A LOWER LIMIT ΔH^Z_{vap} FOR THE LATENT HEAT OF VAPORIZATION ΔH_{vap} WITH RESPECT TO THE PRESSURE AND THE VOLUME CHANGE OF THE PHASE TRANSITION.

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ABSTRACT. We derive a lower limit ΔH^Z_{vap} for the latent heat of vaporization ΔH_{vap} with respect to the pressure and the volume change of the phase transition from the study of a heat engine using water as working fluid with an infinitesimal variation of the temperature δT and an infinitesimal variation of the pressure δP and in the vanishing limit of the massive flow rate Q_m. We calculate the latent heat index $h^Z = \Delta H^Z_{vap}/\Delta H_{vap}$ for few gas at $P = 100\ kPa$.

We derive a lower limit expression for the latent heat of vaporization ΔH_{vap} from the study of a heat engine using water as working fluid with an infinitesimal variation of the temperature δT and an infinitesimal variation of the pressure δP and in the vanishing limit of the massive flow rate Q_m:

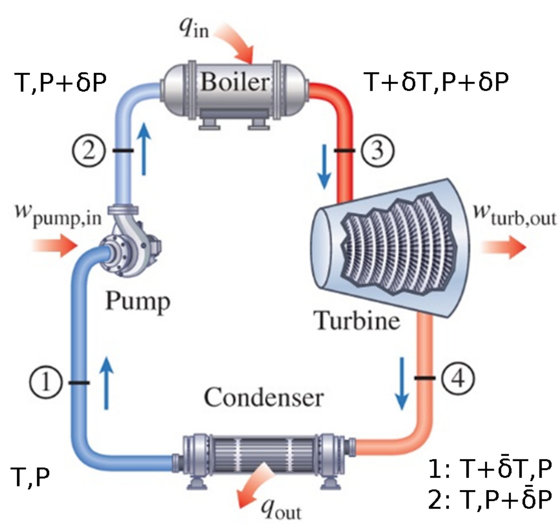

FIGURE 1. Heat engine with $\bar{\delta T} < \delta T$ and $\bar{\delta} P < \delta P$.

Case: 2: $\bar{\delta P} = 0$

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The efficiency of that heat engine is:

$$\eta = 1 - \frac{T}{T+\delta T} = \frac{\delta T}{T} + \mathcal{O}\left(\delta^2 T\right)$$

Since the irreversible processes are negligible with infinitesimal variations for the temperature and the volume and .

The input power of that heat engine is: $P_{in} = Q_m \Delta H_{vap} + \mathcal{O}\left(\delta P\right)$

The output power of that heat engine is: $P_{out} = \frac{Q_m}{\rho_g}\delta P$

Therefore, we derive the efficiency of that heat engine in a second way:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\delta P}{\rho_g \Delta H_{vap}} + \mathcal{O}\left(\delta^2 P\right)$$

Finally the case 1 is impossible since it breaks the well known Clausius–Clapeyron relation:

$$\frac{\delta P}{\delta T} = \frac{\Delta H_{vap}}{T(\nu_g - \nu_l)}$$

Case: 1: $\bar{\delta} T = 0$

We start from $\bar{\delta} P < \delta P$:

$$1 > \frac{\bar{\delta} P}{\delta P} = \frac{\delta T}{\delta P}\frac{\bar{\delta} P}{\delta T} = \frac{T(\nu_g - \nu_l)}{\Delta H_{vap}}\frac{\bar{\delta} P}{\delta T}$$

From there, we express $\frac{\bar{\delta} P}{\delta T}$ with respect to $\frac{\delta V}{\delta P}$ with the help of the following equation of state:

$$\delta V = \left(\frac{\partial V}{\partial T}\right)_P \delta T + \left(\frac{\partial V}{\partial P}\right)_T \bar{\delta} P = V\left(\alpha \delta T - \beta_T \bar{\delta} P\right)$$

$$\frac{\bar{\delta} P}{\delta V} = \frac{P\alpha}{\frac{P}{V}\frac{\delta V}{\delta P} + P\beta_T}$$

$$\frac{\bar{\delta} P}{\delta V} = \frac{P}{T}\frac{\gamma_a T \alpha}{\gamma_a P \beta_T - 1}$$

, where γ_a is the adiabatic index such that $V\bar{\delta}P = -\gamma_a P\delta V$.

Finally, we derive a lower limit expression for the latent heat of vaporization ΔH_{vap}:

$$\Delta H_{vap}^{Z} = \frac{\gamma_a T\alpha}{\gamma_a P\beta_T - 1} P\left(\nu_g - \nu_l\right) < \Delta H_{vap}$$

We can define some latent heat index h^Z ranging between 0 and 1:

$$h^Z = \frac{\Delta H_{vap}^{Z}}{\Delta H_{vap}} = \frac{\gamma_a T\alpha}{\gamma_a P\beta_T - 1} \frac{P(\nu_g - \nu_l)}{\Delta H_{vap}}$$

From the above equation and the definition of an adiabatic expansion:

$$\frac{dV}{V} = \alpha dT - \beta_T dP$$
$$-P\frac{dV}{V} = \left(c_P - P\alpha\right) dT + \left(P\beta_T - T\alpha\right) dP$$

, we derive an expression for the adiabatic index γ_a with respect to the heat capacity ratio γ:

$$\gamma_a = -\frac{P}{V}\frac{dV}{P} = \frac{1}{P\beta_T - \frac{P}{c_p}T\alpha^2} = \frac{\gamma}{P\beta_T}$$

To conclude the theoretical part, we develop γ_a with respect to the heat capacity ratio γ inside the previous results:

$$\Delta H_{vap}^{Z} = \frac{\gamma}{\gamma - 1}\frac{T\alpha}{P\beta_T} P\left(\nu_g - \nu_l\right) < \Delta H_{vap}$$

$$h^Z = \frac{\Delta H_{vap}^{Z}}{\Delta H_{vap}} = \frac{\gamma}{\gamma - 1}\frac{T\alpha}{P\beta_T}\frac{P(\nu_g - \nu_l)}{\Delta H_{vap}}$$

To conclude this paper, we calculate the latent heat index h^Z for the following gas:

$$h_{H_2O}^{Z} = 1/2256540 \times 1.69402 \times 100000 \times 1.324/0.324 = 0.306774$$

$$h_{CO_2}^{Z} = 1/574000/1.977 \times 100000 \times 1.310/0.310 = 0.372384$$

$h^Z_{N_2O_4} = 1/415000/2.853 \times 100000 \times 1.262/0.262 = 0.4068262$

$h^Z_{C_2H_6O} = 1/855000/1.627 \times 100000 \times 1.135/0.135 = 0.604378$

$h^Z_{He} = 1/20500/16.9 \times 100000 \times 1.66/0.66 = 0.725978$

, at $P = 100\ kPa$ with the approximations: $v_g - v_l \cong v_g$ and $T\alpha \cong P\beta_T \cong 1$.

References

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