## Proof of Goldbach's Conjecture

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#### Abstract

- In this paper we had taken a unique approach and tried to solve Goldbach conjecture, the famously known conjecture which mathematician throughout the centuries are trying to solve it but always get failed. We have also highlighted a new theory on finding prime numbers, prime number appears to have patterns etc.

Introduction - Goldbach conjecture states that - Every even integer greater than 2 is the sum of two primes.

Until now many efforts have been made to prove the above statement to be true or false, but no one has yet come up with the satisfying solutions, but we have 100 \% proof that the Goldbach conjecture statement is true.


## Explanation \& Details -

## Arrangement of Numbers -

As we all know that natural number line contains odd and even numbers such as $1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19 \ldots$.

Using the above single number line, we arrange all numbers below in sets of three named as 'triple sets of numbers line'

Arranging of each line is done by assigning name to it as
L1 - Line 1, L2 - Line 2, L3 - Line 3.
L1 L2 L3
123

495051

Note - Above triple sets of numbers line goes towards infinity.

Notice all the above three sets of number line.
L1 - Contains even numbers and numbers such as prime and composites e.g. 1, 4, 7, 10....

L2 - Also contains even numbers and numbers such as prime and composites e.g. 2, 5, 8, 11...

L3 - Contains only even numbers and odd numbers arranged in alternate order e.g. $3,6,9,12$...

Note - Since L3 does not contain prime numbers therefore for below equations we must consider taking and adding prime numbers either from L1 or L2.

The above 'triple sets of numbers' is used as a base to prove Goldbach conjecture Next-

## Case of L1 \& L1-

By using the above triple sets of numbers we derive a set of equation for L1.
$\mathrm{pL1}$ is any prime numbers from L1.
Where $p$ is prime and $p>3$ then
$\mathrm{pL1}+\mathrm{pL1}=\mathrm{L} 2 \quad$ (Sum L2 must be even number).
$\mathrm{pL1}+\mathrm{pL1}+\mathrm{pL1}=\mathrm{L} 3 \quad$ (Sum L3 must be odd number).
pL1 + pL1 + pL1 + pL1 = L1 (Sum L1 must be even number).
$\mathrm{pL1}+\mathrm{pL1}+\mathrm{pL1}+\mathrm{pL} 1+\mathrm{pL1}=\mathrm{L} 2$ (Sum L2 must be odd number).
$\mathrm{pL1}+\mathrm{pL1}+\mathrm{pL1}+\mathrm{pL1}+\mathrm{pL1}+\mathrm{pL1}=\mathrm{L} 3$ (Sum L2 must be even number).
$\mathrm{pL1}+\mathrm{pL1}+\mathrm{pL1}+\mathrm{pL1}+\mathrm{pL1}+\mathrm{pL1}+\mathrm{pL1}=\mathrm{L} 1$ (Sum L1 must be odd number).
Note - As we go on adding pL1, these equations goes towards infinity and their sums will always be odd, even numbers. Prime number added below is in sequence, but one can try taking any random prime numbers from L1 the result will be same.

By getting prime numbers from L1 of 'triplet sets of numbers line' and inputting it in the equation, we get solution for equations series shown on the left hand side.

## Equations Series

$7+13=20$ even
$7+13+19=39$ odd
$7+13+19+31=70$ even
$7+13+19+31+37=107$ odd
$7+13+19+31+37+43=150$ even
$7+13+19+31+37+43+67=217$ odd

Natural Number Line
2 even
3 odd
4 even
5 odd
6 even
7 odd

Note - Equation series and natural number line, both goes toward infinity.
Notice above as we use the equation and input two or more prime numbers, we get sums as even, odd, even, odd in sequence. Comparing equations with natural number line on the right side, we can see that equations series and natural number line both follows same odd even rule. In other words, sum of all equations are in odd even order and natural numbers on the right hand side are also in the form of odd even order.

