Quantum Metrology: The Hubble Constant

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Abstract

The international System of Measurements (SI) gathers a small number of fundamental units that define certain physical quantities.In this article we intend to make a discretization of two of the SI units , the meter (length measurement) and the kilogram (mass measurement).We will apply the value obtained from this discretization to a controversial topic in Cosmology: the Hubble constant.The term *quantum* in the title of the article has been used as a synonym for minimum quantity or smallest value of a specific physical system.The value we have obtained for the Hubble constant is $\sim 75.200\ m\ s^{-1}/M_{pc}$.

Keywords. *International system of units.Discrete metrology.Hubble constant.*

Introduction

The international system of units (SI) consists of a small set of quantities or magnitudes to each of which a unit is assigned.
For example, the meter is used for the length quantity, the second for the time quantity and the kilogram for the mass.
Our work has consisted in looking for a way to discretize some of the SI units.
Specifically, the unit of length, the meter and the unit of mass or kilogram.
As for the term *quantum* that appears in the title of this article, it alludes to the primordial, etymological concept of a minimum indivisible quantity, whether of matter, energy, etc.
How to *discretize* SI units?
We have been lucky to find a formula that will be very useful for that purpose.
To our surprise, the value given by said formula is equal to twice the value of the so-called Avogadro's number.
Avogadro's number [2] is used in Chemistry and represents the number of atoms or molecules in a mole of any substance.

The Italian physicist Amadeo Avogadro proposed for the first time, at the beginning of the 19th century, that a volume of any gas contains, in principle and at a specific temperature and pressure, the same number of particles, regardless of the nature of the gas.
Almost a century later, the French physicist Jean Perrin [3] managed to experimentally calculate the precise value of Avogadro's number.
The value he obtained was $6.023\ x\ 10^{23}\ \ mol^{-1}$
Avogadro's number tells us how many particles, atoms, or molecules there are in a mole of a certain substance.
We are looking for a discrete metrology, derived from the international measurement system (SI).
For this, we have searched for a parameter that may be useful when formulating a discretization of some of the SI units.

Method and results

The mathematical method used is purely arithmetic.
Some time ago we were looking for some kind of numerical parameter that could be useful when analyzing the physical units of dimension length from a *discrete* point of view.
And we focussed our attention on two physical constants of length.Namely: the Bohr radius and the Planck length.

The Bohr radius [4] in the hydrogen atom, which is the longitudinal measure of the radius of the hydrogen atom's orbital at its fundamental energy level.
The Planck length [5] as a limit of magnitude length, below which Physics as such is meaningless.
Therefore the first thing we did was divide both constants.
The result is orders of magnitude of 10^{24} .Which was intriguing, as we quickly remembered that Avogadro's number was on an order of magnitude of 10^{23} .
The next thing we did was multiply the Planck length by Euler's constant $e = 2.71828...$, which is the base of natural logarithms [6] , with the intention of making the Planck length *grow* exponentially.
In summary, when calculating it we saw that

$$\frac{a_0}{e\, l_P} = 1.2045\ x\ 10^{24} \tag{1}$$

And it was immediately evident that , by dividing this result by two , the figure obtained was equal to Avogadro's number.
We have symbolized that like this

$$(N...) = \frac{a_0}{2\, e\, l_P} = 6.0225\ x\ 10^{23} \tag{2}$$

Parameter $(N...)$ is a dimensionless parameter, since it is a division between two quantities that have identical units of length.
In this way, this simple and suggestive formula encouraged us to try to introduce it into various physical equations.
But first we are going to define several parameters derived directly from the dimensionless parameter $(N...)$
Let's divide the unit of length in the international system, one meter , by the dimensionless parameter $(N...)$.
We denote this discrete unit of length with the symbol l_u .

$$l_u = \frac{1m}{(N...)} = 1.66044 \, x \, 10^{-24} m \qquad (3)$$

Now let's consider a system with a quantity of matter within a spherical volume.
The volume of a sphere [7] is equal to $\frac{4}{3}\pi r^3$
Let us apply equation (3) and substitute r for l_u in such a way that

$$V_u = \frac{4}{3}\pi \, l_u^3 = 1.9176 \, x \, 10^{-71} \, m^3 \qquad (4)$$

The density value of a substance , ρ , is calculated by dividing the mass by the volume.
Regarding the value of the mass, we follow the same criteria as with the length.

Namely: we divide the unit of mass of the international system, kilogram, by the metrological discretization parameter $(N...)$

$$m_u = \frac{1kg}{(N...)} = 1.66044\ x\ 10^{-24} kg \tag{5}$$

therefore

$$\rho_u = \frac{m_u}{V_u} = 8.66\ x\ 10^{46}\ kg\ m^{-3} \tag{6}$$

Analogously to how if we increase the temperature of a gas there is an increase in volume , we will assume that a force operates in the system under consideration that expands the spatial volume of the system.
We can study this phenomenon arithmetically.

