

The Euler Product Formula derived from the Sum of the Power of Primes

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Abstract: Relate the sum of powers of multiple primes to the sum of powers of natural numbers.

Key words: Euler product formula, Riemann conjecture.

If I hadn't calculated the sum of the products of different prime powers, I'm afraid I would never have anything to do with Euler, because knowing how magic works, it would be very simple.

$$\forall m, n, k, d, n_k \in \mathbb{N}, \forall p, p_k, \in \text{prime numbers}, \forall p^n = \frac{p^n - 1}{1 - p^{-n}}, \forall \prod_{k=0}^{\infty} p_k^n = \prod_{k=0}^{\infty} \frac{p_k^{n+1} - 1}{1 - p_k^{-n}},$$

$$\text{Divisor of } \forall p^n, \text{ Such as, } p^0, p^1, p^2, p^3, p^4, \dots, p^n, \text{ Sum of divisors of } \forall p^n, S_n = \sum_{m=0}^n p^m = \frac{p^{n+1} - p^0}{p - 1},$$

The reciprocal of the divisor of $\forall p^n$, Such as, $p^0, p^{-1}, p^{-2}, p^{-3}, p^{-4}, \dots, p^{-n}$,

$$\text{The reciprocal sum of the divisor of } \forall p^n, S_{-n} = \sum_{m=0}^n p^{-m} = \frac{p^{-n-1} - p^0}{p^{-1} - 1} = \frac{p^{n+1} - p^0}{p^n(p^1 - 1)},$$

$$\Rightarrow \forall p^n = \frac{S_n}{S_{-n}} = \frac{\sum_{m=0}^n p^m}{\sum_{m=0}^n p^{-m}} = \frac{p^{n+1} - p^0}{p^1 - 1} / \frac{p^{-n-1} - p^0}{p^{-1} - 1}, \prod_{k=0}^{\infty} p_k^n = \prod_{k=0}^{\infty} \frac{\sum_{m=0}^n p_k^m}{\sum_{m=0}^n p_k^{-m}} = \prod_{k=0}^{\infty} \left(\frac{p_k^{n+1} - p_k^0}{p_k^1 - 1} / \frac{p_k^{-n-1} - p_k^0}{p_k^{-1} - 1} \right),$$

$$n \rightarrow \infty, \Rightarrow \prod_{k=0}^{\infty} p_k^n = \prod_{k=0}^{\infty} \left(\frac{p_k^{n+1} - p_k^0}{p_k^1 - 1} / \frac{p_k^{-n-1} - p_k^0}{p_k^{-1} - 1} \right) = \frac{\sum_{d=1}^{\infty} d^{n+1}}{\sum_{d=1}^{\infty} d^{-n-1}}, \sum_{d=1}^{\infty} d^{\pm 1} = \prod_{k=0}^{\infty} \frac{p_k^{\pm(n+1)} - p_k^0}{p_k^{\pm 1} - 1},$$

$$n \rightarrow \infty, n_k \rightarrow \infty, \Rightarrow \prod_{k=0}^{\infty} p_k^{s+n_k} = \frac{\sum_{d=1}^{\infty} d^{s+1}}{\sum_{d=1}^{\infty} d^{-s-1}}, \sum_{d=1}^{\infty} d^{\pm s} = \prod_{k=0}^{\infty} \frac{p_k^{\pm s(n_k+1)} - p_k^0}{p_k^{\pm s} - 1}.$$

$$\Rightarrow \sum_{d=1}^{\infty} d^{-s} = \prod_{k=0}^{\infty} \frac{p_k^{-s(n_k+1)} - p_k^0}{p_k^{-s} - 1} = \left(\sum_{d=1}^{\infty} d^{+s} \right) \prod_{k=0}^{\infty} \frac{1 - p_k^{-s+n_k}}{p_k^{-s+n_k} - 1},$$

So, the Euler product formula is not accurate, that is, $\sum_{d=1}^{\infty} d^{-s} \neq \prod_{k=0}^{\infty} \frac{1}{1 - p_k^{-s}}$.

Therefore, all papers quoting the Euler product formula are problematic, and the Riemann conjecture related to the Euler product formula is wrong.

Reference: none.