Derivation of general Doppler effect equations (II)

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February 6, 2021

Abstract

In the manuscript [1] we derived the general Doppler effect equations. In order to prove the correctness of the equations, it remains to define an adequate coordinate system. We have argued that such a coordinate system cannot be chosen arbitrarily but is determined by the direction between the receiver at the time when the signal is received and the sender at the time when the signal is emitted. In this manuscript, several experiments have been proposed to prove the existence of such a coordinate system. In addition, we will determine the velocities at which the sender and receiver of the signal move and the distance between them.

Keywords: *Doppler effect, stellar distance, stellar velocity, absolute velocity*

1. Introduction

Suppose that at time τ signal has been emitted from sender Z and at time t is received by the observer A, Fig. (1). The coordinate system of the telescope noted by (T) at the time t is defined in the following way. The origin is determined by point A and direction AZ represents its positive $z-axis$. The positive x and y axes will be chosen so that (x, y, z) forms a right-handed Cartesian coordinate system. The paper [2] gives a much more detailed description of the coordinate system (T). The sender Z and receiver A of the signal move uniformly at velocities $\mathbf{u}$ and $\mathbf{v}$, respectively, regarding to the coordinate system (T).
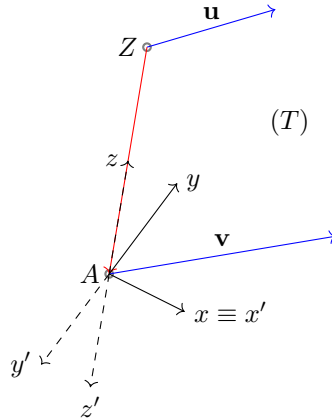

Figure 1: The sender Z and the observer A move uniformly with respect to the coordinate system (T)

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Referring to Fig. (1) we have that

$$\mathbf{a} = \frac{\mathbf{AZ}}{||\mathbf{AZ}||} \tag{1}$$

$$\mathbf{a} = [a_x, a_y, a_z] = [0, 0, 1] \tag{2}$$

$$\mathbf{u} = [u_x, u_y, u_z] \ \wedge \ || \ \mathbf{u} \ || < c \tag{3}$$

$$\mathbf{v} = [v_x, v_y, v_z] \ \wedge \ || \ \mathbf{v} \ || < c \tag{4}$$

$$\mathbf{a} \cdot \mathbf{u} = a_x * u_x + a_y * u_y + a_z * u_z = u_z \tag{5}$$

$$\mathbf{a} \cdot \mathbf{v} = a_x * v_x + a_y * v_y + a_z * v_z = v_z \tag{6}$$

We already found [1] that:

$$\lambda' \approx \frac{c + \mathbf{a} \cdot \mathbf{u}}{c + \mathbf{a} \cdot \mathbf{v}} * \lambda \tag{7}$$

Where λ is wavelength of the emitted light, λ' is the wavelength we observe and c is constant speed of light in vacuum.

It is easy to prove that if instead of the direction AZ, we chose the direction ZA for the positive $z - axis$, formula (7) would have the same form.

Suppose the (xyz) coordinate system is rotated around its x axis through an angle Π. The $x - axis$ is unchanged, the positive $y - axis$ is rotated into $y' - axis$ and the positive $z - axis$ is rotated into $z' - axis$. For example, a vector $\mathbf{v}$ described in coordinate system (T) as $\mathbf{v} = [v_x, v_y, v_z]$ would be written regarding the new coordinate system (x', y', z') as $\mathbf{v} = [v_x, -v_y, -v_z]$.

