the fine-structure constant and Feigenbaum constant as follows, but without any explanations to its physical meanings.

$$\delta' = (1/(2\pi\alpha))^{1/2} = 4.670114 \approx \delta = 4.669201609$$

$$\delta' - \delta = 0.000912$$

α : the fine-structure constant, $\alpha \approx 1/137.036$

2. Integrated Formulas of α_1 , δ and 2π

A Concise Deduction

The Fine-structure Constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7 \cdot (2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037435$$

Feigenbaum Constants: $\delta = 4.66920160910299$

$$\begin{aligned} \alpha_1 &= \frac{36}{7 \cdot (2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} \approx \frac{36}{7 \cdot (2\pi)} \frac{1}{112} = \left(\frac{3}{14}\right)^2 \frac{1}{(2\pi)} \approx \frac{1}{\delta^2(2\pi)} \\ &= \frac{1}{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)} \approx \frac{1}{136.982} \end{aligned}$$

So it should be reasonable to assume the following approximate formulas:

$$\alpha_1 \delta^2(2\pi) \approx 1 \text{ or } \frac{1}{\alpha_1 \delta^2(2\pi)} \approx 1$$

$$\text{Numerically: } \alpha_1 \delta^2(2\pi) = \frac{4 \cdot 6692^2 \times 6.2832}{137.036} = 0.99961 \approx 1$$

2021/2/1-3

The above approximate formula $\alpha_1 \delta^2(2\pi) \approx 1$ is assumed to be the brief form of integrated formulas of α_1 , δ and 2π . There should be some corresponding accurate forms of integrated formulas of α_1 , δ and 2π as follows.

$$\begin{aligned} \alpha_1 \delta^2(2\pi)_{Chen-25-17} &= \frac{4.66920160910299^2 \cdot \left(\frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}} \right)}{137.035999037435} \\ &= \frac{4.66920160910299^2 \cdot 6.28564399787948}{137.035999037435} \\ &= 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717 \approx 1 \end{aligned}$$

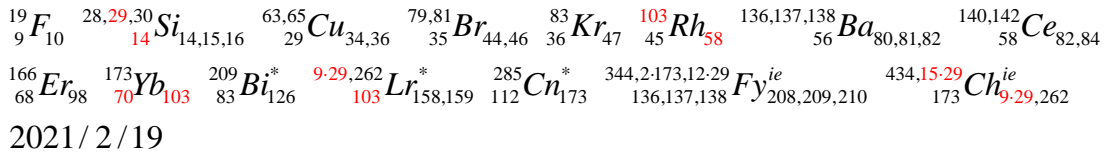
${}_{7}^{14,15}N_{7,8}$ ${}_{8}^{16,17,18}O_{8,9,10}$ ${}_{11}^{23}Na_{12}$ ${}_{17}^{35,37}Cl_{18,20}$ ${}_{23}^{50,51}V_{27,28}$ ${}_{30}^{64,66,67,68}Zn_{34,36,37,38}$ ${}_{34}^{80}Se_{46}$ ${}_{35}^{79,81}Br_{44,46}$
 ${}_{36}^{82,83,84,86}Kr_{46,47,48,50}$ ${}_{37}^{85,87}Rb_{48,50}$ ${}_{46}^{105,110}Pd_{59,64}$ ${}_{48}^{111,112}Cd_{63,64}$ ${}_{50}^{112,114,115-120,122,124}Sn_{62,64,65-70,72,74}$
 ${}_{51}^{121,123}Sb_{70,72}$ ${}_{52}^{126}Te_{74}$ ${}_{54}^{128}Xe_{74}$ ${}_{56}^{136,137,138}Ba_{80,81,82}$ ${}_{64}^{157}Gd_{93}$ ${}_{68}^{168,170}Er_{100,102}$ ${}_{70}^{171,173}Yb_{101,103}$
 ${}_{69}^{169}Tm_{100}$ ${}_{71}^{175,176}Lu_{104,105}$ ${}_{75}^{185,187}Re_{110,112}$ ${}_{82}^{208}Pb_{126}$ ${}_{83}^{209}Bi_{126}^*$ ${}_{84}^{209}Po_{125}^*$ ${}_{85}^{210}At_{125}^*$ ${}_{103}^{261,262}Lr_{158,159}^*$
 ${}_{100}^{257}Fm_{157}^*$ ${}_{112}^{285}Cn_{173}^*$ ${}_{113}^{4-71,286}Nh_{171,173}^{ie}$ ${}_{128}^{326}Ch_{18-11}^{ie}$ ${}_{136}^{344,2-173,348}Fy_{208,209,210}^{ie}$ ${}_{169}^{6-71}Ch_{257}^{ie}$ ${}_{173}^{434,435}Ch_{261,262}^{ie}$

