Is the course of the Planck's radiation-function the result of the existence of an upper cut-off frequency of the vacuum?

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Abstract

This work is based on the model published in viXra:1310.0189. Because the CMBR follows the PLANCK's radiation rule more or less exactly, it should, because of the indistinguishability of individual photons, apply to a whatever black emitter. Therefrom arises the guess, that the existence of an upper cut-off frequency of the vacuum could be the cause for the decrease in the upper frequency range. Since the lower-frequent share of the curve correlates with the frequency response of an oscillating circuit with the Q-factor 1/2, it is examined, whether it succeeds to approximate the Planck curve by multiplication of the initial curve with the dynamic, time-dependent frequency response of the above mentioned model. Reason of the time-dependence is the expansion of the universe. Deutsche Version verfügbar in viXra "Ist der Verlauf der Planckschen Strahlungsfunktion das Resultat der Existenz einer oberen Grenzfrequenz des Vakuums".

1. Fundamentals

This article is based on a model I published in [1], the idea stems from Prof. Cornelius LANCZOS. It defines the expansion of the universe as a consequence of the existence of a metric wave-field. The time-function is based on the Hankel function, which consists of the sum of two Bessel functions (J_0 and Y_0) in turn. The particular qualities of the Bessel function lead to an increase of the wavelength, which is defined by the spacing between two zero-transits. Thus, the model leads to a quantization of the universe into discrete line-elements with particular physical characteristics. An individual line-element can be described by the model of a loss-affected oscillatory circuit with shunt-resistor. A special quality of the model consists in the fact, that the Q-factor of this oscillatory circuit is equal to the phase-angle $2\omega_0 t$ of the above-mentioned Bessel function. It applies $Q_0=2\omega_0 t$. The value ω_0 corresponds to the PLANCK's frequency on this occasion.

A special solution of the MAXWELL equations was found for the Hankel function with overlaid interference function, which describes the wave-propagation in the vacuum and co-includes the expansion. This special solution owns an inherent propagation-velocity in reference to the empty space (subspace) which is almost zero to the current point of time. Main-idea of the model is, that this propagation-velocity adds up geometrically to the propagation-velocity of an overlaid wave, at which point the total-velocity always amounts to exactly c in reference to the subspace. Thus, the cosmologic red-shift exactly can be described.

One conclusion from the model is the existence of an upper cut-off frequency of the vacuum, which could not be detected until now, because its value is about magnitudes greater than the technically feasible. Another conclusion of the model is the supposition that each photon is connected really or/and virtually with an origin at $Q_0=1/2$ That is the frequency, at which the excessive energy after the shape of the metric wave-function has been coupled into the very same one, as an overlaid wave, where it can be observed until now as cosmic background-radiation. Furthermore could be determined, that the bandwidth in the lower frequency range exactly matches the one of an oscillatory circuit with the Q-factor 1/2, which equals the conditions to the point of time of the input coupling. Hence the intention of this article is, to determine, whether the PLANCK's graph can be approximated by application of the frequency response given by the model, upon the spectrum of an oscillatory circuit with the Q-factor ½, furthermore to compare the calculated radiation-temperature with the measured one.

Since the cosmic background-radiation exactly follows the PLANCK's radiation-rule more or less, it should, because of the indistinguishability of individual photons, apply to a whatever black emitter. Therefrom arises the guess, that the existence of an upper cut-off frequency of the vacuum could be the cause for the decrease in the upper frequency range. In [1] already a simple attempt of an approximation has been taken up, at which point several values of the time-dependent frequency response $A(\omega)\cdot\cos\varphi$ have been multiplied with the source-function, which led to a quite good match, as measured by the simple procedure.

Another aim of this article is, to improve the proceeding any farther in order to make more precise statements. Attention should be paid to with the model that with some many exceptions (c μ_0, ε_0, κ_0, k) most of the fundamental physical constants are time- and reference-frame-dependent (~). And there is a conductivity of the subspace κ_0 different from zero. If you know these 5 values, you are able to calculate all other ones. The model is based on the PLANCK's units (e.g. ω_0) which can be obtained from the locally measured values. That points into one direction to the values of the universe as a whole (e.g. H_0), into the opposite direction to the (constant) values of the so called subspace (e.g. r_1). That is the medium the metric wave-field is propagating in. The proportionality factor is the phase angle of the temporal function $Q_0=2\omega_0 t$.

2. The WIEN displacement law and the source-function

During the examination of the WIEN displacement law meets the eye, that the displacement happens exactly at the lower wing pass of the PLANCK's radiation-function, which coincides with the wing pass of an oscillatory circuit with the Q-factor 1/2 in this section. Quite often in publications the curve is shown in another manner. I prefer the duplicate logarithmic presentation, then the curve turns into a straight line.

Considering the WIEN displacement law (902)[1] more exactly, the factor $\tilde{x}$ = 2.821439372 attracts attention particularly. With an oscillatory circuit of the Q-factor 1/2 rather the factor $2\sqrt{2}$ would be applicable for this, at which point the 2 stems from the source-frequency $2\omega_1$. The expression $\sqrt{2}$ arises from the rotation of the coordinate-system about $\pi/4$.

Now the validity of the WIEN displacement law in the time short after BB does not have been examined yet and neither PLANCK's radiation-rule nor the WIEN displacement law contain any information about the way, temperature varies, when it varies. In [1] I found the following relations for the calculation of temperature:

$$T_k = \frac{\hbar\omega_k}{\tilde{x}k} = \frac{\varepsilon_v}{\tilde{x}}\frac{\hbar_1\omega_1}{6k}Q^{-\frac{5}{2}} = 0.055693\frac{\hbar_1\omega_1}{k}Q^{-\frac{5}{2}} \qquad \tilde{x} = \begin{cases} 2.821439372 & \text{Exactly} \\ 2\sqrt{2} & \text{Approximation} \end{cases}$$

([1] 405)

$$T_k = \frac{\hbar\omega_k}{\tilde{x}k} \approx \frac{1}{3}\frac{\hbar_1\omega_1}{6k}Q^{-\frac{5}{2}} = \frac{\hbar_1\omega_1}{18k}Q^{-\frac{5}{2}} \qquad \varepsilon_v = \tfrac{2}{3}\sqrt{2} = 0.9428090416$$

$$T_k = \frac{\hbar_1\omega_1}{18k}Q_0^{-\frac{5}{2}} = \frac{\hbar\omega_0}{18k}Q_0^{-\frac{1}{2}} \qquad \omega_1 = \frac{\kappa_0}{\varepsilon_0} \qquad \text{([1] 902)}$$

Expression ε_v is the *vacuum coefficient of absorption*. The calculation of T_k according to [1] turns out a value of 2.79146K, which is 0.06598K higher than the measured temperature of the CMBR (2.7250K).

During an investigation in the Internet, I found a detailed deduction of the WIEN displacement law [2]. The value of the proportionality-factor can be obtained by the identification of the maximum of PLANCK's radiation-rule as follows. We start from (382):

$$d\mathbf{S}_k = \frac{1}{4\pi^2}\frac{\hbar\omega^3}{c^2}\frac{1}{e^{\frac{\hbar\omega}{kT}}-1}\mathbf{e}_s\,d\omega \qquad \text{PLANCK's radiation rule} \qquad \text{([1] 382)}$$

$$d\mathbf{S}_k = \frac{1}{4\pi^2}\frac{k^3T^3}{\hbar^2c^2}\left(\frac{\hbar\omega}{kT}\right)^3\frac{1}{e^{\frac{\hbar\omega}{kT}}-1}\mathbf{e}_s\,d\omega \qquad x = \frac{\hbar\omega}{kT} \qquad d\omega = \frac{kT}{\hbar}dx \qquad (1)$$

$$d\mathbf{S}_k = \frac{1}{4\pi^2}\frac{k^4T^4}{\hbar^3c^2}\frac{x^3}{e^x-1}\mathbf{e}_s\,dx \qquad \frac{d}{dx}\frac{x^3}{e^x-1} = 0 \qquad (2)$$

$$3\frac{x^2}{e^x-1} - \frac{x^3e^x}{(e^x-1)^2} = \frac{3x^2(e^x-1)-x^3e^x}{(e^x-1)^2} = 0 \qquad (3)$$

$$3x^2(e^x-1)-x^3e^x = 0 \qquad x^3e^x = 3x^2(e^x-1) \qquad (4)$$

$$e^x(x-3) = -3 \qquad y = x-3 \qquad x = 3+y \qquad (5)$$

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[1] Three-digit numerations always refer to [1]

$$ye^{y+3} = ye^y e^3 = -3 \qquad\qquad ye^y = -3e^{-3} \tag{6}$$

$$x = 3 + \mathrm{lx}(-3e^{-3}) = 2.821439372 \qquad\qquad \mathrm{lx}(xe^x) = x \tag{7}$$

lx is LAMBERT's W-function (ProductLog [#]). Finally, after insertion into the middle expression of (1) WIEN's displacement law turns out:

$$\hbar\omega_{max} = 2.821439372\ \mathrm{k}T \qquad \text{WIEN's displacement law} \tag{8}$$