Now it follows that

$$\mathbf{v} \cdot \mathbf{a} = [v_x, v_y, v_z] \cdot [0, 0, 1] = v_z * 1 \tag{8}$$

$$\mathbf{v} \cdot \mathbf{a} = [v_x, -v_y, -v_z] \cdot [0, 0, -1] = (-v_z) * (-1) = v_z * 1 \tag{9}$$

It is obvious that in the equations (8) and (9) one can substitute the velocity $\mathbf{v}$ by the velocity $\mathbf{u}$

Making a certain error we have the following equations:

$$\lambda' = \frac{c + \mathbf{a} \cdot \mathbf{u}}{c + \mathbf{a} \cdot \mathbf{v}} * \lambda \tag{10}$$

$$\frac{\lambda'}{\lambda} * \mathbf{a} \cdot \mathbf{v} = -\frac{\lambda' - \lambda}{\lambda} * c + \mathbf{a} \cdot \mathbf{u} \tag{11}$$

$$z = \frac{\lambda' - \lambda}{\lambda} \tag{12}$$

$$\frac{\lambda'}{\lambda} * \mathbf{a} \cdot \mathbf{v} = -z * c + \mathbf{a} \cdot \mathbf{u} \tag{13}$$

In Equation (13) velocities $\mathbf{u}$ and velocities $\mathbf{v}$ are unknown while the vector $\mathbf{a}$ and the wavelengths λ and λ' are obtained by direct measurement.

2.   The stationary coordinate system (K)

The coordinate system (T) Fig. (1) is temporary because we only need it to perform the necessary measurements in a short period of time. Therefore, it is necessary to define a fixed coordinate system, denoted by (K). After that, we will transform the coordinates from the coordinate system (T) into the coordinate system (K). The way how to measure the direction AZ by eliminating the error caused by stellar aberration and how to transform coordinates from one coordinate system to another is given in [2]. We will first give a description of the coordinate system (K).
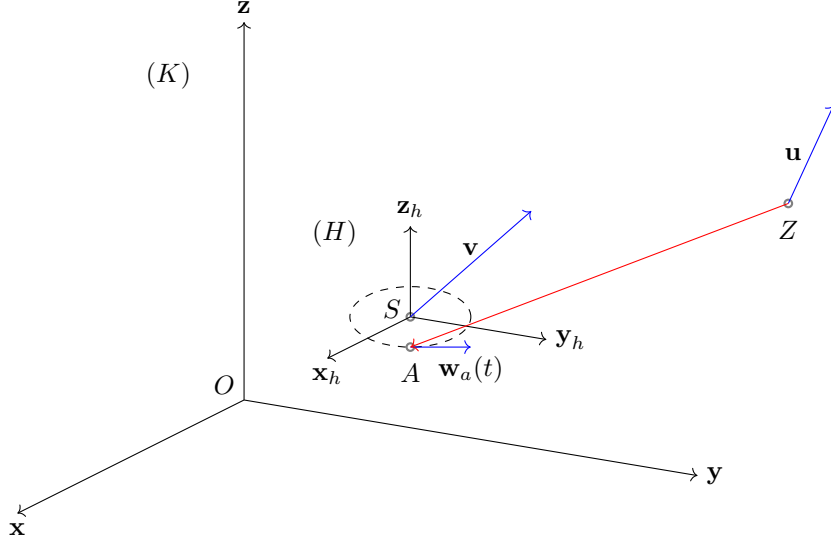
Figure 2: The ecliptic coordinate system (H) and the stationary coordinate system (K)

Let (H) denote the eliptic coordinate system. The origin S is at barycenter of the solar system, the plane of reference is the ecliptic plane and coordinate axes are determined in accordance with the $(ICRS)$ standard Fig. (2). The $ICRS$ coordinates are approximately the same as equatorial coordinates, therefore it is necessary to make transformation from the equatorial into ecliptic coordinates. The sender and receiver of the signal are denoted by Z and A respectively.

The sender Z and receiver A move uniformly at velocities $\mathbf{u}$ and $\mathbf{v} + \mathbf{w}_a(t)$ respectively, with respect to the coordinate system (T), where $\mathbf{v}$ denotes the velocity at which the solar system moves relative to the (T). The velocity $\mathbf{w}_a(t)$ consists of two velocities. The first component is the velocity at which the Earth moves relative to the barycenter S at the time t and second one is the velocity at which observer moves relative to Earth-centered inertial (ECI) coordinate system (its origin is at the center of mass of Earth and coordinate axes are fixed with respect to the stars).

