

# Formulas of Feigenbaum Constants and Their Physical Meanings

Gang Chen<sup>†</sup>, Tianman Chen, Tianyi Chen

*Guangzhou Huifu Research Institute Co., Ltd., Guangzhou, P. R. China*

*7-20-4, Greenwich Village, Wangjianglu 1, Chengdu, P. R. China*

<sup>†</sup>Correspondence to: gang137.chen@connect.polyu.hk

Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70<sup>th</sup> birthday

## Abstract

This paper supposes that Feigenbaum constants should be rational numbers in the world of nuclides, gives their formulas in fractional number format and exhibits the physical meanings of the factors in the formulas, especially their relationships with nuclides, the fine-structure constant and  $2\pi$ . This paper also supposes that there would be the third Feigenbaum constant and gives its two possible approximate values.

**Keywords:** Feigenbaum constants; the fine-structure constant; nuclides.

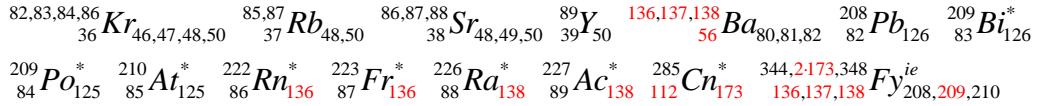
## 1. Introduction

Feigenbaum constants are characterizing constants in chaotic systems, it is assumed by scientific community that they would characterize chaos just as  $2\pi$  stands for periodicity. The nuclei of nuclides except proton should be multiple-body systems and hence chaos should be the common state in the world of nuclei. So Feigenbaum constants should express some functions in nuclides.

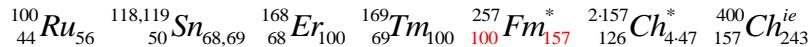
There are two Feigenbaum constants, i. e.,  $\delta=4.6692\dots$  and  $\alpha=2.5029\dots$ , and mathematically they should be irrational numbers with infinite digits.

In our previous papers<sup>1,2,3</sup>, we had already given formulas of the fine-structure constant and given two values of it, i. e.,  $\alpha_1=1/137.035999037435$  and  $\alpha_2=1/137.035999111818$  which are rational numbers with 15 digits. The values and formulas of the fine-structure constant show strong relationships with nuclides, and hereby some relevant examples of these relationships are listed as follows. And it

shows that the values of the fine-structure constant express as integer numbers of 136, 137 and 138 in the world of nuclides.



In our previous paper<sup>1,2,3</sup>, we also exhibited the relationships of  $2\pi$  with nuclides, for example,  $2\pi \approx 6.28 = (4 \times 157)/100$  relates to nuclides as follows.



In this paper, we suppose Feigenbaum constants should also be rational numbers in the world of nuclides, give their formulas and exhibit their relationships with nuclides and hence with the fine-structure constant and  $2\pi$ .

## 2. Formulas of Feigenbaum Constants in Fractional Numbers

It is supposed that Feigenbaum constants should be rational numbers with 15 digits and have the following fractional formulas and relationships with nuclides.

