Special Beta Distribution for Big Data Analysis

$X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$

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Abstract

This book discusses the special case of Beta distribution as $\alpha = \lambda + 1$ and $\beta = 2 - \lambda$. To compare with the continuous Bernoulli distribution, the change of λ affected the pdf of the special Beta distribution. Then find out the sufficient statistic, the point estimator, the confidence interval, the test statistic, and the goodness of fit. The special Beta distribution at the case of $\lambda = 0.5$ is different from the continuous Bernoulli distribution. The special Beta distribution pdf is changed in smoothing but the Continuous Bernoulli distribution pdf has a big wave when λ is from small to large. As the sample size becomes large, two distributions are approximated to Normal distribution with different relationships between λ and the sum of samples.

Contents

Chapter 1. Special case of beta distribution

This probability density function is a special case of beta distribution, that is $X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$,

$$f_X(x;\lambda) = \frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)} x^{\lambda}(1-x)^{1-\lambda}, 0 \le x \le 1, 0 < \lambda < 1$$

The random variable could be a set of probability which will follow the character of this distribution. The idea of this probability distribution is from the Continuous Bernoulli distribution. This book is about the probability distribution and the test statistic of λ.

Section1, The probability density function

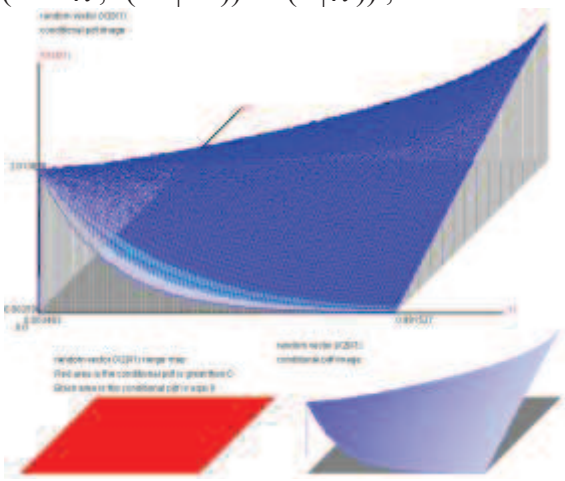
$f_X(x;\lambda) = \frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)} x^{\lambda}(1-x)^{1-\lambda}, 0 \le x \le 1, 0 < \lambda < 1,$

$F_X(x;\lambda) = \int_0^x \frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)} x^{\lambda}(1-x)^{1-\lambda}\, dx$ is the cumulative distribution function.

$E(X) = \frac{\lambda+1}{3}, Var(X) = \frac{(\lambda+1)(2-\lambda)}{36}$, λ is the shape parameter,

$E(X^k) = \frac{2}{\Gamma(\lambda+1)} \frac{\Gamma(\lambda+k+1)}{\Gamma(3+k)}, k > 0$.

The diagram of (X1= λ , f(X2|X1)) = f(X| λ)) ,



When $\lambda = 0.5, f_X(x;\lambda) = \frac{8}{\pi}\sqrt{x(1-x)} = \frac{8}{\pi}\sqrt{0.25-(x-0.5)^2}, 0 \le x \le 1,$

the semi-circle distribution, $R = 0.5, \mu = 0.5$.

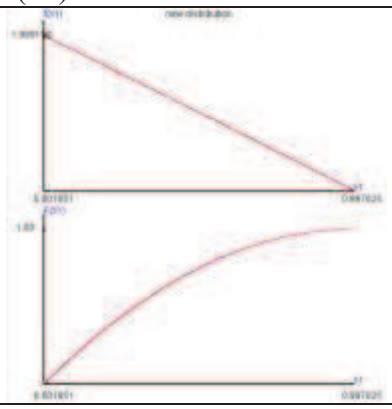
The simulator is the database which from numerical analysis.

$$f_i = f_X(x_i + \Delta x;\lambda) \times \Delta x, \Delta x \to 0, x_i = 0, \frac{1}{k}, \frac{2}{k}, ..., 1 - \frac{1}{k}, k \to \infty, \quad F_X(x_i;\lambda) \approx \frac{\sum_{i=1}^{x_i} f_i}{\sum_{i=1}^{k} f_i},$$

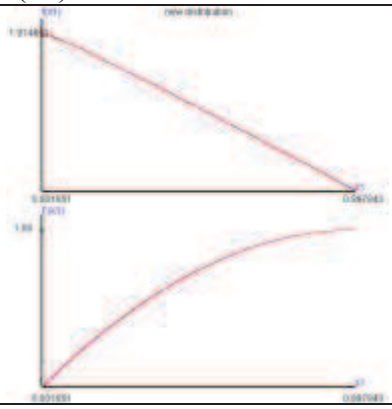
$F_X(x_i;\lambda)$=Random number $\sim Unifom(0,1)$.

Section 2, The diagram of parameter changed

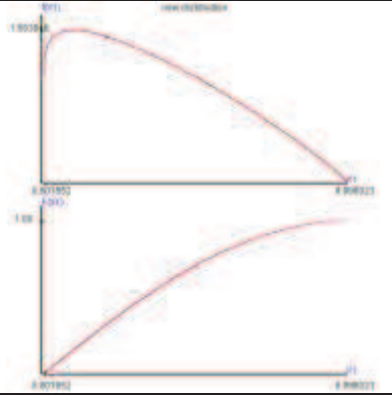
(1) λ=0.001,

f(x1),F(x1)	Ceofficient	
	Mathematical Mean:	0.33377
	Geometrical Mean :	0.22355
	Harmonic Mean :	0.02759
	Variance :	0.05560
	S.D. :	0.23580
	Skewed Coef. :	0.56372
	Kurtosis Coef. :	2.39716
	MAD :	0.19764
	Range :	0.99968
	Mid_range :	0.49984
	Median :	0.29348
	Q1 :	0.13427
	Q2 :	0.29348
	Q3 :	0.50060
	IQR :	0.36633
	C.V. :	0.70648

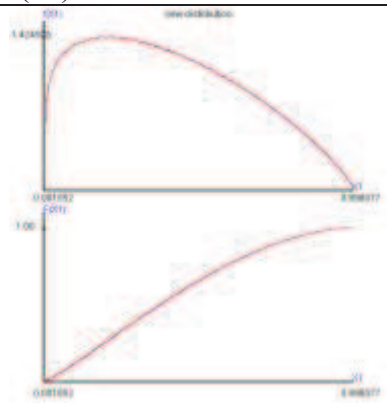
(2) λ=0.01,

f(x1),F(x1)	Ceofficient	
	Mathematical Mean:	0.33674
	Geometrical Mean :	0.22679
	Harmonic Mean :	0.03093
	Variance :	0.05585
	S.D. :	0.23633
	Skewed Coef. :	0.55265
	Kurtosis Coef. :	2.38178
	MAD :	0.19817
	Range :	0.99969
	Mid_range :	0.49985
	Median :	0.29712
	Q1 :	0.13685
	Q2 :	0.29712
	Q3 :	0.50457
	IQR :	0.36773
	C.V. :	0.70183

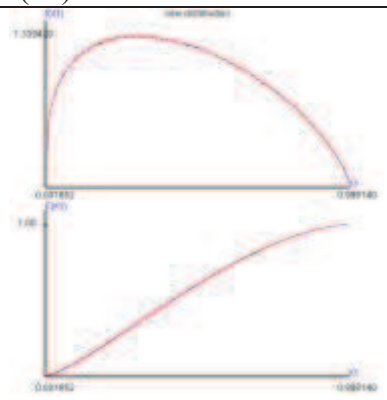
(3) λ=0.1,

f(x1),F(x1)	Ceofficient	
	Mathematical Mean:	0.36670
	Geometrical Mean :	0.26018
	Harmonic Mean :	0.05909
	Variance :	0.05807
	S.D. :	0.24098
	Skewed Coef. :	0.44275
	Kurtosis Coef. :	2.24492
	MAD :	0.20295
	Range :	0.99988
	Mid_range :	0.49994
	Median :	0.33404
	Q1 :	0.16346
	Q2 :	0.33404
	Q3 :	0.54353
	IQR :	0.38007
	C.V. :	0.65715

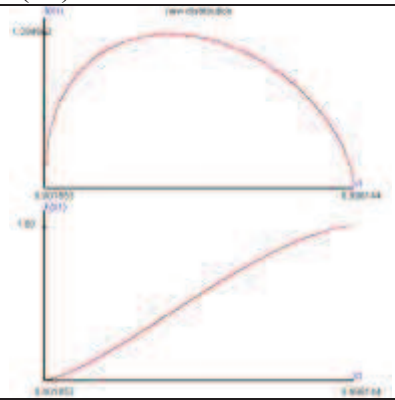
(4) λ =0.2,

f(x1),F(x1)	Ceofficient	
	Mathematical Mean:	0.40012
	Geometrical Mean :	0.29773
	Harmonic Mean :	0.10478
	Variance :	0.06003
	S.D. :	0.24501
	Skewed Coef. :	0.32658
	Kurtosis Coef. :	2.13316
	MAD :	0.20708
	Range :	0.99993
	Mid_range :	0.49996
	Median :	0.37542
	Q1 :	0.19483
	Q2 :	0.37542
	Q3 :	0.58555
	IQR :	0.39072
	C.V. :	0.61233

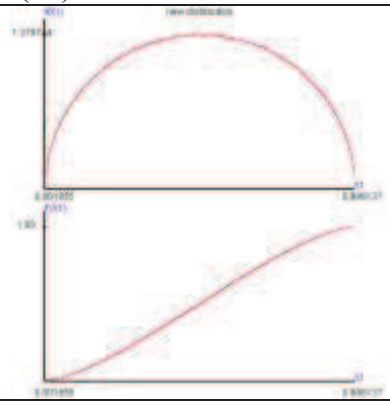
(5) λ =0.3,

f(x1),F(x1)	Ceofficient	
	Mathematical Mean:	0.43335
	Geometrical Mean :	0.33558
	Harmonic Mean :	0.15148
	Variance :	0.06139
	S.D. :	0.24778
	Skewed Coef. :	0.21517
	Kurtosis Coef. :	2.05744
	MAD :	0.20995
	Range :	0.99999
	Mid_range :	0.50000
	Median :	0.41685
	Q1 :	0.22760
	Q2 :	0.41685
	Q3 :	0.62577
	IQR :	0.39817
	C.V. :	0.57178

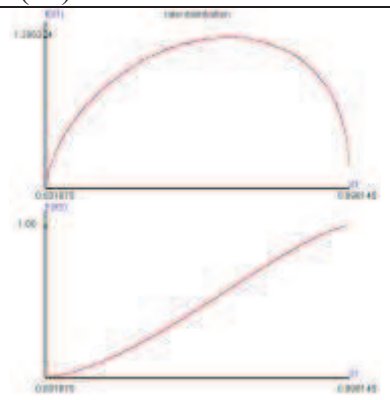
(6) λ =0.4,

f(x1),F(x1)	Ceofficient	
	Mathematical Mean:	0.46667
	Geometrical Mean :	0.37375
	Harmonic Mean :	0.19768
	Variance :	0.06222
	S.D. :	0.24943
	Skewed Coef. :	0.10650
	Kurtosis Coef. :	2.01401
	MAD :	0.21163
	Range :	0.99999
	Mid_range :	0.50000
	Median :	0.45845
	Q1 :	0.26203
	Q2 :	0.45845
	Q3 :	0.66461
	IQR :	0.40259
	C.V. :	0.53449

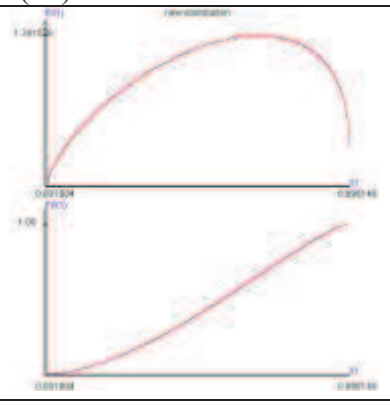
(7) λ =0.5,

f(x1),F(x1)	Ceofficient	
	Mathematical Mean:	0.50004
	Geometrical Mean :	0.41214
	Harmonic Mean :	0.24897
	Variance :	0.06253
	S.D. :	0.25007
	Skewed Coef. :	-0.00028
	Kurtosis Coef. :	1.99962
	MAD :	0.21227
	Range :	0.99998
	Mid_range :	0.50000
	Median :	0.50002
	Q1 :	0.29793
	Q2 :	0.50002
	Q3 :	0.70211
	IQR :	0.40418
	C.V. :	0.50009

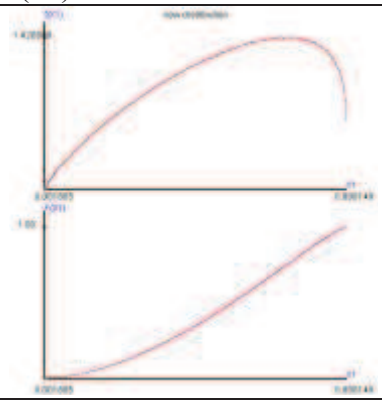
(8) λ =0.6,

f(x1),F(x1)	Ceofficient	
	Mathematical Mean:	0.53334
	Geometrical Mean :	0.45070
	Harmonic Mean :	0.29982
	Variance :	0.06226
	S.D. :	0.24951
	Skewed Coef. :	-0.10719
	Kurtosis Coef. :	2.01455
	MAD :	0.21169
	Range :	0.99997
	Mid_range :	0.50001
	Median :	0.54157
	Q1 :	0.33540
	Q2 :	0.54157
	Q3 :	0.73803
	IQR :	0.40263
	C.V. :	0.46783

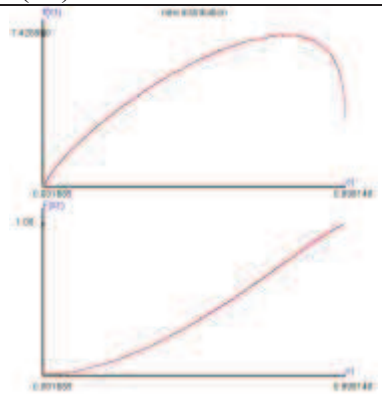
(9) λ =0.7,

f(x1),F(x1)	Ceofficient	
	Mathematical Mean:	0.56670
	Geometrical Mean :	0.48959
	Harmonic Mean :	0.34984
	Variance :	0.06139
	S.D. :	0.24778
	Skewed Coef. :	-0.21538
	Kurtosis Coef. :	2.05817
	MAD :	0.20992
	Range :	0.99995
	Mid_range :	0.50003
	Median :	0.58316
	Q1 :	0.37431
	Q2 :	0.58316
	Q3 :	0.77243
	IQR :	0.39812
	C.V. :	0.43723

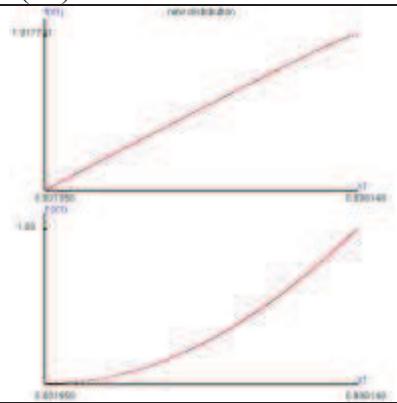
(10) λ =0.8,

f(x1),F(x1)	Ceofficient	
	Mathematical Mean:	0.60000
	Geometrical Mean :	0.52845
	Harmonic Mean :	0.39959
	Variance :	0.06001
	S.D. :	0.24496
	Skewed Coef. :	-0.32674
	Kurtosis Coef. :	2.13302
	MAD :	0.20705
	Range :	0.99997
	Mid_range :	0.50002
	Median :	0.62469
	Q1 :	0.41454
	Q2 :	0.62469
	Q3 :	0.80531
	IQR :	0.39077
	C.V. :	0.40826

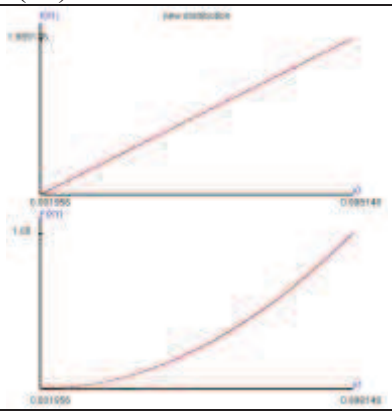
(11) λ =0.9,

f(x1),F(x1)	Ceofficient	
	Mathematical Mean:	0.63340
	Geometrical Mean :	0.56753
	Harmonic Mean :	0.45024
	Variance :	0.05806
	S.D. :	0.24095
	Skewed Coef. :	-0.44248
	Kurtosis Coef. :	2.24489
	MAD :	0.20292
	Range :	0.99994
	Mid_range :	0.50003
	Median :	0.66596
	Q1 :	0.45658
	Q2 :	0.66596
	Q3 :	0.83661
	IQR :	0.38002
	C.V. :	0.38041

(12) λ =0.99,

f(x1),F(x1)	Ceofficient	
	Mathematical Mean:	0.66339
	Geometrical Mean :	0.60271
	Harmonic Mean :	0.49525
	Variance :	0.05582
	S.D. :	0.23626
	Skewed Coef. :	-0.55295
	Kurtosis Coef. :	2.38237
	MAD :	0.19811
	Range :	0.99989
	Mid_range :	0.50005
	Median :	0.70305
	Q1 :	0.49568
	Q2 :	0.70305
	Q3 :	0.86320
	IQR :	0.36753
	C.V. :	0.35614

(13) λ =0.999,

f(x1),F(x1)	Ceofficient
	Mathematical Mean: 0.66634 Geometrical Mean : 0.60617 Harmonic Mean : 0.49946 Variance : 0.05558 S.D. : 0.23575 Skewed Coef. : -0.56410 Kurtosis Coef. : 2.39756 MAD : 0.19760 Range : 0.99990 Mid_range : 0.50005 Median : 0.70669 Q1 : 0.49956 Q2 : 0.70669 Q3 : 0.86574 IQR : 0.36618 C.V. : 0.35379

Chapter 2. The sufficient statistic and the pointer estimator of parameter

Section 1,The likelihood function

Let $X_1, X_2, ..., X_n \overset{iid}{\sim} Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$,

$$f_{X_i}(x_i;\lambda) = \frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)} x_i^{\lambda}(1-x_i)^{1-\lambda}, 0 \le x_i \le 1, 0 < \lambda < 1,$$

The likelihood function is

$$f(x_1, x_2, ..., x_n;\lambda) = \prod_{i=1}^{n} f_{X_i}(x_i;\lambda) = \left(\frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)}\right)^n \left(\prod_{i=1}^{n} x_i^{\lambda}\right)\left(\prod_{i=1}^{n}(1-x_i)\right)^{1-\lambda}$$

$$= \left(\frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)}\right)^n exp\left(\lambda\sum_{i=1}^{n} ln(x_i)\right) exp\left((1-\lambda)\sum_{i=1}^{n} ln(1-x_i)\right)$$

$$= \left(\frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)}\right)^n exp\left(\lambda\sum_{i=1}^{n} ln\left(\frac{x_i}{1-x_i}\right)\right) exp\left(\sum_{i=1}^{n} ln(1-x_i)\right)$$

Section 2, The sufficient statistic of λ

The sufficient statistic of λ cannot be found from the likelihood function.

$$f_X(x;\lambda) = \frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)}\left(\frac{x}{1-x}\right)^{\lambda}(1-x), 0 \le x \le 1, 0 < \lambda < 1,$$

$$Y = \frac{X}{1-X}, Y - YX = X, X = \frac{Y}{1+Y}, 1 - X = \frac{1}{1+Y}, 0 < y < \infty,$$

$$\frac{dx}{dy} = \frac{1}{1+y} - \frac{y}{(1+y)^2} = \frac{1}{(1+y)^2}$$

$$f_Y(y;\lambda) = f_X\left(x = \frac{y}{1+y};\lambda\right) \times \left|\frac{dx}{dy}\right| = \frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)} y^{\lambda} \frac{1}{(1+y)^3},$$

$$W = ln(Y) = ln\left(\frac{X}{1-X}\right), -\infty < w < \infty, \frac{dy}{dw} = exp(w),$$

$$f_W(w;\lambda) = f_Y(y = exp(w);\lambda)\left|\frac{dy}{dw}\right| = \frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)} exp(\lambda w)\frac{exp(w)}{(1+exp(w))^3},$$

The sufficient statistic of λ.

$$X_1, X_2, ..., X_n \overset{iid}{\sim} Beta(\alpha = \lambda + 1, \beta = 2 - \lambda),$$

$$T = \sum_{i=1}^{n} ln\left(\frac{X_i}{1 - X_i}\right) = \sum_{i=1}^{n} W_i, W_i = ln\left(\frac{X_i}{1 - X_i}\right), i = 1,2..., n,$$

$$f(w_1, w_2, ..., w_n) = \left(\frac{2}{\Gamma(\lambda + 1)\Gamma(2 - \lambda)}\right)^n exp\left(\lambda \sum_{i=1}^{n} w_i\right) \prod_{i=1}^{n}\left(\frac{exp(w_i)}{(1 + exp(w_i))^3}\right),$$

$$f\left(w_1, w_2, ..., w_n \middle| T = \sum_{i=1}^{n} W_i = t\right) = \frac{f\left(w_1, w_2, ..., w_n = t - \sum_{i=1}^{n-1} w_i\right)}{f_T(t)}$$

$$= \frac{\prod_{i=1}^{n}\left(\frac{exp(w_i)}{(1 + exp(w_i))^3}\right)}{\int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} \prod_{i=1}^{n-1}\left(\frac{exp(w_i)}{(1 + exp(w_i))^3}\right) \times \frac{exp\left(t - \sum_{i=1}^{n-1} w_i\right)}{\left(1 + expt - \sum_{i=1}^{n-1} w_i\right)^3} dw_1 ... dw_{n-1}},$$

$$f_T(t) = \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} f\left(w_1, w_2, ..., w_n = t - \sum_{i=1}^{n-1} w_i\right) dw_1 ... dw_{n-1}$$

$$= \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} \left(\frac{2}{\Gamma(\lambda + 1)\Gamma(2 - \lambda)}\right)^n exp(\lambda t) K\left(w_1, w_2, ..., w_n = t - \sum_{i=1}^{n-1} w_i\right) dw_1 ... dw_{n-1}$$

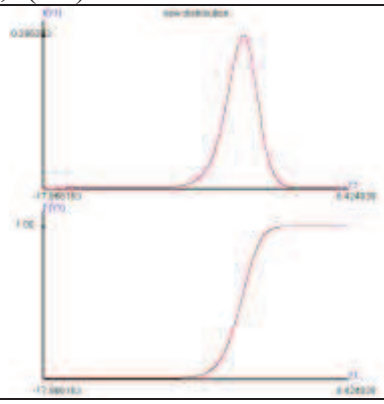
$$K\left(w_1, w_2, ..., w_n = t - \sum_{i=1}^{n-1} w_i\right) = \prod_{i=1}^{n-1}\left(\frac{exp(w_i)}{(1 + exp(w_i))^3}\right) \times \frac{exp\left(t - \sum_{i=1}^{n-1} w_i\right)}{\left(1 + expt - \sum_{i=1}^{n-1} w_i\right)^3},$$

$f\left(w_1, w_2, ..., w_n \middle| T = \sum_{i=1}^{n} W_i = t\right)$ is independent with λ.