The coordinate axes $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$ of the coordinate system (K) are defined in the same way as the coordinate axes $\mathbf{x}_h, \mathbf{y}_h$ and $\mathbf{z}_h$ of the coordinate system (H) respectively. We assume that the coordinate system (K) is fixed (stationary) regarding the coordinate system (T). The origin O of the (K) is not uniquely determined.

Therefore the coordinate system (K) is defined so that the solar system and the sender Z move uniformly at velocities $\mathbf{v}$ and $\mathbf{u}$ respectively, regarding the (K). In order to determine the origin O of the (K) it is necessary to determine the velocity $\mathbf{v}$ and define some initial time.

3. Determining the absolute velocity v

Suppose that from a point denoted by A' we observe some arbitrarily chosen star Z , whose position is marked by Z' Fig. (3). The unit vector determined by the direction $A'Z'$ regarding the coordinate system (H) is denoted by $\mathbf{a}'$, and the wavelength of the signal sent by Z' and measured at point A' is denoted by λ'_a. The velocity at which the receiver of the signal moves with respect to the barycenter S' is denoted by $\mathbf{w}'_a$. Suppose we performed the same measurement six months after that. The unit vector denoted by $\mathbf{a}''$ is determined by the direction $A''Z''$ regarding the coordinate system (H) , the wavelength of the signal sent by Z'' and measured at point A'' is denoted by λ''_a and the velocity at which the receiver of the signal moves with respect to the barycenter S'' is denoted by $\mathbf{w}''_a$.
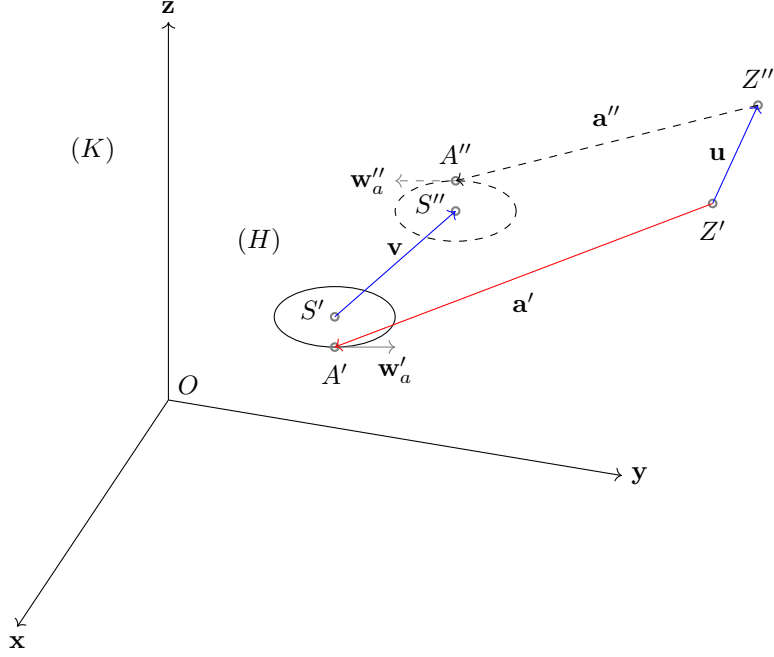
Figure 3: The solar system moves uniformly with velocity $\mathbf{v}$ regarding the stationary coordinate system (K)

Employing (13) and (12) one finds that:

$$\frac{\lambda_a'}{\lambda_a} * \mathbf{a}' \cdot (\mathbf{v} + \mathbf{w}_a') = -z_a' * c + \mathbf{a}' \cdot \mathbf{u} \tag{14}$$

$$\frac{\lambda_a''}{\lambda_a} * \mathbf{a}'' \cdot (\mathbf{v} + \mathbf{w}_a'') = -z_a'' * c + \mathbf{a}'' \cdot \mathbf{u} \tag{15}$$

$$\frac{\lambda_a'}{\lambda_a} * \mathbf{a}' \cdot \mathbf{v} = -\left(\frac{\lambda_a'}{\lambda_a} * \mathbf{a}' \cdot \mathbf{w}_a' + z_a' * c\right) + \mathbf{a}' \cdot \mathbf{u} \tag{16}$$