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$$\frac{1}{\alpha_1 \delta^2 (2\pi)_{Chen-25-17}} = \frac{137.035999037435}{4.66920160910299^2 \cdot \left(e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}} \right)}$$

$$= \frac{137.035999037435}{4.66920160910299^2 \cdot 6.28564399787948}$$

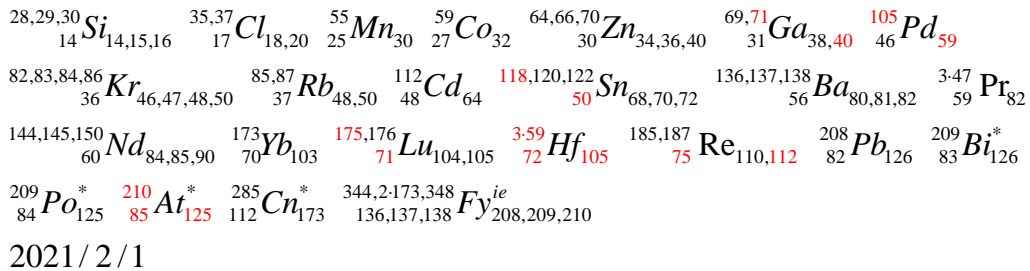
$$= 1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \approx 1$$



$$\alpha_1 \delta^2 (2\pi)_{Wallis-9-71} = \frac{4.66920160910299^2 \cdot 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{1278}{1279} \frac{1280}{2 \cdot 9 \cdot 71 + 1} \right)}{137.035999037435}$$

$$= \frac{4.66920160910299^2 \cdot 6.28564015562186}{137.035999037435}$$

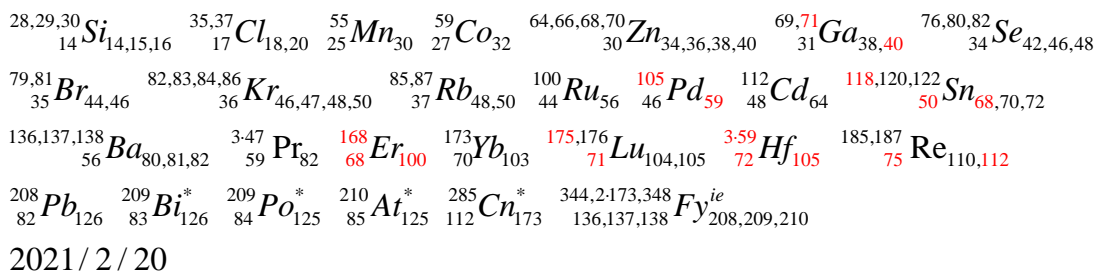
$$= 1 + \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) - \frac{5}{17}} = 1.00000022419606 \approx 1$$



$$\frac{1}{\alpha_1 \delta^2 (2\pi)_{Wallis-9-71}} = \frac{137.035999037435}{4.66920160910299^2 \cdot 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{1278}{1279} \frac{1280}{2 \cdot 9 \cdot 71 + 1} \right)}$$

$$= \frac{137.035999037435}{4.66920160910299^2 \cdot 6.28564015562186}$$

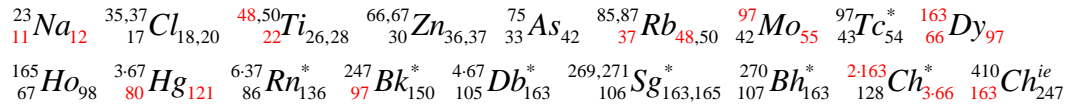
$$= 1 - \frac{1}{2 \cdot 9 \cdot (8 \cdot 3 \cdot 25 \cdot 7 \cdot 59 - 1) + \frac{3 \cdot 4}{17}} = 0.999999775803991 \approx 1$$



$$\alpha_1 \delta^2 (2\pi)_{GL-22,37} = \frac{4.66920160910299^2 \cdot 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1}\right)}{137.035999037435}$$

$$= \frac{4.66920160910299^2 \cdot 6.28563929398602}{137.035999037435}$$

$$= 1 + \frac{1}{163 \cdot (6 \cdot 11^2 \cdot 97 + 1) + \frac{9}{10}} = 1.00000008711598 \approx 1$$