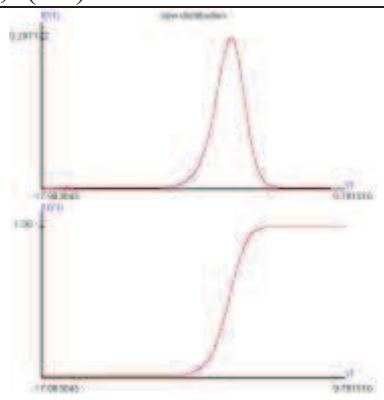
The sufficient of λ is not existed, because $\sum_{i=1}^{n} ln\left(\frac{X_i}{1 - X_i}\right) = \sum_{i=1}^{n} W_i$, but the range is $(-\infty, \infty)$.

(2-1) $X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$, Y1= $ln\left(\frac{X}{1-X}\right)$,

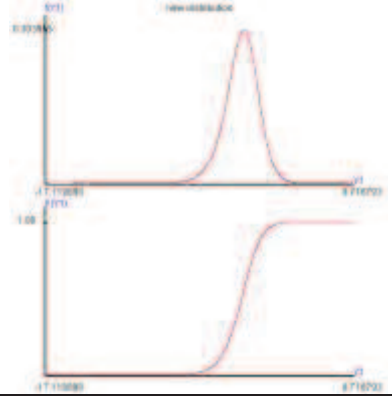
(2-1-1) λ =0.001, Y1= $ln\left(\frac{X}{1-X}\right)$,

f(Y1),F(Y1)	Ceofficient	
	Mathematical Mean:	-0.99708
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	2.28747
	S.D. :	1.51244
	Skewed Coef. :	-0.57667
	Kurtosis Coef. :	4.33222
	MAD :	1.16381
	Range :	26.48831
	Mid_range :	-4.77106
	Median :	-0.87894
	Q1 :	-1.86295
	Q2 :	-0.87894
	Q3 :	0.00229
	IQR :	1.86524
	C.V. : none	

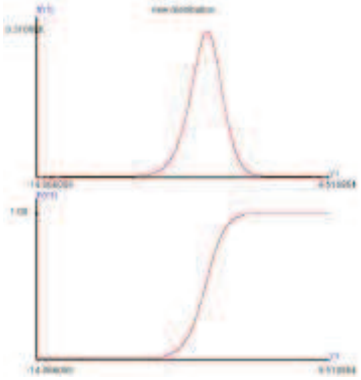
(2-1-2) λ =0.01, Y1= $ln\left(\frac{X}{1-X}\right)$,

f(Y1),F(Y1)	Ceofficient	
	Mathematical Mean:	-0.97713
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	2.27103
	S.D. :	1.50699
	Skewed Coef. :	-0.56461
	Kurtosis Coef. :	4.30950
	MAD :	1.16002
	Range :	27.84830
	Mid_range :	-4.09106
	Median :	-0.86161
	Q1 :	-1.84205
	Q2 :	-0.86161
	Q3 :	0.01807
	IQR :	1.86012
	C.V. : none	

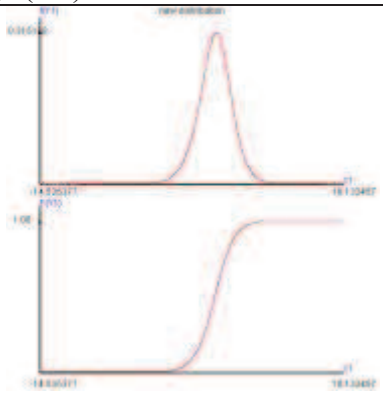
(2-1-3) λ =0.1, Y1= $ln\left(\frac{X}{1-X}\right)$,

f(Y1),F(Y1)	Ceofficient	
	Mathematical Mean:	-0.77969
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	2.12323
	S.D. :	1.45713
	Skewed Coef. :	-0.45489
	Kurtosis Coef. :	4.13592
	MAD :	1.12396
	Range :	25.93474
	Mid_range :	-4.20055
	Median :	-0.68982
	Q1 :	-1.63266
	Q2 :	-0.68982
	Q3 :	0.17489
	IQR :	1.80755
	C.V. : none	

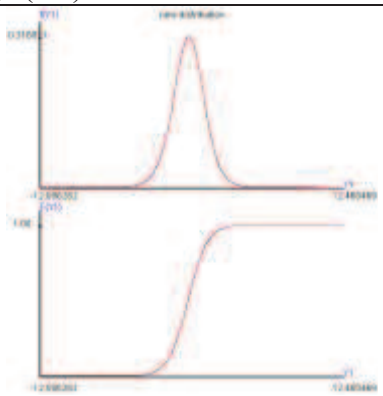
(2-1-4) λ =0.2, Y1= $ln\left(\frac{X}{1-X}\right)$,

f(Y1),F(Y1)	Ceofficient	
	Mathematical Mean:	-0.57378
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	2.00663
	S.D. :	1.41656
	Skewed Coef. :	-0.33768
	Kurtosis Coef. :	3.98925
	MAD :	1.09469
	Range :	23.66456
	Mid_range :	-2.27760
	Median :	-0.50903
	Q1 :	-1.41932
	Q2 :	-0.50903
	Q3 :	0.34549
	IQR :	1.76480
	C.V. : none	

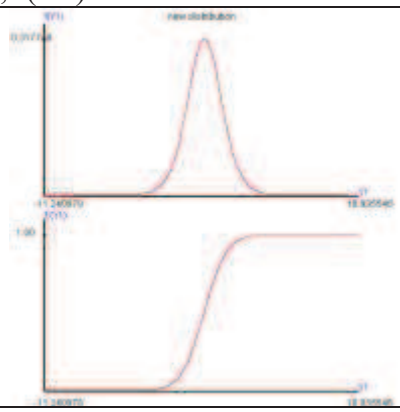
(2-1-5) λ =0.3, Y1= $ln\left(\frac{X}{1-X}\right)$,

f(Y1),F(Y1)	Ceofficient	
	Mathematical Mean:	-0.37742
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	1.92768
	S.D. :	1.38841
	Skewed Coef. :	-0.22270
	Kurtosis Coef. :	3.89052
	MAD :	1.07443
	Range :	24.75954
	Mid_range :	-2.20146
	Median :	-0.33560
	Q1 :	-1.22134
	Q2 :	-0.33560
	Q3 :	0.51419
	IQR :	1.73553
	C.V. : none	

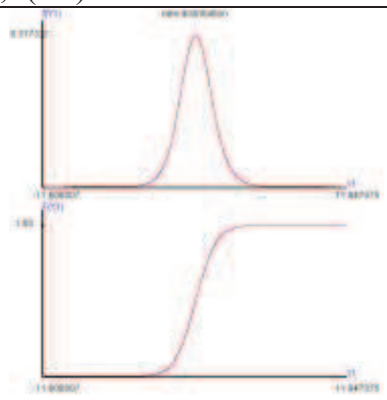
(2-1-6) λ =0.4, Y1= $ln\left(\frac{X}{1-X}\right)$,

f(Y1),F(Y1)	Ceofficient	
	Mathematical Mean:	-0.18730
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	1.88434
	S.D. :	1.37271
	Skewed Coef. :	-0.11132
	Kurtosis Coef. :	3.82735
	MAD :	1.06312
	Range :	24.63798
	Mid_range :	0.18710
	Median :	-0.16651
	Q1 :	-1.03508
	Q2 :	-0.16651
	Q3 :	0.68372
	IQR :	1.71880
	C.V. : none	

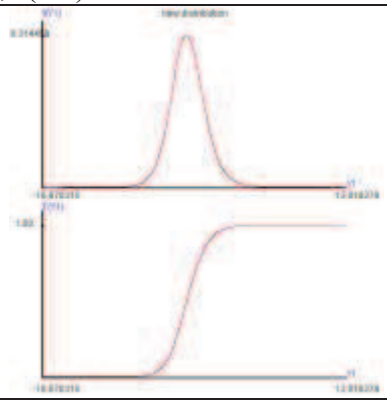
(2-1-7) λ =0.5, Y1= $ln\left(\frac{X}{1-X}\right)$,

f(Y1),F(Y1)	Ceofficient	
	Mathematical Mean:	0.00015
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	1.86805
	S.D. :	1.36677
	Skewed Coef. :	-0.00061
	Kurtosis Coef. :	3.80335
	MAD :	1.05903
	Range :	22.25896
	Mid_range :	-0.15272
	Median :	0.00016
	Q1 :	-0.85677
	Q2 :	0.00016
	Q3 :	0.85663
	IQR :	1.71340
	C.V. : none	

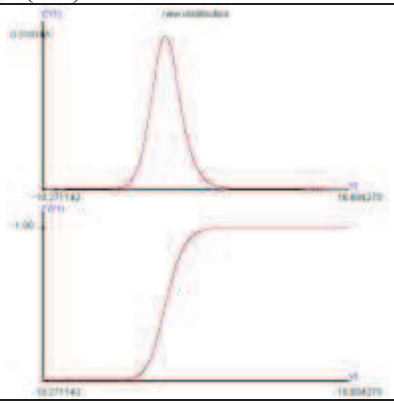
(2-1-8) λ =0.6, Y1= $ln\left(\frac{X}{1-X}\right)$,

f(Y1),F(Y1)	Ceofficient	
	Mathematical Mean:	0.18797
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	1.88530
	S.D. :	1.37306
	Skewed Coef. :	0.11014
	Kurtosis Coef. :	3.82045
	MAD :	1.06348
	Range :	23.54228
	Mid_range :	0.11953
	Median :	0.16696
	Q1 :	-0.68342
	Q2 :	0.16696
	Q3 :	1.03598
	IQR :	1.71939
	C.V. :	7.30464

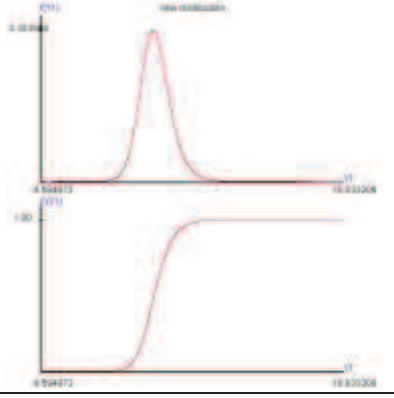
(2-1-9) λ =0.7, Y1= $ln\left(\frac{X}{1-X}\right)$,

f(Y1),F(Y1)	Ceofficient	
	Mathematical Mean:	0.37824
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	1.92948
	S.D. :	1.38906
	Skewed Coef. :	0.22127
	Kurtosis Coef. :	3.88609
	MAD :	1.07486
	Range :	23.77665
	Mid_range :	0.97398
	Median :	0.33633
	Q1 :	-0.51364
	Q2 :	0.33633
	Q3 :	1.22265
	IQR :	1.73629
	C.V. :	3.67238

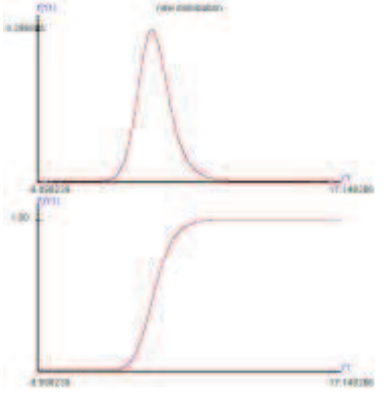
(2-1-10) λ =0.8, Y1= $ln\left(\frac{X}{1-X}\right)$,

f(Y1),F(Y1)	Ceofficient	
	Mathematical Mean:	0.57401
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	2.00493
	S.D. :	1.41596
	Skewed Coef. :	0.33623
	Kurtosis Coef. :	3.98624
	MAD :	1.09437
	Range :	27.05562
	Mid_range :	3.20656
	Median :	0.50936
	Q1 :	-0.34505
	Q2 :	0.50936
	Q3 :	1.41948
	IQR :	1.76453
	C.V. :	2.46679

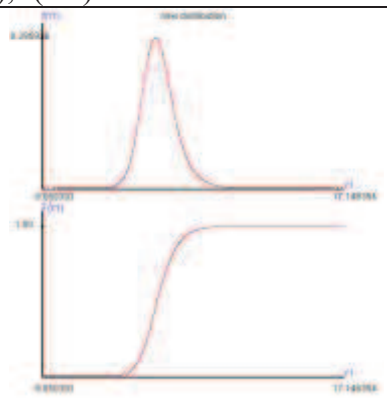
(2-1-11) λ =0.9, Y1= $ln\left(\frac{X}{1-X}\right)$,

f(Y1),F(Y1)	Ceofficient	
	Mathematical Mean:	0.77993
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	2.12133
	S.D. :	1.45648
	Skewed Coef. :	0.45214
	Kurtosis Coef. :	4.12550
	MAD :	1.12377
	Range :	27.73079
	Mid_range :	4.21917
	Median :	0.69000
	Q1 :	-0.17480
	Q2 :	0.69000
	Q3 :	1.63316
	IQR :	1.80796
	C.V. :	1.86745

(2-1-12) λ =0.99, Y1= $ln\left(\frac{X}{1-X}\right)$,

f(Y1),F(Y1)	Ceofficient	
	Mathematical Mean:	0.97808
	Geometrical Mean : none	
	Harmonic Mean : none	
	Variance :	2.27241
	S.D. :	1.50745
	Skewed Coef. :	0.56629
	Kurtosis Coef. :	4.31537
	MAD :	1.16007
	Range :	26.34408
	Mid_range :	4.02501
	Median :	0.86204
	Q1 :	-0.01705
	Q2 :	0.86204
	Q3 :	1.84230
	IQR :	1.85935
	C.V. :	1.54124

(2-1-13) λ =0.999, Y1= $ln\left(\frac{X}{1-X}\right)$,

F(Y1),F(Y1)	Ceofficient
	Mathematical Mean: 0.99849 Geometrical Mean : none Harmonic Mean : none Variance : 2.29042 S.D. : 1.51341 Skewed Coef. : 0.57644 Kurtosis Coef. : 4.33026 MAD : 1.16439 Range : 26.29614 Mid_range : 4.04898 Median : 0.87981 Q1 : -0.00126 Q2 : 0.87981 Q3 : 1.86446 IQR : 1.86571 C.V. : 1.51569

Section 3, The point estimator of λ

Let $X_1, X_2, ..., X_n \overset{iid}{\sim} Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$.

(1) UMVU(Uniformly Minimum Variance Unbiased)

The sufficient statistic of λ is not existed, the UMVUE of λ cannot be found.

(2) MLE

$$f(x_1, x_2, ..., x_n; \lambda) = \left(\frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)}\right)^n exp\left(\lambda \sum_{i=1}^{n} ln\left(\frac{x_i}{1-x_i}\right)\right) exp\left(\sum_{i=1}^{n} ln(1-x_i)\right)$$

$$= L(\lambda | x_1, x_2, ..., x_n),$$

$$ln\, L(\lambda | x_1, x_2, ..., x_n) = n\, ln\, 2 - n\, ln(\Gamma(\lambda+1)\Gamma(2-\lambda)) + \lambda \sum_{i=1}^{n} ln\left(\frac{x_i}{1-x_i}\right) + \sum_{i=1}^{n} ln(1-x_i)$$

$$\frac{d\, ln\, L(\lambda | x_1, x_2, ..., x_n)}{d\lambda} = \frac{n\left(\frac{d\Gamma(\lambda+1)\Gamma(2-\lambda)}{d\lambda}\right)}{\Gamma(\lambda+1)\Gamma(2-\lambda)} + \sum_{i=1}^{n} ln\left(\frac{x_i}{1-x_i}\right) = 0,$$

$$\frac{\frac{d\Gamma(\lambda+1)\Gamma(2-\lambda)}{d\lambda}}{\Gamma(\lambda+1)\Gamma(2-\lambda)} = H(\lambda) = -\frac{\sum_{i=1}^{n} ln\left(\frac{x_i}{1-x_i}\right)}{n}, \hat{\lambda} = H^{-1}\left(-\frac{\sum_{i=1}^{n} ln\left(\frac{x_i}{1-x_i}\right)}{n}\right),$$

$H(\lambda)$ is unkown, MLE of λ is not existed.

(3) MME

$$E(X) = \frac{\lambda+1}{3}, \overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{\hat{\lambda}+1}{3}, \hat{\lambda} = 3 \times \overline{X} - 1,$$

$$\mathrm{E}(\hat{\lambda}) = \lambda, \mathrm{Var}(\hat{\lambda}) = \frac{9(\lambda+1)(2-\lambda)}{36n},$$

Chapter 3. The sampling distribution about sample mean

This chapter is about the sampling distribution of $\sum_{i=1}^{n} X_i$, $\frac{\sqrt{n}(\overline{X}-\mu)}{\sigma}$ and $\frac{\sqrt{n}(\overline{X}-\mu)}{S}$.

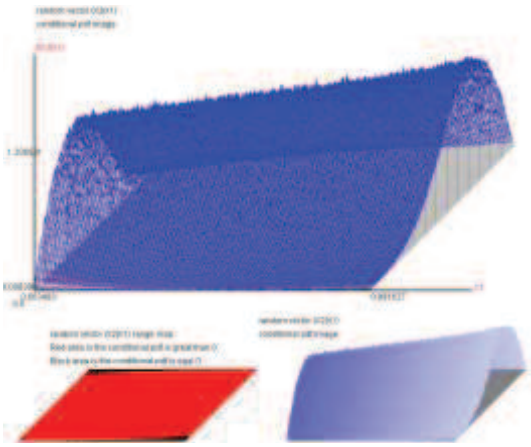
Let $X_1, X_2, ..., X_n \overset{iid}{\sim} Beta(\alpha=\lambda+1, \beta=2-\lambda)$, with $\mu=\frac{\lambda+1}{3}, \sigma^2=\frac{(\lambda+1)(2-\lambda)}{36}$.

Section 1, The sampling distribution of $\sum_{i=1}^{n} X_i$

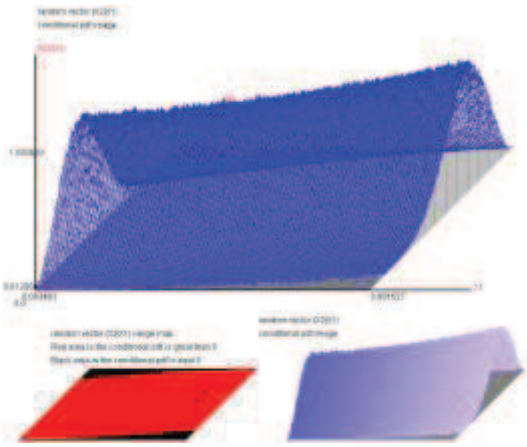
(1) $f\left(X2=\sum_{i=1}^{n} X_i \middle| X1=\lambda\right)$=?

$$X=\sum_{i=1}^{n} X_i \xrightarrow[n\to\infty]{} Normal\left(E(X)=n\mu, Var(X)=n\sigma^2\right).$$