Note:  $136=8 \cdot 17$ ,  $138=6 \cdot 23$

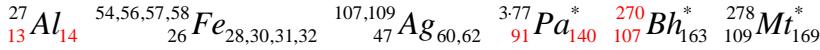
$$\begin{aligned} \frac{1}{\delta} &= \frac{1}{4.66920160910299} = 0.214169377062326 \\ &= \frac{1}{4} - \frac{1}{27} + \frac{1}{4 \cdot 9 \cdot 23} - \frac{1}{2 \cdot 3 \cdot 7 \cdot 23 \cdot (2 \cdot 3 \cdot (4 \cdot 3 \cdot 11 - 1) + 1) + \frac{2 \cdot 23}{3 \cdot 19}} \\ &\quad 9 Be_5 \quad 19 F_{10} \quad 23 Na_{12} \quad 27 Al_{14} \quad 36,38,40 Ar_{18,20,22} \quad 39,40,41 K_{20,21,22} \quad 45 Sc_{24} \quad 46,47,48,49,50 Ti_{24,25,26,27,28} \\ &\quad 23 V_{27,28} \quad 54,56,57,58 Fe_{28,30,31,32} \quad 59 Co_{32} \quad 64,66 Zn_{34,36} \quad 69,71 Ga_{38,40} \quad 75 As_{42} \quad 80,82,83,84 Kr_{44,46,47,48} \\ &\quad 84,86,88 Sr_{46,48,50} \quad 92,96,98 Mo_{50,54,56} \quad 100,101 Ru_{56,57} \quad 102,105,108,110 Pd_{56,59,62,64} \quad 119 Sn_{69} \quad 137,138 Ba_{81,82} \\ &\quad 138,139 La_{81,82} \quad 169 Tm_{100} \quad 209 Bi_{126}^* \quad 209 Po_{125}^* \quad 226 Ra_{138}^* \quad 238 U_{146}^* \quad 257 Fm_{157}^* \quad 285 Cn_{173}^* \quad 2-173,348 Fy_{209,210}^{ie} \\ \frac{1}{\alpha} &= \frac{1}{2.50290787509589} = 0.399535280523135 \\ &= \frac{1}{2} - \frac{1}{9} + \frac{1}{3 \cdot 31} - \frac{1}{23 \cdot (8 \cdot 3 \cdot 17 + 1)} + \frac{1}{17 \cdot 23 \cdot (8 \cdot 3 \cdot 11^4 - 1)} \\ &\quad 23 Na_{12} \quad 31 P_{16} \quad 35,37 Cl_{18,20} \quad 36,38,40 Ar_{18,20,22} \quad 46,48,50 Ti_{24,26,28} \quad 50,51 V_{27,28} \quad 56,57 Fe_{30,31} \quad 69,71 Ga_{38,40} \\ &\quad 64,66,68 Zn_{34,36,38} \quad 80,82,83,84 Kr_{44,46,47,48} \quad 84,86,87,88 Sr_{46,48,49,50} \quad 89 Y_{50} \quad 90,91,92 Zr_{50,51,52} \quad 93 Nb_{52} \quad 100 Ru_{56} \\ &\quad 112,114,116,118,119,120,124 Ra_{62,64,66,68,69,70,74} \quad 121,123 Sb_{70,72} \quad 136,137,138 Ba_{80,81,82} \quad 150,152 Sm_{88,90} \quad 168 Er_{100} \\ &\quad 156,157 Gd_{92,93} \quad 169 Tm_{100} \quad 201,204 Hg_{121,124} \quad 208 Pt_{126} \quad 209 Bi_{126}^* \quad 209 Po_{125}^* \quad 210 At_{125}^* \quad 222 Rn_{136}^* \quad 223 Fr_{136}^* \\ &\quad 226 Ra_{138}^* \quad 227 Ac_{138}^* \quad 14,17 U_{146}^* \quad 237 Np_{144}^* \quad 257 Fm_{157}^* \quad 285 Cn_{173}^* \quad 2-151 Ch_{181}^{ie} \quad 344,2-173,348 Fy_{208,209,210}^{ie} \end{aligned}$$

### 3. Formulas of Feigenbaum Constants in Continued Fractional Numbers

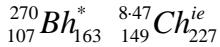
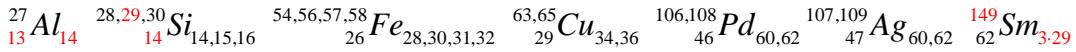
$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \cfrac{3}{14 + \cfrac{1}{131 + \cfrac{1}{2 + \cfrac{1}{54 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{1}{5 + \cfrac{1}{18}}}}}}}}$$