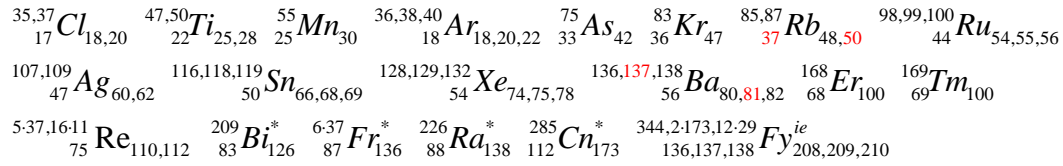


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$$\alpha_1 \delta^2 (2\pi)_{GL-22,37} = \frac{1}{4.66920160910299^2 \cdot 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 37 + 1}\right)}$$

$$= \frac{137.035999037435}{4.66920160910299^2 \cdot 6.28563929398602}$$

$$= 1 - \frac{1}{9 \cdot 11 \cdot 47 \cdot (2 \cdot 9 \cdot 137 + 1) - \frac{2}{25}} = 0.999999912884025 \approx 1$$

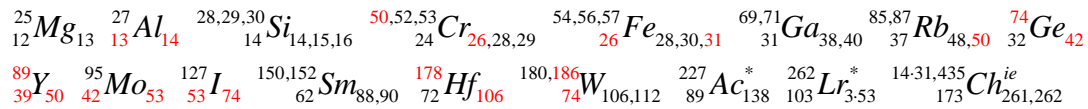


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$$\alpha_1 \delta^2 (2\pi)_{NC-3} = \frac{4.66920160910299^2 \cdot \left(6 + \sum_{n=1}^3 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)}\right)}{137.035999037435}$$

$$= \frac{4.66920160910299^2 \cdot 6.29047619047619}{137.035999037435}$$

$$= 1 + \frac{1}{3 \cdot (2 \cdot 7 \cdot 31 + 1)} - \frac{1}{13 \cdot 89 \cdot (2 \cdot 37 \cdot 53 + 1) - \frac{2 \cdot 9}{25}} = 1.00076960262352 \approx 1$$

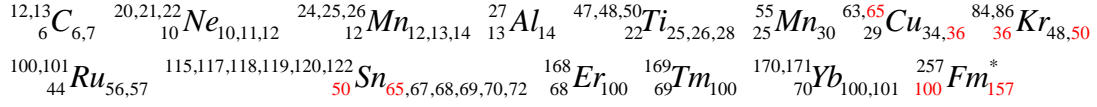


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$$\alpha_1 \delta^2 (2\pi)_{NC-3} = \frac{1}{4.66920160910299^2 \cdot \left(6 + \sum_{n=1}^3 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)}\right)}$$

$$= \frac{137.035999037435}{4.66920160910299^2 \cdot 6.29047619047619}$$

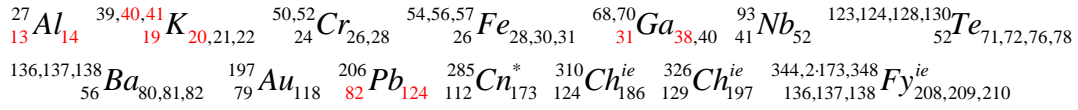
$$= 1 - \frac{1}{4 \cdot 25 \cdot 13} + \frac{1}{4 \cdot 9 \cdot 25 \cdot (2 \cdot 25 \cdot (4 \cdot 25 + 1) + 1) + \frac{2}{7}} = 0.999230989209198 \approx 1$$