$$\frac{\lambda_a''}{\lambda_a} * \mathbf{a}'' \cdot \mathbf{v} = -\left(\frac{\lambda_a''}{\lambda_a} * \mathbf{a}'' \cdot \mathbf{w}_a'' + z_a'' * c\right) + \mathbf{a}'' \cdot \mathbf{u} \tag{17}$$

$$\frac{\lambda_a'' * \mathbf{a}'' - \lambda_a' * \mathbf{a}'}{\lambda_a} \cdot \mathbf{v} = -\left(\frac{\lambda_a'' * \mathbf{a}'' \cdot \mathbf{w}_a'' - \lambda_a' * \mathbf{a}' \cdot \mathbf{w}_a' + (\lambda_a'' - \lambda_a')c}{\lambda_a}\right) + (\mathbf{a}'' - \mathbf{a}') \cdot \mathbf{u} \tag{18}$$

In general, we have that:

$$\| (\mathbf{a}'' - \mathbf{a}') \cdot \mathbf{u} \| \leq \| \mathbf{a}'' - \mathbf{a}' \| * \| \mathbf{u} \| < \| \mathbf{a}'' - \mathbf{a}' \| c \tag{19}$$

The star Z should be chosen so that it lies as close as possible to the eliptic plane , and that $\| (\mathbf{a}'' - \mathbf{a}' \| \to 0$. The following inequality must hold:

$$\left| \frac{\lambda_a'' * \mathbf{a}'' \cdot \mathbf{w}_a'' - \lambda_a' * \mathbf{a}' \cdot \mathbf{w}_a' + (\lambda_a'' - \lambda_a')c}{\lambda_a} \right| >> \| \mathbf{a}'' - \mathbf{a}' \| c \tag{20}$$

One can make the following substitutions:

$$k_a' = \frac{\lambda_a'}{\lambda_a} \tag{21}$$

$$k_a'' = \frac{\lambda_a''}{\lambda_a} \tag{22}$$

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By making a certain error, Equation (18) can be written as follows:

$$(k_a'' * \mathbf{a}'' - k_a' * \mathbf{a}') \cdot \mathbf{v} = -k_a'' * \mathbf{a}'' \cdot \mathbf{w}_a'' + k_a' * \mathbf{a}' \cdot \mathbf{w}_a' - (k_a'' - k_a')c \tag{23}$$

We can repeat the whole procedure by observing some two other arbitrarily chosen stars, and thus obtain the following equations:

$$(k_b'' * \mathbf{b}'' - k_b' * \mathbf{b}') \cdot \mathbf{v} = -k_b'' * \mathbf{b}'' \cdot \mathbf{w}_b'' + k_b' * \mathbf{b}' \cdot \mathbf{w}_b' - (k_b'' - k_b')c \tag{24}$$

$$(k_c'' * \mathbf{c}'' - k_c' * \mathbf{c}') \cdot \mathbf{v} = -k_c'' * \mathbf{c}'' \cdot \mathbf{w}_c'' + k_c' * \mathbf{c}' \cdot \mathbf{w}_c' - (k_c'' - k_c')c \tag{25}$$

Equations (23),(24) and (25) can be written in the following way:

$$(k_a''a_x'' - k_a'a_x') * v_x + (k_a''a_y'' - k_a'a_y') * v_y + (k_a''a_z'' - k_a'a_z') * v_z = -k_a'' * \mathbf{a}'' \cdot \mathbf{w}_a'' + k_a' * \mathbf{a}' \cdot \mathbf{w}_a' - (k_a'' - k_a')c \tag{26}$$

$$(k_b''b_x'' - k_b'b_x') * v_x + (k_b''b_y'' - k_b'b_y') * v_y + (k_b''b_z'' - k_b'b_z') * v_z = -k_b'' * \mathbf{b}'' \cdot \mathbf{w}_b'' + k_b' * \mathbf{b}' \cdot \mathbf{w}_b' - (k_b'' - k_b')c \tag{27}$$

$$(k_c''c_x'' - k_c'c_x') * v_x + (k_c''c_y'' - k_c'c_y') * v_y + (k_c''c_z'' - k_c'c_z') * v_z = -k_c'' * \mathbf{c}'' \cdot \mathbf{w}_c'' + k_c' * \mathbf{c}' \cdot \mathbf{w}_c' - (k_c'' - k_c')c \tag{28}$$

In this way we have got a system of three linear equations with three unknowns v_x, v_y and v_z, which can be easily solved.