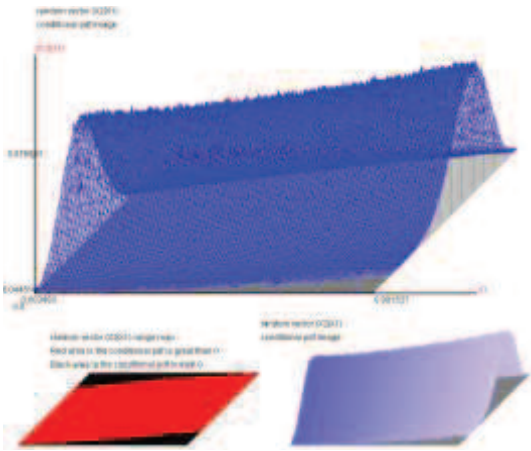
(1-1)n=2,



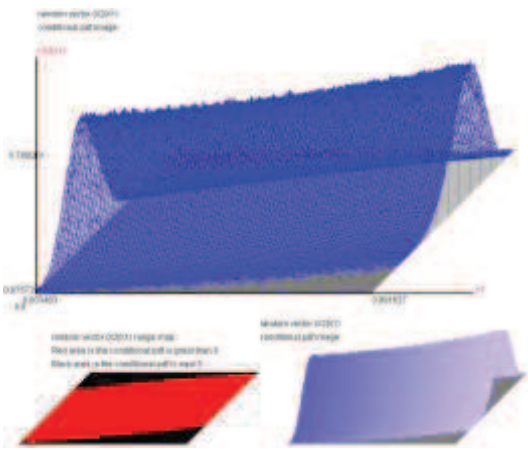
(1-2)n=3,



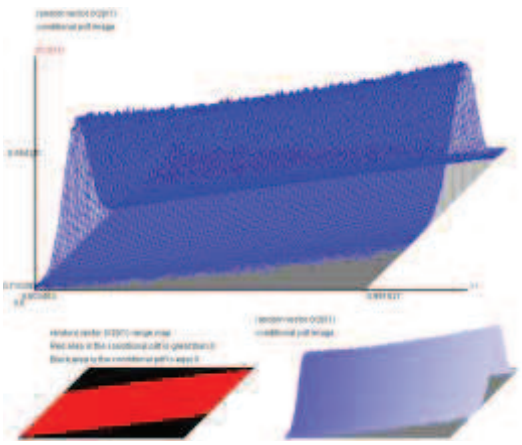
(1-3)n=4,



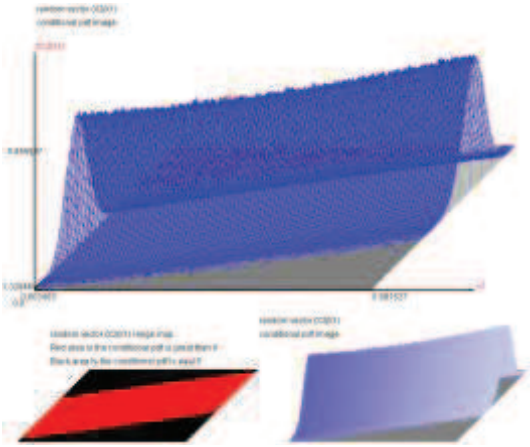
(1-4)n=5,



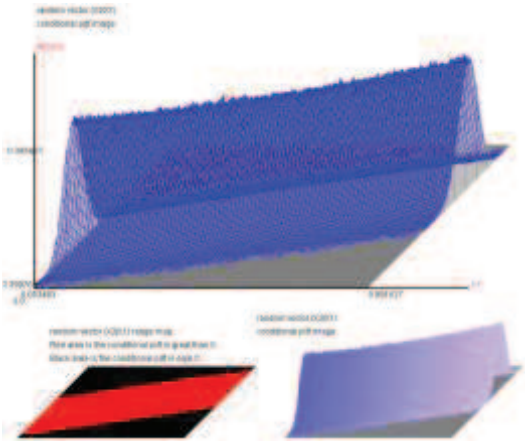
(1-5)n=10,



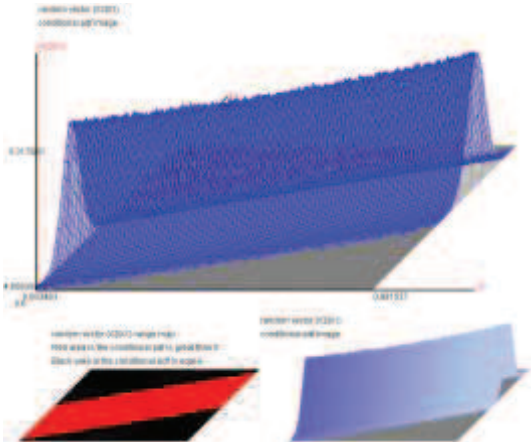
(1-6)n=15,



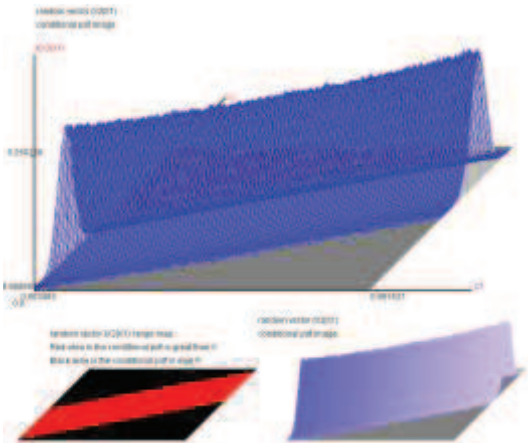
(1-7)n=20,



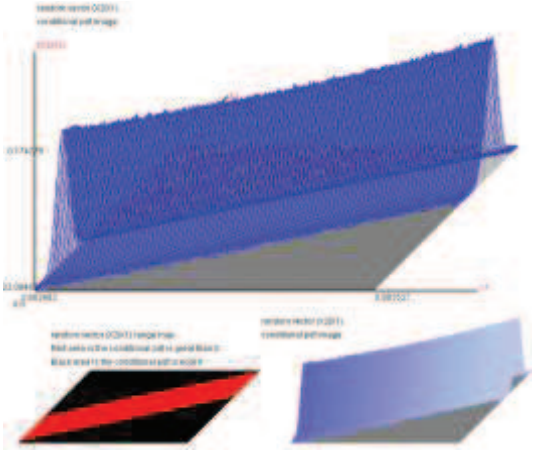
(1-8)n=30,



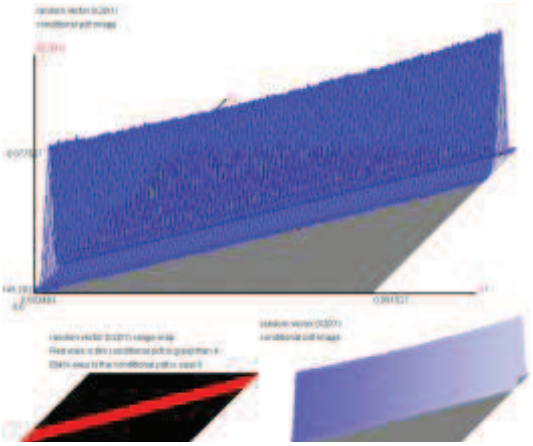
(1-9)n=50,



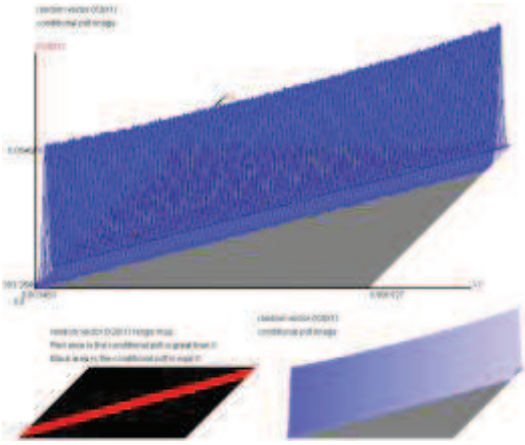
(1-10)n=100,



(1-11)n=500,



(1-12)n=1,000,



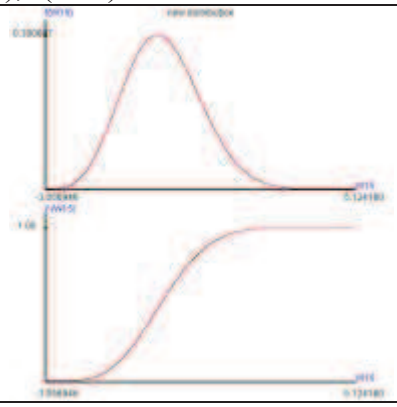
Section 2, Sampling distribution of $\frac{\sqrt{n}\left(\overline{X}-\mu\right)}{\sigma}$

Let $\frac{\sqrt{n}\left(\overline{X}-\mu\right)}{\sigma}=\frac{\sqrt{n}\left(3\overline{X}+1-\left(3\mu-1\right)\right)}{3\sigma}=\frac{\sqrt{n}\left(\hat{\lambda}-\lambda\right)}{3\sigma}$=W15,

$\frac{\sqrt{n}\left(\overline{X}-\mu\right)}{\sigma}\xrightarrow[n\leftarrow\infty]{}Normal(0,1)$

(2-1)n=5,

(2-1-1) λ=0.001,

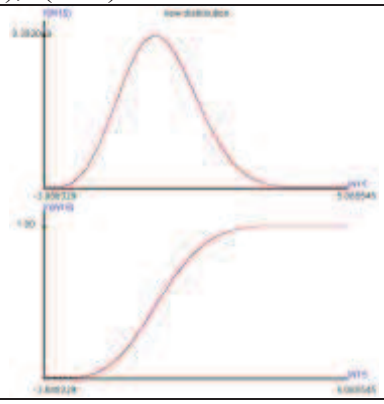
f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00011 Geometrical Mean : none Harmonic Mean : none Variance : 1.00080 S.D. : 1.00040 Skewed Coef. : 0.25264 Kurtosis Coef. : 2.88052 MAD : 0.80455 Range : 8.21154 Mid_range : 1.03362 Median : -0.04403 Q1 : -0.71344 Q2 : -0.04403 Q3 : 0.66458 IQR : 1.37801 C.V. : none

Z0~standard normal distribution,

E(| W15 distribution - Z0 distribution |^2)=0.0045232614
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0001307608,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 0.563537,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.240987,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.044943,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.022230,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.004479,

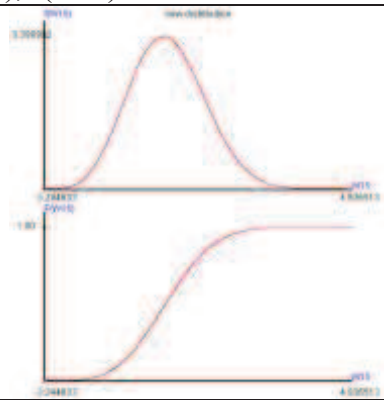
Note:
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000, ,
W15→ standard normal distribution.

(2-1-2) λ=0.01,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00012 Geometrical Mean : none Harmonic Mean : none Variance : 0.99984 S.D. : 0.99992 Skewed Coef. : 0.24769 Kurtosis Coef. : 2.87607 MAD : 0.80418 Range : 8.20426 Mid_range : 0.99861 Median : -0.04354 Q1 : -0.71233 Q2 : -0.04354 Q3 : 0.66489 IQR : 1.37721 C.V. : none

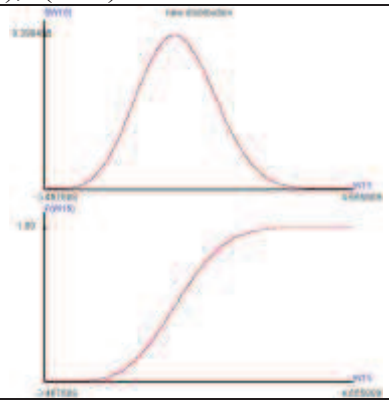
E(| W15 distribution - Z0 distribution |^2)=0.0043774040
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0001247348,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 0.569812,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.245337,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.045207,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.022086,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.004518,

(2-1-3) λ=0.1,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00029 Geometrical Mean : none Harmonic Mean : none Variance : 1.00047 S.D. : 1.00023 Skewed Coef. : 0.19807 Kurtosis Coef. : 2.84984 MAD : 0.80464 Range : 8.21156 Mid_range : 0.84594 Median : -0.03424 Q1 : -0.70796 Q2 : -0.03424 Q3 : 0.67045 IQR : 1.37841 C.V. : none

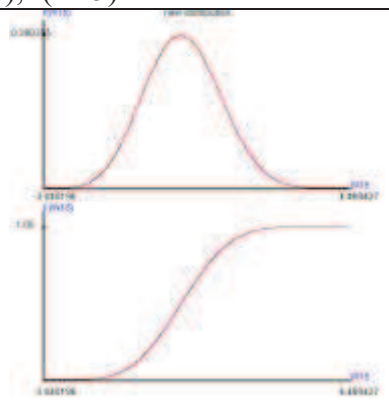
E(| W15 distribution - Z0 distribution |^2)=0.0029626380
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000826839,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 0.641511,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.315465,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.056572,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.028071,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.005404,

(2-1-4) λ =0.2,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00012 Geometrical Mean : none Harmonic Mean : none Variance : 0.99982 S.D. : 0.99991 Skewed Coef. : 0.14671 Kurtosis Coef. : 2.82660 MAD : 0.80449 Range : 8.15362 Mid_range : 0.59410 Median : -0.02567 Q1 : -0.70298 Q2 : -0.02567 Q3 : 0.67528 IQR : 1.37826 C.V. : none

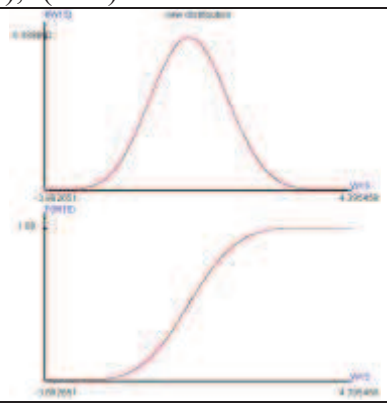
E(| W15 distribution - Z0 distribution |^2)=0.0018557352
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000509577,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 0.763454,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.426020,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.073665,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.036613,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.007285,

(2-1-5) λ =0.3,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00012 Geometrical Mean : none Harmonic Mean : none Variance : 1.00033 S.D. : 1.00017 Skewed Coef. : 0.09629 Kurtosis Coef. : 2.81122 MAD : 0.80490 Range : 8.13977 Mid_range : 0.41462 Median : -0.01689 Q1 : -0.69866 Q2 : -0.01689 Q3 : 0.68034 IQR : 1.37901 C.V. : none

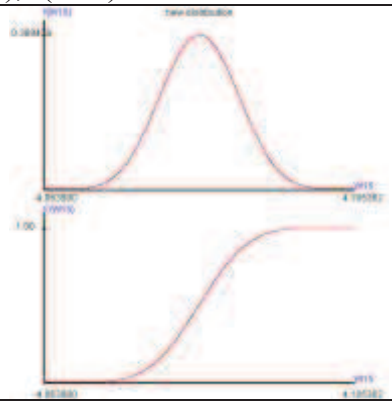
E(| W15 distribution - Z0 distribution |^2)=0.0010694255
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000285415,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.615372,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.100541,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.051406,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.011603,

(2-1-6) λ =0.4,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00013 Geometrical Mean : none Harmonic Mean : none Variance : 0.99996 S.D. : 0.99998 Skewed Coef. : 0.04868 Kurtosis Coef. : 2.80407 MAD : 0.80468 Range : 8.30888 Mid_range : 0.25640 Median : -0.00836 Q1 : -0.69398 Q2 : -0.00836 Q3 : 0.68483 IQR : 1.37881 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0006357276
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000149101,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.756577,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.140045,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.073029,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.013311,

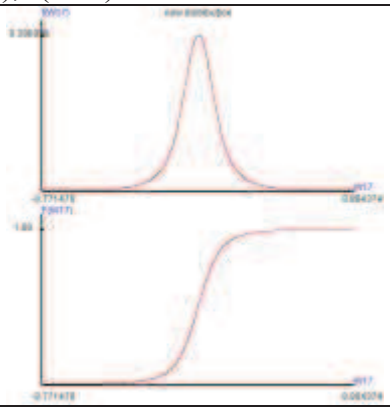
(2-1-7) λ =0.5,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00013 Geometrical Mean : none Harmonic Mean : none Variance : 1.00028 S.D. : 1.00014 Skewed Coef. : -0.00022 Kurtosis Coef. : 2.79924 MAD : 0.80496 Range : 8.19963 Mid_range : 0.02073 Median : 0.00022 Q1 : -0.68944 Q2 : 0.00022 Q3 : 0.68983 IQR : 1.37927 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0005026929
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000114996,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.154279,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.073209,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.014871,

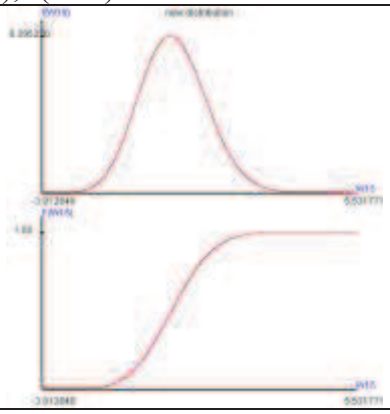
(2-2)n=10,

(2-2-1) λ=0.001,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00001 Geometrical Mean : none Harmonic Mean : none Variance : 1.00019 S.D. : 1.00010 Skewed Coef. : 0.17838 Kurtosis Coef. : 2.93885 MAD : 0.80110 Range : 9.31540 Mid_range : 0.74548 Median : -0.03042 Q1 : -0.69830 Q2 : -0.03042 Q3 : 0.66471 IQR : 1.36301 C.V. : none

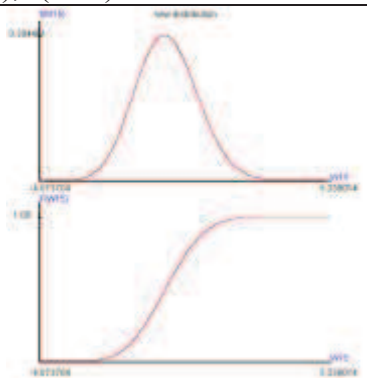
E(| W15 distribution - Z0 distribution |^2)=0.0019893315
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000590326,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 0.711815,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.395783,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.066997,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.033787,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.006751,

(2-2-2) λ=0.01,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00025 Geometrical Mean : none Harmonic Mean : none Variance : 1.00006 S.D. : 1.00003 Skewed Coef. : 0.17366 Kurtosis Coef. : 2.93652 MAD : 0.80111 Range : 9.47973 Mid_range : 0.80946 Median : -0.02928 Q1 : -0.69756 Q2 : -0.02928 Q3 : 0.66553 IQR : 1.36309 C.V. : none

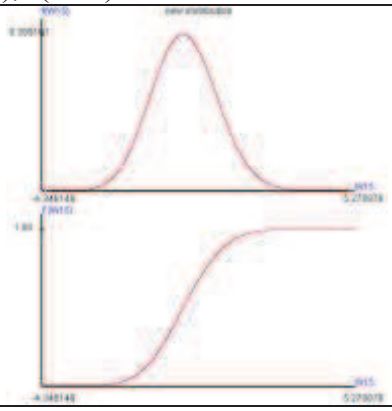
E(| W15 distribution - Z0 distribution |^2)=0.0018874919
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000559598,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 0.729760,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.423013,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.067141,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.033519,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.006498,

(2-2-3) λ =0.1,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00013 Geometrical Mean : none Harmonic Mean : none Variance : 0.99992 S.D. : 0.99996 Skewed Coef. : 0.14026 Kurtosis Coef. : 2.92554 MAD : 0.80104 Range : 9.44771 Mid_range : 0.63266 Median : -0.02414 Q1 : -0.69444 Q2 : -0.02414 Q3 : 0.66844 IQR : 1.36288 C.V. : none

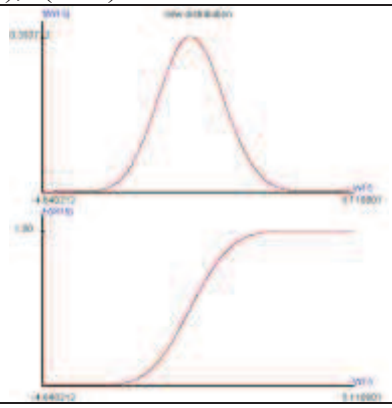
E(| W15 distribution - Z0 distribution |^2)=0.0012750800
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000387646,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 0.914072,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.543300,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.083806,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.041650,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.008731,

(2-2-4) λ =0.2,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00021 Geometrical Mean : none Harmonic Mean : none Variance : 1.00017 S.D. : 1.00009 Skewed Coef. : 0.10205 Kurtosis Coef. : 2.91184 MAD : 0.80131 Range : 9.65278 Mid_range : 0.46236 Median : -0.01736 Q1 : -0.69126 Q2 : -0.01736 Q3 : 0.67271 IQR : 1.36397 C.V. : none

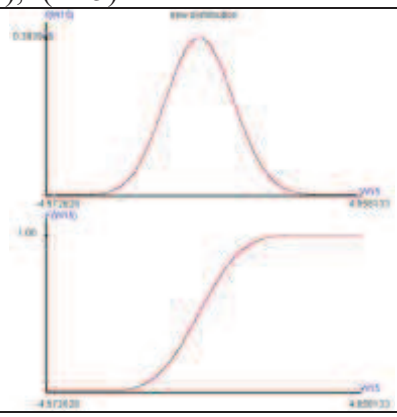
E(| W15 distribution - Z0 distribution |^2)=0.0007312314
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000205522,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.643576,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.116639,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.056746,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.011978,

(2-2-5) λ =0.3,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00003 Geometrical Mean : none Harmonic Mean : none Variance : 1.00029 S.D. : 1.00014 Skewed Coef. : 0.06847 Kurtosis Coef. : 2.90550 MAD : 0.80139 Range : 9.78726 Mid_range : 0.23529 Median : -0.01171 Q1 : -0.68824 Q2 : -0.01171 Q3 : 0.67543 IQR : 1.36368 C.V. : none

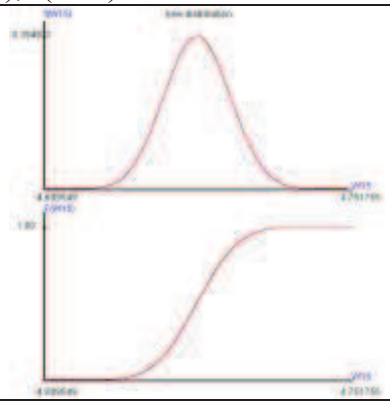
E(| W15 distribution - Z0 distribution |^2)=0.0003868715
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000111720,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.805792,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.163450,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.083268,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.017084,

(2-2-6) λ =0.4,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00018 Geometrical Mean : none Harmonic Mean : none Variance : 1.00004 S.D. : 1.00002 Skewed Coef. : 0.03248 Kurtosis Coef. : 2.90094 MAD : 0.80132 Range : 9.46582 Mid_range : 0.14275 Median : -0.00523 Q1 : -0.68474 Q2 : -0.00523 Q3 : 0.67907 IQR : 1.36381 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0001752376
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000045347,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.253891,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.133044,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.026251,

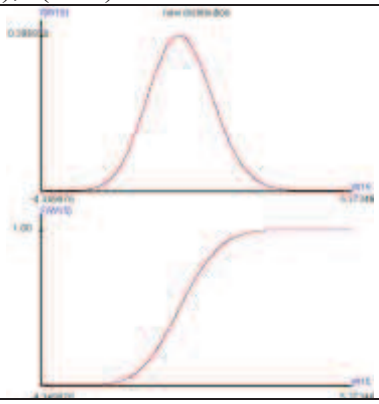
(2-2-7) λ =0.5,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00012 Geometrical Mean : none Harmonic Mean : none Variance : 0.99978 S.D. : 0.99989 Skewed Coef. : 0.00054 Kurtosis Coef. : 2.90036 MAD : 0.80115 Range : 9.39610 Mid_range : 0.07110 Median : -0.00006 Q1 : -0.68182 Q2 : -0.00006 Q3 : 0.68180 IQR : 1.36362 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0001128391
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000026481,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.356106,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.172022,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.031527,

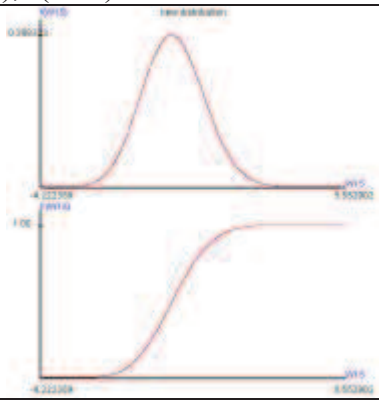
(2-3)n=20,

(2-3-1) λ=0.001,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00009 Geometrical Mean : none Harmonic Mean : none Variance : 1.00035 S.D. : 1.00017 Skewed Coef. : 0.12574 Kurtosis Coef. : 2.96922 MAD : 0.79956 Range : 9.75949 Mid_range : 0.51180 Median : -0.02108 Q1 : -0.68975 Q2 : -0.02108 Q3 : 0.66605 IQR : 1.35580 C.V. : none

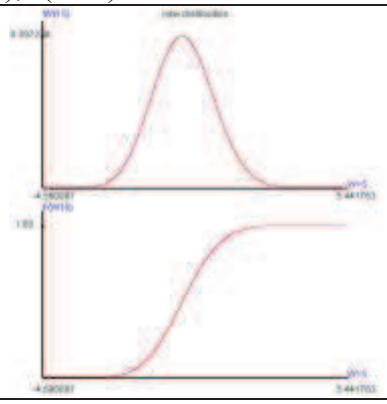
E(| W15 distribution - Z0 distribution |^2)=0.0009300977
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000282702,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.572344,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.094966,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.046115,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.008643,

(2-3-2) λ=0.01,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00009 Geometrical Mean : none Harmonic Mean : none Variance : 1.00011 S.D. : 1.00006 Skewed Coef. : 0.12405 Kurtosis Coef. : 2.96853 MAD : 0.79945 Range : 9.81160 Mid_range : 0.66527 Median : -0.02112 Q1 : -0.68951 Q2 : -0.02112 Q3 : 0.66637 IQR : 1.35588 C.V. : none

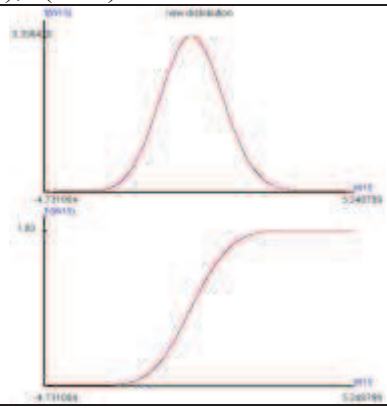
E(| W15 distribution - Z0 distribution |^2)=0.0009094687
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000283925,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.576018,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.094538,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.045486,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.009279,

(2-3-3) λ =0.1,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00006 Geometrical Mean : none Harmonic Mean : none Variance : 0.99995 S.D. : 0.99997 Skewed Coef. : 0.09969 Kurtosis Coef. : 2.96305 MAD : 0.79944 Range : 10.06914 Mid_range : 0.42584 Median : -0.01683 Q1 : -0.68731 Q2 : -0.01683 Q3 : 0.66886 IQR : 1.35617 C.V. : none

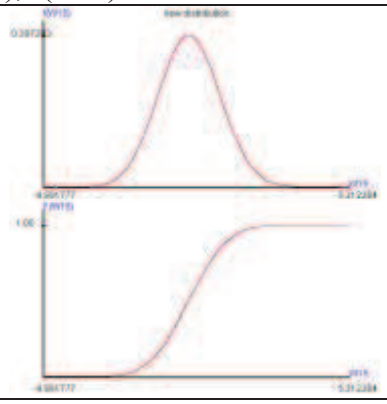
E(| W15 distribution - Z0 distribution |^2)=0.0005978293
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000181325,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.655479,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.121208,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.060752,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.011370,