So if we agree $2,4,6 \ldots$ is even in the natural number line just because they follow odd even sequence. So one must also agree that all above equations
$7+13=20$ even
$7+13+19=39$ odd
$7+13+19+31=70$ even.....
is following odd even sequence and the first equation is having sum as even.
$\mathrm{pL} 1+\mathrm{pL} 1=\mathrm{L} 2$
$7+13=20$ even

Therefore this equation sum must always be even no matter which two prime numbers are added.

Goldbach conjecture in the form of equation.
$\mathrm{pL1}+\mathrm{pL1}=\mathrm{L} 2$
So from the above explanation we proved that Goldbach conjecture is true,

SUM OF TWO PRIME NUMBERS FROM L1 SET OF NUMBER IS ALLWAYS BE EVEN NUMBERS.

Now let's move further and check whether these equation gives the same result if we add two or more prime numbers from L2 \& L2 or from L1 \& L2?

## Case of L2 \& L2 -

By using the above triple sets of numbers we derive a set of equation for L .
pL 2 is any prime numbers from L 2 .
Where $p$ is prime \& $p>3$ then
$\mathrm{pL} 2+\mathrm{pL} 2=\mathrm{L} 1 \quad$ (Sum L1 must be even number).
$\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2=\mathrm{L} 3 \quad$ (Sum L3 must be odd number).
$\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2=\mathrm{L} 2$ (Sum L2 must be even number).
$\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2=\mathrm{L} 1$ (Sum L1 must be odd number).
$\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2=\mathrm{L} 3$ (Sum L3 must be even number).
$\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2+\mathrm{pL} 2=\mathrm{L} 2$ (Sum L2 must be odd number).

By inputting prime numbers in sequence from L1 of triplet sets of numbers we get $5+11=16$ even.
$5+11+17=33$ odd.
$5+11+17+23=56$ even.
$5+11+17+23+29=85$ odd .
$5+11+17+23+29+41=126$ even.
$5+11+17+23+29+41+47=173$ odd .
So above equation of L 2 also yield the same result as we input two or more prime numbers from L2. We get sums as even, odd, even, odd in sequence order.

Therefore it also proves that
SUM OF TWO PRIME NUMBERS FROM L2 SET OF NUMBER IS ALLWAYS EVEN.

## Case of L1 \& L2 -

Now lets check addition of pL1 \& pL2.
By using the above 'triple sets of numbers' we derive a set of equation for L1 \& L2. $\mathrm{pL1}, \mathrm{pL} 2$ is any prime numbers from L1 \& L2.

Where $p$ is prime \& $p>3$ then
pL1 + pL2 = L3 (Sum L3 must be even number).
$\mathrm{pL1}+\mathrm{pL2}+\mathrm{pL1}=\mathrm{L} 1 \quad$ (Sum L1 must be odd number).
$\mathrm{pL1}+\mathrm{pL2}+\mathrm{pL1}+\mathrm{pL} 2$ = L3 (Sum L3 must be even number).
pL1 + pL2 + pL1 + pL2 + pL1 = L1 (Sum L1 must be odd number).
$\mathrm{pL1}+\mathrm{pL} 2+\mathrm{pL} 1+\mathrm{pL} 2+\mathrm{pL} 1+\mathrm{pL} 2=\mathrm{L} 3$ (Sum L3 must be even number).
$\mathrm{pL1}+\mathrm{pL} 2+\mathrm{pL} 1+\mathrm{pL} 2+\mathrm{pL1}+\mathrm{pL} 2+\mathrm{pL1}=\mathrm{L} 1$ (Sum L1 must be odd number).
$\mathrm{pL2}+\mathrm{pL1}=\mathrm{L} 3 \quad$ (Sum L3 must be even number).
$\mathrm{pL2}+\mathrm{pL1}+\mathrm{pL2}=\mathrm{L} 2$ (Sum L2 must be odd number).
$\mathrm{pL2}+\mathrm{pL1}+\mathrm{pL2}+\mathrm{pL1}=\mathrm{L} 3$ (Sum L3 must be even number).
$\mathrm{pL2}+\mathrm{pL1}+\mathrm{pL2}+\mathrm{pL1}+\mathrm{pL2}$ = L2 (Sum L1 must be odd number).