Suppose that by selecting three different stars we repeated the whole procedure n times. We will get a sequence noted by υ of n velocities $\upsilon = \{\mathbf{v}_1, \mathbf{v}_2, .., \mathbf{v}_n\}$. Let $\mathbf{V}$ denote the arithmetic mean of this sequence.

One can consider the following cases:

1° $\mathbf{V} = \mathbf{0}$
The coordinate system (K) is identical to the coordinate system (H) .

2° $\mathbf{V} \neq \mathbf{0}$
Depending on the value of $\mathbf{V}$, we will also determine the origin O of the coordinate system K. If $\mathbf{V}$ is equal to the velocity at which the solar system moves relative to the center of the galaxy, then the origin O would be determined by the center of the galaxy.

3°   The sequence υ represents sequence of random numbers (velocities) . The velocity $\mathbf{V}$ cannot be determined and the whole idea should be rejected.

So far it has been assumed that all the stars we have observed are located in the Milky Way. Now we can repeat the whole procedure taking into account extra-galactic objects (the galaxy groups,the galaxy clusters,...).

One possibility is that as a final result we have got a sequence of k ($k \geq 1$) different velocities denoted by $\mathcal{V}$. $\mathcal{V} = \{\mathbf{V}_1, \mathbf{V}_2, .., \mathbf{V}_k\}$.

We can consider following cases:

1° $\mathbf{V}_i = \mathbf{0}$ $i = \{1, 2, ..., k\}$
This means that by applying this method we are not able to detect absolute velocity.

2° $\mathbf{V}_i \neq \mathbf{0}$ (for some i)
The sequence $\mathcal{V}$ could be be written in the following form:
$\mathcal{V} = \{\mathbf{V}_1, \mathbf{V}_2, .., \mathbf{V}_k\}$ where $||\mathbf{V}_i|| < ||\mathbf{V}_{i+1}||$. This means that we were able to detect and measure absolute velocity, which is not uniquely determined.
In this way, all objects in the universe would be divided into k ($k \geq 1$) disjoint classes, denoted by $\mathcal{K}_i$. In fact, we have proved that this statement applies only to those objects for which certain conditions are met.

Let denote the set of these classes by $\mathcal{K}$.
$\mathcal{K} = \{ \mathcal{K}_1, \mathcal{K}_2, ..., \mathcal{K}_k \}$

To each class $\mathcal{K}_i$ one can assign a corresponding stationary coordinate system denoted by K_i.
The sequences $\mathcal{V}$ and $\mathcal{K}$ can be extended as follows:

$\mathcal{V} = \{\mathbf{V}_0, \mathbf{V}_1, \mathbf{V}_2, .., \mathbf{V}_k\}$

$\mathcal{K} = \{\, \mathcal{K}_0,\, \mathcal{K}_1, \mathcal{K}_2, ..., \mathcal{K}_k\}$.

Where velocity $\mathbf{V}_0\ (= \mathbf{0})$ is determined by observing objects from the solar system. Actually, a coordinate system K_0 assigned to the class $\mathcal{K}_0$ is identical to the ecliptic coordinate system H.