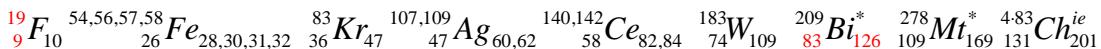
$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \cfrac{3}{14 + \cfrac{1}{131 + \cfrac{1}{2 + \cfrac{1}{54 + \cfrac{1}{6 + \cfrac{7 \cdot 13}{109}}}}}}$$



$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \cfrac{3}{14 + \cfrac{1}{131 + \cfrac{1}{2 + \cfrac{5 \cdot 149}{13 \cdot 29 \cdot 107}}}}$$



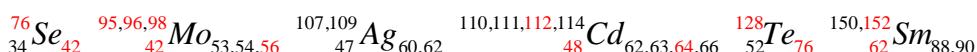
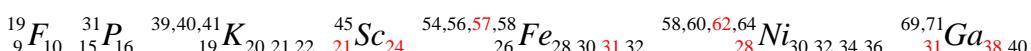
$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326 = \cfrac{3}{14 + \cfrac{9 \cdot 83 \cdot 109}{64 \cdot 7 \cdot (6 \cdot 7 \cdot (6 \cdot 5 \cdot 19 - 1) + 1)}}$$



$$\frac{1}{\delta} = \frac{1}{4.66920160910299} = 0.214169377062326$$

$$= \frac{64 \cdot 3 \cdot 7 \cdot (2 \cdot 3 \cdot 7 \cdot (2 \cdot 3 \cdot 5 \cdot 19 - 1) + 1)}{(128 - 1) \cdot (2 \cdot 3 \cdot 7 \cdot (2 \cdot 3 \cdot 7 \cdot 31 \cdot (4 \cdot 227 - 1) - 1)}$$

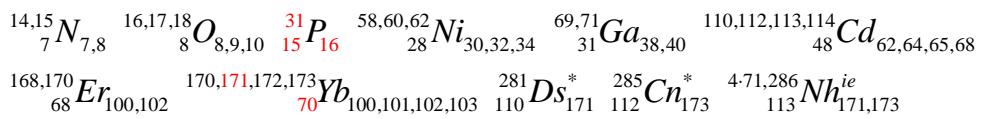
$$= \frac{64 \cdot 3 \cdot 7 \cdot (2 \cdot 3 \cdot 7 \cdot (2 \cdot 3 \cdot 5 \cdot 19 - 1) + 1)}{(128 - 1) \cdot (2 \cdot 3 \cdot 7 \cdot (2 \cdot 3 \cdot 7 \cdot 31 \cdot (2 \cdot 3 \cdot (8 \cdot 19 - 1) + 1) - 1)} = \frac{32120256}{149975951}$$



$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

$$= \cfrac{2}{5 + \cfrac{1}{171 + \cfrac{1}{1 + \cfrac{1}{17 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{15}}}}}}}}}}}}$$

$$= \cfrac{2}{5 + \cfrac{1}{171 + \cfrac{1}{1 + \cfrac{1}{17 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{7 + \cfrac{1}{1 + \cfrac{16}{31}}}}}}}}}}$$



$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

$$= \cfrac{2}{5 + \cfrac{1}{171 + \cfrac{1}{1 + \cfrac{1}{17 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{47}{8 \cdot 9 \cdot 5}}}}}}}}$$

$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

$$= \cfrac{2}{5 + \cfrac{1}{171 + \cfrac{1}{1 + \cfrac{1}{17 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{11 \cdot 37}{13 \cdot 59}}}}}}}$$

$^{23}_{11}Na_{12}$     $^{27}_{13}Al_{14}$     $^{35,37}_{17}Cl_{18,20}$     $^{46,47,48,50}_{22}Sc_{24,25,27,28}$     $^{59}_{27}Co_{32}$     $^{82,83,84}_{36}Kr_{46,47,48}$     $^{85,87}_{37}Rb_{48,50}$   
 $^{100,101,102,104}_{44}Ru_{56,57,58,60}$     $^{103}_{45}Rh_{58}$     $^{102,104,105,106,108,110}_{46}Pd_{56,58,59,60,62,64}$     $^{107,109}_{47}Ag_{60,62}$     $^{3,47}_{59}Pr_{82}$