(2-3-4) λ =0.2,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00008 Geometrical Mean : none Harmonic Mean : none Variance : 1.00040 S.D. : 1.00020 Skewed Coef. : 0.07275 Kurtosis Coef. : 2.95477 MAD : 0.79971 Range : 10.01698 Mid_range : 0.25886 Median : -0.01230 Q1 : -0.68513 Q2 : -0.01230 Q3 : 0.67135 IQR : 1.35648 C.V. : none

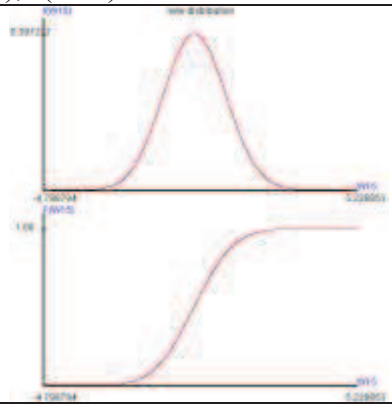
E(| W15 distribution - Z0 distribution |^2)=0.0003336759
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000102920,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.860678,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.166341,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.078873,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.018313,

(2-3-5) λ =0.3,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00005 Geometrical Mean : none Harmonic Mean : none Variance : 0.99974 S.D. : 0.99987 Skewed Coef. : 0.04862 Kurtosis Coef. : 2.95308 MAD : 0.79941 Range : 9.94086 Mid_range : 0.26024 Median : -0.00823 Q1 : -0.68232 Q2 : -0.00823 Q3 : 0.67342 IQR : 1.35574 C.V. : none

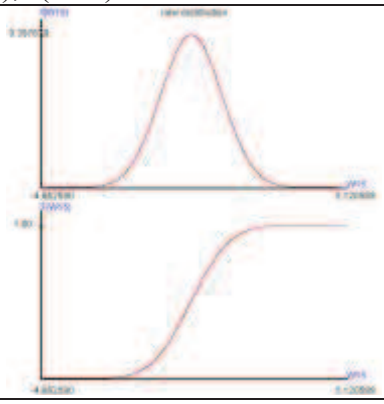
E(| W15 distribution - Z0 distribution |^2)= 0.0001629978
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)= 0.0000049427,
Pr(| W15 distribution function - Z0 distribution function|< 0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|< 0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|< 0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|< 0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|< 0.0010000000)= 0.248616,
Pr(| W15 distribution function - Z0 distribution function|< 0.0005000000)= 0.119518,
Pr(| W15 distribution function - Z0 distribution function|< 0.0001000000)= 0.021641,

(2-3-6) λ =0.4,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00008 Geometrical Mean : none Harmonic Mean : none Variance : 1.00054 S.D. : 1.00027 Skewed Coef. : 0.02354 Kurtosis Coef. : 2.95011 MAD : 0.79975 Range : 10.06292 Mid_range : 0.21403 Median : -0.00397 Q1 : -0.68068 Q2 : -0.00397 Q3 : 0.67593 IQR : 1.35661 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0000595989
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000016667,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.461943,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.206538,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.048030,

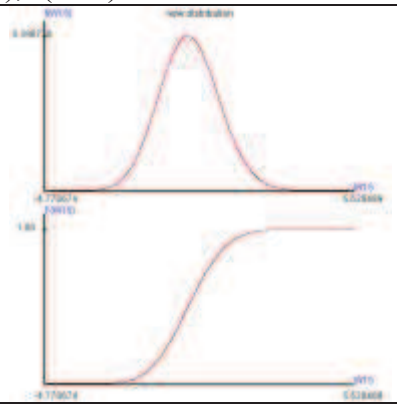
(2-3-7) λ =0.5,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00016 Geometrical Mean : none Harmonic Mean : none Variance : 1.00005 S.D. : 1.00003 Skewed Coef. : 0.00014 Kurtosis Coef. : 2.94996 MAD : 0.79958 Range : 10.02029 Mid_range : 0.12900 Median : 0.00017 Q1 : -0.67810 Q2 : 0.00017 Q3 : 0.67818 IQR : 1.35628 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0000275767
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000006948,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.626310,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.343050,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.065313,

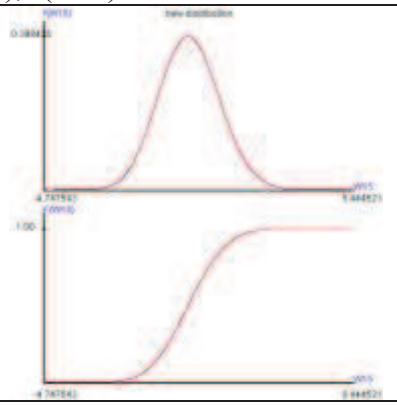
(2-4)n=30,

(2-4-1) λ=0.001,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00022 Geometrical Mean : none Harmonic Mean : none Variance : 0.99954 S.D. : 0.99977 Skewed Coef. : 0.10346 Kurtosis Coef. : 2.98063 MAD : 0.79873 Range : 10.32942 Mid_range : 0.37491 Median : -0.01723 Q1 : -0.68587 Q2 : -0.01723 Q3 : 0.66732 IQR : 1.35319 C.V. : none

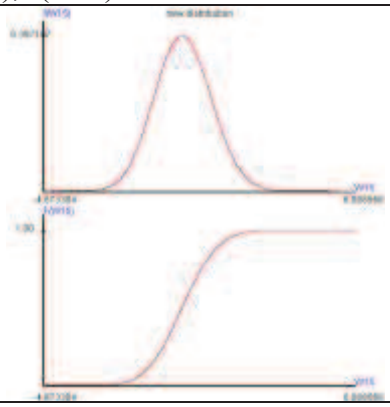
E(| W15 distribution - Z0 distribution |^2)=0.0006158355
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000184410,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.655305,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.118137,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.058605,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.012052,

(2-4-2) λ=0.01,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00018 Geometrical Mean : none Harmonic Mean : none Variance : 0.99988 S.D. : 0.99994 Skewed Coef. : 0.10074 Kurtosis Coef. : 2.97779 MAD : 0.79887 Range : 10.22995 Mid_range : 0.34849 Median : -0.01704 Q1 : -0.68634 Q2 : -0.01704 Q3 : 0.66716 IQR : 1.35350 C.V. : none

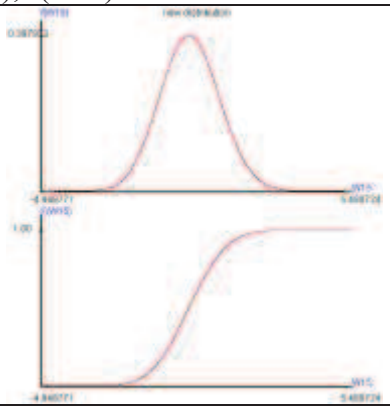
E(| W15 distribution - Z0 distribution |^2)=0.0005925953
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000183233,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.653523,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.122123,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.060286,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.011675,

(2-4-3) λ =0.1,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00018 Geometrical Mean : none Harmonic Mean : none Variance : 0.99995 S.D. : 0.99997 Skewed Coef. : 0.08135 Kurtosis Coef. : 2.97589 MAD : 0.79890 Range : 11.00262 Mid_range : 0.60763 Median : -0.01382 Q1 : -0.68443 Q2 : -0.01382 Q3 : 0.66908 IQR : 1.35350 C.V. : none

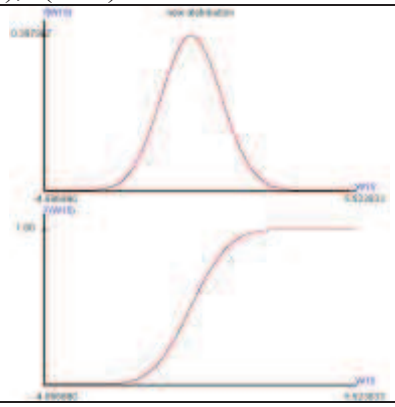
E(| W15 distribution - Z0 distribution |^2)=0.0003856007
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000121224,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.780322,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.154052,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.074123,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.013432,

(2-4-4) λ =0.2,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00018 Geometrical Mean : none Harmonic Mean : none Variance : 1.00023 S.D. : 1.00011 Skewed Coef. : 0.06030 Kurtosis Coef. : 2.97077 MAD : 0.79907 Range : 10.46827 Mid_range : 0.26598 Median : -0.01015 Q1 : -0.68248 Q2 : -0.01015 Q3 : 0.67128 IQR : 1.35376 C.V. : none

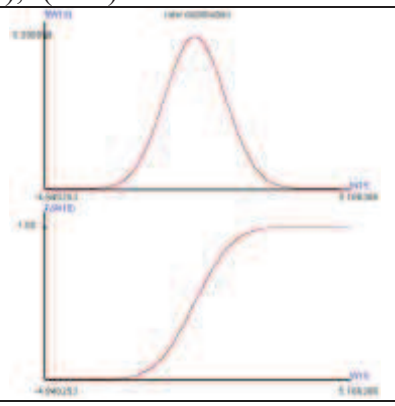
E(| W15 distribution - Z0 distribution |^2)=0.0002181689
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000068741,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.208669,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.098050,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.019560,

(2-4-5) λ =0.3,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00016 Geometrical Mean : none Harmonic Mean : none Variance : 1.00052 S.D. : 1.00026 Skewed Coef. : 0.03868 Kurtosis Coef. : 2.96773 MAD : 0.79919 Range : 10.44942 Mid_range : 0.31847 Median : -0.00671 Q1 : -0.68037 Q2 : -0.00671 Q3 : 0.67344 IQR : 1.35381 C.V. : none

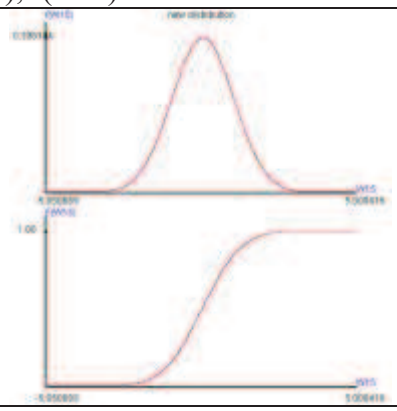
E(| W15 distribution - Z0 distribution |^2)=0.0000995329
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000031533,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.335133,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.156240,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.029978,

(2-4-6) λ =0.4,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00015 Geometrical Mean : none Harmonic Mean : none Variance : 1.00014 S.D. : 1.00007 Skewed Coef. : 0.02005 Kurtosis Coef. : 2.96743 MAD : 0.79906 Range : 10.14421 Mid_range : 0.11307 Median : -0.00321 Q1 : -0.67880 Q2 : -0.00321 Q3 : 0.67545 IQR : 1.35425 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0000348610
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000009892,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.624607,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.278548,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.049944,

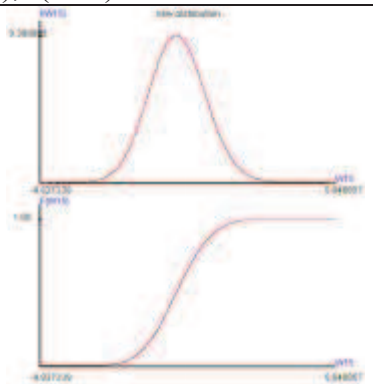
(2-4-7) λ =0.5,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00014 Geometrical Mean : none Harmonic Mean : none Variance : 0.99998 S.D. : 0.99999 Skewed Coef. : -0.00011 Kurtosis Coef. : 2.96737 MAD : 0.79895 Range : 10.08847 Mid_range : -0.02514 Median : 0.00022 Q1 : -0.67662 Q2 : 0.00022 Q3 : 0.67698 IQR : 1.35360 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0000114172
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000002804,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.507760,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.114440,

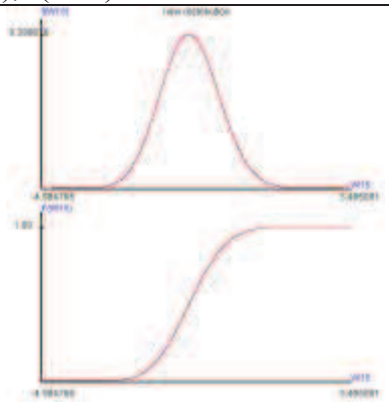
(2-5)n=40,

(2-5-1) λ=0.001,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00003 Geometrical Mean : none Harmonic Mean : none Variance : 1.00024 S.D. : 1.00012 Skewed Coef. : 0.08897 Kurtosis Coef. : 2.98461 MAD : 0.79874 Range : 10.50701 Mid_range : 0.40681 Median : -0.01492 Q1 : -0.68437 Q2 : -0.01492 Q3 : 0.66825 IQR : 1.35261 C.V. : none

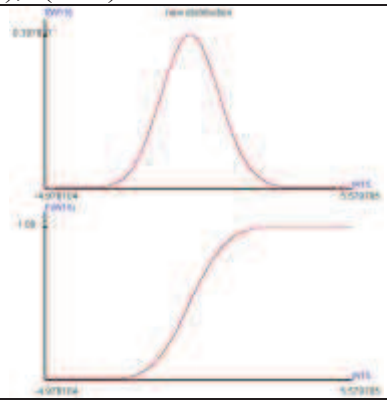
E(| W15 distribution - Z0 distribution |^2)=0.0004543842
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000142668,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.719801,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.138781,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.066510,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.014518,

(2-5-2) λ=0.01,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00003 Geometrical Mean : none Harmonic Mean : none Variance : 0.99975 S.D. : 0.99988 Skewed Coef. : 0.08782 Kurtosis Coef. : 2.98493 MAD : 0.79852 Range : 10.51881 Mid_range : 0.25516 Median : -0.01482 Q1 : -0.68404 Q2 : -0.01482 Q3 : 0.66779 IQR : 1.35183 C.V. : none

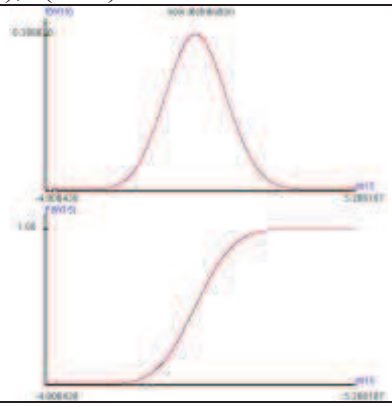
E(| W15 distribution - Z0 distribution |^2)=0.0004416971
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000140078,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.725845,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.139138,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.066422,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.013815,

(2-5-3) λ=0.1,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00009 Geometrical Mean : none Harmonic Mean : none Variance : 1.00008 S.D. : 1.00004 Skewed Coef. : 0.06905 Kurtosis Coef. : 2.98034 MAD : 0.79869 Range : 10.59513 Mid_range : 0.30184 Median : -0.01160 Q1 : -0.68263 Q2 : -0.01160 Q3 : 0.66981 IQR : 1.35244 C.V. : none

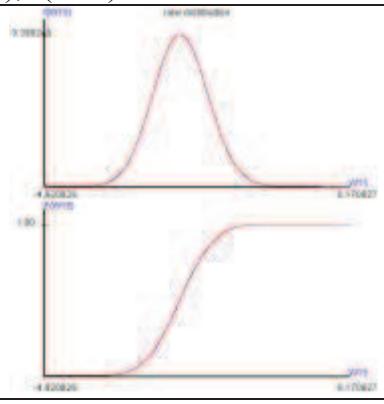
E(| W15 distribution - Z0 distribution |^2)=0.0002751811
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000084042,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.188005,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.089288,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.016322,

(2-5-4) λ=0.2,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00010 Geometrical Mean : none Harmonic Mean : none Variance : 1.00027 S.D. : 1.00013 Skewed Coef. : 0.05082 Kurtosis Coef. : 2.97730 MAD : 0.79880 Range : 10.21044 Mid_range : 0.17987 Median : -0.00862 Q1 : -0.68095 Q2 : -0.00862 Q3 : 0.67146 IQR : 1.35240 C.V. : none

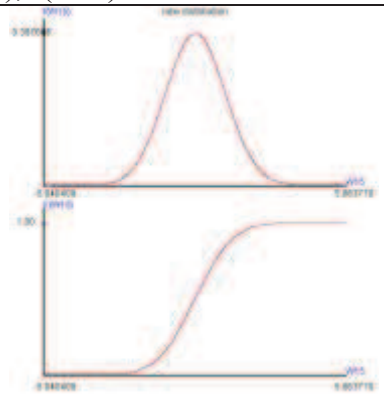
E(| W15 distribution - Z0 distribution |^2)=0.0001525387
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000049084,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.251641,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.112318,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.020450,

(2-5-5) λ =0.3,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00010 Geometrical Mean : none Harmonic Mean : none Variance : 1.00048 S.D. : 1.00024 Skewed Coef. : 0.03393 Kurtosis Coef. : 2.97728 MAD : 0.79885 Range : 11.13289 Mid_range : 0.62500 Median : -0.00588 Q1 : -0.67958 Q2 : -0.00588 Q3 : 0.67287 IQR : 1.35245 C.V. : none

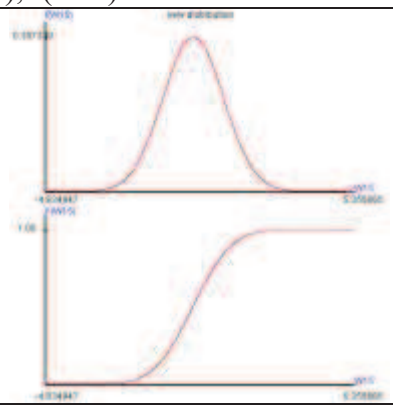
E(| W15 distribution - Z0 distribution |^2)=0.0000743758
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000024520,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.416055,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.174013,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.037476,

(2-5-6) λ =0.4,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 0.99965 S.D. : 0.99983 Skewed Coef. : 0.01636 Kurtosis Coef. : 2.97515 MAD : 0.79859 Range : 10.14174 Mid_range : 0.01168 Median : -0.00292 Q1 : -0.67771 Q2 : -0.00292 Q3 : 0.67491 IQR : 1.35262 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0000226956
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000006701,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.688375,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.342863,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.058314,

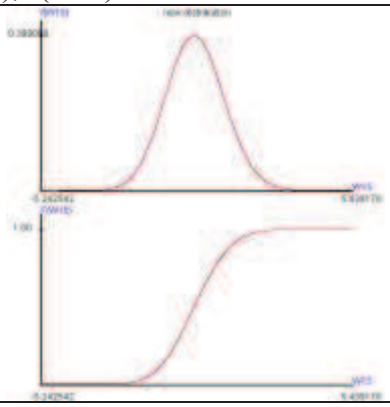
(2-5-7) λ =0.5,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00031 Geometrical Mean : none Harmonic Mean : none Variance : 0.99997 S.D. : 0.99999 Skewed Coef. : 0.00065 Kurtosis Coef. : 2.97673 MAD : 0.79869 Range : 10.22850 Mid_range : 0.26036 Median : 0.00003 Q1 : -0.67596 Q2 : 0.00003 Q3 : 0.67660 IQR : 1.35256 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0000063659
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000001900,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.592351,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.114399,

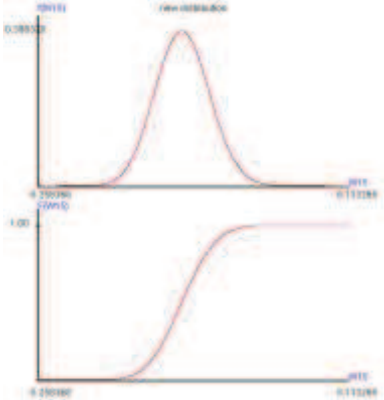
(2-6)n=50,

(2-6-1) λ=0.001,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00021 Geometrical Mean : none Harmonic Mean : none Variance : 0.99996 S.D. : 0.99998 Skewed Coef. : 0.08050 Kurtosis Coef. : 2.98849 MAD : 0.79847 Range : 10.72143 Mid_range : 0.09832 Median : -0.01324 Q1 : -0.68296 Q2 : -0.01324 Q3 : 0.66863 IQR : 1.35159 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0003668727
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000111036,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.828425,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.161668,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.075545,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.013113,

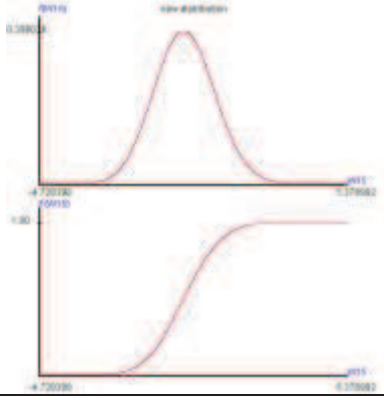
(2-6-2) λ=0.01,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: 0.00001 Geometrical Mean : none Harmonic Mean : none Variance : 1.00036 S.D. : 1.00018 Skewed Coef. : 0.07816 Kurtosis Coef. : 2.98764 MAD : 0.79864 Range : 11.43398 Mid_range : 0.43745 Median : -0.01267 Q1 : -0.68329 Q2 : -0.01267 Q3 : 0.66880 IQR : 1.35209 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0003498694
********** | W15 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W15 distribution function - Z0 distribution function|^2)=0.0000104501,
Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 0.890551,
Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.158077,
Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.077019,
Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.014847,

(2-7)n=60,

(2-7-1) λ=0.001,

f(w15),F(w15)	Ceofficient
	Mathematical Mean: -0.00000 Geometrical Mean : none Harmonic Mean : none Variance : 0.99997 S.D. : 0.99999 Skewed Coef. : 0.07267 Kurtosis Coef. : 2.98974 MAD : 0.79842 Range : 10.14496 Mid_range : 0.32530 Median : -0.01199 Q1 : -0.68228 Q2 : -0.01199 Q3 : 0.66913 IQR : 1.35141 C.V. : none

E(| W15 distribution - Z0 distribution |^2)=0.0002999880

********** | W15 distribution function - Z0 distribution function| **********

The almost surely limiting theory

E(| W15 distribution function - Z0 distribution function|^2)=0.0000092146,

Pr(| W15 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,

Pr(| W15 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,

Pr(| W15 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,

Pr(| W15 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,

Pr(| W15 distribution function - Z0 distribution function|<0.0010000000)= 0.179265,

Pr(| W15 distribution function - Z0 distribution function|<0.0005000000)= 0.086407,

Pr(| W15 distribution function - Z0 distribution function|<0.0001000000)= 0.017940,

Section 3, The sampling distribution of $\frac{\sqrt{n}\left(\overline{X}-\mu\right)}{S}$

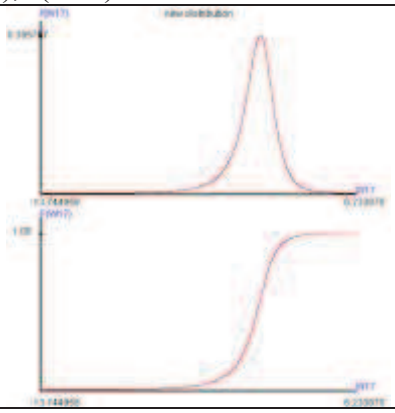
$$\frac{\sqrt{n}\left(\overline{X}-\mu\right)}{S}=\text{W17},\quad S=\sqrt{\frac{\sum_{i=1}^{n}\left(X_i-\overline{X}\right)^2}{n-1}},$$

$$\frac{\sqrt{n}\left(\overline{X}-\mu\right)}{S}\xrightarrow[n\leftarrow\infty]{} Normal(0,1).$$