4.   Determining the velocity of the sender

Suppose that from a point denoted by A we observe some arbitrarily chosen star Z ($Z \in \mathcal{K}_i$) Fig. (4). It means that coordinate system (K) $((K) \equiv (K_i))$ is defined and therefore the velocity $\mathbf{v}$ ($\mathbf{v} = \mathbf{V}_i$) at which the solar system moves in relation to (K) is known. It remains to determine the velocity $\mathbf{u}$ at which the sender moves in relation to (K).
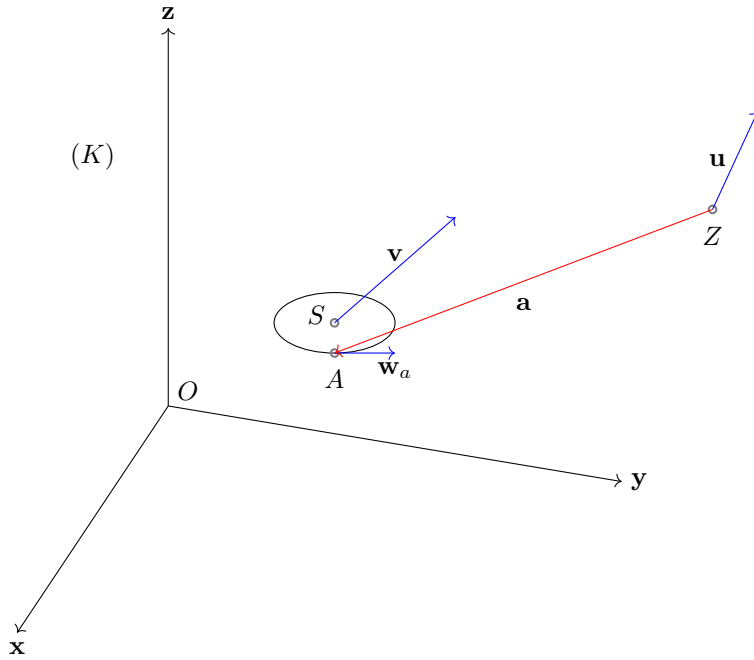

Figure 4: The sender Z moves uniformly by velocity $\mathbf{u}$ with respect to the coordinate system (K)

The unit vector determined by the direction AZ regarding the coordinate system (K) is denoted by $\mathbf{a}$, and the wavelength of the signal sent by Z and measured at point A is denoted by λ_a. The velocity at which the receiver of the signal moves with respect to the barycenter S is denoted by $\mathbf{w}_a$, Section (2).
    Employing formula (12) we have the following equations:

$$\frac{\lambda_a}{\lambda} * \mathbf{a} \cdot (\mathbf{v} + \mathbf{w}_a) = -z_a * c + \mathbf{a} \cdot \mathbf{u} \tag{29}$$

$$k_a = \frac{\lambda_a}{\lambda} \tag{30}$$

$$\mathbf{a} \cdot \mathbf{u} = k_a * \mathbf{a} \cdot (\mathbf{v} + \mathbf{w}_a) + z_a * c \tag{31}$$

Suppose we repeated the same procedure two more times. It follows that:

$$\mathbf{b} \cdot \mathbf{u} = k_b * \mathbf{b} \cdot (\mathbf{v} + \mathbf{w}_b) + z_b * c \tag{32}$$

$$\mathbf{c} \cdot \mathbf{u} = k_c * \mathbf{c} \cdot (\mathbf{v} + \mathbf{w}_c) + z_c * c \tag{33}$$

We finally got the a system of three linear equations with three unknowns u_x, u_y and u_z, which can easily be solved.

$$a_x u_x + a_y u_y + a_z u_z = k_a * \mathbf{a} \cdot (\mathbf{v} + \mathbf{w}_a) + z_a * c \tag{34}$$

$$b_x u_x + b_y u_y + b_z u_z = k_b * \mathbf{b} \cdot (\mathbf{v} + \mathbf{w}_b) + z_b * c \tag{35}$$

$$c_x u_x + c_y u_y + c_z u_z = k_c * \mathbf{c} \cdot (\mathbf{v} + \mathbf{w}_c) + z_c * c \tag{36}$$

In this way we determined the velocity $\mathbf{u}$ of the star regarding the coordinate system (K).

