MILLIKAN'S OIL DROP EXPERIMENT

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Abstract

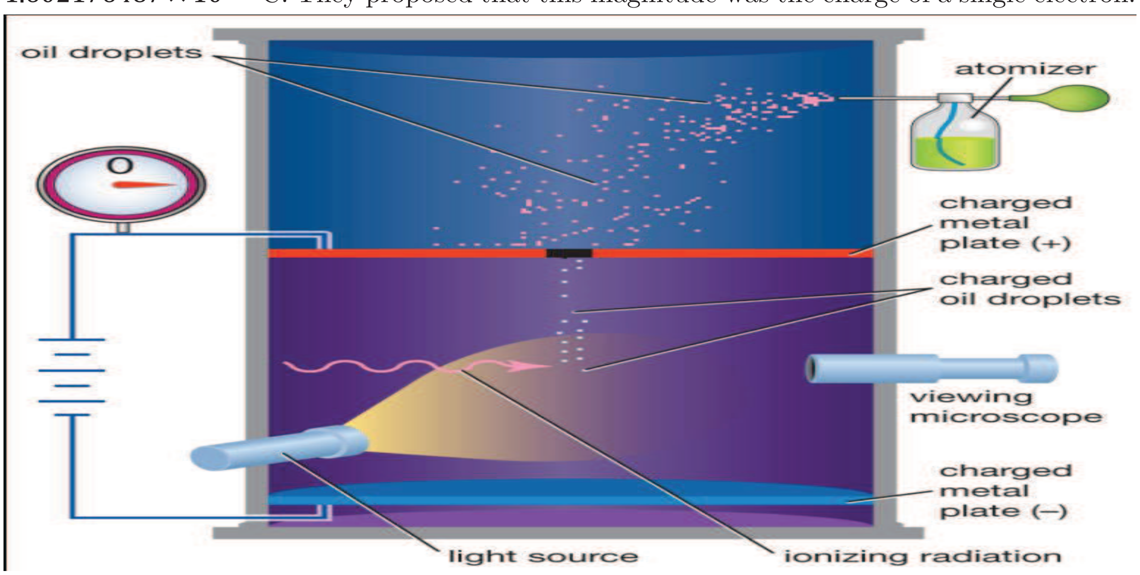
In this paper we will discuss Millikan's oil drop experiment in a detailed way. Here We see combination of many different fields of Physics such as Particle Physics, Electrodynamics, Fluid Mechanics etc...We verify the theory using interactive simulations and MATLAB.

RESOURCES USED

- oPhysics simulation - https://ophysics.com/em2.html
- https://iwant2study.org/lookangejss/06QuantumPhysics/ejss_model_millikan/
- MS Word, LaTeX, MATLAB

HISTORY

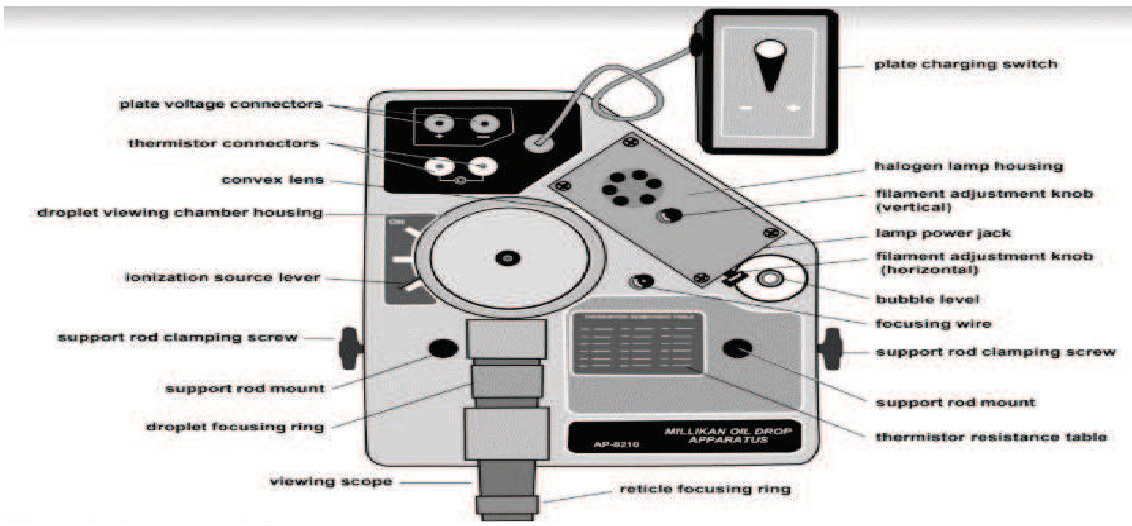
In 1909, Robert Millikan and Harvey Fletcher held the oil drop experiment to determine the charge of an electron. By balancing downward gravitational force with upward drag and electric forces, they suspended tiny charged droplets (particles) of oil between two metal electrodes. Millikan and Fletcher could determine the droplets' masses from their observed radii (since from the radii they could calculate the volume and in effect, the mass) since the density of the oil was a known quantity. When the oil droplets were in mechanical equilibrium Using the values of electric field, gravity and mass, Millikan and Fletcher determined charge on them. By repeating the experiment, they confirmed that the charges were all integral multiples of some fundamental value. They calculated this fundamental value to be 1.5924×10^{-19}Coulombs (C), which showed only 1% of deviation from the currently accepted value of $1.602176487 \times 10^{-19}$C. They proposed that this magnitude was the charge of a single electron.