(2-1)n=5

(2-1-1) λ=0.001

The left and right extremely probability are removing 0.001,

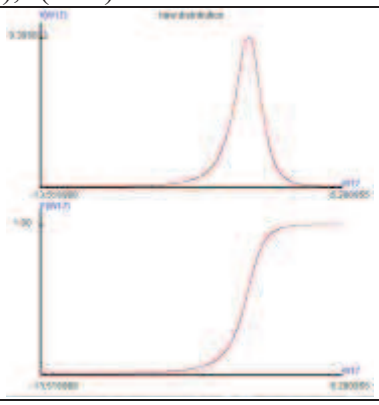
f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.25955 Geometrical Mean : none Harmonic Mean : none Variance : 3.06289 S.D. : 1.75011 Skewed Coef. : -3.61815 Kurtosis Coef. : 91.43554 MAD : 1.10915 Range : 291.37947 Mid_range : -64.88348 Median : -0.04375 Q1 : -0.84299 Q2 : -0.04375 Q3 : 0.62297 IQR : 1.46596 C.V. : none

Z0~standard normal distribution,

E(| W17 distribution - Z0 distribution |^2)=0.9857472476
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0013022936,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 0.799909,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.121759,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.056964,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.011158,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.005544,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.001114,

(2-1-2) λ =0.01,

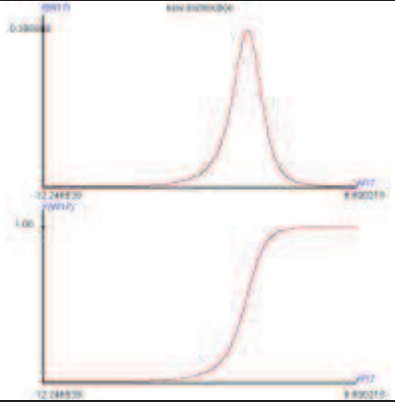
The left and right extremely probability are removing 0.001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.25306 Geometrical Mean : none Harmonic Mean : none Variance : 3.02459 S.D. : 1.73914 Skewed Coef. : -3.83451 Kurtosis Coef. : 150.30731 MAD : 1.10462 Range : 398.18514 Mid_range : -110.21538 Median : -0.04330 Q1 : -0.83862 Q2 : -0.04330 Q3 : 0.62434 IQR : 1.46297 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.9571762012
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0012625026,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 0.804620,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.123537,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.057791,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.011169,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.005616,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.001059,

(2-1-3) λ =0.1,

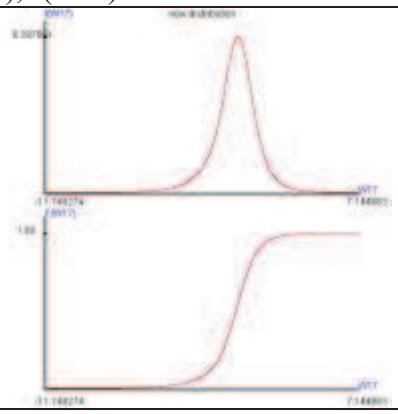
The left and right extremely probability are removing 0.001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.19872 Geometrical Mean : none Harmonic Mean : none Variance : 2.77980 S.D. : 1.66727 Skewed Coef. : -2.75432 Kurtosis Coef. : 73.38604 MAD : 1.07580 Range : 288.22184 Mid_range : -44.76597 Median : -0.03382 Q1 : -0.80727 Q2 : -0.03382 Q3 : 0.63917 IQR : 1.44644 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.7675198078
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0009497982,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 0.842805,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.159759,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.071934,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.014057,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.006937,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.001418,

(2-1-4) λ =0.2,

The left and right extremely probability are removing 0.001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.14420 Geometrical Mean : none Harmonic Mean : none Variance : 2.59422 S.D. : 1.61066 Skewed Coef. : -2.18561 Kurtosis Coef. : 92.00361 MAD : 1.05191 Range : 367.29147 Mid_range : -81.56084 Median : -0.02530 Q1 : -0.77712 Q2 : -0.02530 Q3 : 0.65505 IQR : 1.43217 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.6290920568

********** | W17 distribution function - Z0 distribution function| **********

The almost surely limiting theory

E(| W17 distribution function - Z0 distribution function|^2)=0.0006974608,

Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 0.888619,

Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.377134,

Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.094307,

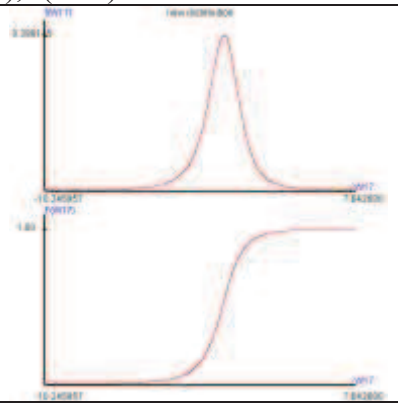
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.017855,

Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.008935,

Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.001800,

(2-1-5) λ =0.3,

The left and right extremely probability are removing 0.001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.09381 Geometrical Mean : none Harmonic Mean : none Variance : 2.47709 S.D. : 1.57388 Skewed Coef. : -1.32907 Kurtosis Coef. : 53.31404 MAD : 1.03698 Range : 295.25546 Mid_range : -37.94720 Median : -0.01658 Q1 : -0.75190 Q2 : -0.01658 Q3 : 0.67239 IQR : 1.42429 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.5413096319

********** | W17 distribution function - Z0 distribution function| **********

The almost surely limiting theory

E(| W17 distribution function - Z0 distribution function|^2)=0.0005294471,

Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 0.943452,

Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.474973,

Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.147288,

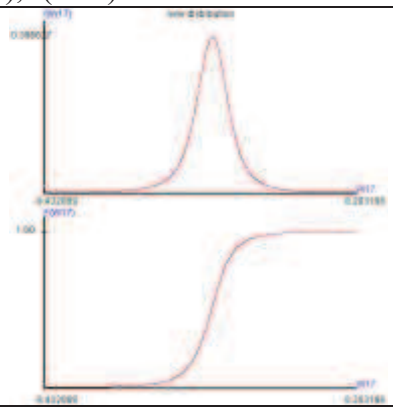
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.024879,

Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.012058,

Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.002346,

(2-1-6) λ =0.4,

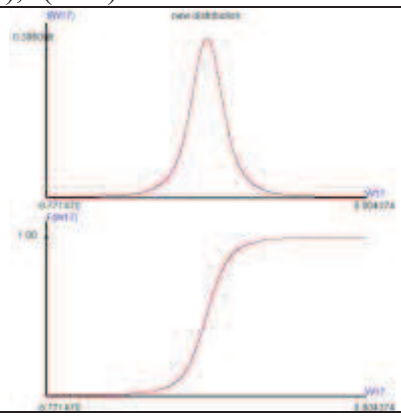
The left and right extremely probability are removing 0.001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.04607 Geometrical Mean : none Harmonic Mean : none Variance : 2.41520 S.D. : 1.55409 Skewed Coef. : -0.63672 Kurtosis Coef. : 52.67204 MAD : 1.02815 Range : 338.37823 Mid_range : -30.79360 Median : -0.00820 Q1 : -0.72892 Q2 : -0.00820 Q3 : 0.68969 IQR : 1.41861 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.4961634496
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0004287037,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.501924,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.364171,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.041190,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.021161,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.003965,

(2-1-7) λ =0.5,

The left and right extremely probability are removing 0.001,

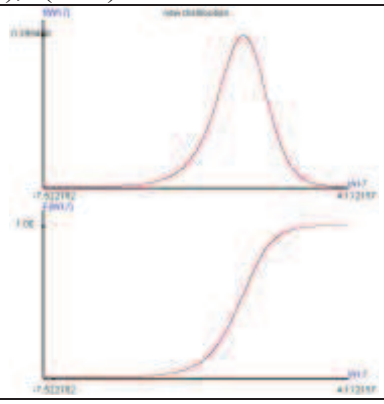
f(w17),F(w17)	Ceofficient
	Mathematical Mean: 0.00080 Geometrical Mean : none Harmonic Mean : none Variance : 2.39302 S.D. : 1.54694 Skewed Coef. : 0.05195 Kurtosis Coef. : 33.42012 MAD : 1.02560 Range : 201.19156 Mid_range : 2.29916 Median : 0.00022 Q1 : -0.70857 Q2 : 0.00022 Q3 : 0.70935 IQR : 1.41792 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.4795264291
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0004002747,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.504417,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.382029,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.194290,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.126359,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.023216,

(2-2)n=10,

(2-2-1) λ =0.001,

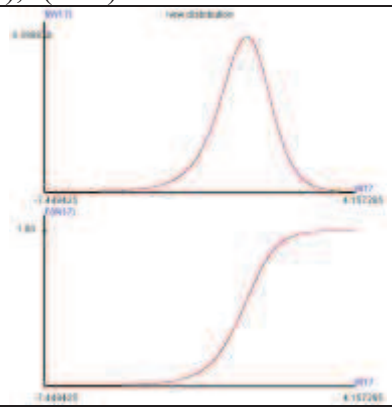
The left and right extremely probability are removing 0.0005,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.12554 Geometrical Mean : none Harmonic Mean : none Variance : 1.46766 S.D. : 1.21147 Skewed Coef. : -0.89015 Kurtosis Coef. : 7.11024 MAD : 0.90027 Range : 48.51953 Mid_range : -9.96177 Median : -0.03026 Q1 : -0.76089 Q2 : -0.03026 Q3 : 0.63171 IQR : 1.39260 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.1093633672
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0004436024,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.128302,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.072621,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.014729,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.007483,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.001379,

(2-2-2) λ =0.01,

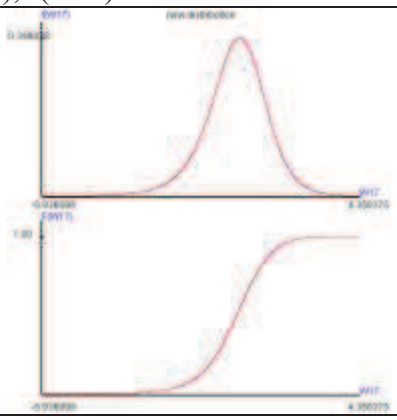
The left and right extremely probability are removing 0.0005,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.12258 Geometrical Mean : none Harmonic Mean : none Variance : 1.46212 S.D. : 1.20918 Skewed Coef. : -0.87408 Kurtosis Coef. : 7.12902 MAD : 0.89908 Range : 54.87775 Mid_range : -9.02091 Median : -0.02912 Q1 : -0.75882 Q2 : -0.02912 Q3 : 0.63294 IQR : 1.39177 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.1065150605
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0004257701,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.133329,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.075304,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.014996,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.007498,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.001491,

(2-2-3) λ =0.1,

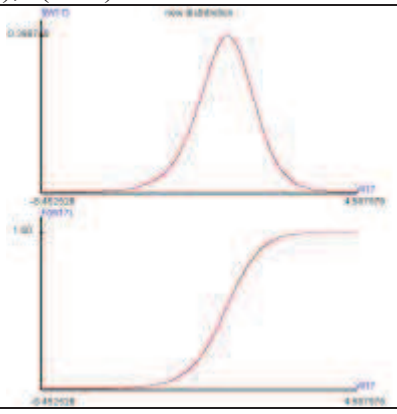
The left and right extremely probability are removing 0.0005,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.09735 Geometrical Mean : none Harmonic Mean : none Variance : 1.41422 S.D. : 1.18921 Skewed Coef. : -0.68398 Kurtosis Coef. : 6.37134 MAD : 0.88939 Range : 51.18115 Mid_range : -5.08580 Median : -0.02398 Q1 : -0.74269 Q2 : -0.02398 Q3 : 0.64236 IQR : 1.38506 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0817208830
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0003013528,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.391936,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.117400,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.018582,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.009251,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.001846,

(2-2-4) λ =0.2,

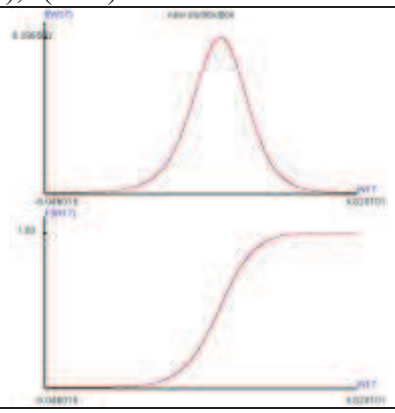
The left and right extremely probability are removing 0.0005,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.07077 Geometrical Mean : none Harmonic Mean : none Variance : 1.37546 S.D. : 1.17280 Skewed Coef. : -0.49671 Kurtosis Coef. : 5.74293 MAD : 0.88163 Range : 42.13634 Mid_range : -4.26275 Median : -0.01719 Q1 : -0.72692 Q2 : -0.01719 Q3 : 0.65390 IQR : 1.38082 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0623546895
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0001959365,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.653249,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.146206,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.025887,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.012751,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.002833,

(2-2-5) λ =0.3,

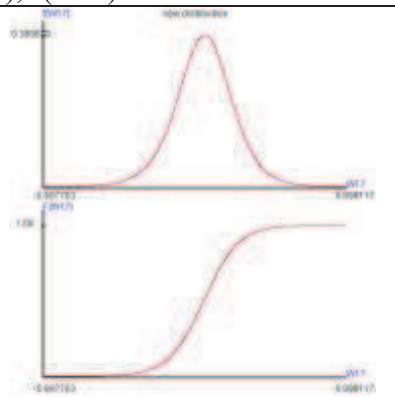
The left and right extremely probability are removing 0.0005,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.04642 Geometrical Mean : none Harmonic Mean : none Variance : 1.35098 S.D. : 1.16231 Skewed Coef. : -0.31963 Kurtosis Coef. : 5.36466 MAD : 0.87663 Range : 39.97708 Mid_range : -3.63309 Median : -0.01163 Q1 : -0.71282 Q2 : -0.01163 Q3 : 0.66439 IQR : 1.37721 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0502182581
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0001277401,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.710312,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.445163,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.037119,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.018275,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.003653,

(2-2-6) λ =0.4,

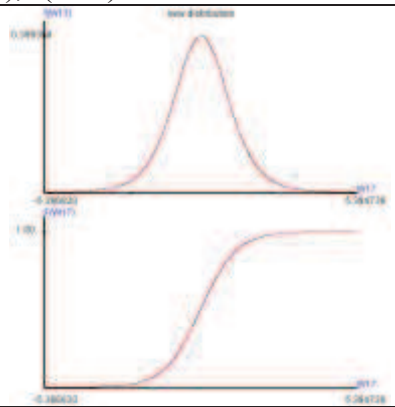
The left and right extremely probability are removing 0.0005,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.02282 Geometrical Mean : none Harmonic Mean : none Variance : 1.33698 S.D. : 1.15628 Skewed Coef. : -0.16255 Kurtosis Coef. : 5.22211 MAD : 0.87353 Range : 46.91051 Mid_range : 0.10643 Median : -0.00515 Q1 : -0.69948 Q2 : -0.00515 Q3 : 0.67604 IQR : 1.37552 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0437026615
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000865414,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.709183,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.533952,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.069535,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.032558,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.006818,

(2-2-7) λ =0.5,

The left and right extremely probability are removing 0.0005,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.00003 Geometrical Mean : none Harmonic Mean : none Variance : 1.33261 S.D. : 1.15439 Skewed Coef. : 0.00399 Kurtosis Coef. : 5.13195 MAD : 0.87250 Range : 38.67819 Mid_range : 1.57228 Median : -0.00006 Q1 : -0.68740 Q2 : -0.00006 Q3 : 0.68719 IQR : 1.37459 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0415911240

********** | W17 distribution function - Z0 distribution function| **********

The almost surely limiting theory

E(| W17 distribution function - Z0 distribution function|^2)=0.0000732414,

Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.711180,

Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.544958,

Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.300378,

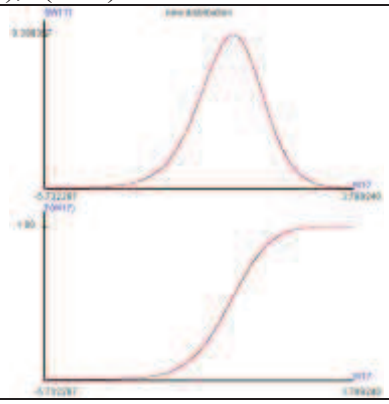
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.223980,

Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.058894,

(2-3)n=20,

(2-3-1) λ=0.001,

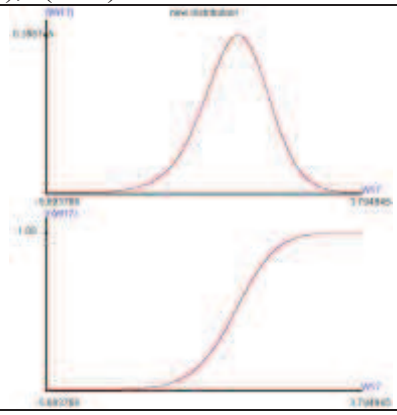
The left and right extremely probability are removing 0.0002,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.07407 Geometrical Mean : none Harmonic Mean : none Variance : 1.16969 S.D. : 1.08152 Skewed Coef. : -0.39164 Kurtosis Coef. : 4.00395 MAD : 0.83996 Range : 22.62283 Mid_range : -3.98493 Median : -0.02103 Q1 : -0.72683 Q2 : -0.02103 Q3 : 0.64124 IQR : 1.36807 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0227835441
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0001840606,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.434199,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.048287,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.013195,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.009540,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.001700,

(2-3-2) λ=0.01,

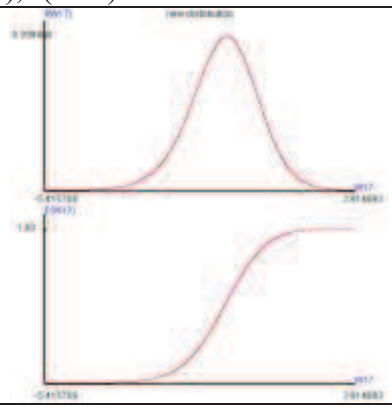
The left and right extremely probability are removing 0.0002,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.07248 Geometrical Mean : none Harmonic Mean : none Variance : 1.16746 S.D. : 1.08049 Skewed Coef. : -0.38303 Kurtosis Coef. : 3.99122 MAD : 0.83933 Range : 21.77095 Mid_range : -3.84515 Median : -0.02106 Q1 : -0.72576 Q2 : -0.02106 Q3 : 0.64206 IQR : 1.36782 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0220381487
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0001771373,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.470111,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.050627,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.014404,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.010599,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.001815,

(2-3-3) λ =0.1,

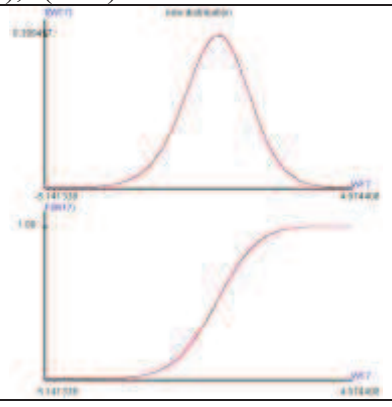
The left and right extremely probability are removing 0.0002,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.05763 Geometrical Mean : none Harmonic Mean : none Variance : 1.15171 S.D. : 1.07318 Skewed Coef. : -0.30143 Kurtosis Coef. : 3.84892 MAD : 0.83551 Range : 21.17344 Mid_range : -2.32135 Median : -0.01675 Q1 : -0.71623 Q2 : -0.01675 Q3 : 0.64920 IQR : 1.36543 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0161222009
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0001187314,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.692566,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.088948,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.030180,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.012034,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.002318,

(2-3-4) λ =0.2,

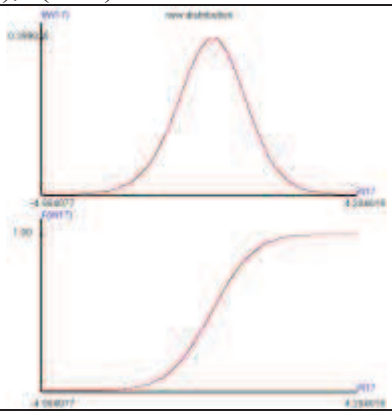
The left and right extremely probability are removing 0.0002,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.04203 Geometrical Mean : none Harmonic Mean : none Variance : 1.13951 S.D. : 1.06748 Skewed Coef. : -0.21956 Kurtosis Coef. : 3.73421 MAD : 0.83252 Range : 19.90045 Mid_range : -1.61901 Median : -0.01204 Q1 : -0.70587 Q2 : -0.01204 Q3 : 0.65722 IQR : 1.36309 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0115633513
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000711888,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.766774,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.317804,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.036616,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.018043,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.003684,

(2-3-5) λ =0.3,

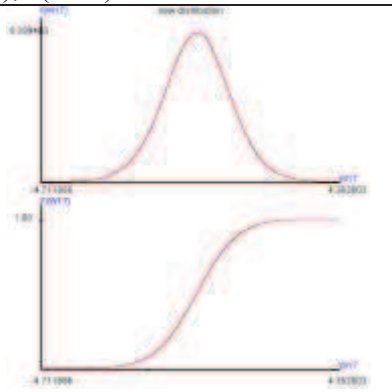
The left and right extremely probability are removing 0.0002,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.02780 Geometrical Mean : none Harmonic Mean : none Variance : 1.13103 S.D. : 1.06350 Skewed Coef. : -0.14320 Kurtosis Coef. : 3.65788 MAD : 0.83037 Range : 20.24103 Mid_range : -1.61526 Median : -0.00818 Q1 : -0.69673 Q2 : -0.00818 Q3 : 0.66426 IQR : 1.36099 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0085687667
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000404949,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.840224,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.677184,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.053333,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.026677,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.005086,

(2-3-6) λ =0.4,

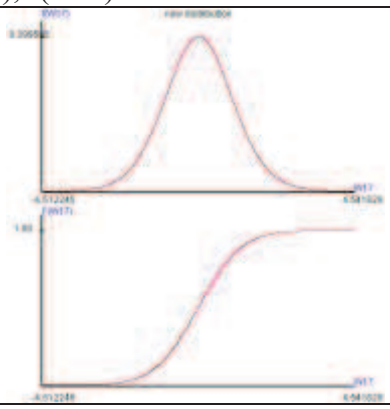
The left and right extremely probability are removing 0.0002,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01389 Geometrical Mean : none Harmonic Mean : none Variance : 1.12753 S.D. : 1.06185 Skewed Coef. : -0.07138 Kurtosis Coef. : 3.61583 MAD : 0.82961 Range : 19.59110 Mid_range : -0.53348 Median : -0.00395 Q1 : -0.68883 Q2 : -0.00395 Q3 : 0.67242 IQR : 1.36125 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0069493827
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000228182,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.929479,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.722082,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.100026,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.049285,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.008982,

(2-3-7) λ =0.5,

The left and right extremely probability are removing 0.0002,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: 0.00017 Geometrical Mean : none Harmonic Mean : none Variance : 1.12526 S.D. : 1.06078 Skewed Coef. : 0.00101 Kurtosis Coef. : 3.60860 MAD : 0.82895 Range : 19.16957 Mid_range : 0.20464 Median : 0.00017 Q1 : -0.68003 Q2 : 0.00017 Q3 : 0.68025 IQR : 1.36028 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0063945945