5. Determining the distance between the sender and the receiver

Suppose we observe some arbitrarily chosen cosmic object denoted by Z ($Z \in \mathcal{K}_i$). Therefore one can assume that the coordinate system (K) $((K) \equiv (K_i))$ is defined. The velocities $\mathbf{v}$ ($\mathbf{v} = \mathbf{V}_i$) and $\mathbf{u}$, at which solar system and cosmic object move regarding the (K) respectively, are known. It remains to find out the distance between the sender and the receiver.

Suppose that at time τ_1 a signal has been sent sent from the point marked by Z_a. The time when the signal was registered by the receiver at point A is denoted by t_1. Let the same measurement be repeated after six (eighteen, thirty, ...) months. The time when a signal has been sent from point Z_b is denoted by τ_2 and the time when the signal was received at point B is denoted by t_2.
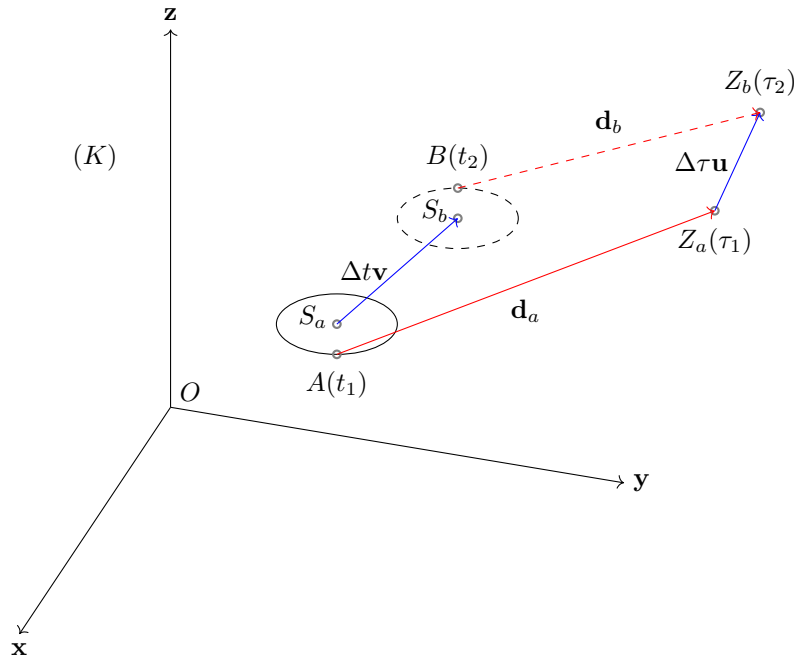

Figure 5: Distance d_a between the observer A the sender Z

We will first define three well-known constants

$$R = AU \quad (an \ astronomical \ unit) \tag{37}$$

$$c = \quad constant \ speed \ of \ light \tag{38}$$

$$year\_sec = 365.25 * 24 * 3600 \tag{39}$$

Referring to Fig. (5) one can find that

$$\Delta\tau = \tau_2 - \tau_1 \tag{40}$$

$$\Delta t = t_2 - t_1 \tag{41}$$

$$\{\Delta t = (0.5 + k) * year\_sec, \quad k = (0, 1, 2, ..)\} \tag{42}$$

$$\mathbf{v} = [v_x, v_y, v_z] \tag{43}$$

$$\mathbf{u} = [u_x, u_y, u_z] \tag{44}$$

$$\mathbf{a} = [a_x, a_y, a_z] = \frac{\mathbf{AZ}}{||\mathbf{AZ}||} \tag{45}$$

$$\mathbf{b} = [b_x, b_y, b_z] = \frac{\mathbf{BZ}}{||\mathbf{BZ}||} \tag{46}$$

$$\tag{47}$$

Let define the angle α and unit vector $\mathbf{e}$ as functions of time t:

$$\alpha(t) = 2\Pi * \frac{t}{year\_sec} \tag{48}$$

$$\mathbf{e}(t) = [\cos(\alpha), \sin(\alpha), 0] \tag{49}$$

$$A = A(t_1) = t_1\mathbf{v} + R\mathbf{e}(t_1) \tag{50}$$

$$B = t_2\mathbf{v} + R\mathbf{e}(t_2) \tag{51}$$

$$\mathbf{AB} = B - A = \Delta t\mathbf{v} + R * (\mathbf{e}(t_2) - \mathbf{e}(t_1)) \tag{52}$$