On success in doing the same even for the source-function with Q=½, obtaining the same result, we would be a step forward in answer to the question: Is the course of the Planck's radiation-function the result of the existence of an upper cut-off frequency of the vacuum? First of all however, we have to bring the output-function into a form, suitable for further processing. We start with (380) with the substitution:

$$P_v = \frac{P_s}{1+v^2Q^2} \qquad v = \frac{\omega}{\omega_s} - \frac{\omega_s}{\omega} \qquad \omega_s = 2\omega_1 \qquad \Omega = \frac{\omega}{\omega_s} = \frac{1}{2}\frac{\omega}{\omega_1} \tag{9}$$

The expression stems from electrotechnics describing the power dissipation P_v of an oscillatory circuit with the Q-factor Q and the frequency ω (see [3]), v is the detuning. The Q-factor is known and amounts to Q=1/2 at $\omega_S=2\omega_1$. The right-hand expression results directly from the sampling-theorem. The cut-off frequency of the subspace ω_1 is the value ω_0 at Q=1. After substitution, we get the following expressions:

$$v = \Omega - \Omega^{-1} \qquad v^2 = \Omega^2 + \Omega^{-2} - 2 \qquad v^2Q^2 = \frac{1}{4}\Omega^2 + \frac{1}{4}\Omega^{-2} - \frac{1}{2} \tag{10}$$

$$P_v = \frac{P_s}{\frac{1}{4}\Omega^2 + \frac{1}{4}\Omega^{-2} + \frac{1}{2}} \cdot \frac{4\Omega^2}{4\Omega^2} = 4P_s \frac{\Omega^2}{\Omega^4 + 2\Omega^2 + 1} = 4P_s \left(\frac{\Omega}{1+\Omega^2}\right)^2 \tag{11}$$

You can find that expression more often in [1], among other things even with the group delay T_{Gr} however for a frequency ω_1. For a frequency $2\omega_1$ applies for T_{Gr} and the energy W_v:

$$T_{Gr} = \frac{dB(\omega)}{d\omega} = \frac{1}{\omega_1}\left(\frac{\Omega}{1+\Omega^2}\right)^2 \qquad W_v = \frac{1}{6}P_s T_{Gr} = \frac{2}{3}\frac{P_s}{\omega_1}\left(\frac{\Omega}{1+\Omega^2}\right)^2 \tag{12}$$

The factor ⅙ comes from the splitting of energy onto 4 line-elements, as well as from the multiplication with the factor ⅔ because of refraction during the in-coupling into the metric transport lattice. It oftenly occurs in thermodynamic relations, which doesn't astonish. Thus, total-energy of the CMBR during input coupling is equal to the product of power dissipation and group delay, that is the average time, the wave stays within the MLE, but only for what it's worth. With the help of (11) we obtain:

$$P_v = 4b\ P_s \left(\frac{\Omega}{1+\Omega^2}\right)^2 \qquad P_v = 512b\ \hbar_1\omega_1^2 \left(\frac{\Omega}{1+\Omega^2}\right)^2 \tag{13}$$

b is a factor, we want to determine later on. Let's equate it to one at first. We determined the value Ps with the help of (394) using the values of the point of time Q=1/2. Interestingly enough, the HUBBLE-parameter H_0 at the time $t_{0.5}$ is greater than ω_1 and ω_0. For an individual line-element applies:

$$\omega_{0.5} = \frac{\omega_1}{Q_{0.5}} = \frac{\omega_1}{\frac{1}{2}} = 2\omega_1 \qquad\qquad H_{0.5} = \frac{\omega_1}{Q_{0.5}^2} = \frac{\omega_1}{\frac{1}{4}} = 4\omega_1 \tag{14}$$

$$P_s = \frac{\hat{\hbar}_i}{4\pi t_{0.5}^2 Q_{0.5}^4} = \frac{\hat{\hbar}_i}{2\pi}\frac{2^5}{4t_{0.5}^2} = 32\hbar_1 H_{0.5}^2 = 128\hbar_1\omega_1^2 \qquad \frac{\hat{\hbar}_i}{2\pi} = \hbar_1 = \frac{\hbar_{0.5}}{2} \tag{15}$$

Expression (13) is very well-suited for the description of the conditions at the signal-source. Here, the power makes more sense than the POYNTING-vector $\mathbf{S}_k$. But for a comparison with (382) we just need an expression for $\mathbf{S}_k$, quasi a sort of PLANCK's radiation-rule for technical signals with the bandwidth $2\omega_1/Q_{0.5} = 4\omega_1$. Then, this would look like this approximately:

$$d\mathbf{S_k} = 4bA\left(\frac{\Omega}{1+\Omega^2}\right)^2 \mathbf{e_s}\, d\Omega \tag{16}$$

We determine the factor A by a comparison of coefficients (3). We assume, the WIEN displacement law (8) would apply and substitute as follows:

$$A = \frac{1}{4\pi^2}\frac{k^4 T^4}{\hbar^3 c^2} \qquad\qquad c = \omega_1 Q^{-1} r_1 Q \tag{17}$$

We put in $2\sqrt{2}\omega_1$ as initial-frequency into the expression k^4T^4 That's advantageous, as we will already see. This frequency is not a metric indeed ($\omega_x \sim Q^{-1}$), but an overlaid frequency ($\omega \sim Q^{-3/2}$). During red-shift of the source-signal, likewise not the factor 2.821439372 but the factor $2\sqrt{2}$ becomes effective. Thus applies:

$$k^4T^4 = \frac{(2\sqrt{2})^4}{(2\sqrt{2})^4}\hbar_1^4 Q^{-4}\omega_1^4 Q^{-6} = \hbar_1^4\omega_1^4 Q^{-10} \qquad\qquad Q^{-10} = \frac{Q^{-8}}{Q^2} \tag{18}$$

$$A = \frac{1}{4\pi^2}\frac{\hbar_1^4\omega_1^4 Q^{-8}}{\hbar_1^3 Q^{-3}\omega_1^2 Q^{-2} r_1^2 Q^4} = \frac{1}{4\pi^2}\frac{\hbar^4\omega_0^4}{\hbar^3\omega_0^2 r_1^2 Q^4} = \frac{1}{\pi}\frac{\hbar\omega_0^2}{4\pi R^2} \tag{19}$$

$$4A = \frac{4}{\pi}\frac{\hbar\omega_0^2}{4\pi r_0^2 Q^2} = \frac{4}{\pi}\frac{\hbar\omega_0^2}{4\pi R^2} \qquad\qquad R \text{ for } Q \gg 1 \tag{20}$$

$$d\mathbf{S_k} = \frac{4b}{\pi}\frac{\hbar\omega_0^2}{4\pi R^2}\left(\frac{\Omega}{1+\Omega^2}\right)^2 \mathbf{e_s}\, d\Omega \qquad\qquad R \text{ for } Q \gg 1 \tag{21}$$

Indeed, that submits only the expression without consideration of red-shift. We determine the actual values to the point of time of input coupling, in that we apply the values for Q=1/2 in turn. It applies:

$$A = \frac{1}{4\pi^2}\frac{\hbar_1^4\omega_1^4 Q^{-8}}{\hbar_1^3 Q^{-3}\omega_1^2 Q^{-2} r_1^2 Q^4} = \frac{2^{8-3-2+4}}{4\pi^2}\frac{\hbar_1^4\omega_1^4}{\hbar_1^3\omega_1^2 r_1^2} = \frac{128}{\pi}\frac{\hbar\omega_1^2}{4\pi r_1^2} \tag{22}$$

$$4A = \frac{512}{\pi}\frac{\hbar_1\omega_1^2}{4\pi r_1^2} \qquad\qquad d\mathbf{S_k} = \frac{512b}{\pi}\frac{\hbar_1\omega_1^2}{4\pi r_1^2} Q^{-7}\left(\frac{\Omega}{1+\Omega^2}\right)^2 \mathbf{e_s}\, d\Omega \tag{23}$$

b will be determined later on. It shows, the POYNTING-vector is equal to the quotient of a power P_k resp. P_s and the surface of a sphere with the radius R (world-radius), exactly as per definition. Omitting the surface, we would get the transmitting-power P_v directly. In the above-mentioned expressions the parametric attenuation of 1Np/R, which occurs during propagation in space, is unaccounted for. This must be considered separately if necessary.