$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

$$= \cfrac{2}{5 + \cfrac{1}{171 + \cfrac{1}{1 + \cfrac{1}{17 + \cfrac{25 \cdot (4 \cdot 5 \cdot 7 - 1)}{2 \cdot 3 \cdot 7 \cdot 101}}}}} = \cfrac{2}{5 + \cfrac{1}{171 + \cfrac{269 \cdot 281}{97 \cdot (6 \cdot 137 + 1)}}}$$

$^{35,37}_{17}Cl_{18,20}$     $^{55}_{25}Mn_{30}$     $^{54,56}_{26}Fe_{28,30}$     $^{94,96,97,98}_{42}Mo_{52,53,55,56}$     $^{137}_{56}Ba_{81}$     $^{170,171,172,173}_{70}Yb_{100,101,102,103}$   
 $^{163}_{66}Dy_{97}$     $^{247}_{97}Bk_{150}^*$     $^{258}_{101}Md_{157}^*$     $^{268}_{105}Db_{163}^*$     $^{269}_{106}Sg_{163}^*$     $^{270}_{107}Bh_{163}^*$     $^{281}_{110}Ds_{171}^*$     $^{285}_{112}Cn_{173}^*$     $^{2,173}_{137}Fy_{209}^{ie}$

$$\frac{1}{\alpha} = \frac{1}{2.50290787509589} = 0.399535280523135$$

$$= \cfrac{2}{5 + \cfrac{97 \cdot (6 \cdot 137 + 1)}{2 \cdot 5 \cdot (2 \cdot 113 + 1) \cdot (32 \cdot 27 \cdot 7 - 1)}} = \cfrac{2}{5 + \cfrac{(2 \cdot 49 - 1) \cdot (6 \cdot 137 + 1)}{2 \cdot 5 \cdot (2 \cdot 113 + 1) \cdot (32 \cdot 27 \cdot 7 - 1)}}$$

$$= \cfrac{4 \cdot 5 \cdot (2 \cdot 113 + 1) \cdot (32 \cdot 27 \cdot 7 - 1)}{9 \cdot 7 \cdot 13 \cdot 53 \cdot (2 \cdot 7 \cdot 113 + 1)} = \cfrac{27453380}{68713281}$$

$^{19}_{9}F_{10}$     $^{27}_{13}Al_{14}$     $^{50,52,53,54}_{24}Cr_{26,28,29,30}$     $^{54,56,58}_{26}Fe_{28,30,32}$     $^{59}_{27}Co_{32}$     $^{95,96,97,98}_{42}Mo_{53,54,55,56}$     $^{97}_{43}Tc_{54}^*$   
 $^{113,115}_{49}In_{64,66}$     $^{137}_{56}Ba_{81}^*$     $^{4,47,27,7}_{76}Os_{112,113}^*$     $^{2,113}_{88}Ra_{138}^*$     $^{285}_{112}Cn_{173}^*$     $^{284,286}_{113}Nh_{171,173}^{ie}$     $^{2,173}_{137}Fy_{209}^{ie}$

2021/1/22

$$\frac{1}{\alpha} = \frac{4 \cdot 5 \cdot (2 \cdot (2 \cdot 3 \cdot 19 - 1) + 1) \cdot (32 \cdot 27 \cdot 7 - 1)}{9 \cdot 7 \cdot 13 \cdot (2 \cdot 27 - 1) \cdot (2 \cdot 7 \cdot (2 \cdot 3 \cdot 19 - 1) + 1)} = \cfrac{27453380}{68713281}$$