Source:https://cdn.britannica.com/82/22482-050-40927428/oil-drop-experiment-Millikan-series-multiples-experiments-American-1910.jpg

APPARATUS

A squeezable pressure pump is attached outside to the Millikan Apparatus for atomising oil droplets. Inside the apparatus there are two parallel metal plates which is connected to a voltmeter. This is useful in supplying electric field. di A ring of insulating material was used to hold the plates apart. Four holes were cut into the ring—three for illumination by a bright light and another to allow viewing through a microscope of appreciable focal length.



Source: https://hepweb.ucsd.edu/2dl/pasco/MillikansOilDropManual (AP-8210).pdf

THEORY

When we squeeze the pump the oil droplets move inside the plate. Suppose it carries negative charge, so it's attracted to positive plate. Then electric force $\vec{F_E}$ acts in upward direction. We know there is a force of gravity $\overrightarrow{F_g}$ due to mass of the oil droplet.

$$q\vec{E} = mg$$

Electric field between two parallel plates, $\vec{E} = \frac{V}{d}$ in NC^{-1}

$$\text{So, } q = \frac{mgd}{V}$$

$$\overrightarrow{F_g} = \frac{4}{3}\pi g r^3 \rho_{oil}$$

Some other forces acting on the particle are buoyant force $\vec{F_B}$ and drag force $\vec{F}_{drag}$

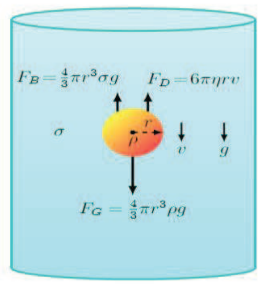
$$\overrightarrow{F_B} = \frac{4}{3}\pi g r^3 \rho_{air}$$

$$\vec{F}_{drag} = 6\pi\eta r v \qquad \text{Where } \eta \text{ is viscosity of air}$$

These two forces are acting upward and it resists the downward motion of particle. At a certain point when the particle reaches equilibrium, it stops moving and start accelerating and acquires a constant velocity, v_t

At equilibrium, $\overrightarrow{F_B} + \vec{F}_{drag} = \overrightarrow{F_g}$

We get terminal velocity, $v_t = \frac{2r^2\rho_{air} - \rho_{oil}}{9\eta}$ in ms^{-1}



Drag force F_D, buoyant force F_B and gravitational force F_G acting on a sphere of radius r and density ρ moving with velocity v in a fluid of density σ and viscosity η.