Now we have framed the essential requirements and can dare the next step, the proof of the validity of the WIEN displacement law in strong gravitational-fields. The basic-idea was just, that the Planck's radiation-rule (382) should emerge as the result of the application of the metrics' cut-off frequency (302) to the function of power dissipation P_v of an oscillatory circuit with the Q-factor Q=1/2 (13) We proceed on the lines of (2), in that we equate the first derivative of the bracketed expression (23) to zero. A substitution like in (1) is not necessary, because the expression is already correct. It applies:

$$\frac{d}{d\Omega}\left(\frac{\Omega}{1+\Omega^2}\right)^2 = \frac{2\Omega}{\left(1+\Omega^2\right)^2} - \frac{4\Omega^3}{\left(1+\Omega^2\right)^3} = \frac{2\Omega\left(1-\Omega^2\right)}{\left(1+\Omega^2\right)^3} = 0 \qquad (24)$$

$$2\Omega\left(1-\Omega^2\right) = 0 \qquad \Omega_1 = 0 \quad \text{Minimum} \qquad \Omega_{2,3} = \pm 1 \quad \text{Maximum} \qquad (25)$$

The first solution is trivial, the second and third is identical, if we tolerate negative frequencies (incoming and outgoing vector). Now, we must only find a substitution for Ω, with which (382) and (23) come to congruence in the lower range. This would be the displacement law for the source-signal then (22). Since the ascend of both functions has the same size in the lower range, there is theoretically an infinite number of superpositions, whereat only one of them is useful. Therefore, as another criterion, we introduce, that both maxima should be settled at the same frequency. The displacement law for the source-signal would be then as follows:

$$\hbar\omega_{max} = a\,kT \qquad \text{Displacement law source-signal} \qquad (26)$$

at which point we still need to determine the factor a. As turns out, we still have to multiply even the output-function itself, with a certain factor b, in order to achieve a congruence. The 4 we had already pulled out. We apply the value $2\sqrt{2}$and 2.821439372 for a one after the other and determine b numerically with the help of the relation and the function FindRoot[#] using the substitution 2x=ay:

$$\frac{\left(a\frac{y}{2}\right)^3}{e^{a\frac{y}{2}}-1} - 4b\left(\frac{\frac{y}{2}}{1+\left(\frac{y}{2}\right)^2}\right)^2 = 0 \quad y = 10^{-5} \qquad \begin{array}{ll} b \to 2 & \text{for } a = 2\sqrt{2} \\ b \to 2.009918917 & \text{for } a = 2.821439372 \end{array} \qquad (27)$$

The maxima overlap accurately in both cases. The lower value a is equal to the factor in (903). Thus it seems, that with references, except for those to the origin of each wave with $2\omega_1$, multiplied with $\sqrt{2}$, which is caused by the rotation of the coordinate-system about π/4, rather the approximative solutions with the factor $2\sqrt{2}$ apply. With lower frequencies, the factor 2.821439372 of the WIEN displacement law applies then again.

But to the exact proof of the validity of the WIEN displacement law in the presence of strong gravitational-fields this ansatz is not enough. We must also show that the maximum of the PLANCK's radiation-function behaves exactly according to the WIEN displacement law, that means the approximation and the target-function must come accurately to the congruence. Since the difference between a factor $2\sqrt{2}$ and 2.821439372 amounts to 0.5% after all, we will execute the examination with both values. Only the relations for b=$2\sqrt{2}$ are depicted. Now, we can set about to write down the individual relations:

$$\hbar\omega_{max} = 2\sqrt{2}\,kT \qquad \text{Displacement law source-signal} \qquad (28)$$

$$\Omega = \frac{1}{2}\frac{\omega}{\omega_1} = \frac{1}{2\sqrt{2}}\frac{\hbar\omega}{kT_k} = \frac{x}{a} = \frac{y}{2} \qquad y = \frac{\omega}{\omega_1} \qquad b = 2 \qquad (29)$$

Thus, we have found our source-function. In y it reads as follows:

$$d\mathbf{S}_{\mathbf{k}} = \frac{16}{\pi}\frac{\hbar\omega_0^2}{4\pi R^2}\left(\frac{\frac{y}{2}}{1+\left(\frac{y}{2}\right)^2}\right)^2 \mathbf{e}_s\, dy \qquad R \text{ for } \qquad Q \gg 1 \tag{30}$$

But we aren't interested in the absolute value but in the relative level only:

$$dS_1 = 8\left(\frac{\frac{y}{2}}{1+\left(\frac{y}{2}\right)^2}\right)^2 dy \tag{31}$$

We want to mark the approximation with dS_2. For the target-function dS_3 we obtain:

$$dS_3 = \frac{\left(2.821439\,\frac{y}{2}\right)^3}{e^{2.821439\frac{y}{2}}-1}\, dy \tag{32}$$

In figure 1 are presented the course of the source-function and the PLANCK's graph.

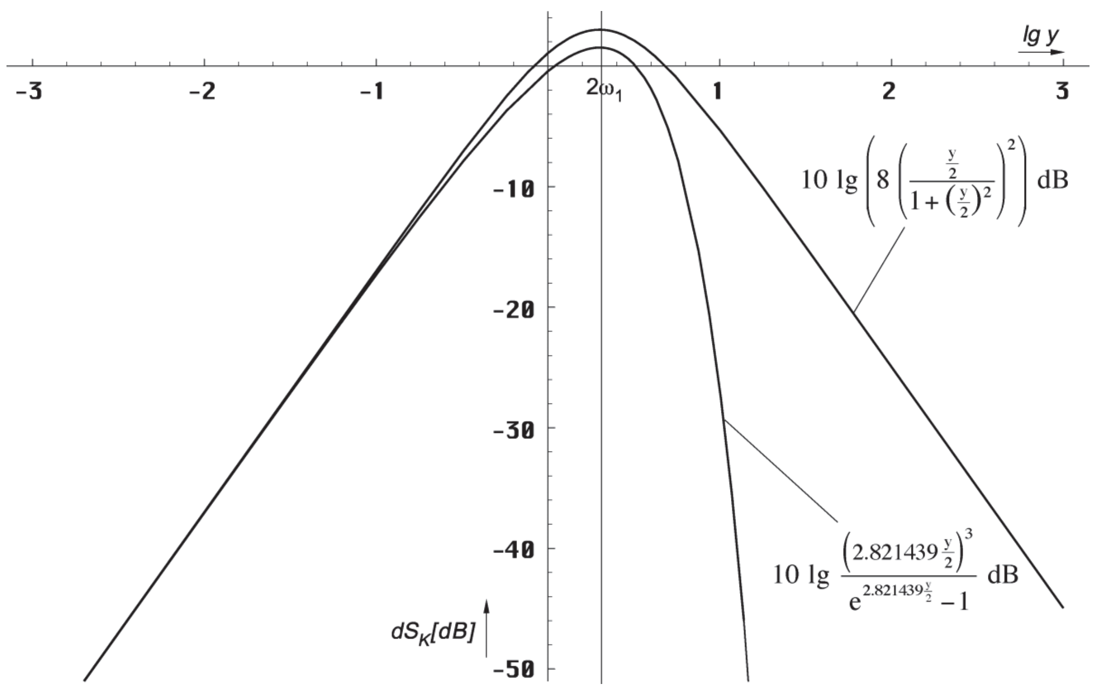

Figure 1
Planck's radiation-rule and source-function
in the superposition (logarithmic, relative level)

3. Solution and evaluation

Of course, there is no shift-information y(Q) contained in these relations. Since the considered system is a minimum phase system, we now have to multiply the source-function dS_1 with the product $A(\omega)\cdot\cos\varphi$ (frequency response). $A(\omega)$ is the amplitude response, the expression $\cos\varphi$ is for the active-share (real-part), because only this is being transferred. The result is our approximation dS_2. The frequency response is merely applied to a single line-element, which is traversed by the signal in the time r_0/c Thereat r_0 is equal to the PLANCK's length and identical to the wavelength of the above-mentioned metric wave-function. That means, we have to execute the multiplication with the frequency response as often as we like, unless the result (almost) no longer changes.

But thereat as well the frequency of the source-function as the cut-off frequency (frequency response) decrease continuously. Therefore it's opportune, to take up the displacement (frequency and amplitude) later on with the result dS_2 (approximation), instead of shifting on and on the location of the source-function. For the proof of our hypothesis indeed this last shift is not of interest, so that we won't take up it in this place.

There is another problem with the amplitude response $A(\omega)$ and with the phase-angle φ. Since the cut-off frequency $\omega_0 = f(Q, \omega_1)$ and the frequency ω are varying according to different functions, it causes difficulties to formulate a practicable algorithm. Thus we use the fact that there is no difference, whether we reduce the frequency of the input-function with constant cut-off frequency or if we shift upward the cut-off frequency with constant input-frequency. We choose this second way incl. the displacement of the approximation at the end of calculation. This all the more, since we would be concerned with two time-dependent quantities (input-frequency and cut-off frequency) otherwise. To the approximation applies:

$$dS_2 = 8\left(\frac{\frac{y}{2}}{1+(\frac{y}{2})^2}\right)^2 \prod_{\frac{1}{2}}^{Q_0} A(y)\cos(\varphi(y))\, đy\, dy \tag{33}$$

Expression (33) looks a little bit strange maybe. It's about a so called product integral, i.e. you have to multiply instead of summate. Then, the letter đ isn't the differential-, but the… let's call it d*ivisional*-operator. I don't want to amplify that, because we anyway have to convert expression (33) to continue. We use $Q_0 = 7.9518 \cdot 10^{60}$ as the current value of the Q-factor and the phase-angle of the metric wave-function[1]. It determines the upper limit of the multiplication resp. summation. Expression (33) possibly appears somewhat strange to the reader. Fortunately the frequency response can be depicted as e-function, so that the product changes into a sum. We simply have to integrate the exponent quite normally then. We obtain the frequency response inclusive phase-correction with the help of the complex transfer-function (150) to:

$$A(\omega)\cdot\cos\varphi = e^{\Psi(\omega)} \qquad \text{Frequency response of a line-element} \tag{34}$$

$$\Psi(\omega) = \frac{1}{2}\ln\left(1+\Omega^2\right) - \frac{\Omega^2}{1+\Omega^2} + \ln\cos\left(\arctan\Omega - \frac{\Omega}{1+\Omega^2}\right) \tag{[1] 302}$$