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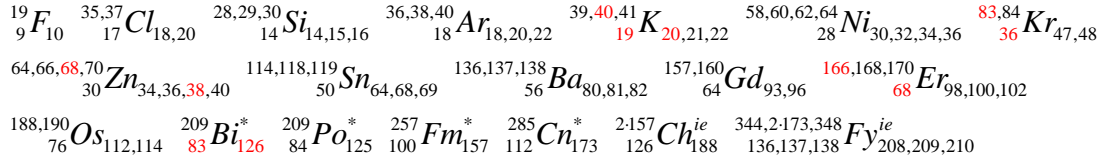
$$\alpha_1 \delta^2 (2\pi) = \frac{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)}{137.035999037435}$$

$$= 1 - \frac{1}{13 \cdot 197} + \frac{1}{2 \cdot 7 \cdot 41 \cdot (4 \cdot 5 \cdot 19 \cdot 31 - 1)} = 0.99960967543223 \approx 1$$



$$\frac{1}{\alpha_1 \delta^2 (2\pi)} = \frac{137.035999037435}{4.66920160910299^2 \cdot (2 \cdot 3.14159265358979)}$$

$$= 1 + \frac{1}{512 \cdot 5} - \frac{1}{4 \cdot 9 \cdot 7 \cdot 17 \cdot 19 \cdot 83} = 1.00039047698053 \approx 1$$



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3. Integrated Formulas of α_2 , δ and α

The Fine-structure Constant:

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13 \cdot (2\pi)_{\text{Chen-278}}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} = 1/137.035999111818$$

Feigenbaum Constants: $\delta = 4.66920160910299$

$$\alpha = 2.50290787509589$$

$$\alpha_2 = \frac{13 \cdot (2\pi)_{\text{Chen-278}}}{100} \frac{1}{112 - \frac{1}{64 \cdot 3 \cdot 29}} \approx \frac{13 \cdot (2\pi)}{100} \frac{1}{112} \approx \frac{(2\pi)}{(\delta\alpha)^2 (2\pi)}$$

$$\approx \frac{1}{(\delta\alpha)^2} = \frac{1}{(4.66920160910299 \cdot 2.50290787509589)^2} \approx 1/136.575$$

So it should be reasonable to assume the following approximate formulas:

$$\alpha_2 (\delta\alpha)^2 \approx 1 \text{ or } \frac{1}{\alpha_2 (\delta\alpha)^2} \approx 1$$

$$\text{Numerically: } \alpha_2 (\delta\alpha)^2 = \frac{(4 \cdot 6692 \times 2.5029)^2}{137.036} = 0.99664 \approx 1$$

2021/2/7

The above approximate formula $\alpha_2(\delta\alpha)^2 \approx 1$ is assumed to be the brief form of integrated formulas of α_2 , δ and α . There should be some corresponding accurate forms of integrated formulas of α_2 , δ and α as follows.

$$\alpha_2(\delta\alpha)^2 = \frac{(4.66920160910299 \cdot 2.50290787509589)^2}{137.035999111818}$$

$$= 1 - \frac{1}{2 \cdot 149} + \frac{1}{29 \cdot 31 \cdot (2 \cdot 3 \cdot 49 \cdot 13 + 1) - \frac{16}{19}} = 0.996644586263908 \approx 1$$

^{12,13}₆C_{6,7} ^{14,15}₇N_{7,8} ¹⁹₉F₁₀ ^{24,25,26}₁₂Mg_{12,13,14} ²⁷₁₃Al₁₄ ^{28,29,30}₁₄Si_{14,15,16} ³¹₁₅P₁₆ ^{39,40,41}₁₉K_{20,21,22}
^{46,47,48,49,50}₂₂Ti_{24,25,26,27,28} ^{50,52,53}₂₄Cr_{26,28,29} ^{54,56,57,58}₂₆Fe_{28,30,31,32} ^{58,60,62,64}₂₈Ni_{30,32,34,36}
^{63,65}₂₉Cu_{34,36} ^{69,71}₃₁Ga_{38,40} ^{70,74,76}₃₂Ge_{38,42,44} ^{76,78}₃₄Se_{42,44} ^{85,87}₃₇Rb_{48,50} ⁸⁹₃₉Y₅₀ ⁹³₄₁Nb₅₂
^{94,95,96,98,100}₄₂Mo_{52,53,54,56,58} ^{112,113}₄₈Cd_{64,65} ^{113,115}₄₉In_{64,66} ⁷⁻¹⁹₅₄Xe_{76,78} ⁷⁻¹⁹₅₅Cs₇₈ ¹⁴⁹₆₂Sm₈₇
^{134,136,137,138}₅₆Ba_{78,80,81,82} ^{155,156,157,160}₆₄Gd_{91,92,93,96} ^{186,187,189,190,192}₇₆Os_{110,112,113,114,116}
^{190,192,194,195,196}₇₈Pt_{112,114,116,117,118} ²²³₈₇Fr₁₃₆ ²²⁷₈₉Ac* ²¹⁻¹¹₉₁Pa* ²³⁷₉₃Np* ¹⁵⁻¹⁹₁₁₂Cn*
²²⁻¹³₁₁₃Nh^{ie}₁₇₃ ^{344,2,173,12-29}_{136,137,138}Fy^{ie}_{208,209,210} ⁸⁻⁴⁷₁₄₉Ch^{ie}₂₂₇ ^{14-31,15-29}₁₇₃Ch^{ie}_{9-19,262}