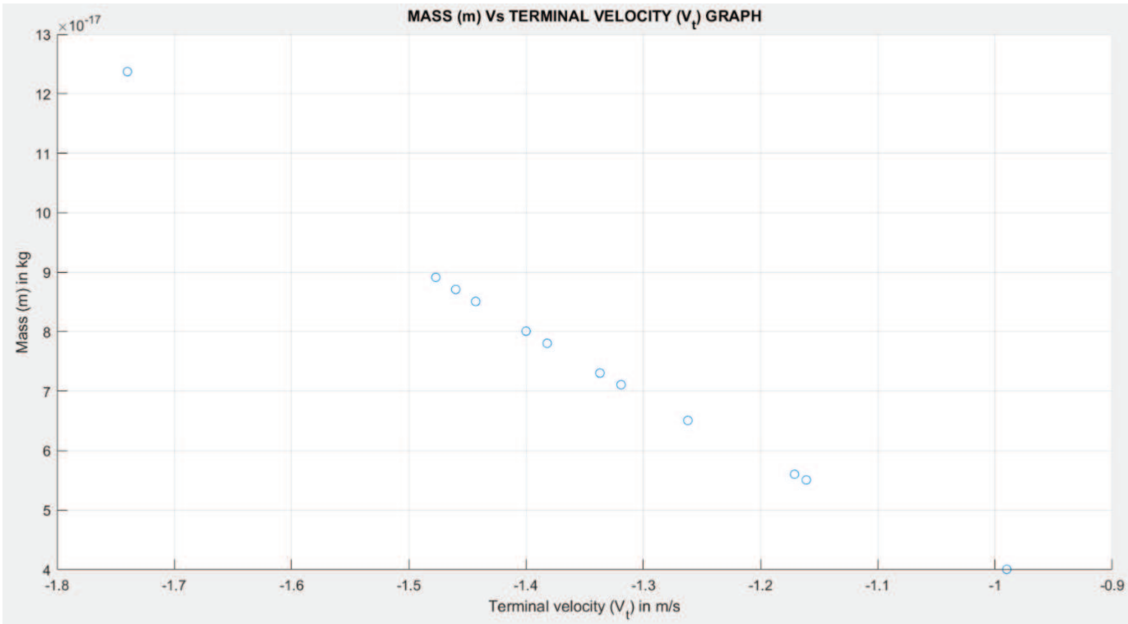
Source: https://www.concepts-of-physics.com/mechanics/stokes-law-and-terminal-velocity.php

EXPERIMENTAL PROCEDURE

1. Spray a fine mist of oil droplets into a chamber above the plates by squeezing the pump. The oil drops become electrically charged through friction with the nozzle as they are sprayed. Alternatively, charge can be induced by including an ionizing radiation source (such as an X-ray tube).
2. A uniform electric field is created in the space between a parallel pair of horizontal metal plates, by applying a potential difference. Motion of droplets can be controlled by changing the voltage.
3. Look inside the apparatus through the hole (opening) of microscope simultaneously follow the procedures listed below.
4. Initially, the electric field between the plates is turned off (zero). The oil droplets will fall down to the plate since they quickly reach terminal velocity due to upward air drag.
5. Turn on the electric field between the plates to a certain amount. If it is high enough, some of the oil droplets move upward under the influence of electric force opposing the downward gravitational force.
6. An identically looking drop is selected and kept in the middle of the line of sight by alternately switching off the voltage until all the other drops fell. The experiment is further continued with this single drop.
7. The oil droplet stops it motion in the middle of line of sight of electric field. This is due to the fact that both the upward electric force as well as downward gravitational force are equal and opposite. Thus, the charged particle is in mechanical equilibrium.
8. Using the values of electric field, gravity and mass, (density and volume found from radii of the oil droplet gives mass) determine the magnitude of the charge.