As next, we substitute Ω by y with the help of (29):

$$\Psi(\omega) = \frac{1}{2}\ln\left(1+\left(\frac{y}{2}\frac{1}{\xi}\right)^2\right) - \frac{\left(\frac{y}{2}\frac{1}{\xi}\right)^2}{1+\left(\frac{y}{2}\frac{1}{\xi}\right)^2} + \ln\cos\left(\arctan\frac{y}{2}\frac{1}{\xi} - \frac{\frac{y}{2}\frac{1}{\xi}}{1+\left(\frac{y}{2}\frac{1}{\xi}\right)^2}\right) \tag{35}$$

The value ω in the numerator of y figures the respective frequency of the cosmic background-radiation, for which we just want to determine the amplitude. It is identical to the ω in PLANCK's radiation-rule. Thereat it's about an overlaid frequency, which is proportional to $Q^{-3/2}$ in the approximation. Instead of the value ω_1 in the denominator actually the PLANCK's frequency ω_0 should be written with the frequency response. That is also the cut-off frequency for the transfer from one line-element to another. But with $Q=1$ the value ω_0 is right equal to ω_1, at which point ω_0 varies with the time; ω_1 on the other hand is strictly defined by quantities of subspace having an invariable value therefore. It applies $\omega_0 = \omega_1/Q$. The frequency ω_0 is exactly proportional to Q^{-1}, which means that even y depends on time, being proportional to $Q^{-1/2}$.

Now we however want to freeze the value ω, at least up to the end of the calculation, which has the consequence, that we must divide y by a supplementary function ξ, which is proportional to $Q^{1/2}$. It applies $\xi = cQ^{1/2}$ and

$$\Psi(\omega) = \frac{1}{2}\ln\left(1+\left(\frac{y}{2}\frac{1}{\xi}\right)^2\right) - \frac{\left(\frac{y}{2}\frac{1}{\xi}\right)^2}{1+\left(\frac{y}{2}\frac{1}{\xi}\right)^2} + \ln\cos\left(\arctan\frac{y}{2}\frac{1}{\xi} - \frac{\frac{y}{2}\frac{1}{\xi}}{1+\left(\frac{y}{2}\frac{1}{\xi}\right)^2}\right) \tag{36}$$

[1] The equality of the Q-factor Q_0 and the phase angle $2\omega_0 t$ is a special property of this function

The factor c arises from the initial conditions at Q=1/2 (resonance-frequency $2\omega_1$, cut-off frequency ω_1) to c=4:

$$y = \frac{\omega}{\omega_0} \sim \frac{2^{-\frac{3}{2}}}{2^{\frac{1}{2}}} = \frac{1}{4} \qquad \xi = 4\sqrt{Q} \quad \text{Approximation} \tag{37}$$

Thus, together with the 2 of y/2, we acquire exactly the same factor 8 as in the source-function (31). Then, the approximation dS_2 calculates as follows:

$$dS_2(y) = 8\left(\frac{\frac{y}{2}}{1+\left(\frac{y}{2}\right)^2}\right)^2 e^{\int_{1/2}^{Q_0} \frac{1}{2}\ln\left(1+\left(\frac{y}{2}\frac{1}{\xi}\right)^2\right) - \frac{\left(\frac{y}{2}\frac{1}{\xi}\right)^2}{1+\left(\frac{y}{2}\frac{1}{\xi}\right)^2} + \ln\cos\left(\arctan\frac{y}{2}\frac{1}{\xi} - \frac{\frac{y}{2}\frac{1}{\xi}}{1+\left(\frac{y}{2}\frac{1}{\xi}\right)^2}\right) dQ} dy \tag{38}$$

For the determination of the integral, a value of 10^3 as upper limit suffices indeed. Over and above this, it changes very little. Therefore, I worked with an upper limit of $3\cdot 10^3$ in the following representations. The integral only can be determined numerically, namely with the help of the function NIntegrate[f(Q), Q, 1/2, 3×10^3]. The quotient of y/2 and ξ expression (37) however describes the dependency y(Q) in the approximation only. There is an exact solution as well. According to [1] (209), (299) and (509) applies:

$$\xi = \frac{a}{b}\frac{1}{Q}\frac{R(Q)}{R(\tilde{Q})}\sqrt{\frac{\beta_\gamma^4 - 1}{\tilde{\beta}_\gamma^4 - 1}} \qquad \text{with } \tilde{Q} = \frac{1}{2} \qquad \text{and} \tag{39}$$

$$R(Q) = 3r_1 Q^{\frac{1}{2}} \int_0^Q \frac{dQ}{\rho_0} \qquad \text{with } \rho_0 = \sqrt[4]{\left(1 - A^2 + B^2\right)^2 + \left(2AB\right)^2} \tag{40}$$

$$A = \frac{J_0(Q)J_2(Q) + Y_0(Q)Y_2(Q)}{J_0^2(Q) + Y_0^2(Q)} \qquad B = \frac{J_2(Q)Y_0(Q) - J_0(Q)Y_2(Q)}{J_0^2(Q) + Y_0^2(Q)} \tag{41}$$

The factor b arises from the demand, that the exact function ξ and its approximation should be of the same size with larger values of Q. The factor a we will determine later on in turn. The functions in (41) are Bessel functions. Problematic in (40) and (45) is the integral, which can be determined even only by numerical methods. In order to avoid the numerical calculation of an integral within the numerical calculation of another integral, it's opportune, to replace the integrand by an interpolation-function (BRQ1), and that inclusive the factor B. The value r_1 cancels itself because of (39). We choose sampling points with logarithmic spacing:

```
brq = {{0, 0}};
For[x = -8; i = 0, x < 25, (++i), x += .1;
 AppendTo[brq, {10^x, N[BRQP[10^x]/BGN/(2.5070314770581117*10^x) ]}]]
BRQ0 = Interpolation[brq];
BRQ1 = Function[If[# < 10^15, BRQ0[#], Sqrt[#]]];
```

(42)

The function BRQP is equal to the product of Q, root-expression and integral in the denominator of (45). The value BGN is equal to the initial value of the same product at Q=1/2. You'll find the complete program in the appendix. The factor b arises to 2.5(0703). According to (211), (482) and (623) applies further:

$$\beta_\gamma = \frac{\sin\alpha}{\sin\gamma_\gamma} \qquad \gamma_\gamma = \arg\underline{c} + \arccos\left(\frac{c_M}{c}\sin\alpha\right) + \frac{\pi}{4} \tag{43}$$

$$\alpha = \frac{\pi}{4} - \arg\underline{c} = \frac{3}{4}\pi + \frac{1}{2}\arg\left(\left(1 - A^2 + B^2\right) + j2AB\right) \qquad c_M = \left|\underline{c}\right| \tag{44}$$

$$\xi \;=\; \frac{3}{0.56408}\,\frac{a}{b}\,Q^{-\frac{1}{2}}\sqrt{\beta_\gamma^4-1}\int_0^Q\frac{dQ}{\rho_0} \;=\; a\,\frac{3}{2}\sqrt{2}\,Q^{-\frac{1}{2}}\sqrt{\beta_\gamma^4-1}\int_0^Q\frac{dQ}{\rho_0} \tag{45}$$

$\underline{c}$ is the complex propagation-velocity of the metric wave-field. As next, we want to take up a comparison of the two functions $Q^{1/2}$ and BRQ1 (figure 2):

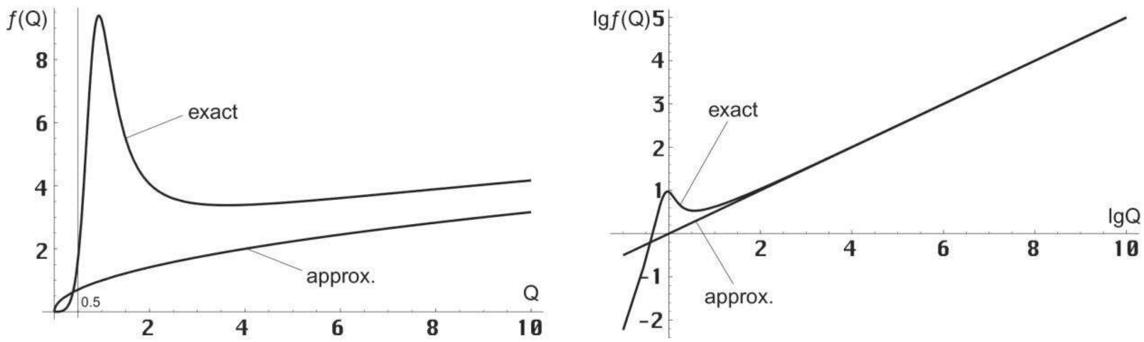

Figure 2
Function BRQ1 exactly and approximation

On the basis of the demand, that the result of both functions must be identical with $Q \gg 1$ we choose the factor a to $\sqrt{\pi}$. In this connection is to be remarked that the exact value is $\sqrt{3,5}$ in fact. But since we finally will not find, in any case, an exact fit in the course of both functions, this small „cheating" in the initial conditions should be allowed. The value $\sqrt{\pi}$ namely leads to the result with the smallest difference, so that we obtain the following final relation for ξ:

$$\xi \;=\; \frac{3}{2}\sqrt{2\pi}\left(Q^{-\frac{1}{2}}\sqrt{\beta_\gamma^4-1}\int_0^Q\frac{dQ}{\rho_0}\right) \qquad\qquad c=\frac{3}{2}\sqrt{2\pi}=3.756 \tag{46}$$