We will first determine the vectors $\mathbf{d}_a$ and $\mathbf{d}_b$

$$\mathbf{d}_a = d_a * \mathbf{a} \tag{53}$$

$$\mathbf{d}_b = \mathbf{BZ}_b = d_b * \mathbf{b} \tag{54}$$

$$Z_a = A + d_a * \mathbf{a} \tag{55}$$

$$Z_b = Z_a + \Delta\tau\mathbf{u} = A + d_a * \mathbf{a} + \Delta\tau\mathbf{u} \tag{56}$$

$$\mathbf{BZ}_b = A + d_a * \mathbf{a} + \Delta\tau\mathbf{u} - B = -(B - A) + d_a * \mathbf{a} + \Delta\tau\mathbf{u} = d_a * \mathbf{a} + (\Delta\tau\mathbf{u} - \mathbf{AB}) \tag{57}$$

$$d_b\mathbf{b} = d_a * \mathbf{a} + (\Delta\tau\mathbf{u} - \mathbf{AB}) \tag{58}$$

$$d_b^2 = d_a^2 + 2(\Delta\tau\mathbf{u} - \mathbf{AB}).\mathbf{a} * d_a + (\Delta\tau\mathbf{u} - \mathbf{AB})^2 \tag{59}$$

$$d_b^2 = d_a^2 + 2(\Delta\tau\mathbf{u}.\mathbf{a} - \mathbf{AB}.\mathbf{a}) * d_a + (\Delta\tau\mathbf{u} - \mathbf{AB})^2 \tag{60}$$

We assume that the following equation is valid:

$$\frac{d_a}{c} + \Delta t = \frac{d_b}{c} + \Delta\tau \tag{61}$$

It follows that:

$$d_b = d_a + c\Delta t - c\Delta\tau = d_a + (c\Delta t - c\Delta\tau) \tag{62}$$

$$d_b^2 = d_a^2 + 2(c\Delta t - c\Delta\tau) * d_a + (c\Delta t - c\Delta\tau)^2 \tag{63}$$

From Equations (60) and (63) one finds that

$$2(\Delta\tau\mathbf{u}.\mathbf{a} - \mathbf{AB}.\mathbf{a}) * d_a + (\Delta\tau\mathbf{u} - \mathbf{AB})^2 = 2(c\Delta t - c\Delta\tau) * d_a + (c\Delta t - c\Delta\tau)^2 \tag{64}$$

$$d_a = d(t_1) = \frac{(c\Delta t - c\Delta\tau)^2 - (\Delta\tau\mathbf{u} - \mathbf{AB})^2}{2\left((\Delta\tau\mathbf{u}.\mathbf{a} - \mathbf{AB}.\mathbf{a}) - c(\Delta t - \Delta\tau)\right)} \tag{65}$$

On the right-hand side of the equation (65) all variables are known except $\Delta\tau$.

Suppose that the interval Δt is divided by n intervals Δt_i and that we have assigned to each of these intervals the frequency f_i. Applying the formula derived in [1] we have following equations:

$$\Delta t = \sum_{i=1}^{n} \Delta t_i \tag{66}$$

$$\Delta \tau = \sum_{i=1}^{n} \Delta \tau_i = \sum_{i=1}^{n} \frac{f^{(i)}}{f} \Delta t_i = \frac{1}{f} \sum_{i=1}^{n} f^{(i)} \Delta t_i \tag{67}$$

Where f is source frequency of light and f_i frequency measured by the observer. In this way we obtained all the necessary parameters needed to determine the distance between the receiver and sender of the signal.

6. Conclusion

All algorithms have been tested using data generated by the computer program. Unfortunately the author could not obtain real data that would be suitable to be used as input parameters (the coordinates of the observer regarding the ecliptic coordinate system, the time when the measurement was performed, wavelength shift,..). Because this is the only way to prove or disprove the correctness of the proposed methods, it would be too early to draw any conclusions at this point.

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