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$$\frac{1}{\alpha_2(\delta\alpha)^2} = \frac{137.035999111818}{(4.66920160910299 \cdot 2.50290787509589)^2}$$

$$= 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (4 \cdot 11 \cdot 47 + 1) - \frac{2}{5}}$$

$$= 1 + \frac{1}{27 \cdot 11} - \frac{1}{2 \cdot 3 \cdot 25 \cdot 11 \cdot (2 \cdot 9 \cdot 5 \cdot 23 - 1) - \frac{2}{5}} = 1.00336671044256 \approx 1$$

^{10,11}₅Be_{5,6} ²³₁₁Na₁₂ ^{24,25}₁₂Mg_{12,13} ^{46,47,49,50}₂₂Ti_{24,25,27,28} ^{50,51}₂₃V_{27,28} ⁵⁵₂₅Mn₃₀ ⁷⁵₃₃As₄₂
^{80,82,83,86}₃₆Kr_{44,46,47,50} ^{98,99,100,104}₄₄Ru_{54,55,56,60} ^{107,109}₄₇Ag_{60,62} ^{115,116,119,120}₅₀Sn_{65,66,69,70}
^{129,131,132}₅₄Xe_{75,77,78} ^{136,137,138}₅₆Ba_{80,81,82} ¹⁶⁹₆₉Tm₁₀₀ ^{185,187}₇₅Re_{110,112} ²²⁶₈₈Ra*₁₃₈
^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}

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4. Integrated Formulas of α_1 , α_2 , δ and α

$$\alpha_1 \delta^2(2\pi) = 0.99961 \approx 1$$

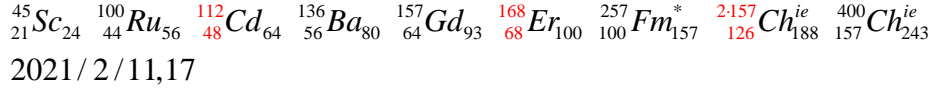
$$\alpha_2(\delta\alpha)^2 = 0.99664 \approx 1$$

$$\frac{\alpha_1 \delta^2(2\pi)}{\alpha_2(\delta\alpha)^2} = \frac{\alpha_1(2\pi)}{\alpha_2 \alpha^2} \approx \frac{2\pi}{\alpha^2} = 1.002975 \approx 1$$

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$$\frac{\alpha_1(2\pi)}{\alpha_2\alpha^2} = \frac{137.035999111818 \cdot (2 \cdot 3.14159265358979)}{137.035999037435 \cdot 2.50290787509589^2}$$

$$= 1 + \frac{1}{16 \cdot 3 \cdot 7} - \frac{1}{2 \cdot 3 \cdot 7 \cdot (4 \cdot 17 \cdot (2 \cdot 157 - 1) - 1) - \frac{1}{4}} = 1.00297507176499 \approx 1$$

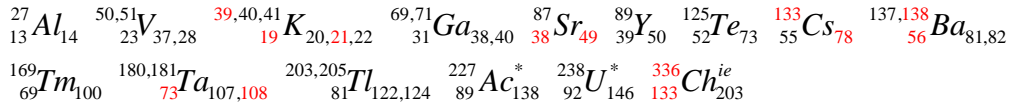


$$\frac{\alpha_2\alpha^2}{\alpha_1(2\pi)} = \frac{137.035999037435 \cdot 2.50290787509589^2}{137.035999111818 \cdot (2 \cdot 3.14159265358979)}$$

$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{7 \cdot 19 \cdot (2 \cdot 3 \cdot 7^2 \cdot 23 - 1) + \frac{16}{3 \cdot 13}}$$