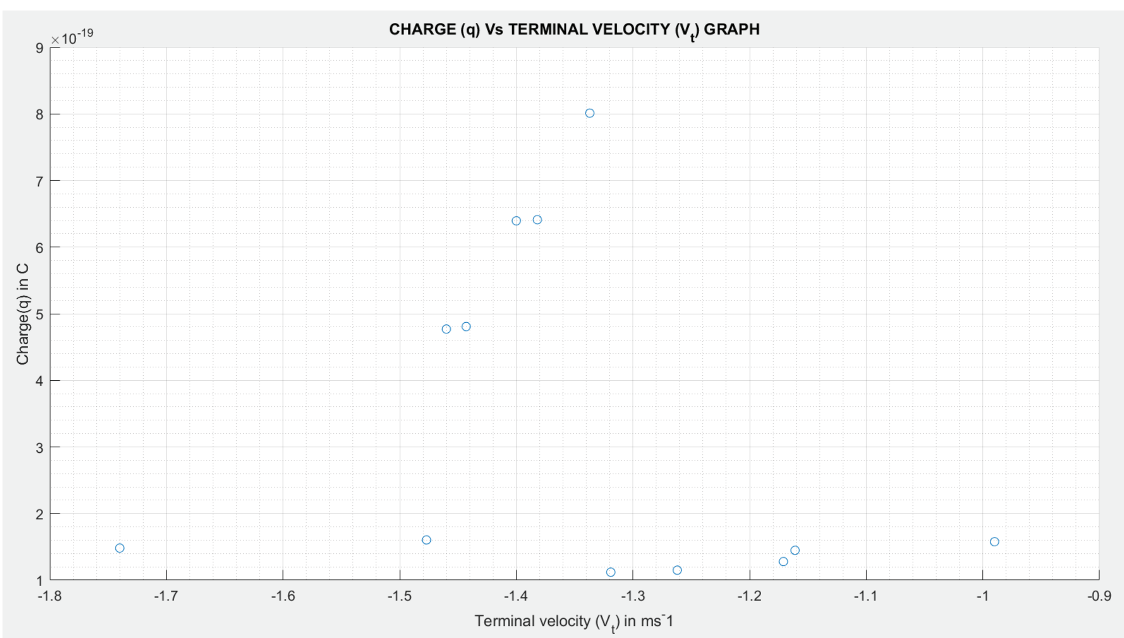
SIMULATION PROCEDURES WITH GRAPHS

(I) Mass Vs Terminal Velocity graph



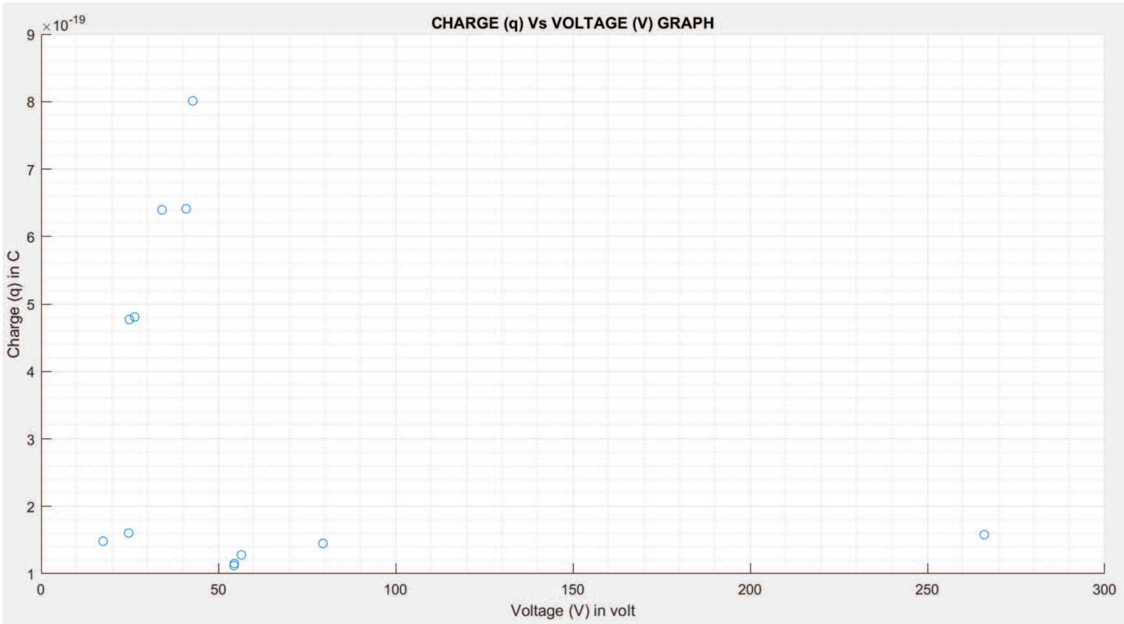
We can find terminal velocity of each particles from the oPhysics simulation. Here we have taken twelve particles. Here we have used the equation given in the simulation for finding mass in Kg. We have equation $m = kv_t^2$ where constant of simulation, $k = 4.086 \times 10^{-17}$ kg s^2/m^2