For $\sqrt{3.5}$ a value of c=4 would arise. The bracketed expression corresponds to the factor $Q^{1/2}$ in the approximation. The course of the integral function in (38) as well as of the dynamic cumulative frequency response $A_{ges}(\omega)=e^{\int\Psi(\omega)dQ}$ you can see in figure 3 and 4. For your information the amount of the complex frequency response $|X_n(j\omega)|$ of subspace is plotted, that's the medium, in which the metric wave field propagates ($\Omega_U=\Omega$).

$$X_n(j\omega) \;=\; \frac{1}{2}\,\frac{1}{1+j\Omega}\left(1+\frac{1}{1+j\Omega}\right) \qquad \text{Complex spectral function} \qquad ([1]\ 459)$$

That applies to EM−waves propagating simultaneously with the metric wave field but not to the metric wave field itself. They achieve the aperiodic borderline case at Q=½.

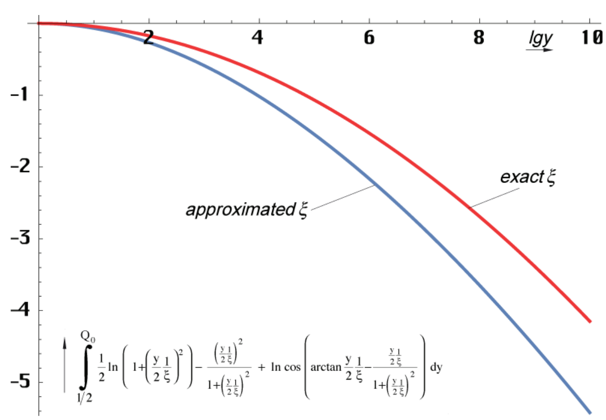

Figure 3
Course of the Integrals $\Psi(\omega)$ in (38) for the approximation and exact function ξ

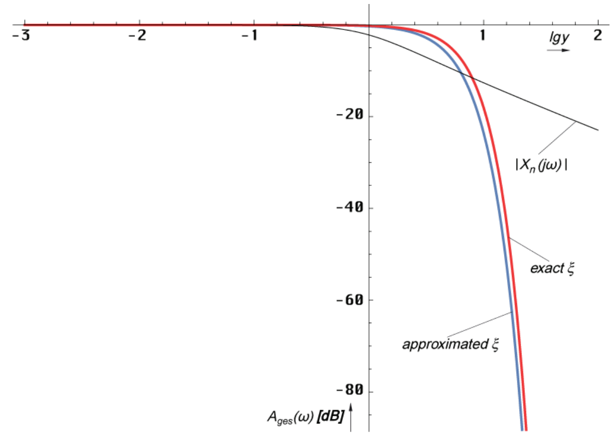

Figure 4
Cumulative frequency response $A_{ges}(\omega)$ and $|X_n(j\omega)|$ of the metric wave field and subspace

Thus, all requirements are filled and we are able to demonstrate the course of the approximation (38) in comparison with the target-function (32) and that as well for the approximation as for the exact function ξ. We use a logarithmic scale with the unit decibel [dB] and, because it's about power per surface, with the factor 10.

Figure 5 shows the shape of the approximation using the approximation (37) for the function ξ (c=4). One can see, both curves doesn't match exactly. The maximum frequency $\Omega_{\pitchfork}$ is downshifted by 18.29% (0.81707). Die maximum deviation of the amplitude $\Delta A_{\doublebarwedge}$ is with +1.20 dB, between both maxima $\Delta A_{\barwedge}$ with +0.4285 dB (+10%). That's comparatively seen, not very much. Altogether the function resembles the shape, shown in [1] section 4.6.4.2.3., obtained by multiplication of the source-function with only 4 choosed values of the frequency response. But there are disparities in the declining branch with higher frequencies.

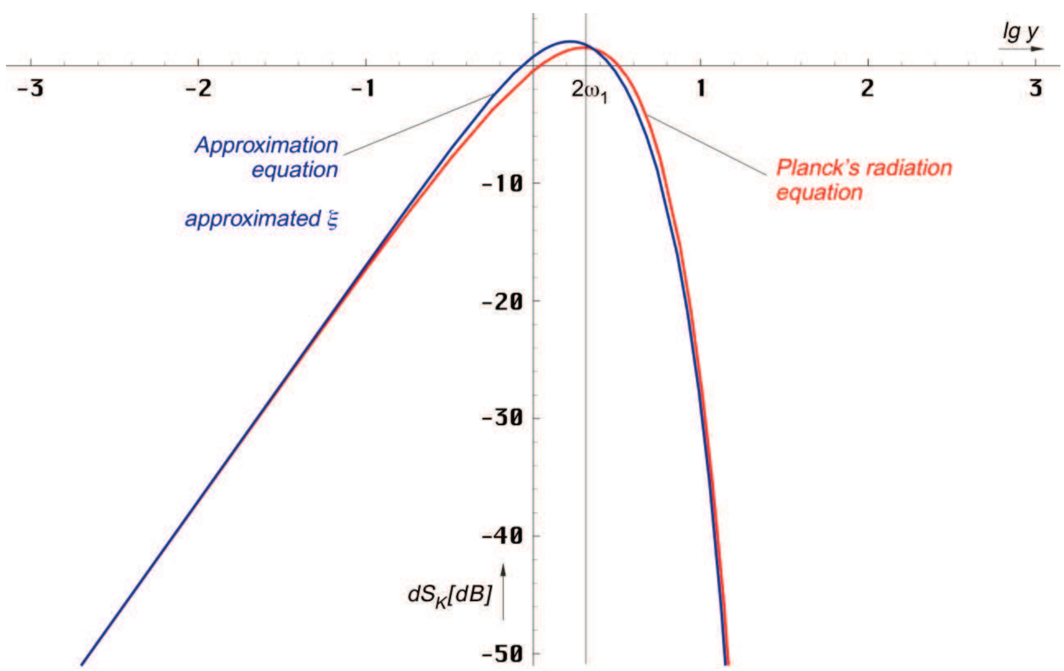

Figure 5
PLANCK's radiation-rule and approximation with approximation for the function ξ (relative level)

Figure 6 presents the course of the approximation under application of the exact function ξ (46) for c=3.756. With it, the best fit (without group delay correction) turns out (With c=4, there is only a minor difference to figure 5). But both functions don't overlap exactly neither in this place. Once again, the maximum frequency $\Omega_{\pitchfork}$ is downshifted by 13.6 % (0.86385). The maximum deviation of amplitude $\Delta A_{\doublebarwedge}$ is about +1.29 dB, between both maxima $\Delta A_{\barwedge}$ with +0.7835 dB (+19.8%).

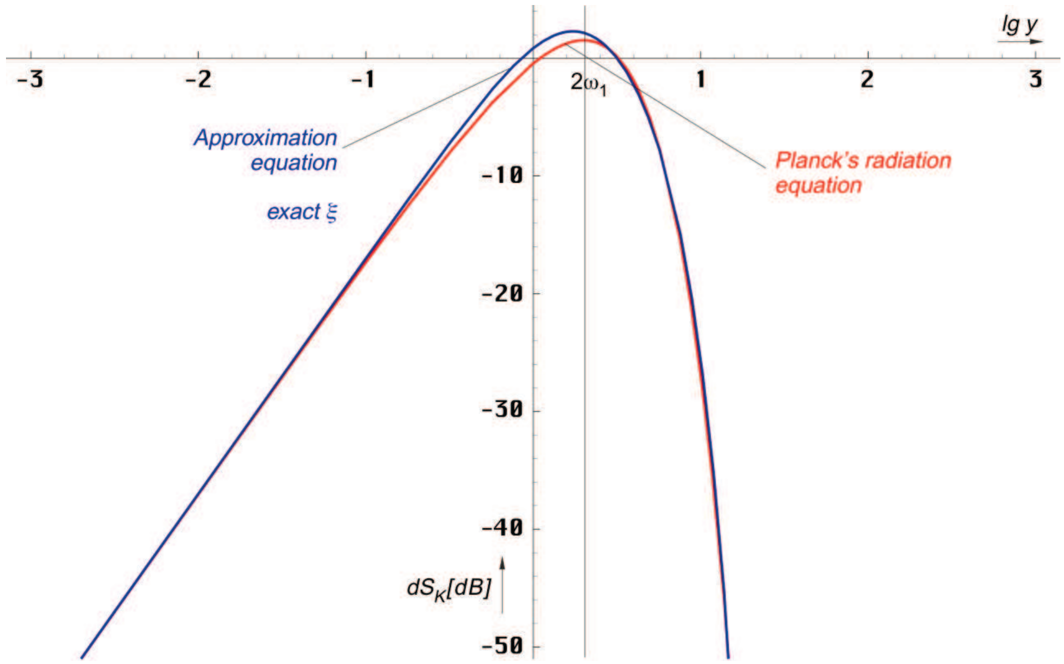

Figure 6
PLANCK's radiation-rule and approximation under application of the exact function ξ (relative level)

The course of deviation (logarithm of the quotient of approximation and PLANCK's radiation-rule) as a function of y is shown in figure 7. One sees, from ca. $10\omega_1$ on the relative deviation between both functions is strongly growing. But since the absolute level in this range is already microscopic (−50dB at the third zero), nobody will take notice of it. Even it seems rather to be about a small frequency shift, than about a deformation of the envelope.