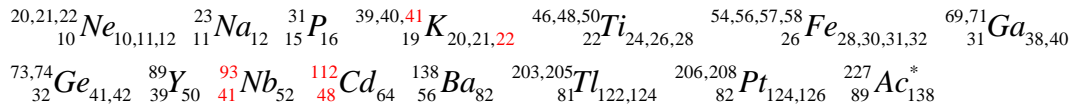
$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{7 \cdot 19 \cdot (8 \cdot 5 \cdot 13^2 + 1) + \frac{16}{3 \cdot 13}}$$

$$= 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{2 \cdot 3 \cdot 7 \cdot (4 \cdot 27 \cdot 19 + 1) - \frac{23}{3 \cdot 13}} = 0.997033753032614 \approx 1$$



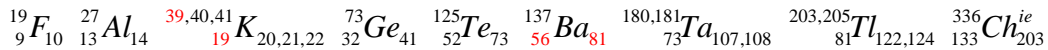
$$\frac{2\pi}{\alpha^2} = \frac{2 \cdot 3.14159265358979}{2.50290787509589^2} = 1 + \frac{1}{16 \cdot 3 \cdot 7} - \frac{1}{3 \cdot 13 \cdot 31 \cdot (2 \cdot 9 \cdot 41 + 1) - \frac{1}{22}}$$

$$= 1.00297507122057 \approx 1$$



$$\frac{\alpha^2}{2\pi} = \frac{2.50290787509589^2}{2 \cdot 3.14159265358979} = 1 - \frac{1}{16 \cdot 3 \cdot 7 + 1} + \frac{1}{8 \cdot 81 \cdot 19 \cdot 73 + \frac{1}{3 \cdot 13}}$$

$$= 0.997033753573803 \approx 1$$



5. Marvelous Coincidences

There are some marvelous coincidences of factors with nuclides in the above formulas. One typical example of these coincidences is listed as follows, which indicates the methodology and the formulas in this paper should be correct.

$$\alpha_1 \delta^2 (2\pi)_{Chen-25:17} = 1 + \frac{1}{128 \cdot 9 \cdot (2 \cdot 3 \cdot 173 + 1) + \frac{7}{10}} = 1.0000008354717 \approx 1$$

$$\frac{1}{\alpha_1 \delta^2 (2\pi)_{Chen-25:17}} = 1 - \frac{1}{2 \cdot 5 \cdot 7 \cdot (2 \cdot 83 \cdot 103 + 1) - \frac{9}{29}} = 0.999999164529037 \approx 1$$

$$(2\pi)_{Chen-25:17} = e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{2 \cdot 3 \cdot 71}{25 \cdot 17}\right)^{23 \cdot 37}}$$

$^{28,29,30}_{14}Si_{14,15,16}$ $^{35,37}_{17}Cl_{18,20}$ $^{50,51}_{23}V_{27,28}$ $^{63,65}_{29}Cu_{34,36}$ $^{80}_{34}Se_{46}$ $^{79,81}_{35}Br_{44,46}$ $^{82,83,84,86}_{36}Kr_{46,47,48,50}$
 $^{85,87}_{37}Rb_{48,50}$ $^{103}_{45}Rh_{58}$ $^{112}_{48}Cd_{64}$ $^{128}_{54}Xe_{74}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{140,142}_{58}Ce_{82,84}$ $^{173}_{70}Yb_{103}$ $^{209}_{83}Bi_{126}^*$
 $^{209}_{84}Po_{125}^*$ $^{210}_{85}At_{125}^*$ $^{9 \cdot 29, 262}_{103}Lr_{158,159}^*$ $^{285}_{112}Cn_{173}^*$ $^{344, 2 \cdot 173, 12 \cdot 29}_{136, 137, 138}Fy_{208, 209, 210}^{ie}$ $^{434, 15 \cdot 29}_{173}Ch_{9 \cdot 29, 262}^{ie}$

6. Formulas of the Fine-structure Constant based on 103

In our previous paper^{1,2,4}, many formulas of the fine-structure constant based on the key numbers 112, 173, 137, 83 and 29 were given. As shown in the above two formulas in **Section 5**, it seems 103 is another key number comparable to the above stated key numbers, so some formulas of the fine-structure constant based on the key number 103 instead of them are constructed as follows.