(II) Charge Vs Terminal Velocity graph



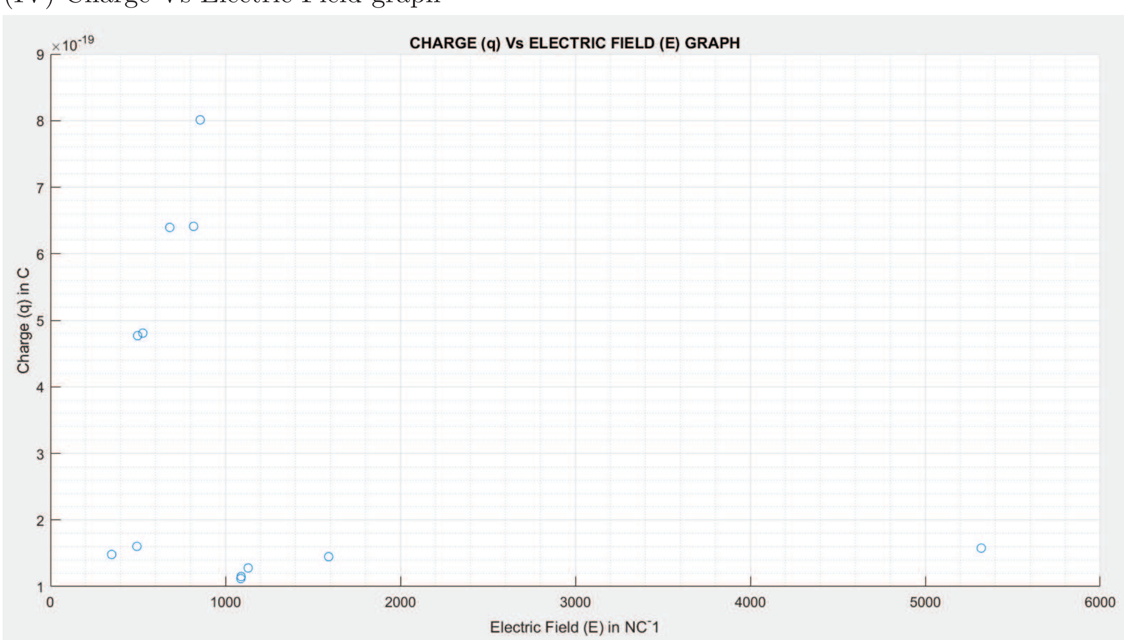
We have already found the terminal velocity using the simulation. Similarly, in order to find the charge of droplets, we firstly find the voltage, V at which terminal velocity, v_t becomes zero (balancing voltage). Using the equation $q = \frac{mgd}{V}$ where m is the mass of particle, g is the acceleration due to gravity d = 0.05 m. We can see that there is no dependency between the charge and terminal velocity. But it's clear that all charges are integral multiples of e.

(III) Charge Vs Voltage graph



Follow the same procedure here to find voltage V and charge, q. Here we can say that from the stationary state Theoretically, amount of voltage to move a charge between two points depends on the magnitude of charge. But the converse is not true. Because, charge is an intrinsic property of matter.