Maybe, the downshift of the approximation's maximum could be a reason for the discrepancy between the CMBR-temperature calculated in 7.5.3. [1] to the measured COBE-value with the amount of +2.42086% (−2.36363% in the reciprocal case). Although, the form of the approximation-graph doesn't correspond to that of a black emitter and the value is too high. But during the COBE-experiment, they just have been ascertained, that the spectrum of the CMBR is exactly? black. Therefore, more forces are required in order to change the form in such a manner, that it equals that of a black emitter. In the next section we will see, which influences may come into consideration for that purpose.

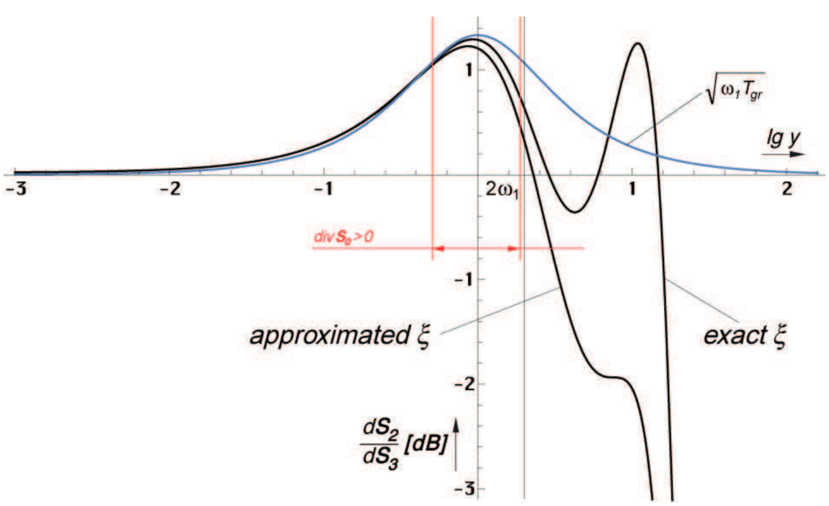

Figure 7
Relative offset between approximation and radiation-rule in dependency of the used function ξ

In figure 7 we can see that we yield an improvement if we use the exact function ξ. Nevertheless a certain left-over difference remains. If we take a look at the course in the 2nd quadrant, we can see a „gap“ where an already known function, multiplied with the factor $\sqrt{2}$, could slot right in there. That's the group delay T_{Gr} of the metric wave field of [1] section 4.3.2. Caution! The variable Ω there is differently defined, namely as $\Omega = \Omega_1 = \omega/\omega_1$. Thus, let's convert the definition to the form used here:

$$T_{Gr} = \frac{dB(\omega)}{d\omega} = 2\frac{\theta^2}{\omega_1} = \frac{2}{\omega_1}\left(\frac{2\Omega}{1+4\Omega^2}\right)^2 \qquad ([1]\ 152)$$

As we can see in figure 7 (blue), the maximum is at ω_1 and not at $2\omega_1$. While group delay is equal to zero across nearly all decades, that's not the case in the proximity of ω_1 respectively ω_0 nowadays. But a frequency-dependent group delay always causes a distortion of the envelope curve. Hitherto, we considered the frequency response $A(\omega)$ and the phase delay $B(\omega)$, but a group delay correction $\Theta(\omega)$ is still missing. Rearranged for θ we obtain:

$$\theta = \frac{2\Omega}{1+4\Omega^2} = \frac{1}{2}\sqrt{2\omega_1 T_{Gr}} \qquad (47)$$

$$\Theta(\omega) = e^{-\sqrt{\omega_1 T_{Gr}}} = e^{-\sqrt{2}\theta} = 10^{-\sqrt{2}\theta\,\lg e} = 10^{-0.614185\,\theta} \qquad (48)$$

We can find the factor $\sqrt{2}$ in that we estimate the maximum deviation of $+1.29393\,\mathrm{dB}$. We have to experiment for a while to find the best match. The decimal power is important, if we want to calculate with dB. The course is depicted in figure 7. The group delay correction $\Theta(\omega)$ on dS_2 is applied only once:

$$dS_2 = 8\left(\frac{\frac{y}{2}}{1+\left(\frac{y}{2}\right)^2}\right)^2 e^{\int_{1/2}^{Q_0}\frac{1}{2}\ln\left(1+\left(\frac{y}{2}\frac{1}{\xi}\right)^2\right)-\frac{\frac{y}{2}\frac{1}{\xi}^2}{1+\frac{y}{2}\frac{1}{\xi}^2}+\ln\cos\left(\arctan\frac{y}{2}\frac{1}{\xi}-\frac{\frac{y}{2}\frac{1}{\xi}}{1+\frac{y}{2}\frac{1}{\xi}^2}\right)dy-\sqrt{\omega_1 T_{Gr}}}\,dy \qquad (49)$$

The resulting functions with group delay correction for both ξ are shown in figure 8 and 9.

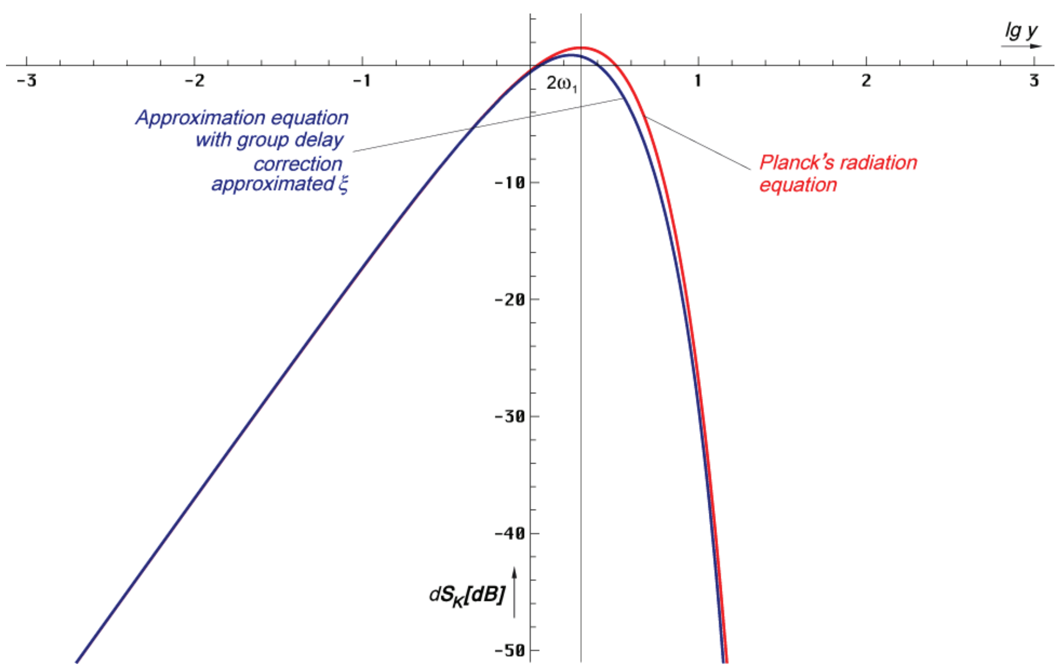

Figure 8
PLANCK's radiation-rule and approximation with group delay correction with approximation of the function ξ (relative level)

There is already a better fit of both graphs in figure 8, as we can see. Now the maximum $\Omega_{\pitchfork}$ of the frequency is downshifted about 14.3% (0.85714). The maximum deviation of amplitude $\Delta A_{\overline{\barwedge}}$ is irrelevant because of the curve progression. The difference between both peaks $\Delta A_{\barwedge}$ is with –0.74601dB (–15.8%).