********** | W17 distribution function - Z0 distribution function| **********

The almost surely limiting theory

E(| W17 distribution function - Z0 distribution function|^2)=0.0000162778,

Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.728668,

Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.409798,

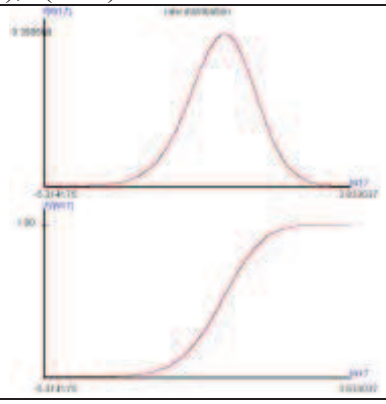
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.318190,

Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.093229,

(2-4)n=30,

(2-4-1) λ=0.001,

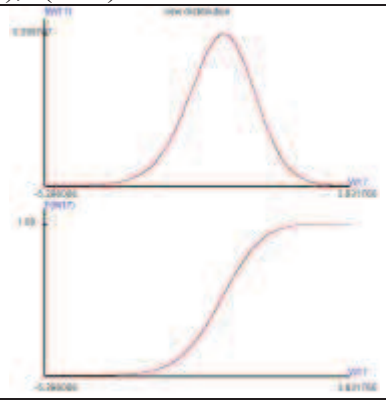
The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.05674 Geometrical Mean : none Harmonic Mean : none Variance : 1.10123 S.D. : 1.04939 Skewed Coef. : -0.27233 Kurtosis Coef. : 3.53946 MAD : 0.82377 Range : 15.81399 Mid_range : -2.06628 Median : -0.01720 Q1 : -0.71420 Q2 : -0.01720 Q3 : 0.64654 IQR : 1.36074 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0107008178
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0001122379,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.698287,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.043961,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.007946,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.004654,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.001864,

(2-4-2) λ=0.01,

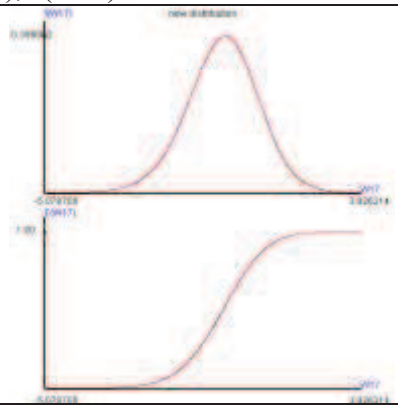
The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.05601 Geometrical Mean : none Harmonic Mean : none Variance : 1.10086 S.D. : 1.04922 Skewed Coef. : -0.26651 Kurtosis Coef. : 3.52572 MAD : 0.82376 Range : 15.67124 Mid_range : -1.82692 Median : -0.01697 Q1 : -0.71424 Q2 : -0.01697 Q3 : 0.64680 IQR : 1.36105 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0104388276
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0001112617,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.701384,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.046656,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.008448,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.004996,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.002124,

(2-4-3) λ =0.1,

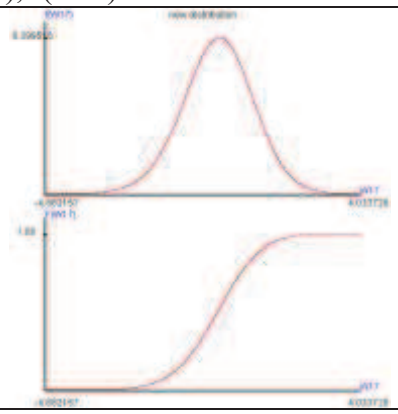
The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.04475 Geometrical Mean : none Harmonic Mean : none Variance : 1.09224 S.D. : 1.04510 Skewed Coef. : -0.21156 Kurtosis Coef. : 3.46158 MAD : 0.82150 Range : 15.48547 Mid_range : -1.65980 Median : -0.01380 Q1 : -0.70652 Q2 : -0.01380 Q3 : 0.65279 IQR : 1.35931 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0075225859
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000739548,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.766051,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.082291,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.017463,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.011763,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.002770,

(2-4-4) λ =0.2,

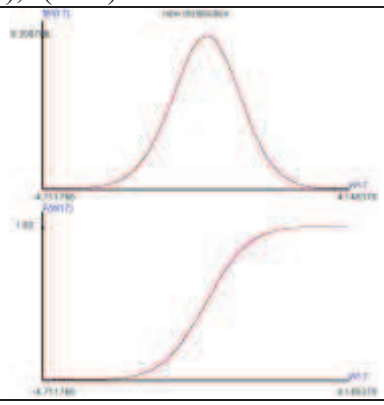
The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.03292 Geometrical Mean : none Harmonic Mean : none Variance : 1.08550 S.D. : 1.04187 Skewed Coef. : -0.15390 Kurtosis Coef. : 3.40440 MAD : 0.81982 Range : 18.21904 Mid_range : -1.10080 Median : -0.01011 Q1 : -0.69899 Q2 : -0.01011 Q3 : 0.65909 IQR : 1.35808 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0051423597
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000439393,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.844123,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.629897,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.045809,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.021302,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.004324,

(2-4-5) λ =0.3,

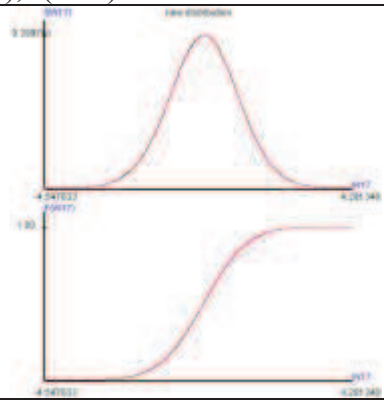
The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.02176 Geometrical Mean : none Harmonic Mean : none Variance : 1.08124 S.D. : 1.03983 Skewed Coef. : -0.10259 Kurtosis Coef. : 3.37022 MAD : 0.81869 Range : 15.39823 Mid_range : -0.88165 Median : -0.00669 Q1 : -0.69133 Q2 : -0.00669 Q3 : 0.66580 IQR : 1.35713 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0036462683
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000232105,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.963893,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.741263,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.064602,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.032957,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.005797,

(2-4-6) λ =0.4,

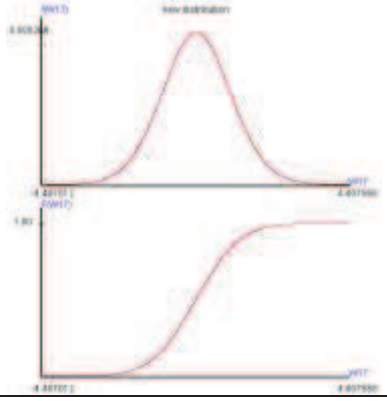
The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01051 Geometrical Mean : none Harmonic Mean : none Variance : 1.07786 S.D. : 1.03820 Skewed Coef. : -0.04974 Kurtosis Coef. : 3.35082 MAD : 0.81775 Range : 14.90535 Mid_range : -0.19144 Median : -0.00319 Q1 : -0.68463 Q2 : -0.00319 Q3 : 0.67208 IQR : 1.35671 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0027113352
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000107560,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.839581,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.147819,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.064676,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.012392,

(2-4-7) λ =0.5,

The left and right extremely probability are removing 0.0001,

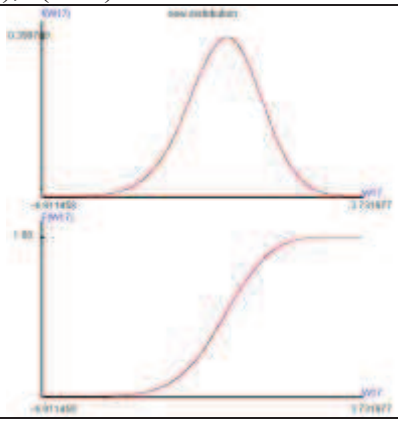
f(w17),F(w17)	Ceofficient
	Mathematical Mean: 0.00018 Geometrical Mean : none Harmonic Mean : none Variance : 1.07685 S.D. : 1.03771 Skewed Coef. : 0.00012 Kurtosis Coef. : 3.34441 MAD : 0.81742 Range : 14.84582 Mid_range : -0.28914 Median : 0.00022 Q1 : -0.67794 Q2 : 0.00022 Q3 : 0.67828 IQR : 1.35622 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0024244457
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000068825,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.911056,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.470892,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.355733,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.218659,

(2-5)n=40,

(2-5-1) λ=0.001,

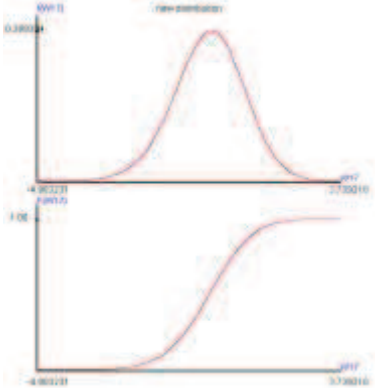
The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.04816 Geometrical Mean : none Harmonic Mean : none Variance : 1.07327 S.D. : 1.03599 Skewed Coef. : -0.21973 Kurtosis Coef. : 3.36284 MAD : 0.81703 Range : 14.69564 Mid_range : -1.37125 Median : -0.01487 Q1 : -0.70825 Q2 : -0.01487 Q3 : 0.65003 IQR : 1.35829 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0068045730
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000835026,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.749128,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.045271,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.006288,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.003319,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.000880,

(2-5-2) λ=0.01,

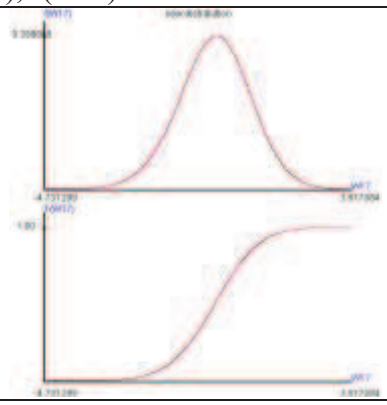
The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.04713 Geometrical Mean : none Harmonic Mean : none Variance : 1.07196 S.D. : 1.03535 Skewed Coef. : -0.21422 Kurtosis Coef. : 3.35857 MAD : 0.81658 Range : 14.59046 Mid_range : -1.71749 Median : -0.01481 Q1 : -0.70755 Q2 : -0.01481 Q3 : 0.64981 IQR : 1.35737 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0065431508
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000803764,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.757288,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.048332,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.006719,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.003549,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.001144,

(2-5-3) λ =0.1,

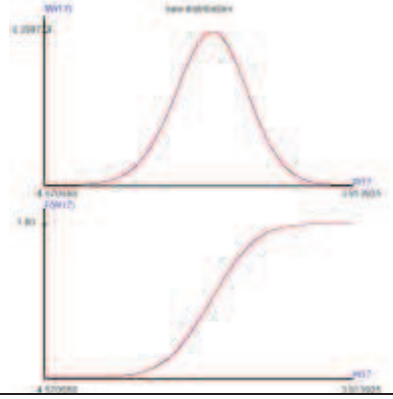
The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.03766 Geometrical Mean : none Harmonic Mean : none Variance : 1.06631 S.D. : 1.03262 Skewed Coef. : -0.17179 Kurtosis Coef. : 3.31623 MAD : 0.81509 Range : 15.11630 Mid_range : -1.14104 Median : -0.01158 Q1 : -0.70121 Q2 : -0.01158 Q3 : 0.65547 IQR : 1.35668 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0046269868
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000521740,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.829229,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.289509,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.013273,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.007563,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.003080,

(2-5-4) λ =0.2,

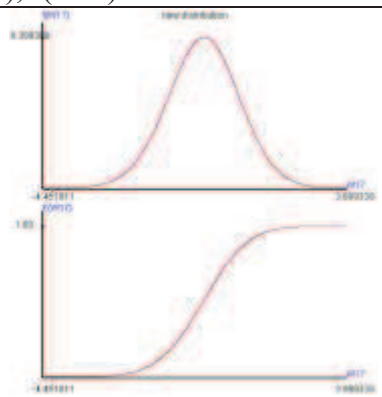
The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.02778 Geometrical Mean : none Harmonic Mean : none Variance : 1.06168 S.D. : 1.03038 Skewed Coef. : -0.12572 Kurtosis Coef. : 3.27877 MAD : 0.81389 Range : 13.66849 Mid_range : -0.78432 Median : -0.00859 Q1 : -0.69464 Q2 : -0.00859 Q3 : 0.66091 IQR : 1.35555 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0030928882
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000304311,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.939541,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.693906,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.033995,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.023151,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.003994,

(2-5-5) λ =0.3,

The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01831 Geometrical Mean : none Harmonic Mean : none Variance : 1.05859 S.D. : 1.02888 Skewed Coef. : -0.08238 Kurtosis Coef. : 3.25914 MAD : 0.81303 Range : 15.25290 Mid_range : -0.11649 Median : -0.00585 Q1 : -0.68875 Q2 : -0.00585 Q3 : 0.66610 IQR : 1.35486 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0020546299

********** | W17 distribution function - Z0 distribution function| **********

The almost surely limiting theory

E(| W17 distribution function - Z0 distribution function|^2)=0.0000155786,

Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.788853,

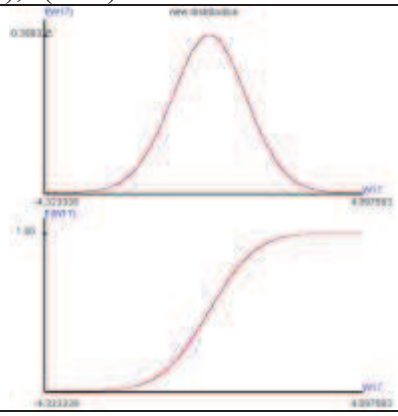
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.082608,

Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.037612,

Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.006871,

(2-5-6) λ =0.4,

The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.00904 Geometrical Mean : none Harmonic Mean : none Variance : 1.05592 S.D. : 1.02758 Skewed Coef. : -0.04167 Kurtosis Coef. : 3.24512 MAD : 0.81228 Range : 13.69390 Mid_range : -0.03794 Median : -0.00290 Q1 : -0.68255 Q2 : -0.00290 Q3 : 0.67181 IQR : 1.35436 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0014614323

********** | W17 distribution function - Z0 distribution function| **********

The almost surely limiting theory

E(| W17 distribution function - Z0 distribution function|^2)=0.0000066799,

Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.904597,

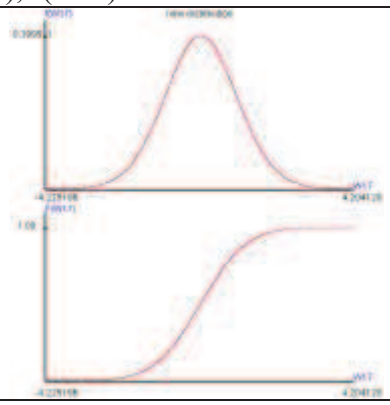
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.174151,

Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.082562,

Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.014510,

(2-5-7) λ =0.5,

The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: 0.00038 Geometrical Mean : none Harmonic Mean : none Variance : 1.05553 S.D. : 1.02739 Skewed Coef. : 0.00080 Kurtosis Coef. : 3.24369 MAD : 0.81216 Range : 14.70574 Mid_range : 0.50126 Median : 0.00003 Q1 : -0.67684 Q2 : 0.00003 Q3 : 0.67754 IQR : 1.35438 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0012815831

********** | W17 distribution function - Z0 distribution function| **********

The almost surely limiting theory

E(| W17 distribution function - Z0 distribution function|^2)=0.0000037921,

Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.520022,

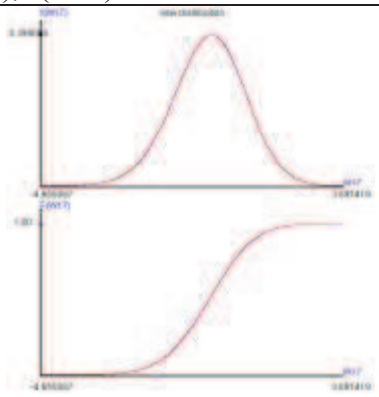
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.408385,

Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.220060,

(2-6)n=50,

(2-6-1) λ=0.001,

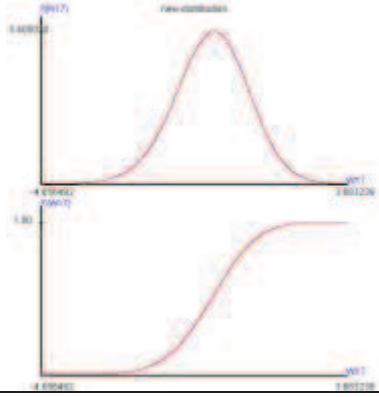
The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.04213 Geometrical Mean : none Harmonic Mean : none Variance : 1.05657 S.D. : 1.02790 Skewed Coef. : -0.18706 Kurtosis Coef. : 3.27364 MAD : 0.81276 Range : 15.11322 Mid_range : -1.73483 Median : -0.01321 Q1 : -0.70373 Q2 : -0.01321 Q3 : 0.65213 IQR : 1.35586 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0048130685
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000645956,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.801715,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.049359,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.005847,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.002769,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.000651,

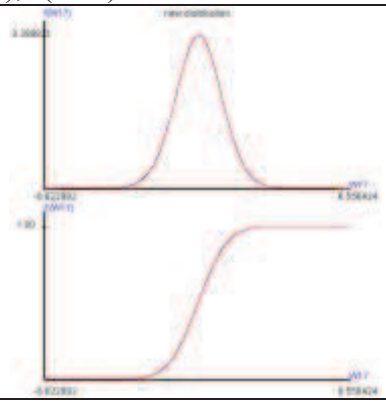
(2-6-2) λ=0.01,

The left and right extremely probability are removing 0.0001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.04153 Geometrical Mean : none Harmonic Mean : none Variance : 1.05663 S.D. : 1.02793 Skewed Coef. : -0.18412 Kurtosis Coef. : 3.27000 MAD : 0.81283 Range : 14.45230 Mid_range : -1.60840 Median : -0.01265 Q1 : -0.70378 Q2 : -0.01265 Q3 : 0.65259 IQR : 1.35637 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0047035207
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000625929,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 0.803315,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.066826,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.006244,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.003025,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.000685,

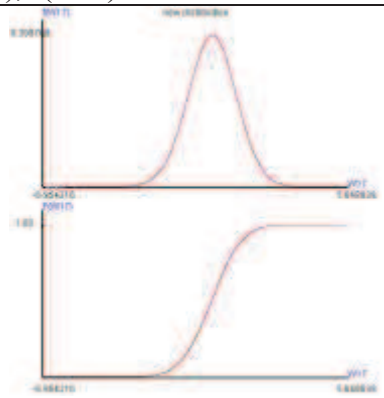
(2-6-3) λ =0.4,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.00810 Geometrical Mean : none Harmonic Mean : none Variance : 1.04384 S.D. : 1.02169 Skewed Coef. : -0.03530 Kurtosis Coef. : 3.18825 MAD : 0.80924 Range : 13.22841 Mid_range : -0.03328 Median : -0.00294 Q1 : -0.68143 Q2 : -0.00294 Q3 : 0.67158 IQR : 1.35300 C.V. : none

E(| W17 distribution - Z0 distribution |^2)= 0.0009329895
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)= 0.0000048840,
Pr(| W17 distribution function - Z0 distribution function|< 0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|< 0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|< 0.0100000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|< 0.0050000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|< 0.0010000000)= 0.207545,
Pr(| W17 distribution function - Z0 distribution function|< 0.0005000000)= 0.080927,
Pr(| W17 distribution function - Z0 distribution function|< 0.0001000000)= 0.018923,

(2-7)n=60,

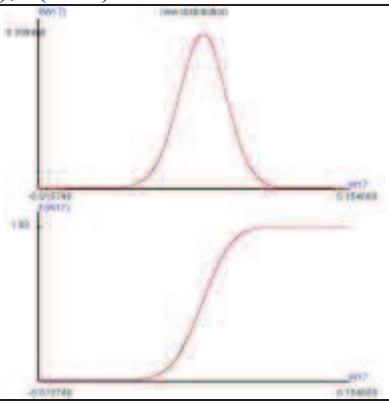
(2-7-1) λ=0.3,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01453 Geometrical Mean : none Harmonic Mean : none Variance : 1.03723 S.D. : 1.01845 Skewed Coef. : -0.06302 Kurtosis Coef. : 3.15956 MAD : 0.80764 Range : 12.64991 Mid_range : -0.65269 Median : -0.00460 Q1 : -0.68537 Q2 : -0.00460 Q3 : 0.66779 IQR : 1.35316 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0009934392
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000094321,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.871952,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.076316,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.049178,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.007225,

(2-8)n=70,

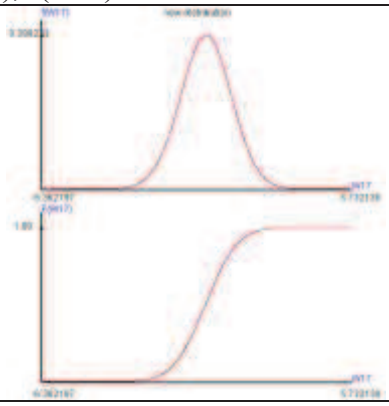
(2-8-1) λ=0.3,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01360 Geometrical Mean : none Harmonic Mean : none Variance : 1.03167 S.D. : 1.01571 Skewed Coef. : -0.05684 Kurtosis Coef. : 3.13703 MAD : 0.80619 Range : 13.02158 Mid_range : -0.33207 Median : -0.00453 Q1 : -0.68478 Q2 : -0.00453 Q3 : 0.66775 IQR : 1.35253 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0007759501
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000080012,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.919120,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.066648,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.044064,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.008999,

(2-9)n=80,

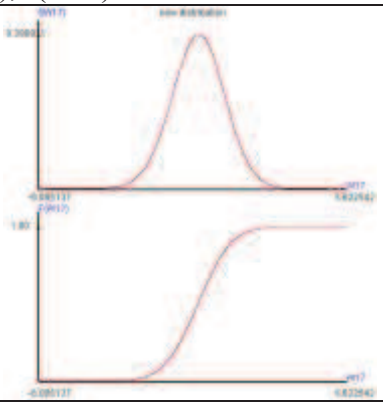
(2-9-1) λ=0.3,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01230 Geometrical Mean : none Harmonic Mean : none Variance : 1.02762 S.D. : 1.01372 Skewed Coef. : -0.05223 Kurtosis Coef. : 3.11676 MAD : 0.80513 Range : 12.13929 Mid_range : -0.31503 Median : -0.00418 Q1 : -0.68335 Q2 : -0.00418 Q3 : 0.66837 IQR : 1.35173 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0006123907
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000065579,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.070467,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.040907,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.010357,

(2-10)n=90,

(2-10-1) λ =0.2,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01778 Geometrical Mean : none Harmonic Mean : none Variance : 1.02534 S.D. : 1.01259 Skewed Coef. : -0.07453 Kurtosis Coef. : 3.10927 MAD : 0.80457 Range : 11.67091 Mid_range : -0.19130 Median : -0.00581 Q1 : -0.68700 Q2 : -0.00581 Q3 : 0.66464 IQR : 1.35164 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0008781972

********** | W17 distribution function - Z0 distribution function| **********

The almost surely limiting theory

E(| W17 distribution function - Z0 distribution function|^2)=0.0000124675,

Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.856004,

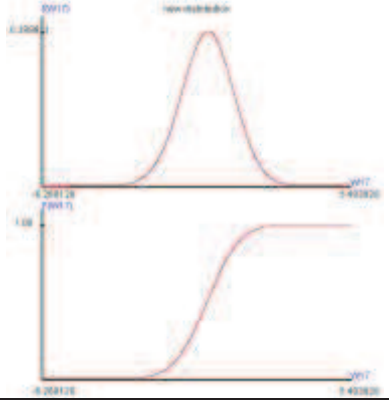
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.020655,

Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.010015,

Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.002481,

(2-11)n=100,

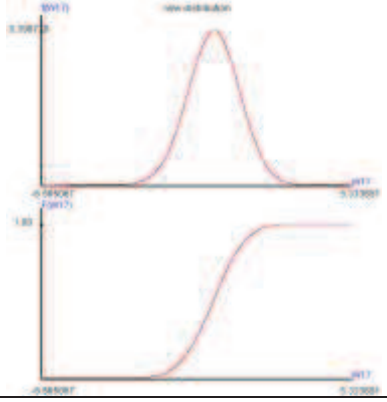
(2-11-1) λ =0.2,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01674 Geometrical Mean : none Harmonic Mean : none Variance : 1.02307 S.D. : 1.01147 Skewed Coef. : -0.06877 Kurtosis Coef. : 3.09688 MAD : 0.80397 Range : 11.79774 Mid_range : -0.38310 Median : -0.00545 Q1 : -0.68627 Q2 : -0.00545 Q3 : 0.66512 IQR : 1.35139 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0007558969
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000112275,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.884186,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.021417,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.009329,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.002415,

(2-12)n=120,

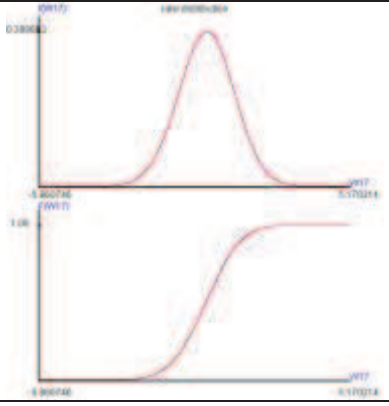
(2-12-1) λ =0.2,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01530 Geometrical Mean : none Harmonic Mean : none Variance : 1.01906 S.D. : 1.00948 Skewed Coef. : -0.06289 Kurtosis Coef. : 3.07755 MAD : 0.80300 Range : 11.94299 Mid_range : -0.61569 Median : -0.00494 Q1 : -0.68530 Q2 : -0.00494 Q3 : 0.66588 IQR : 1.35118 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0005929880
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000091790,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.963197,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.020280,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.008847,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.001652,

(2-13)n=160,

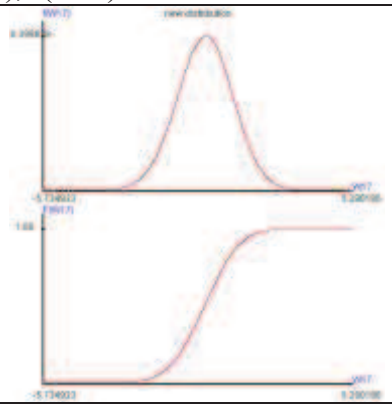
(2-13-1) λ =0.1,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01771 Geometrical Mean : none Harmonic Mean : none Variance : 1.01476 S.D. : 1.00735 Skewed Coef. : -0.07278 Kurtosis Coef. : 3.06274 MAD : 0.80188 Range : 11.17234 Mid_range : -0.39527 Median : -0.00595 Q1 : -0.68663 Q2 : -0.00595 Q3 : 0.66445 IQR : 1.35107 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0006907513
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000120177,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.900877,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.010742,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.004109,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.000624,

(2-14)n=180,

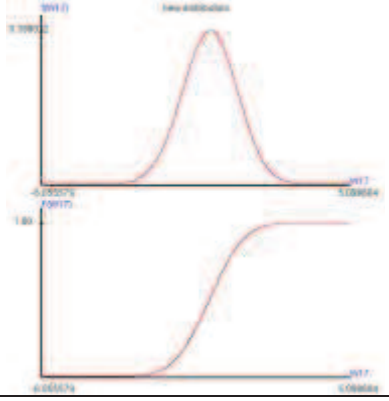
(2-14-1) λ =0.1,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01669 Geometrical Mean : none Harmonic Mean : none Variance : 1.01298 S.D. : 1.00647 Skewed Coef. : -0.06801 Kurtosis Coef. : 3.05711 MAD : 0.80130 Range : 11.06609 Mid_range : -0.22237 Median : -0.00525 Q1 : -0.68593 Q2 : -0.00525 Q3 : 0.66445 IQR : 1.35038 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0006027734
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000107409,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.011744,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.004461,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.000665,

(2-15)n=230,

(2-15-1) λ =0.01,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01849 Geometrical Mean : none Harmonic Mean : none Variance : 1.01130 S.D. : 1.00563 Skewed Coef. : -0.07521 Kurtosis Coef. : 3.04924 MAD : 0.80096 Range : 11.19573 Mid_range : -0.47845 Median : -0.00589 Q1 : -0.68708 Q2 : -0.00589 Q3 : 0.66359 IQR : 1.35067 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0007073336

********** | W17 distribution function - Z0 distribution function| **********

The almost surely limiting theory

E(| W17 distribution function - Z0 distribution function|^2)=0.0000130788,

Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 0.888145,

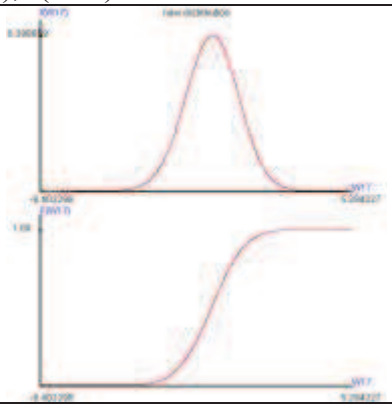
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.008531,

Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.003257,

Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.000439,

(2-16)n=260,

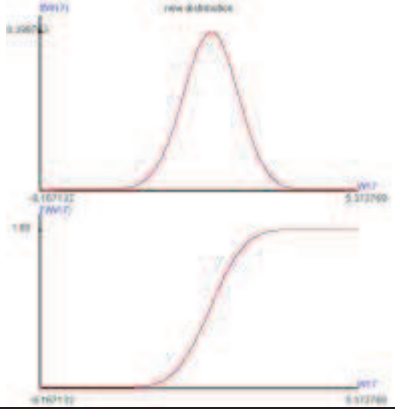
(2-16-1) λ =0.01,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01737 Geometrical Mean : none Harmonic Mean : none Variance : 1.00987 S.D. : 1.00492 Skewed Coef. : -0.07092 Kurtosis Coef. : 3.04237 MAD : 0.80059 Range : 11.74001 Mid_range : -0.55404 Median : -0.00569 Q1 : -0.68617 Q2 : -0.00569 Q3 : 0.66417 IQR : 1.35034 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0006128612
********** | W17 distribution function - Z0 distribution function| **********
The almost surely limiting theory
E(| W17 distribution function - Z0 distribution function|^2)=0.0000112640,
Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,
Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.008909,
Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.003372,
Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.000442,

(2-17)n=310,

(2-17-1) λ =0.001,

f(w17),F(w17)	Ceofficient
	Mathematical Mean: -0.01617 Geometrical Mean : none Harmonic Mean : none Variance : 1.00830 S.D. : 1.00414 Skewed Coef. : -0.06570 Kurtosis Coef. : 3.03447 MAD : 0.80021 Range : 11.58280 Mid_range : -0.39718 Median : -0.00539 Q1 : -0.68538 Q2 : -0.00539 Q3 : 0.66498 IQR : 1.35035 C.V. : none

E(| W17 distribution - Z0 distribution |^2)=0.0005270327

********** | W17 distribution function - Z0 distribution function| **********

The almost surely limiting theory

E(| W17 distribution function - Z0 distribution function|^2)=0.0000098547,

Pr(| W17 distribution function - Z0 distribution function|<0.1000000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0500000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0100000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0050000000)= 1.000000,

Pr(| W17 distribution function - Z0 distribution function|<0.0010000000)= 0.009552,

Pr(| W17 distribution function - Z0 distribution function|<0.0005000000)= 0.003544,

Pr(| W17 distribution function - Z0 distribution function|<0.0001000000)= 0.000476,

Chapter 4. The test statistic and confidence interval

Section 1, One population

1. The test statistic

$$X_1, X_2, ..., X_n \overset{iid}{\sim} Beta(\alpha = \lambda + 1, \beta = 2 - \lambda), \overline{X} = \frac{\sum_{i=1}^{n} X_i}{n},$$

$$H_0 : \lambda = \lambda_0,$$

The large sample,

λ =0.001, n $\geq$ 60,
λ =0.01, n $\geq$ 50,
λ =0.1, n $\geq$ 40,
λ =0.2, n $\geq$ 30,
λ =0.3, n $\geq$ 20,
λ =0.4, n $\geq$ 10,
λ =0.5, n $\geq$ 4,
λ =0.6, n $\geq$ 10,
λ =0.7, n $\geq$ 20,
λ =0.8, n $\geq$ 30,
λ =0.9, n $\geq$ 40,
λ =0.99, n $\geq$ 50,
λ =0.999, n $\geq$ 50,

$$\mu = \frac{\lambda_0 + 1}{3}, \sigma^2 = \frac{(\lambda_0 + 1)(2_0 - \lambda)}{36},$$

the test statistic $Z = \frac{\sqrt{n}(\overline{X} - \mu)}{\sigma}$,

Z ~standard normal distribution.

2. The confidence interval

$$X_1, X_2, ..., X_n \overset{iid}{\sim} Beta(\alpha = \lambda + 1, \beta = 2 - \lambda), \quad S = \sqrt{\frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2}{n-1}},$$

$\hat{\lambda} = 3 \times \overline{X} - 1,$

The large sample,

$\hat{\lambda} = 0.001$, $n \geq 310$,
$\hat{\lambda} = 0.01$, $n \geq 260$,
$\hat{\lambda} = 0.1$, $n \geq 180$,
$\hat{\lambda} = 0.2$, $n \geq 120$,
$\hat{\lambda} = 0.3$, $n \geq 80$,
$\hat{\lambda} = 0.4$, $n \geq 50$,
$\hat{\lambda} = 0.5$, $n \geq 40$,
$\hat{\lambda} = 0.6$, $n \geq 50$,
$\hat{\lambda} = 0.7$, $n \geq 80$,
$\hat{\lambda} = 0.8$, $n \geq 120$,
$\hat{\lambda} = 0.9$, $n \geq 180$,
$\hat{\lambda} = 0.99$, $n \geq 260$,
$\hat{\lambda} = 0.999$, $n \geq 310$,

$(1-\alpha) \times 100\%$ CI. for λ

$$P\left(\left|\frac{\sqrt{n}\left(\overline{X} - \mu\right)}{S}\right| \leq Z_{\alpha/2}\right) = 1 - \alpha, P\left(Z > Z_{\alpha}\right) = \alpha,$$

$$\overline{X} - Z_{\alpha/2}\frac{S}{\sqrt{n}} \leq \mu \leq \overline{X} + Z_{\alpha/2}\frac{S}{\sqrt{n}}$$

$$3\left(\overline{X} - Z_{\alpha/2}\frac{S}{\sqrt{n}}\right) - 1 \leq 3\mu - 1 \leq 3\left(\overline{X} + Z_{\alpha/2}\frac{S}{\sqrt{n}}\right) - 1$$

$$3\left(\overline{X} - Z_{\alpha/2}\frac{S}{\sqrt{n}}\right) - 1 \leq \lambda \leq 3\left(\overline{X} + Z_{\alpha/2}\frac{S}{\sqrt{n}}\right) - 1,$$

Section 2, Two independent populations

There are two independent populations, $Beta(\alpha=\lambda_i+1,\beta=2-\lambda_i),i=1,2$.

1.The test statistic

$$X_{1,1},X_{1,2},...,X_{1,n_1}\overset{iid}{\sim}Beta(\alpha=\lambda_1+1,\beta=2-\lambda_1),\overline{X}_1=\frac{\sum_{j=1}^{n_1}X_{1,j}}{n_1},S_1=\sqrt{\frac{\sum_{j=1}^{n_1}\left(X_{1,j}-\overline{X}_1\right)^2}{n_1-1}},$$

$$X_{2,1},X_{2,2},...,X_{2,n_2}\overset{iid}{\sim}Beta(\alpha=\lambda_2+1,\beta=2-\lambda_2),\overline{X}_2=\frac{\sum_{j=1}^{n_2}X_{2,j}}{n_2},S_2=\sqrt{\frac{\sum_{j=1}^{n_2}\left(X_{2,j}-\overline{X}_2\right)^2}{n_2-1}}$$

(1) $H_0:\lambda_1-\lambda_2=\lambda_0,\lambda_0\neq 0$,

$$\lambda_1=3\mu_1-1,\lambda_2=3\mu_2-1,\lambda_1-\lambda_2=\lambda_0=3(\mu_1-\mu_2),\mu_1-\mu_2=\frac{\lambda_0}{3},$$

$H_0:\mu_1-\mu_2=\frac{\lambda_0}{3}$,

$\hat{\lambda}_i=3\overline{X}_i-1.i=1,2$

The large sample,

$\hat{\lambda}_i$=0.001, $n_i\geq 310$,

$\hat{\lambda}_i$=0.01, $n_i\geq 260$,

$\hat{\lambda}_i$=0.1, $n_i\geq 180$,

$\hat{\lambda}_i$=0.2, $n_i\geq 120$,

$\hat{\lambda}_i$=0.3, $n_i\geq 80$,

$\hat{\lambda}_i$=0.4, $n_i\geq 50$,

$\hat{\lambda}_i$=0.5, $n_i\geq 40$,

$\hat{\lambda}_i$=0.6, $n_i\geq 50$,

$\hat{\lambda}_i$=0.7, $n_i\geq 80$,

$\hat{\lambda}_i$=0.8, $n_i\geq 120$,

$\hat{\lambda}_i$=0.9, $n_i\geq 180$,

$\hat{\lambda}_i$=0.99, $n_i\geq 260$,

$\hat{\lambda}_i$=0.999, $n_i\geq 310$,

the test statistic $Z=\frac{\left(\overline{X}_1-\overline{X}_2-\frac{\lambda_0}{3}\right)}{\sqrt{S_1^2/n_1+S_2^2/n_2}}$,

Z ~standard normal distribution.

(2) $H_0 : \lambda_1 - \lambda_2 = 0$,

$$\lambda_1 = 3\mu_1 - 1, \lambda_2 = 3\mu_2 - 1, \lambda_1 - \lambda_2 = \lambda_0 = 3(\mu_1 - \mu_2), \mu_1 - \mu_2 = 0,$$

$H_0 : \mu_1 - \mu_2 = 0,$

$$\overline{X}_1 = \frac{\sum_{j=1}^{n_1} X_{1,j}}{n_1}, \ \overline{X}_2 = \frac{\sum_{j=1}^{n_2} X_{2,j}}{n_2}, \ \overline{\overline{X}} = \frac{\sum_{j=1}^{n_1} X_{1,j} + \sum_{j=1}^{n_2} X_{2,j}}{n_1 + n_2},$$

$$S_p = \sqrt{\frac{\sum_{j=1}^{n_1} \left(X_{1,j} - \overline{X}_1\right)^2 + \sum_{j=1}^{n_2} \left(X_{2,j} - \overline{X}_2\right)^2}{n_1 + n_2 - 1}}$$

$\hat{\lambda}_i = 3\overline{X}_i - 1.i = 1,2$

The large sample,

$\hat{\lambda}_i$=0.001, $n_i \geq 310$,

$\hat{\lambda}_i$=0.01, $n_i \geq 260$,

$\hat{\lambda}_i$=0.1, $n_i \geq 180$,

$\hat{\lambda}_i$=0.2, $n_i \geq 120$,

$\hat{\lambda}_i$=0.3, $n_i \geq 80$,

$\hat{\lambda}_i$=0.4, $n_i \geq 50$,

$\hat{\lambda}_i$=0.5, $n_i \geq 40$,

$\hat{\lambda}_i$=0.6, $n_i \geq 50$,

$\hat{\lambda}_i$=0.7, $n_i \geq 80$,

$\hat{\lambda}_i$=0.8, $n_i \geq 120$,

$\hat{\lambda}_i$=0.9, $n_i \geq 180$,

$\hat{\lambda}_i$=0.99, $n_i \geq 260$,

$\hat{\lambda}_i$=0.999, $n_i \geq 310$,

the test statistic $Z = \frac{\left(\overline{X}_1 - \overline{X}_2\right)}{\sqrt{S_p^2 / n_1 + S_p^2 / n_{\backslash 2}}}$,

Z ~standard normal distribution.

2. The confidence interval

$\hat{\lambda}_i = 3\overline{X}_i - 1.i = 1,2$

The large sample,

$\hat{\lambda}_i$=0.001, $n_i \geq 310$,

$\hat{\lambda}_i$=0.01, $n_i \geq 260$,

$\hat{\lambda}_i$=0.1, $n_i \geq 180$,

$\hat{\lambda}_i$=0.2, $n_i \geq 120$,

$\hat{\lambda}_i$=0.3, $n_i \geq 80$,

$\hat{\lambda}_i$=0.4, $n_i \geq 50$,

$\hat{\lambda}_i$=0.5, $n_i \geq 40$,

$\hat{\lambda}_i$=0.6, $n_i \geq 50$,

$\hat{\lambda}_i$=0.7, $n_i \geq 80$,

$\hat{\lambda}_i$=0.8, $n_i \geq 120$,

$\hat{\lambda}_i$=0.9, $n_i \geq 180$,

$\hat{\lambda}_i$=0.99, $n_i \geq 260$,

$\hat{\lambda}_i$=0.999, $n_i \geq 310$,

$(1-\alpha)\times 100\%$ CI. for $\lambda_1 - \lambda_2$

$$P\left(\left|\frac{\left(\overline{X}_1 - \overline{X}_2 - \frac{(\lambda_1 - \lambda_2)}{3}\right)}{\sqrt{S_1^2/n_1 + S_2^2/n_2}}\right| \leq Z_{\alpha/2}\right) = 1-\alpha, P(Z > Z_\alpha) = \alpha,$$

$$\overline{X}_1 - \overline{X}_2 - Z_{\alpha/2}\sqrt{S_1^2/n_1 + S_2^2/n_2} \leq \frac{\lambda_1 - \lambda_2}{3} \leq \overline{X}_1 - \overline{X}_2 + Z_{\alpha/2}\sqrt{S_1^2/n_1 + S_2^2/n_2}$$

$$3\left(\overline{X}_1 - \overline{X}_2 - Z_{\alpha/2}\sqrt{S_1^2/n_1 + S_2^2/n_2}\right) - 1 \leq \lambda_1 - \lambda_2 \leq 3\left(\overline{X}_1 - \overline{X}_2 + Z_{\alpha/2}\sqrt{S_1^2/n_1 + S_2^2/n_2}\right)$$

Chapter 5. Goodness of fit

Section 1, λ is known

Let $X_1, X_2, ..., X_n \overset{iid}{\sim} Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$,

H_0 :X~ $Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$ and λ is known,

H_1 :against H_0,

<u>The test process</u>

The frequency distribution setting,

(i)The class number and the probability of each class

The class number= $k = log_2(n) + 1$, each class probability is setting to $\frac{1}{k}$.

(ii)The class limit

The first class lower limit=0 and the last class upper limit=1.

$$F_X(c_j;\lambda) = \int_0^{c_j} \frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)} x^{\lambda}(1-x)^{1-\lambda}\,dx = \frac{j}{n},\ j = 1,2,...,k-1,$$

The first class upper limit= c_1 =the second class lower limit,…..,

The $j-th$ class upper limit= c_j =the $(j+1)-th$ class lower limit, $j = 1,2,...,k-1$.

(iii)The frequency table for testing and computing the observed number and expected number

class	class limit	frequency= O	$E = n \times \frac{1}{k}$
1	$0 \sim c_1$	O_1	E_1
2	$c_1 \sim c_2$	O_2	E_2
…			
k	$c_{k-1} \sim 1$	O_k	E_k

The chi square test statistic,

$\chi^2_{k-1} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} > \chi^2_{\alpha,k-1}$, rejected H_0.

Section 2, λ is unknown

Let $X_1, X_2, ..., X_n \overset{iid}{\sim} Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$,

H_0 :X~ $Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$ and λ is unknown.

H_1 :against H_0,

$\hat{\lambda} = 3\overline{X} - 1$,

<u>The test process</u>

The frequency distribution setting:

(i)The class number and the probability of each class

The class number= $k = log_2(n) + 1$, each class probability is setting to $\frac{1}{k}$.

(ii)The class limit

The first class lower limit=0 and the last class upper limit=1.

$$F_X(c_j; \hat{\lambda}) = \int_0^{c_j} \frac{2}{\Gamma(\hat{\lambda}+1)\Gamma(2-\hat{\lambda})} x^{\hat{\lambda}}(1-x)^{1-\hat{\lambda}} dx = \frac{j}{n}, j = 1,2,..., k-1,$$

The first class upper limit= c_1 =the second class lower limit,…..,

The $j-th$ class upper limit= c_j =the $(j+1)-th$ class lower limit, $j = 1,2,..., k-1$.

(iii)The frequency table for testing and computing the observed number and expected number

class	class limit	frequency= O	$E = n \times \frac{1}{k}$
1	$0 \sim c_1$	O_1	E_1
2	$c_1 \sim c_2$	O_2	E_2
…			
k	$c_{k-1} \sim 1$	O_k	E_k

The chi square test statistic,

$\chi^2_{k-2} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} > \chi^2_{\alpha,k-2}$, rejected H_0.