(IV) Charge Vs Electric Field graph

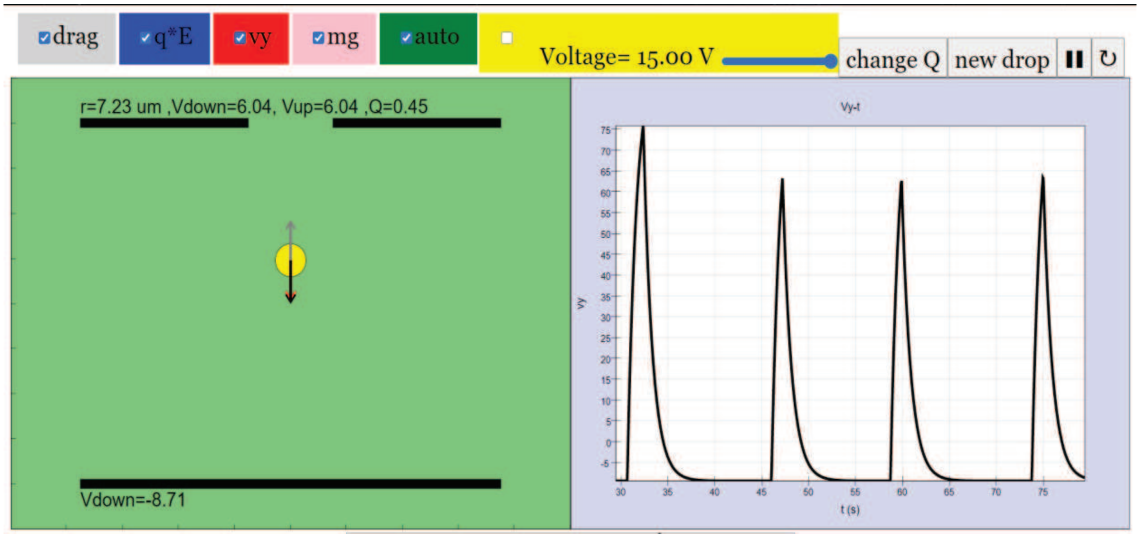


We can find electric field by applying equilibrium condition, $\vec{E}q = mg \Rightarrow \vec{E} = \frac{mg}{q}$

Similarly, here also we can say that amount of applied electric field depends on the charge. But the converse is not true, since charge will not vary with these parameters.

The above four graphs are drawn using Matlab Codes.

(V) Terminal Velocity Vs Time graph



When the droplet enters the space between the plates, its velocity increases and reaches to a certain constant velocity (terminal velocity) until it almost stops in the simulation. This can be graphically represented by a plot between charge and equilibrium voltage bottom plate. But just before it is about to stop, the electric field or voltage in the simulation is applied, which gives the resulting sudden increase in velocity. This process continues till the simulation stops.

PROBLEM

A charged oil drop of mass 3.2×10^−14 kg is held stationary between parallel plates 6 mm apart, by applying a potential difference of 1200 V between them. How many electrons does the oil drop carry?

We know the expression of charge for a particle having mass m separated by distance of separation d staying stationary under the influence of a potential difference V.

$$q = \frac{mgd}{V}$$

$$\Rightarrow \qquad \frac{3{\cdot}2\times10^{-14}\times10\times6\times10^{-3}}{1200}$$

Therefore, $q = 1 \cdot 6 \times 10^{-18}$C

$$q = ne \quad n = \frac{q}{e} \quad ; \quad e = 1 \cdot 6 \times 10^{-19}\text{C}$$

$$\text{n} = \frac{1{\cdot}6\times10^{-18}}{1{\cdot}6\times10^{-19}} = 10 \text{ electrons}$$

MATLAB CODE USED FOR PLOTTING GRAPHS

```
% *Mass Vs Terminal Velocity Graph*
%  All quantities are in SI Units unless stated
%  m= kv_t^2; k = 4.086*10^-17kgs^2*m^-2;
   Velocity V_t in x axis; mass m in y axis;
   x = [-1.74 -1.477 -1.46 -1.443 -1.4 -1.382 -1.337 -1.319 -1.262 -1.171 -1.161 -0.99];
   y = (4.086*10^-17)*x.^2;
   scatter(x,y)
   xlabel('Terminal velocity (V_t) in ms^-1'),
   ylabel('Mass (m) in kg'),
   title('MASS (m) Vs TERMINAL VELOCITY (V_t) GRAPH');
   grid on; grid minor
```

```
% *Charge Vs Terminal Velocity Graph*
% All quantities are in SI Units unless stated
% E = V/d; d = 0.05m; q = mg/E;
x = [-1.74 -1.477 -1.46 -1.443 -1.4 -1.382 -1.337 -1.319 -1.262 -1.171 -1.161 -0.99];
y = [1.48198E-19 1.60374E-19 4.76991E-19 4.80755E-19
     6.39522E-19 6.41121E-19 8.01083E-19 1.12E-19
     1.15071E-19 1.27912E-19 1.44818E-19 1.57765E-19];
scatter(x,y)
xlabel('Terminal velocity (V_t) in ms^-1'),
ylabel('Charge(q) in C'),
title('CHARGE (q) Vs TERMINAL VELOCITY (V_t) GRAPH'), grid on, grid minor
```

```
% *Charge Vs Electric Field Graph*
% All quantities are in SI Units unless stated
Voltage V in volt; Field E in x axis; Charge q in y axis
V = [17.5 24.7 24.9 26.4 34.1 40.9 42.8 54.4 54.5 56.5 79.5 266];
y = [1.48198E-19 1.60374E-19 4.76991E-19 4.80755E-19
     6.39522E-19 6.41121E-19 8.01083E-19 1.12E-19
     1.15071E-19 1.27912E-19 1.44818E-19 1.57765E-19];
x = V/0.05; scatter(x,y)
xlabel('Electric Field (E) in NC^-1'),
ylabel('Charge (q) in C'),
title('CHARGE (q) Vs ELECTRIC FIELD (E) GRAPH'), grid on, grid rinor
% *Charge Vs Voltage Graph*
% All quantities are in SI Units unless stated
  Voltage V in x axis; charge q in y axis;
  x = [17.5 24.7 24.9 26.4 34.1 40.9 42.8 54.4 54.5 56.5 79.5 266];
  y = [1.48198E-19 1.60374E-19 4.76991E-19 4.80755E-19
       6.39522E-19 6.41121E-19 8.01083E-19 1.12E-19
       1.15071E-19 1.27912E-19 1.44818E-19 1.57765E-19];
  scatter(x,y)
  xlabel('Voltage (V) in volt'),
  ylabel('Charge (q) in C'),
  title('CHARGE (q) Vs VOLTAGE (V) GRAPH'), grid on, grid minor,
```