$^{19}_{9}F_{10}$     $^{27}_{13}Al_{14}$     $^{36,38,40}_{18}Ar_{18,20,22}$     $^{39,40,41}_{19}K_{20,21,22}$     $^{50,52,53,54}_{24}Cr_{26,28,29,30}$     $^{54,56,57,58}_{26}Fe_{28,30,31,32}$   
 $^{59}_{27}Co_{32}$     $^{70,74,76}_{32}Ge_{38,42,44}$     $^{86}_{38}Sr_{48}$     $^{96,98}_{42}Mo_{54,56}$     $^{112,114}_{48}Cd_{64,66}$     $^{130,132}_{54}Xe_{76,78}$     $^{138,139}_{57}La_{81,82}$

2021/1/28

#### 4. The Third Feigenbaum Constant

There would be the third Feigenbaum constant  $\gamma$ , and we could only give some guesses at present stage as follows.

Note:  $136=8\cdot17$ ,  $138=6\cdot23$

$$\begin{aligned}\frac{1}{\alpha} &= \frac{1}{2.50290787509589} = 0.399535280523135 = f(2,3,17,23,11,\dots) \\ &= \frac{1}{2} - \frac{1}{9} + \frac{1}{3\cdot31} - \frac{1}{23\cdot(8\cdot3\cdot17+1)} + \frac{1}{17\cdot23\cdot(8\cdot3\cdot11^4-1)} \\ &= \frac{2}{5 + \frac{1}{171 + \frac{269\cdot281}{97\cdot(6\cdot137+1)}}} \\ \frac{1}{\delta} &= \frac{1}{4.66920160910299} = 0.214169377062326 = f(2,3,23,11,\dots) \\ &= \frac{1}{4} - \frac{1}{27} + \frac{1}{4\cdot9\cdot23} - \frac{1}{2\cdot3\cdot7\cdot23\cdot(2\cdot3\cdot(4\cdot3\cdot11-1)+1) + \frac{2\cdot23}{3\cdot19}} \\ &= \frac{3}{14 + \frac{1}{131 + \frac{13\cdot29\cdot107}{9\cdot83\cdot109}}} \\ \frac{1}{\gamma} &= f(2,3,17,\dots) \approx 1 - \frac{1}{3} + \frac{1}{8\cdot17} = \frac{25\cdot11}{8\cdot3\cdot17} \approx \frac{2}{3 - \frac{1}{31 + \frac{1}{2}}} = \frac{2\cdot9\cdot7}{11\cdot17} \approx \frac{1}{1.484}\end{aligned}$$

or

$$\frac{1}{\gamma} = f(2,3,17,\dots) \approx \frac{1}{8} - \frac{1}{81} + \frac{1}{8\cdot3\cdot17} = \frac{317}{2\cdot81\cdot17} \approx \frac{3}{26 + \frac{1}{15}} = \frac{9\cdot5}{17\cdot23} \approx \frac{1}{8.689}$$

2021/1/27–28

$$\frac{1}{\gamma} = f(2,3,17,\dots) \approx \frac{1}{8} - \frac{1}{101} = \frac{3\cdot31}{8\cdot101} \approx \frac{3}{26 + \frac{1}{151}} = \frac{151}{7\cdot11\cdot17} \approx \frac{1}{8.6689}$$