By inputting prime numbers in sequence from L1 \& L2 of triplet sets of numbers we get
$7+5=12$ even.
$7+5+13=25$ odd.
$7+5+13+17=42$ even.
$7+5+13+17+19=61$ odd.
$7+5+13+17+19+23=84$ even.
$7+5+13+17+19+23+31=115$ odd.
$5+7=12$
$5+7+11=23$
$5+7+11+13=36$
$5+7+11+13+17=53$
So above equation of L1 \& L2 also yield the same result that all sums of two or more primes is in odd even order as we input prime numbers from L1 \& L2.

Therefore it also proves that.
SUM OF TWO PRIME NUMBERS OF L1 \& L2 SET OF NUMBER WILL ALLWAYS BE EVEN NUMBER.

## Addition of two prime from all possible ways -

Finally, all first equations from case of L1 \& L1, case of L2 \& L2, case of L1 \& L2 shows that if we take any two primes numbers from either L1 or L2 or L3 in sequence or randomly and add it, their sum will always be even and this proves Goldbach conjecture to be true.
pL1 + pL1 = L2 (Sum L3 must be even number).
$\mathrm{pL2}+\mathrm{pL2}=\mathrm{L} 1$ (Sum L3 must be even number).
$\mathrm{pL1}+\mathrm{pL2}=\mathrm{L} 3$ (Sum L3 must be even number).

The ternary Goldbach conjecture or three primes problem -
States - Every odd number greater than 5 can be expressed as the sum of three primes.

Notice above all equations having addition of three primes.
They all have odd number as the sums.
$\mathrm{pL1}+\mathrm{pL2}+\mathrm{pL1}=\mathrm{L} 1 \quad$ (Sum L1 must be odd number).
$7+5+13=25$ odd .
pL2 + pL2 + pL2 = L3 (Sum L3 must be odd number).
$5+11+17=33$ odd.
pL1 + pL1 + pL1 = L3 (Sum L3 must be odd number).
$7+13+19=39$ odd .
$\mathrm{pL2}+\mathrm{pL1}+\mathrm{pL2}=\mathrm{L} 1 \quad$ (Sum L1 must be odd number).
So, these equations also proves ternary Goldbach conjecture to be true.

## Prime Number Theory -

A new method to find prime number and their factors -
For any given prime numbers or semi prime numbers there are additive property along with multiplicative property which are responsible for existence of prime numbers and composite numbers in natural number line. Explained below is the 'Series Multiples of 5' which shows clearly how adding one specific natural number 16 and prime number 7 creates a series and using this series one can find composites numbers and prime numbers.

First thing to notice in this series is the patterns, like numbers $3,5,7 \ldots$. Is repeatedly arise in the series exactly in the pattern. This pattern helps us to

1) Know what prime number is the divisible factor is for given composite numbers.
2) What composite numbers would come next in the line etc.
3) Unlike Mersenne prime number finding method, one can use this method to find composite numbers along with their two or more prime factors.

Follow below instructions-
Generating series multiples of 5 .
$7+16=23,23+16=39,39+16=55,55+16=71,71+16=87,87+16=103$, $103+16=119 \ldots$.