<u>Hubble constant</u>

According to the standard model of Cosmology, confirmed by a diverse set of observations, the universe is expanding [8].
It is actually space itself, the one that contains galaxies, stars, and planets, that is expanding.
How big is the expansion rate?
Various techniques have been used for some time to measure the value of the expansion rate.
The Hubble constant [9] represents, simply, the speed at which the universe is expanding.

Various instrumental methods have been used to determine its value.
One of these methods uses **Ia** supernova redshift data from distant galaxies [10].
And the other method is based on data from the cosmic microwave background [11].
There are significant discrepancies and tensions between the values obtained by each of the two methods.
The Hubble constant H_0 is measured in kilometers per second per megaparsec.
A megaparsec [12], is an astronomical measure of longitude

$$M_{pc} = 3.0857\, x\, 10^{22} m$$

Will apply our discrete metrology to the ideas above.
Let's first calculate the scale factor ϖ of the system, or observable universe that we are considering, whose units are given in s^{-2} using the following equation

$$\varpi = \frac{H_0^2}{M_{pc}^2} = G_N \rho_u \frac{n}{d} \tag{7}$$

H_0 symbolizes the current value of the Hubble constant that we will obtain by applying the discrete metrology mentioned above.
And $M_{pc} = 3.0857\, x\, 10^{22} m$ the astronomical magnitude of a megaparsec.

$G_N = 6.674\ x\ 10^{-11} m^3 kg^{-1} s^{-2}$ is the value of Newton 's constant of gravitation [13].
Regarding the parameter $n \sim 10^{78}$ it symbolizes the number of discretized particles of mass m_u that we will introduce into the universe that we are studying.

So by multiplying V_u by a parameter that we call d, we will get the size of the observable universe model, at a given time , once expansion has taken place over a certain time interval.
Current cosmology estimates the age of our universe [14] at about $t_0 \sim 10^{10}\, yr$, ten giga years ; or in units of time of the international system, the second , $t_0 \sim 10^{17} s$
Let's define d as

$$d = e^{3(t_0\ c\ \triangle x)} \tag{8}$$

factor 3 means that expansion occurs along all three dimensions of space.

$t_0 \sim 3.84\ x\ 10^{17} s$ represents the time elapsed since the beginning of the expansion.

$c = 299792458\ m\ s^{-1}$ is the speed of light in vacuum.

$\triangle x$ represents a *scalar field constant* that computes arithmetically according to the discrete metrology that inspires our work.
Its dimension is given by L^{-1} and the units of this parameter in the international system are the reciprocal of the length dimension , m^{-1} .
Its estimated value is

$$\triangle x \sim \frac{1}{10^{24} m}$$

which has the same order of magnitude as the metrological quantum l_u , totally congruent with the arithmetic method of discrete metrology that we have proposed and applied.

Once the pertinent calculations have been made, we can deduce from equation (7) the approximate value of the Hubble constant in a model of the observable universe that is expanding in time.

The value that we have obtained by applying arithmetics of discrete metrology is : $H_0 \sim 75.180\ m\ s^{-1} / M_{pc}$.

Discussion

The value that we have obtained for the Hubble constant in an observable universe-model system is very close to that obtained by measuring the redshift of supernovae **Ia** from distant galaxies.
The result of these measurements gives a value for the Hubble constant of about $74\ km\ s^{-1}\ /M_{pc}$ [15].
Equation (7) with which we study the scale factor incorporates several terms that make it possible to obtain a plausible numerical value for the Hubble constant.
The main thing is the use of a discrete metrology.A discrete metrology that we have applied to both the length dimension (L) and the mass dimension (M).
On the other hand, we have introduced an arithmetic term that computes as an expansion factor over time. This is the parameter d.
The exponential that defines the parameter d incorporates the three dimensions of space, the time variable, the constant of the speed of light in vacuum and a constant Δx which computes as the constant value of a discrete metrology scalar field, whose dimension is L^{-1} ,its unit is m^{-1} and its value, according to discrete metrology of the international system of units that we have used, computes as $\sim 10^{-24} m^{-1}$.

We have verified that a very slight arithmetic variation in the terms that define the exponential of the parameter d can yield different values for the Hubble constant.
Once the value of $\triangle x$ is set to $\sim 10^{-24} m^{-1}$, the variable time t_0 will determine the value of H_0.

Conclusion

A discrete metrology of the international system of units has been investigated.
We have applied this discretization to a system-model of the observable universe described by an equation from which the value of the Hubble constant is derived.
The value obtained is about $\sim 75.200\ m\ s^{-1}/M_{pc}$.
This value is in strong agreement with the experimental value obtained in various astrophysical observations.
The use of a discrete metrology of the international measurement system, applied arithmetically in various concepts of physics, can bring us curious results.

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