$$\alpha_1 = \frac{137}{29 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{11 \cdot 47}{4 \cdot 3 \cdot 43}\right)^{1033}}} \frac{1}{103 + \frac{1}{32 \cdot (32 \cdot 29 + 1) - \frac{3}{2 \cdot 17}}}$$

$$= \frac{137}{29 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{11 \cdot 47}{4 \cdot 3 \cdot 43}\right)^{1033}}} \frac{1}{103 + \frac{1}{81 \cdot (2 \cdot 3 \cdot 61 + 1) + \frac{31}{2 \cdot 17}}}$$

$$= \frac{137}{29 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{11 \cdot 47}{4 \cdot 3 \cdot 43}\right)^{1033}}} \frac{1}{103 + \frac{1}{81 \cdot (16 \cdot 23 - 1) + \frac{31}{2 \cdot 17}}}$$

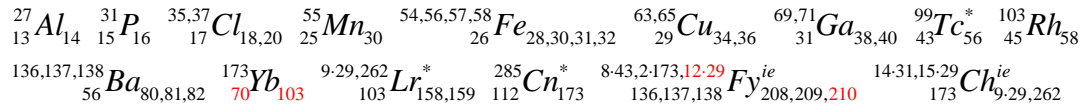
$$= 1/137.035999037435$$

$^{23}_{11}Na_{12}$ $^{35,37}_{17}Cl_{18,20}$ $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{50,51}_{23}V_{27,28}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{63,65}_{29}Cu_{34,36}$
 $^{69,71}_{31}Ga_{38,40}$ $^{70,72,76}_{32}Ge_{38,40,44}$ $^{77,78}_{34}Se_{43,44}$ $^{79,81}_{35}Br_{44,46}$ $^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ $^{97,98,99}_{43}Tc_{54,55,56}^*$
 $^{103}_{45}Rh_{58}$ $^{107,109}_{47}Ag_{60,62}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{5 \cdot 29, 146, 147}_{61}Pm_{84,85,86}^*$ $^{173}_{70}Yb_{103}$ $^{7 \cdot 29, 205}_{81}Tl_{122,124}$
 $^{9 \cdot 29, 262}_{103}Lr_{158,159}^*$ $^{285}_{112}Cn_{173}^*$ $^{8 \cdot 43, 2 \cdot 173, 12 \cdot 29}_{136, 137, 138}Fy_{208, 209, 210}^{ie}$ $^{14 \cdot 31, 15 \cdot 29}_{173}Cl_{9 \cdot 29, 262}^{ie}$

2021/2/25

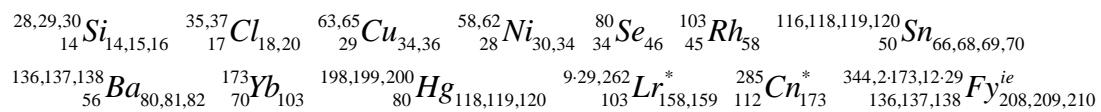
$$\alpha_1 = \frac{137}{29 \cdot 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{1548}{1549} \frac{2 \cdot 25 \cdot 31}{2 \cdot 2 \cdot 9 \cdot 43 + 1}\right)} \frac{1}{103 + \frac{1}{2 \cdot 3 \cdot 5 \cdot (2 \cdot 3 \cdot 5 \cdot 7 \cdot 13 - 1) - \frac{3}{17}}}$$

$$= 1/137.035999037435$$



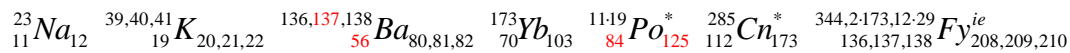
$$\alpha_1 = \frac{137}{29 \cdot 8 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 17 \cdot 29 + 1}\right)} \frac{1}{103 + \frac{1}{7 \cdot (4 \cdot 7 \cdot 199 + 1) + \frac{4}{7}}}$$

$$= 1/137.035999037435$$



$$\alpha_1 = \frac{137}{29 \cdot \left(6 + \sum_{n=1}^7 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)}\right)} \frac{1}{103 + \frac{1}{3 \cdot 19} - \frac{1}{125 \cdot (8 \cdot 7 \cdot 11 + 1) + \frac{1}{4}}}$$

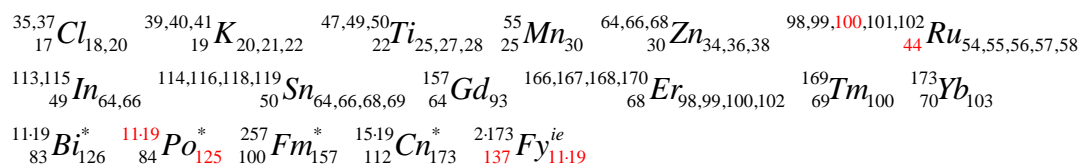
$$= 1/137.035999037435$$



2021/2/26

$$\alpha_2 = \frac{25 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2500}{3 \cdot 49 \cdot 17}\right)^{4999}}}{11 \cdot 19} \frac{1}{103 - \frac{1}{32 \cdot (512 \cdot 25 - 1) + \frac{3}{10}}}$$