| 55 (5 * 11) | 487 | 919 |
| :---: | :---: | :---: |
| 71 | 503 | 935 (11*85)(5*17=85) |
| 87 (3*29) | 519 (3*173) | 951 (3*317) |
| 103 | 535 ( $5^{*} 107$ ) |  |
| 119 ( 7*17) | 551 |  |
| 135 ( $5 * 35$ (3*45) | 567 (7* 81) (3*189) |  |
| 151 | 583 (11*53) |  |
| 167 | 599 |  |
| 183 (3*61) | 615 (5*123)(3*205)(5*41=205) |  |
| 199 | 631 |  |
| 215 (5 * 43) | 647 |  |
| $231(7 * 33)(11 * 3=33)$ | 663 (13*51)(17*39) (3*21) |  |
| 247 (13*19) | 679 (7*97) |  |
| 263 | 695 (5*139) |  |
| 279 (3*93) | 711 (3*237)(3*79=237) |  |
| 295 (5*59) | 727 |  |
| 311 | 745 |  |
| 327 (3*109) | 759 (11*69)(3*253) |  |
| 343 ( $7 * 49)(7 * 7=49)$ | $775(5 * 155)(5 * 31=155)$ |  |
| 359 | 791 (7*119) $(7 * 17=119)$ |  |
| 375 (5*75)(5*15)3) | 807 (3*269) |  |
| 391 (17* 23) | 823 |  |
| 407 (11*37) | 839 |  |

Note -
This series goes towards infinity. After series numbers 951 one can continue the series by adding 16 to each sums in sequence and find more results.

Step to follow-
As we know 7, 2, 3 is prime so start checking from number 39.
Check number 39 by Dividing it by 3, 5, 7
39 is divisible by 3 . We get prime factor 13 . Remember that from number 39, every $3 r d$ number is a composite number in a column and must be divisible by 3 . This shows number 3 is having pattern in this series and will reveal many prime factors.

Check number 55 by Dividing it by 3, 5, 7
55 is divisible by 5 . We get prime factor 11 . Remember that starting from number 55 , every 5th number in a column must be divisible by 5 and starting from number 55 , every 11th number in a column must be divisible by 11.

Check number 71
Check number 71 by Dividing it by 3, 5, 7
It is not divisible by any of this number therefore it is prime.

Check number 87
After number 39 , number 87 is third number in the column.
Therefore 87 is divisible by 3 . Dividing 87 by 3 we get prime factor 29. Now remember that starting from number 11, every 11th number is a composite
number in a column and must be divisible by 11 .

Check number 87 dividing it by $3,5,7$
It is not divisible by any of this number therefore it is prime.

Check number 103 dividing it by 3,5, 7
It is not divisible by any of this number therefore it is prime.

Check number 119 by Dividing it by 3, 5, 7

119 is divisible by 7. Dividing 119 by 7 we get prime factor 17 . Remember that starting from number 119, every 17th number is a composite number in a column and must be divisible by 17. Also Remember that from number 119, every 7th number is a composite number in a column and must be divisible by 7 .

Check number 135

After number 55, number 135 is 5 th number in the column and after number 87, number 135 is 3 rd number in the column. Therefore 135 is divisible by both by 5 and 3 . Dividing 87 by 5 \& we get prime factor 45 and composite 35 which is also divisible by 5. Also Remember that starting from number 135, every 45th number is a composite number in a column and must be divisible by 45 .

Check number 151,167 by Dividing it by 3, 5, 7
None of the number is divisible 3,5,7 therefore it is prime.

Check number 183
After number 135 , number 183 is third number in the column.
Therefore 183 is divisible by 3 . Dividing 183 by 3 we get prime factor 61 . Now remember that starting from number 183, every 61th number is a composite number in a column must be divisible by 61 .

Notice how the whole series follows pattern each after the other. This pattern is the real proof that prime numbers are not randomly distributed in natural number line. Continue generating series and follow the above explained process to find more results. Any doubt? contact us.

## Conclusion -

Goldbach conjecture is not just a statement but also an equation that follows odd even order which we have proven in this paper, also using the same equations we also so proved Goldbach weak conjecture. About Prime number theory - This is how one can further count the series, follow the process and find as many prime numbers and composite with $100 \%$ guarantee. Notice how the whole series follows pattern each after the other. This pattern is the real proof that prime numbers are not randomly distributed in natural number line. Therefore we conclude that prime number distribution is the result of additive property along with multiplicative property.

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