CALCULATIONS

K in kg s^2/m^2	V_t in ms^-1	Kv_t^2 in Kg	q = mgd/V in C	mg in N	E = V/d in N/C	V= E*d in V	
4.806*10^-17	Terminal Velocity	Mass	Charge	Weight	Electric Field	Voltage	
Voltage (V) in V	Distance of Separation (d) in m	Electric Field Intensity (E) in V/m	Terminal velocity (v_t) in m/s	Mass (m) in Kg	Acceleration due to gravity (g) in m/s^2	Weight (mg) in N	Charge (q) in C
40.9	0.05	818	-1.74	1.237	9.8	12.1226	1.48198E-19
266	0.05	5320	-1.46	8.706	9.8	85.3188	1.60374E-19
56.5	0.05	1130	-1.161	5.5	9.8	53.9	4.76991E-19
79.5	0.05	1590	-1.382	7.8	9.8	76.44	4.80755E-19
54.4	0.05	1088	-1.319	7.1	9.8	69.58	6.39522E-19
42.8	0.05	856	-1.171	5.6	9.8	54.88	6.41121E-19
54.5	0.05	1090	-1.477	8.91	9.8	87.318	8.01083E-19
17.5	0.05	350	-0.99	4	9.8	39.2	1.12E-19
34.1	0.05	682	-1.4	8.008	9.8	78.4784	1.15071E-19
24.9	0.05	498	-1.262	6.5	9.8	63.7	1.27912E-19
24.7	0.05	494	-1.337	7.3	9.8	71.54	1.44818E-19
26.4	0.05	528	-1.443	8.5	9.8	83.3	1.57765E-19

ERRORS

1. There may be a chance in error in the defined constant K of the simulation.
2. There may be some errors that can occur while taking readings of terminal velocity.

CONCLUSIONS

1. The charge is quantised. Charges are integral multiple of a fundamental quantity. And this is a charge of an electron, $e = 1 \cdot 6 \times 10^{-19}\mathrm{C}$.
2. Currently accepted value of charge is very close to the Millikan's value. It shows only 1%of deviation from the magnitude of charge. Millikan's value is 1.5924 $\times 10^{-19}$Coulombs (C).
3. Terminal of velocity of a particle is dependent on its mass m. It is directly proportional to the mass m.
4. Charge is an intrinsic property of matter. It is independent of any other parameters like voltage v or electric field E etc....
5. Equilibrium voltage V as well as applied electric field E are dependent on the magnitude of charge q. They are inversely proportional to charge. But the reverse relation is false. Since, the charge of a particle is an intrinsic property.

   So, we can express it as $V \propto \frac{1}{q}$ and $E \propto \frac{1}{q}$

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