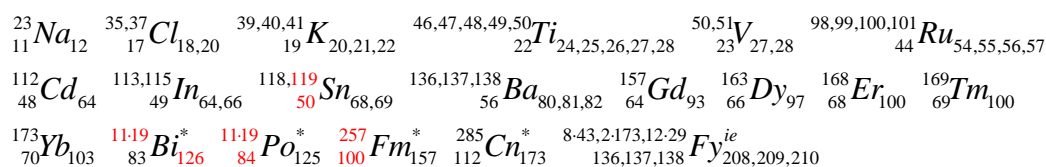
$$= 1/137.035999111818$$



2021/2/25

$$\alpha_2 = \frac{25 \cdot 4 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{7496}{7497} \frac{2 \cdot 23 \cdot 163}{9 \cdot 49 \cdot 17}\right)}{11 \cdot 19} \frac{1}{103 - \frac{1}{7 \cdot (16 \cdot 3 \cdot 19 \cdot 257 - 1)}}$$

$$= 1/137.035999111818$$



$$\alpha_2 = \frac{25 \cdot 8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{5 \cdot 23 \cdot 83})}{11 \cdot 19} \frac{1}{103 - \frac{1}{2 \cdot 3 \cdot 5 \cdot 13 \cdot 23 \cdot (2 \cdot 11 \cdot 17 - 1)}}$$

$$= 1/137.035999111818$$

$$^{23}_{11}\text{Na}_{12} \quad ^{27}_{13}\text{Al}_{14} \quad ^{35,37}_{17}\text{Cl}_{18,20} \quad ^{39,40,41}_{19}\text{K}_{20,21,22} \quad ^{50,51}_{23}\text{V}_{27,28} \quad ^{55}_{25}\text{Mn}_{30} \quad ^{168}_{68}\text{Er}_{100} \quad ^{169}_{69}\text{Tm}_{100} \quad ^{11-19}_{83}\text{Bi}_{126}^*$$

$$\alpha_2 = \frac{25 \cdot (6 + \sum_{n=1}^{11} \frac{(-1)^{n+1}}{n(n+1/2)(n+1)})}{11 \cdot 19} \frac{1}{103 - \frac{1}{4 \cdot 5 \cdot 23} + \frac{1}{2 \cdot (2 \cdot 3 \cdot 5 \cdot 7 + 1) \cdot (2 \cdot 9 \cdot 17 + 1) + \frac{2}{3}}}$$

$$= 1/137.035999111818$$

$$^{23}_{11}\text{Na}_{12} \quad ^{35,37}_{17}\text{Cl}_{18,20} \quad ^{39,40,41}_{19}\text{K}_{20,21,22} \quad ^{50,51}_{23}\text{V}_{27,28} \quad ^{55}_{25}\text{Mn}_{30} \quad ^{103}_{45}\text{Rh}_{58} \quad ^{136,137,138}_{56}\text{Ba}_{80,81,82} \quad ^{173}_{70}\text{Yb}_{103}$$

$$^{11-19}_{83}\text{Bi}_{126}^* \quad ^{11-19}_{84}\text{Po}_{125}^* \quad ^{210}_{85}\text{At}_{125}^* \quad ^{238}_{92}\text{U}_{143}^* \quad ^{15-19}_{112}\text{Cn}_{173}^* \quad ^{344,2173,348}_{136,137,138}\text{Fy}_{208,209,210}^{ie}$$

2021/2/26

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Appendix I: Research History

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2	3-6	2021/2/2-3	Chengdu
		2021/2/8	Chengdu
		2021/2/19-20	Hanyuan
3	6-7	2021/2/7	Chengdu
	7	2021/2/9	Chengdu
4	7-8	2021/2/11	Chengdu – Hanyuan
		2021/2/17,21	Hanyuan
5	8-9	2021/2/25	Chengdu
6	9-11	2021/2/25-26	Chengdu
Preparing this paper	1-12	2021/1/31-2/26	

Note: Time was recorded according to Beijing Time.