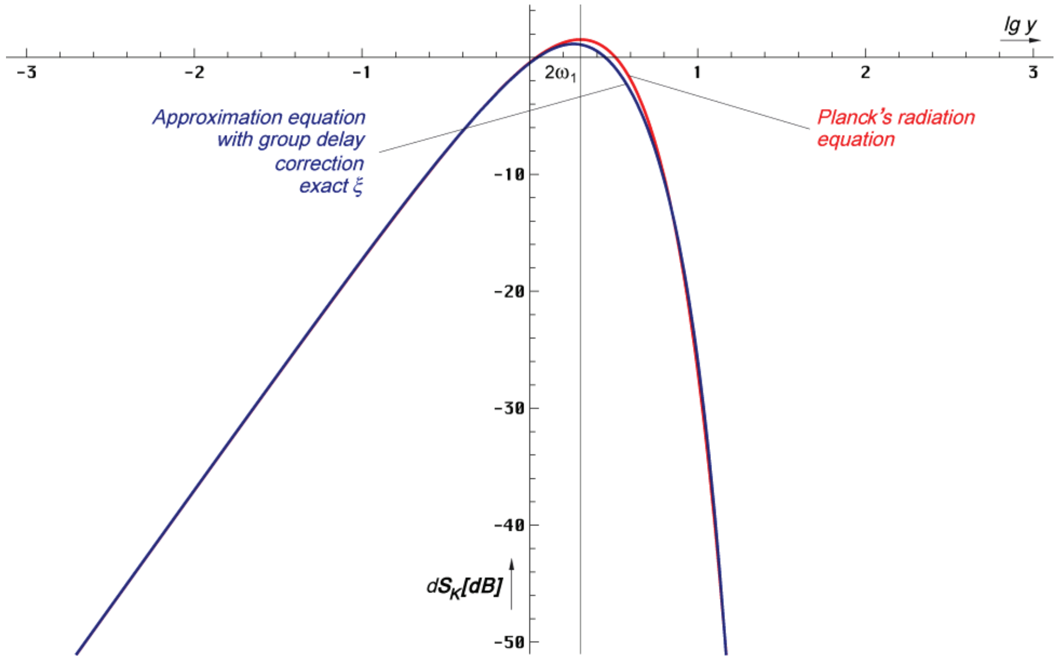

Figure 9
PLANCK's radiation-rule and approximation
with group delay correction under application
of the exact function ξ (relative level)

The best result we have got for the case exact ξ with group delay correction (figure 9). Now the maximum $\Omega_{\pitchfork}$ of frequency is downshifted about −8.831% (0.91169) only. That value is far in excess of the −2.36% deviation between measured and calculated CMBR-temperature. The maximum amplitude deviation $\Delta A_{\overline{\barwedge}}$ is at about +1.01 dB, between both maxima $\Delta A_{\barwedge}$ at −0.38246 dB (−8.430% i.e. 0.9157).

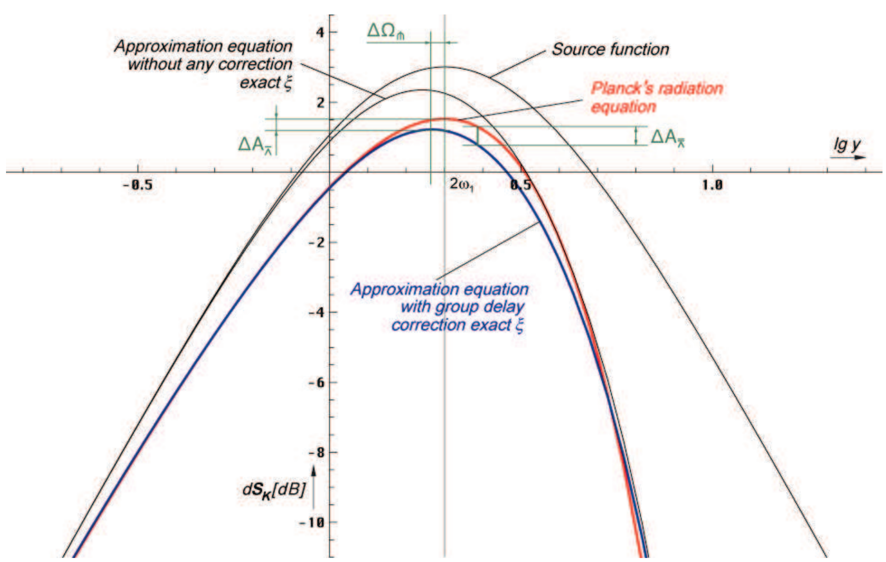

Figure 10
PLANCK's radiation-rule and approximation
with group delay correction under application of
the exact function ξ (relative level) high resolution

To the better clarity, the last case is depicted in figure 10 with higher resolution. You can find the exact results in table 1. Figure 11 shows a summary of the relative deviations of all solutions in comparison with the course of the absolute value of the complex frequency response $|X_n(j\omega)|$ of subspace.

Value	$\Omega_{\pitchfork}$	$\Delta\Omega_{\pitchfork}$	$A_{\barwedge}$	$\Delta A_{\barwedge}$	$\Omega_{\doublebarwedge}$	$\Delta A_{\doublebarwedge}$	$\Omega_{\doublebarwedge}$	$\Delta A_{\doublebarwedge}$
	[1]	[%]	[dB]	[dB]	[1]	[dB]	[1]	[dB]
Planck	1.00000	± 0.00	1.52727	±0.00000	--	--	--	--
Figure 5	0.81707	−18.29	1.95578	+0.42851	0.41943	+1.20007	--	--
Figure 6	0.86385	−13.61	2.28562	+0.75835	0.46495	+1.29393	5.43512	+1.25614
Figure 8	0.85714	−14.28	0.78126	−0.74601	0.05906	+0.04271	--	--
Figure 9	0.91169	− 8.83	1.14481	−0.38246	1.90966	−0.98101	5.50581	+1.01438

Table 1
Extreme values of PLANCK's radiation-function and approximation according to the function ξ used without and with group delay correction

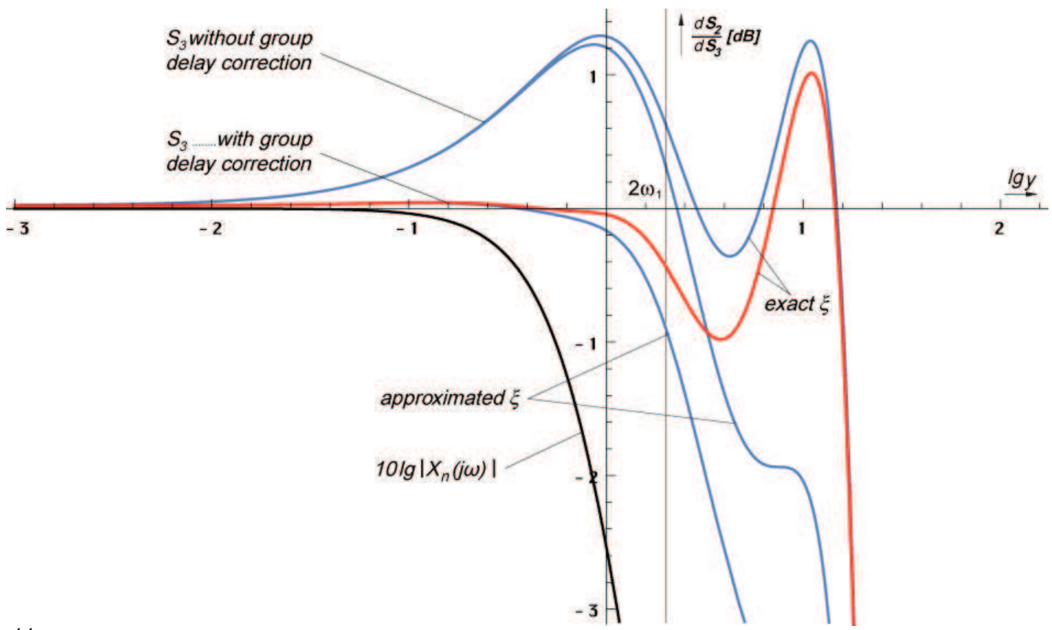

Figure 11
Relative deviation between approximation and radiation-rule according to the function ξ used without and with group delay correction

4. WIEN's Displacement

The solution as per figure 9 seems to best fit the observations, if it weren't for the unsightly dent. Let's suppose, that the ±1dB are „healed up" during the many billion years or have been „ironed out" by other influences not considered here – at the end, we must carry out, as promised, a WIEN-displacement. Starting with the in-coupling frequency $2\omega_1$, with the help of the expressions given in [1] section 2, we are able to calculate the temperature of the CMBR to compare it with the COBE-measuring:

Values from [1] $Q_0=7.9518\cdot10^{60}$, $\hbar_1=8.38572\cdot10^{26}$Js, $\omega_1=1.47506\cdot10^{104}\text{s}^{-1}$, ω_0=PLANCK's frequency

$$T_k = \frac{\hbar\omega_k}{\tilde{x}k} = \frac{\varepsilon_\nu}{\tilde{x}}\frac{\hbar_1\omega_1}{6k}Q^{-\frac{5}{2}} = 0.055693\frac{\hbar_1\omega_1}{k}Q^{-\frac{5}{2}} \qquad \tilde{x} = \begin{cases} 2.821439372 & \text{Exactly} \\ 2\sqrt{2} & \text{Approximation} \end{cases} \qquad ([1]\,405)$$

$$T_k = \frac{\hbar\omega_k}{\tilde{x}k} \approx \frac{1}{3}\frac{\hbar_1\omega_1}{6k}Q^{-\frac{5}{2}} = \frac{\hbar_1\omega_1}{18k}Q^{-\frac{5}{2}} \qquad \varepsilon_\nu = \tfrac{2}{3}\sqrt{2} = 0.9428090416 \qquad ([1]\ 405)$$

$$T_k = \frac{\hbar_1\omega_1}{18k}Q_0^{-\frac{5}{2}} = (1.002476662)\frac{\hbar\omega_0}{18k}Q_0^{-\frac{1}{2}} \qquad \omega_1 = \frac{\kappa_0}{\varepsilon_0} \qquad ([1]\ 902)$$

Substituting the values specified above, we obtain in terms of figures a value of 2.79146K for the temperature T_k, exactly calculated (bracketed expression) even 2.79837K. But the measured value was 2.72548K$\pm$0.00057K. That yields a deviation of +0.06598K (+0.07289K) respectively +2.421% (2.675%). Now one could mean, that's an acceptable result, the model is quite accurate – far wrong. Not for nothing great efforts are being made in order to determine ω_K to decimal places as many as possible, since it's about a flat curve progression there and that takes significant effects on other values. Therefore, from now on, we will calculate with the exact numbers.

