Addendum to “A Bifurcation Model of the Quantum Field”

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Abstract

The purpose of this brief Addendum is to elaborate on a couple of relationships introduced in Physica A 165: 399-419 (1990).

Key words: Standard Model parameters, Period doubling bifurcations, fermion generation problem, fermion chirality, CPT theorem, neutrino helicity.

We bring here additional clarifications on two relationships introduced in [1], namely,

1) relation (26) expressing the time-reversal operator applied to fermion doublets,

2) relations (30) and (31) concerning the breaking of chiral symmetry in the neutrino sector.

The reader is also directed to [2], which complements [1] with details on the Feigenbaum route to chaos in unimodal maps.

1) Consider a pair of fields $\varphi(x), \varphi(y)$ defined at points separated by a space-like interval

$$x = (x^0, \mathbf{x}) \, ; \qquad y = (y^0, \mathbf{y}) \tag{1}$$

The *time-ordered product* of fields is customarily represented as [3]

$$\tau\{\varphi(x)\varphi(y)\} = \begin{cases} \varphi(y)\varphi(x); & y^0 > x^0 \\ \varphi(x)\varphi(y); & y^0 < x^0 \end{cases} \tag{2}$$

Reversing the sign of time follows from the action of the *time-reversal operator* T given by

$$T(\varphi(x)) = \varphi(\mathbf{x}, -x^0) \tag{3a}$$

$$T(\varphi(y)) = \varphi(\mathbf{y}, -y^0) \tag{3b}$$

It is apparent that changing the sign of time swaps the top and bottom rows of (2) as $-y^0 < -x^0$ in the first row and $-y^0 > -x^0$ in the second.

2) In the chiral basis, the Dirac spinor is assembled from a left and right-handed Weyl spinor and reads, in a column vector format,

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{4}$$

Defining the projection operators acting on Dirac spinor enables one to identify the left and right-handed components of (4) as in

$$P_L \psi = \frac{1-\gamma_5}{2} \psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} \tag{5a}$$

$$P_R \psi = \frac{1+\gamma^5}{2} \psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} \tag{5b}$$

As shown in [1], the neutrino-antineutrino branch develops in a triplicated pattern at the first fermionic vertex of the bifurcation diagram (V^1) and is described by the trio of states

$$\left(\nu_e, \overline{\nu_e}\right), (\nu_\mu, \overline{\nu_\mu}), (\nu_\tau, \overline{\nu_\tau}) \tag{6}$$

Applying the CPT operator to the electron-neutrino doublet leads to

$$(\nu_e, \overline{\nu_e})_L \xrightarrow{C} (\overline{\nu_e}, \nu_e)_L \xrightarrow{P} (\overline{\nu_e}, \nu_e)_R \xrightarrow{T} (\nu_e, \overline{\nu_e})_R \tag{7}$$

which means that the left-handed and right-handed doublets are physically indistinguishable. In turn, this finding indicates that the only meaningful representation of (4) - (5) is in terms of a *broken doublet* comprising a pair of symmetrical singlets. In symbolic form, this result can be presented as

$$\nu = \begin{pmatrix} \nu_L \\ \overline{\nu_R} \end{pmatrix} = \begin{pmatrix} \nu_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \overline{\nu_R} \end{pmatrix} \tag{8}$$

It follows that neutrinos exist in a single polarized state, either as left-handed fermions or right-handed antifermions, a conclusion validated by all experiments carried out to date. But what happens if the neutrino is a Majorana field instead of a Dirac field? One recalls that a Majorana spinor is *still* a Dirac spinor whose chiral components are not independent but coupled through the connection [3]

$$\psi_M = \begin{pmatrix} \psi_L \\ i\sigma^2 \psi_L^* \end{pmatrix} \tag{9}$$

where σ^2 stands for the square of the Pauli matrix. As the only physical distinction between the two components of (9) is the helicity sign, one can formally break (9) into a pair of symmetrical singlets, namely,

$$\psi_M = \begin{pmatrix} \psi_L \\ i\sigma^2 \psi_L^* \end{pmatrix} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ i\sigma^2 \psi_L^* \end{pmatrix} \tag{10}$$

As argued in [1], chiral asymmetry manifest in the right-hand side of either (8) or (10) carries over to the massive fermion sector including charged leptons and quarks.

References

1. E. Goldfain, "A Bifurcation Model of the Quantum Field", Physica A 165: 399-419 (1990). A copy of this reference is available at the following sites:
https://www.academia.edu/38767540/A_bifurcation_model_of_the_quantum_field
https://vixra.org/pdf/1309.0015v1.pdf

2. Available at the following sites:
https://www.researchgate.net/publication/343686626_Three_Generations_and_Fermion_Chirality_from_Universal_Bifurcations
https://vixra.org/pdf/2008.0120v4.pdf

3. M. Maggiore, "A Modern Introduction to Quantum Field Theory", Oxford Univ. Press, 2005.