Chapter 6. The comparison of special Beta distribution and Continuous Bernoulli distribution

Section1, The probability distribution, E(X) and Var(X) and diagram

(1) Probability density function and Cumulative probability distribution function

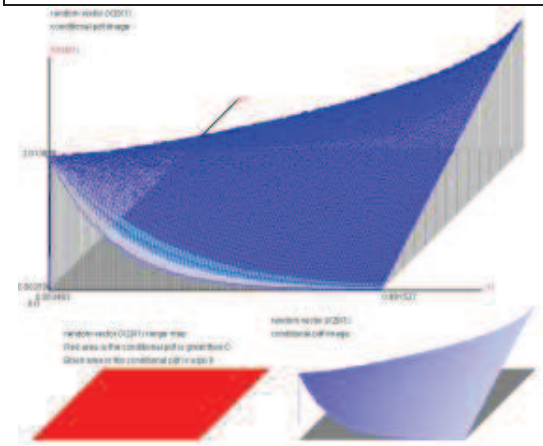
$X \sim Beta(\alpha = \lambda+1, \beta = 2-\lambda)$	$X \sim CB(\lambda),$
$f_X(x;\lambda) = \frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)} x^{\lambda}(1-x)^{1-\lambda},$ $0 \le x \le 1, 0 < \lambda < 1,$	$f_X(x;\lambda) = C(\lambda)\lambda^x(1-\lambda)^{1-x},$ $0 \le x \le 1, 0 < \lambda < 1,$ $C(\lambda) = \begin{cases} \frac{2\tanh^{-1}(1-2\lambda)}{1-2\lambda}, \lambda \neq \frac{1}{2} \\ 2, \lambda = \frac{1}{2} \end{cases}$
$F_X(x;\lambda)$ $= \int_0^x \frac{2}{\Gamma(\lambda+1)\Gamma(2-\lambda)} x^{\lambda}(1-x)^{1-\lambda} dx$	$F_X(x;\lambda) = \begin{cases} \frac{\lambda^x(1-\lambda)^{1-x}+\lambda-1}{2\lambda-1}, \lambda \neq \frac{1}{2}, \\ x, \lambda = \frac{1}{2} \end{cases}$ $0 < x < 1$

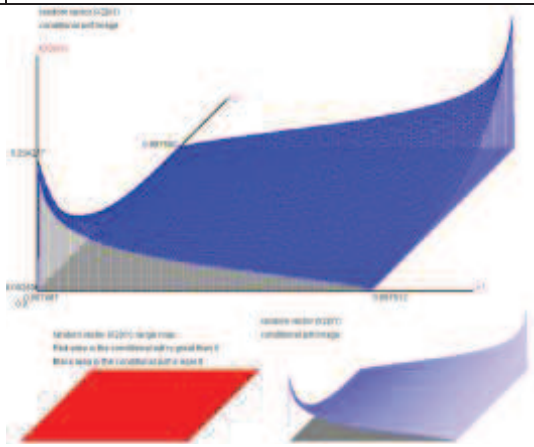
(2)The E(X) and Var(X)

$X \sim Beta(\alpha = \lambda+1, \beta = 2-\lambda)$	$X \sim CB(\lambda),$
$E(X) = \frac{\lambda+1}{3},$	$E(X) = \begin{cases} \frac{\lambda}{2\lambda-1} + \frac{1}{2\tan^{-1}(1-2\lambda)} & if\ \lambda \neq \frac{1}{2} \\ \frac{1}{2} & if\ \lambda = \frac{1}{2} \end{cases}$
$Var(X) = \frac{(\lambda+1)(2-\lambda)}{36},$	$Var(X)$ $= \begin{cases} \frac{(1-\lambda)\lambda}{(1-2\lambda)^2} + \frac{1}{\left(2\tan^{-1}(1-2\lambda)\right)^2} & if\ \lambda \neq \frac{1}{2} \\ \frac{1}{12} & if\ \lambda = \frac{1}{2} \end{cases}$
$E(X^k) = \frac{2}{\Gamma(\lambda+1)} \frac{\Gamma(\lambda+k+1)}{\Gamma(3+k)}, k > 0$	$E(X^k)$ is not existed.

(3)The diagram of The diagram of (X1=λ, *f*(X2|X1)) = *f*(X|λ))

$X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$	$X \sim CB(\lambda),$





When $\lambda = 0.5,$

$$f_X(x;\lambda) = \frac{8}{\pi}\sqrt{0.25-(x-0.5)^2},$$

the semi circle distribution when
$R = 0.5, \mu = 0.5, 0 < x < 1,$

When $\lambda = 0.5,$

$$f_X(x;\lambda) = 1,$$

the Uniform distribution when
$\alpha = 0, \beta = 1, 0 < x < 1,$

The special Beta distribution pdf is changed in smoothing and Continuous Bernoulli distribution pdf has big wave when λ is from small to large.

Section2, The sufficient statistic and pointer estimator of parameter

(1) The sufficient statistic

$X \sim Beta(\alpha = \lambda+1, \beta = 2-\lambda)$	$X \sim CB(\lambda),$
$X_1, X_2, ..., X_n \overset{iid}{\sim} Beta(\alpha = \lambda+1, \beta = 2-\lambda),$	$X_1, X_2, ..., X_n \overset{iid}{\sim} CB(\lambda),$
The sufficient of λ is not existed.	The sufficient of λ is $\sum_{i=1}^{n} X_i$, $\sum_{i=1}^{n} X_i$ ~Continuous Binomial distribution.

Note: Continuous Binomial distribution

$X \sim$ Continuous Binomial distribution(λ),

$X_1, X_2, ..., X_n \overset{iid}{\sim}$ Uniform($\alpha = 0, \beta = 1$),

$X = X_1 + X_2 + ... + X_n, h(x)$ is irwin-hall distribution and parameter n.

The pdf of Continuous Binomial distribution (λ) is

$$f_X(x; \lambda, n) = h(x)(C(\lambda))^n \lambda^x (1-\lambda)^{n-x}, 0 \le x \le n, 0 < \lambda < 1.$$

and $X = \sum_{i=1}^{n} X_i \xrightarrow[n\to\infty]{} Normal(E(X) = n\mu, Var(X) = n\sigma^2)$.

$$\mu = E(X) = \begin{cases} \frac{\lambda}{2\lambda - 1} + \frac{1}{2\tan^{-1}(1-2\lambda)} & if\ \lambda \neq \frac{1}{2} \\ \frac{1}{2} & if\ \lambda = \frac{1}{2} \end{cases}$$

$$\sigma^2 = Var(X) = \begin{cases} \frac{(1-\lambda)\lambda}{(1-2\lambda)^2} + \frac{1}{\left(2\tan^{-1}(1-2\lambda)\right)^2} & if\ \lambda \neq \frac{1}{2} \\ \frac{1}{12} & if\ \lambda = \frac{1}{2} \end{cases}$$

(2)The point estimator of λ

$X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$	$X \sim CB(\lambda),$
$X_1, X_2, ..., X_n \overset{iid}{\sim} Beta(\alpha = \lambda + 1, \beta = 2 - \lambda),$	$X_1, X_2, ..., X_n \overset{iid}{\sim} CB(\lambda),$
UMVUE and MLE are not exited.	UMVUE and MLE are not exited.
MME, $\hat{\lambda}$ =3× $\overline{X}$ -1,	The point estimator equation,
	$\hat{\lambda} = \phi(\overline{X}),$

Note:

X= -0.596698+2.193196× $\overline{X}$,

$\phi(\overline{X})$=0.49997386580423608+1.36802409685464270*(X-0.5056)^1+
-0.00092474767006933 6890*(X-0.5056)^2+-2.73607823707760640*(X-0.5056)^3+
0.095109043642878532*(X-0.5056)^4+5.7483773675921839*(X-0.5056)^5+
-1.8419988453388214*(X-0.5056)^6+-12.357242575206328*(X-0.5056)^7+
16.361405849456787*(X-0.5056)^8+26.41792850010097*(X-0.5056)^9+
-80.02126121520996*(X-0.5056)^10+-48.621550429612398*(X-0.5056)^11+
228.76872253417969*(X-0.5056)^12+64.702439151704311*(X-0.5056)^13+
-380.75874328613281*(X-0.5056)^14+-51.895506033673882*(X-0.5056)^15+
341.66360473632812*(X-0.5056)^16+18.360968290828168*(X-0.5056)^17+
-127.70810317993164*(X-0.5056)^18,

ANOVA

Source	df	SS	MS
Regression	18	83.0834922851	4.6157495714
Error	980	0.0000077149	0.0000000079
Total	998	83.0835000000	

H0:slope1=....=slope18=0, test statistic=586328245.808614, p value=0.000000,
sample size=999, R2=1.000000, R2(adj)=1.000000,MSE=0.000000,

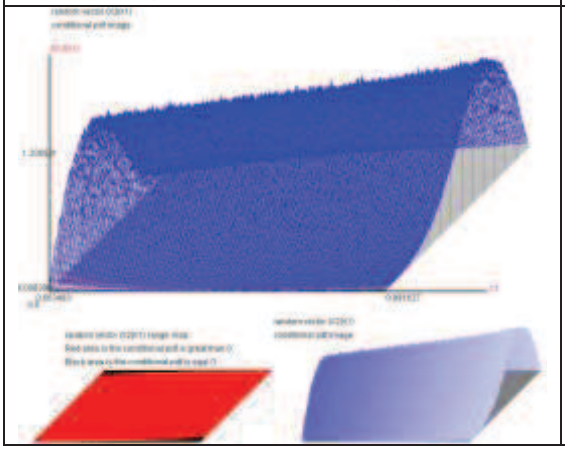
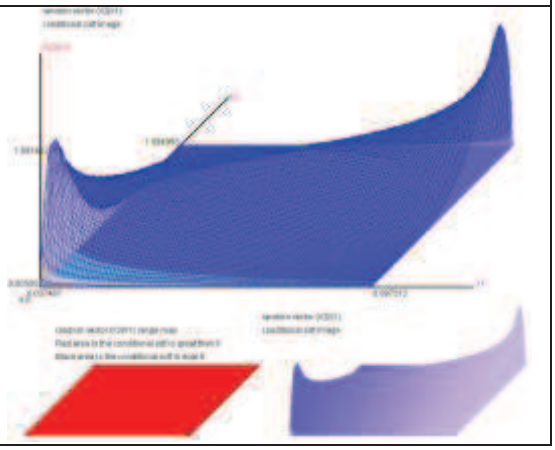
(3) The sampling distribution of $\sum_{i=1}^{n} X_i$

$X_1, X_2, ..., X_n \overset{iid}{\sim} Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$, $X_1, X_2, ..., X_n \overset{iid}{\sim} CB(\lambda)$,

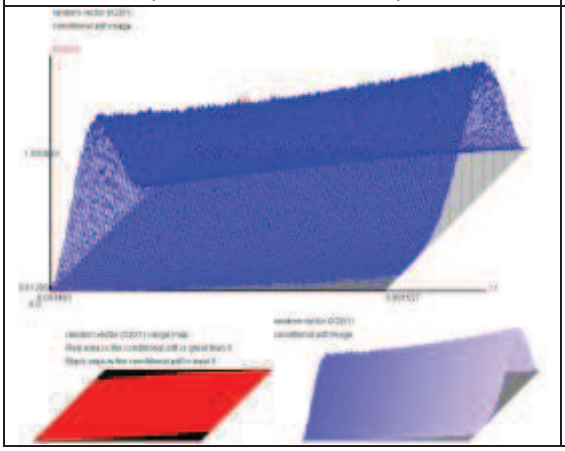
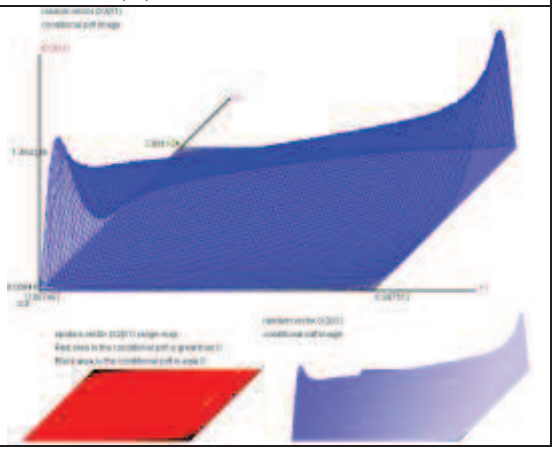
The pdf of $\sum_{i=1}^{n} X_i$ is unkown, $\sum_{i=1}^{n} X_i$ ~Continuous Binomial distribution,

Let X2=$\sum_{i=1}^{n} X_i$, the diagram is (X1=λ, *f*(X2|X1)),

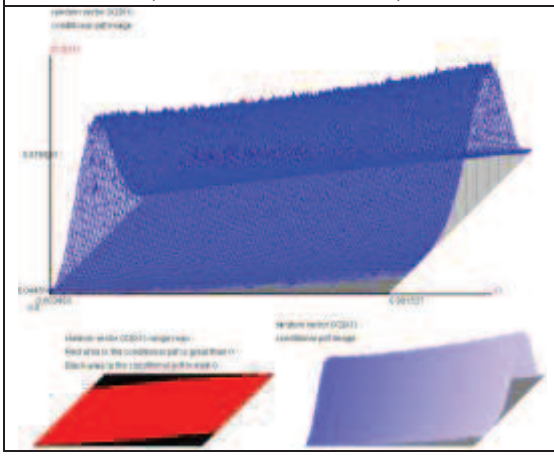
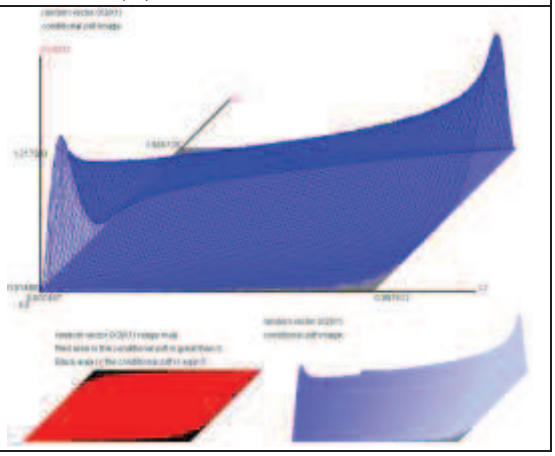
n=2

$X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$	$X \sim CB(\lambda)$,
	

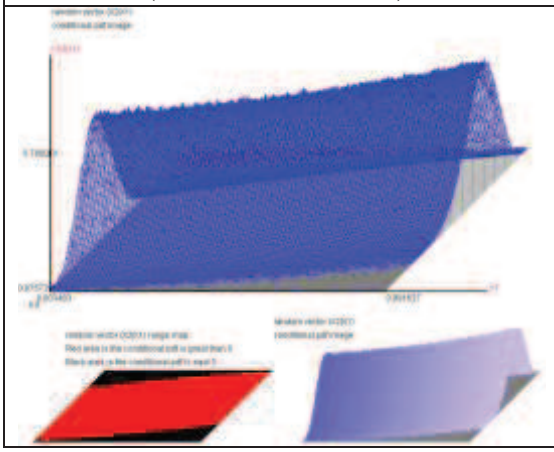
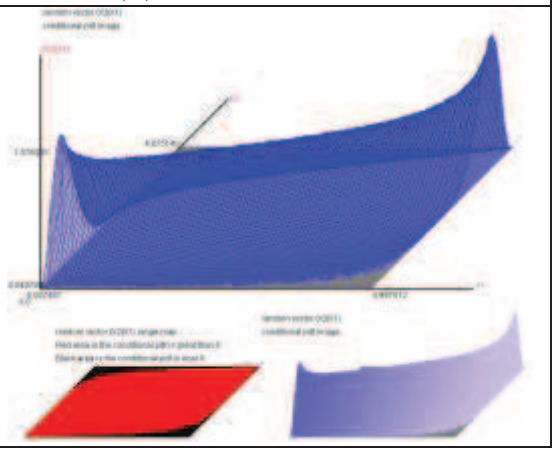
n=3

$X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$	$X \sim CB(\lambda)$,
	

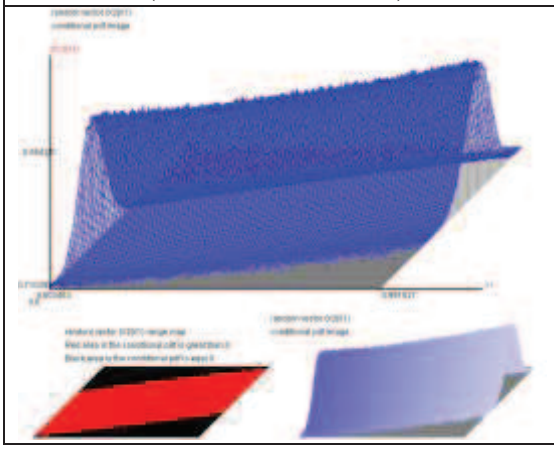
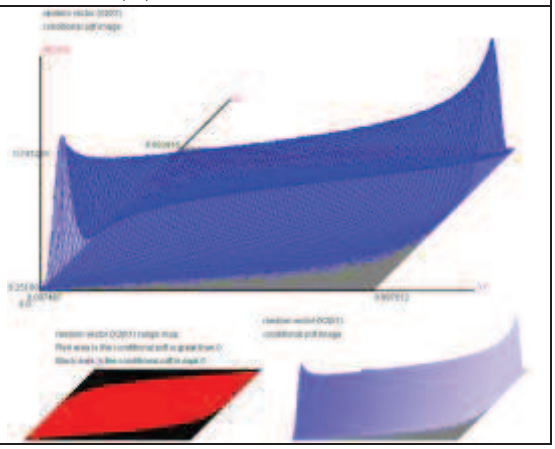
n=4

$X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$	$X \sim CB(\lambda)$,
	

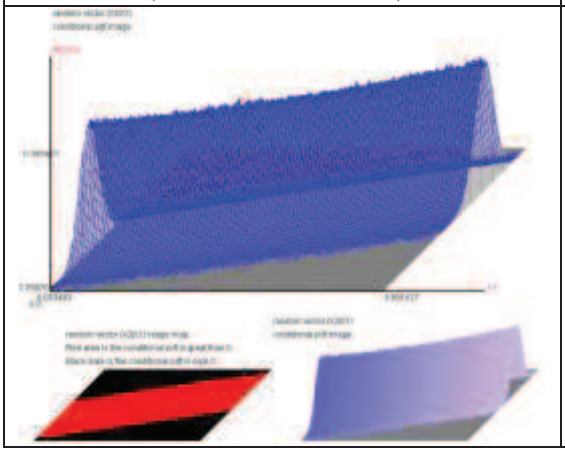
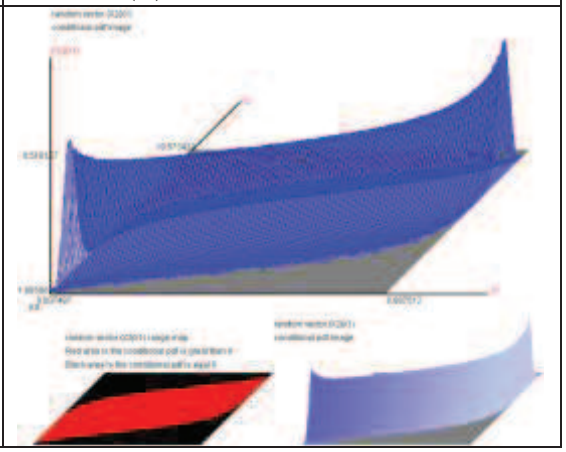
n=5

$X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$	$X \sim CB(\lambda)$,
	

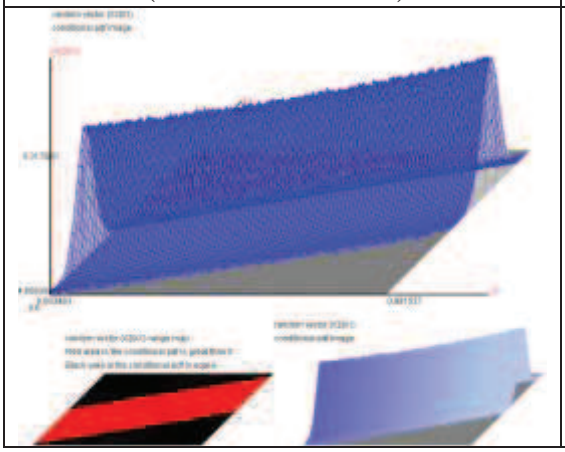
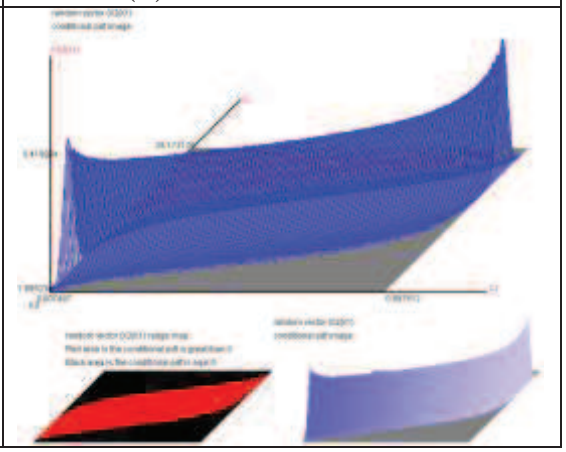
n=10

$X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$	$X \sim CB(\lambda)$,
	

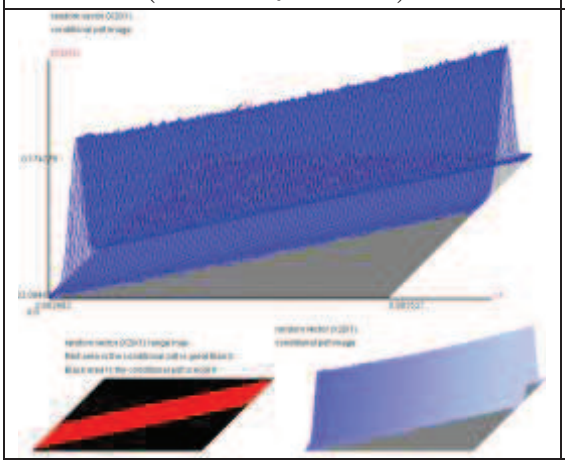
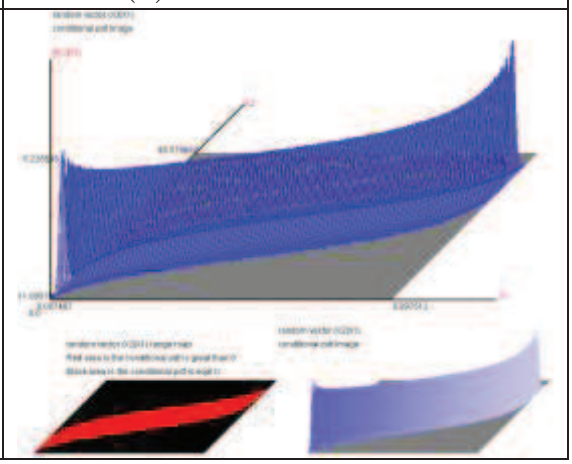
n=20

$X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$	$X \sim CB(\lambda)$,
	

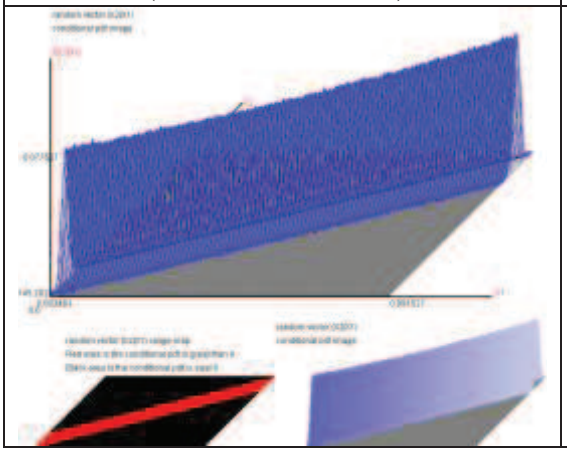
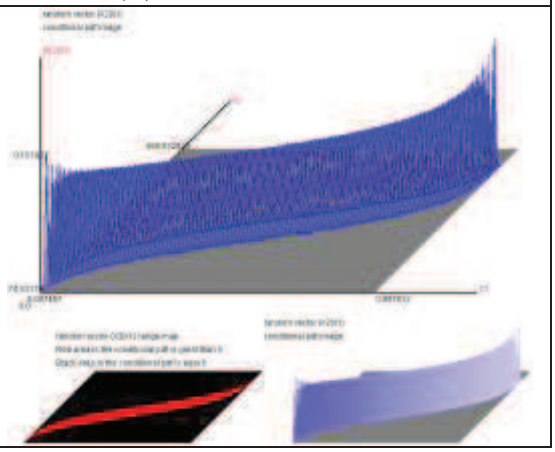
n=30

$X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$	$X \sim CB(\lambda)$,
	

n=100

$X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$	$X \sim CB(\lambda)$,
	

n=500

$X \sim Beta(\alpha = \lambda + 1, \beta = 2 - \lambda)$	$X \sim CB(\lambda),$
	

$$\sum_{i=1}^{n} X_i \xrightarrow[n\to\infty]{} Normal\left(E\left(\sum_{i=1}^{n} X_i \right), Var\left(\sum_{i=1}^{n} X_i \right) \right).$$