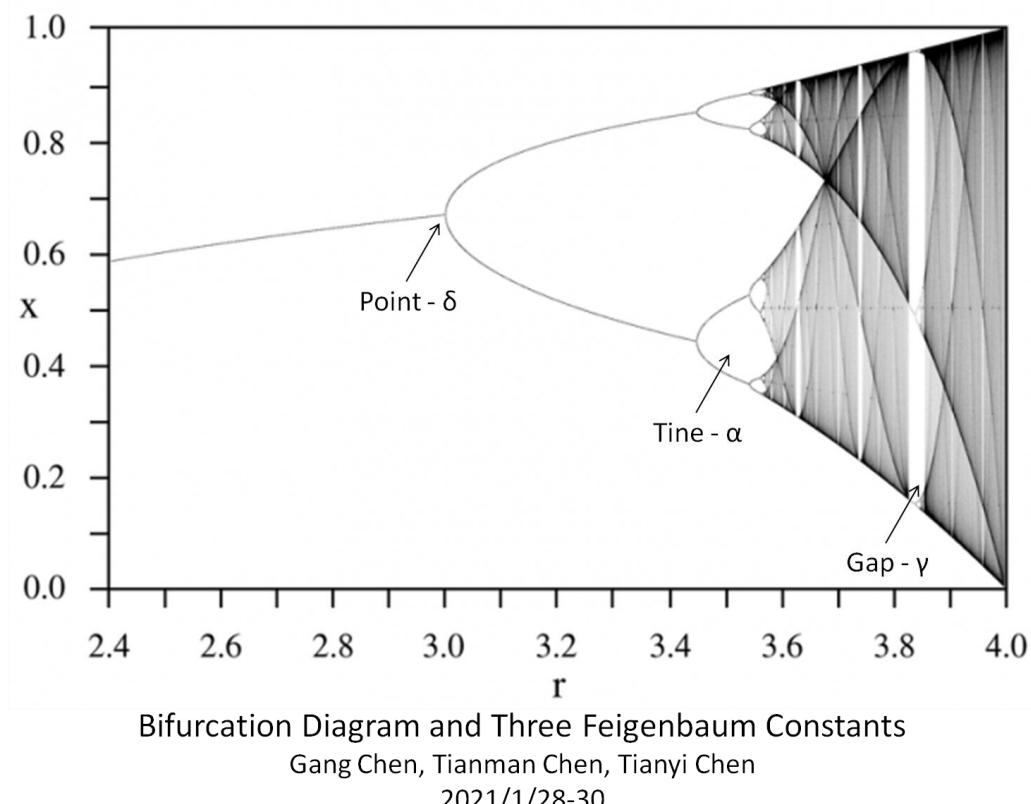
$$\frac{1}{\gamma} = f(2,3,17,\dots) \approx \frac{1}{8} - \frac{1}{103} = \frac{5\cdot19}{8\cdot103} \approx \frac{3}{26 + \frac{1}{157}} = \frac{157}{6\cdot227-1} \approx \frac{1}{8.6688}$$

2021/1/30–31

Formula of  $\alpha$  in fractional number format contains the factors of 2, 3, 17, 23 and 11, Formula of  $\delta$  in fractional number format contains the factors of 2, 3, 23 and 11, so it is supposed that there would be the third Feigenbaum constant  $\gamma$  which should relate to factors of 2, 3, 17 and 11.

## 5. Bifurcation Diagram and Feigenbaum Constants

There are three features in a typical bifurcation diagram, which are “point”, “tine” and “gap”. These three features should correspond to three Feigenbaum constants  $\delta$ ,  $\alpha$  and  $\gamma$  (**Fig. 1**), so there should be three Feigenbaum constants.



**Fig. 1**

## **References:**

1. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2002.0203.
2. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2008.0020.
3. G. Chen, T-M. Chen, T-Y. Chen. viXra e-prints, viXra:2012.0107.

## **Acknowledgements**

Yichang Huifu Silicon Material Co., Ltd., Guangzhou Huifu Research Institute Co., Ltd. and Yichang Huifu Nanometer Material Co., Ltd. have been giving Dr. Gang Chen a part-time employment since Dec. 2018. Thank these companies for their financial support. Specially thank Dr. Yuelin Wang and other colleagues of these companies for their appreciation, support and help.

Thank Prof. Wenhao Hu, the dean of School of Pharmaceutical Sciences, Sun Yet-Sen University, for providing us an apartment in Shanghai since January of 2021 and hence facilitating the process of writing this paper.

## **Appendix I: Research History**

Section	Page	Date	Remarks
2	2	2021/1/23-24	
3	3-5	2021/1/20-22,28	
4	6-7	2021/1/27-28,30-31	
5	7	2021/1/28-30	
Preparing this paper	1-8	2021/1/20-31	