From ([1] 902) arises, that Q_0 depends on T_k first of all, $\hbar$ and ω_0 can be determined and calculated with the help of measurements. And most of the other quantities are strongly affected by Q_0. Obtaining a value of $Q_0 = 7.9518\cdot10^{60}$ for the calculated 2.79837K we would get $Q_0 = 8{,}38287\cdot10^{60}$ for 2.72548K. But Q_0 even affects the value of the HUBBLE-parameter:

$$H_0 \approx \omega_0\left(\frac{\hbar\omega_0}{18kT_k}\right)^{-2} \qquad H_0 = 322.4(010652877)\,\omega_0\left(\frac{\hbar\omega_0}{kT_k}\right)^{-2} \qquad ([1]\ 905)$$

H_0 would amount to 71.9843 $\mathrm{kms^{-1}Mpc^{-1}}$ for the calculated temperature of 2.79837K and only 68.2829 $\mathrm{kms^{-1}Mpc^{-1}}$ for 2.72548K. That's quite a significant difference, which neither cannot be solved by number games with the values from table 1. Thus, there must be another reason of deviation.

5. Possible reasons of deviation

Next we want to discuss possible reasons, which may lead to the deviation. The simplest and mostly unpleasant one would be, that this model is wrong. But at least, the result, somewhat well, coincides the predictions, so that we cannot approve it with sufficient certainty. But then there must be another reason. Therefore, the most probable shall be discussed as next.

Since the line-element is a minimum phase system, we computed the approximation function, by an iterative multiplication of the source-function with the just significant amplitude characteristic $A(\omega)$, as long as the result changes essentially. At the point the frequency of the signal-function has dropped far below the cut-off frequency, there is no more change to be observed. The factor $\cos\varphi$ emerges from the fact, that only the real-part is being transferred ($\varphi = B(\omega)$).

That's the procedure with minimum phase systems in general. But according to [3] p. 340 it applies for *stable* minimum phase systems only! Because only with these, an explicit correlation exists between amplitude- and phase response curve, so that we can calculate with the amplitude response exclusively. At the line-element just after the input coupling ($Q\approx1$), that is shortly after big bang however, it's not about a stable system at all. Rather, it shows its largest dynamics to that point of time, so that our approach may lead to an *inexact* result, as we can see.

If we want to get an exact result, we must also introduce a reference between amplitude and phase, quasi a phase-correction, because a phase-lag appears with unstable systems. At the observer the phase-lag manifests itself in the form, that the spectral shares with lower frequency are more redshifted, than the higher frequent ones. Indeed, the lower-frequent shares aren't older than the higher-frequent ones (we observe always the same point of time at the in-coupling with $Q_0=1/2$), but they have covered a longer distance. And that automatically leads to a higher redshift. But how this longer way can be explained? The lower-frequent shares simply took a different route, than the higher-frequent ones (different angle of emission). Because the lower-frequent shares, taking the same way, already have passed us. That leads to a kind of achromatism at the observer, which is hard to be detected, since the radiation arrives from all directions at once. Even with the propagation-function (306) such a phase-lag occurred, characterized by the term $\Phi(\omega)$. We considered that term and we also took a group delay correction. Hence, it cannot be that.

Let's go to talk about the high dynamics during the in-coupling process. Figure 12 shows the course of the energy flux-density vector div$\mathbf{S_0}$ of the metric wave field at that point of time. One sees, it's positive in the range $0.52549<Q<1.5975$. Thus, energy is radiated. The range is depicted even in figure 7. In the range below 0.52549 the field is been established, above 1.5975 the effect of parametric attenuation for overlaid waves can be seen.

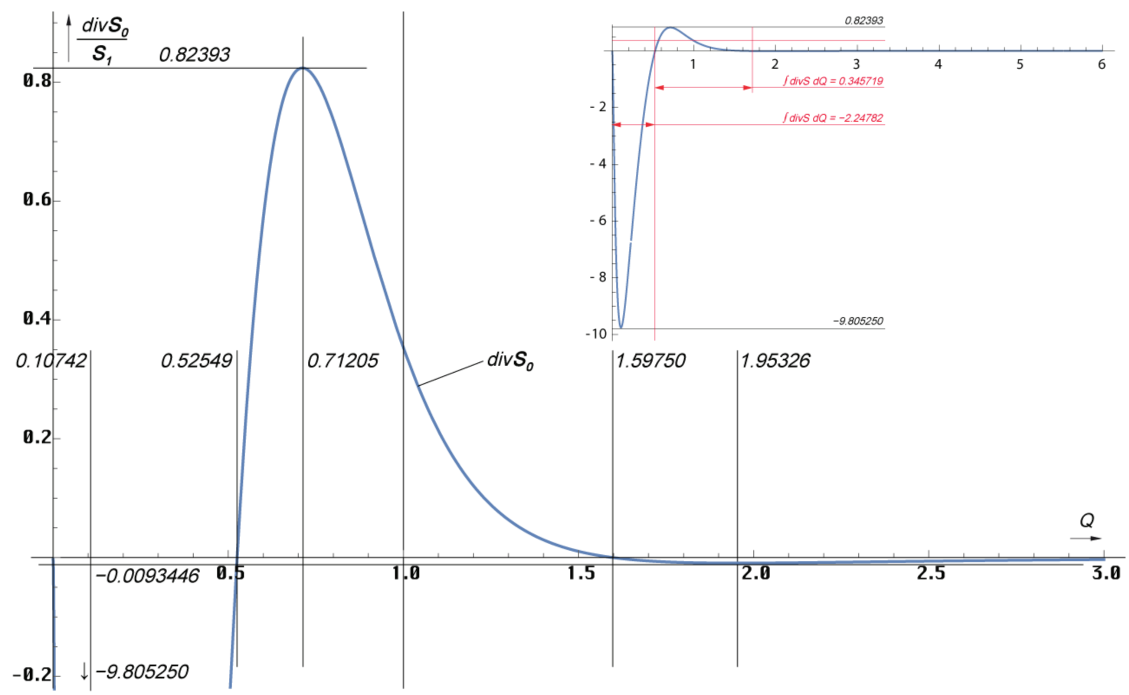

Figure 12
Course of the energy flux-density vector of the metric wave field as a function of Q

Hence, with the in-coupling process it's not about a sudden act with before → after, but it's a dynamic process. Energy is absorbed and partially re-emitted, deferred by the group delay time. At the same time the CMBR is coupled in, according to the frequency at different moments. Concerning the partial re-emission the share of absorbed energy depends on the area ratio of both left-hand sections. The numerical integration yields a value of 2.24784 for the absorbed, as well as of 0.345719 for the re-emitted energy share. The calculation 2.24784/(0.345719+2.24784) a value of 0.866700931 turns out in reference to Q. But we need the value in reference to the time t. Because $t^2 \sim Q$, we must resolve the substitution t^2 on the x-axis in that we extract the root of the result. We obtain a value of 0.930967739. It corresponds, except for a deviation of 0.0118413026, to our *vacuum coefficient of absorption* $\varepsilon_v=0{,}9428090416$.

Thus, the deviation has something to do with the *gray body* [4]. Now, once we already considered ε_v indeed, but only as a constant and with the value at the time of in-coupling. But with the *gray body* ε_v depends on the frequency ω. If we want to consider that, we have to calculate an $\varepsilon_T(\omega)$ respectively a correction term $\varepsilon_K(\omega)$ to multiply ([1] 902) with, since ε_v is already included there. In [4] the following is denoted for ε_T: » Thereby ε_T correlates with the weighted averages of ε_v resp. ε_λ, which are equal:

$$\varepsilon_T = \frac{\int\int_0^\infty \varepsilon_\nu \cdot I(\nu)\cdot \mathrm{d}\nu \cdot \mathrm{d}\Omega}{\int\int_0^\infty I(\nu)\cdot \mathrm{d}\nu \cdot \mathrm{d}\Omega} = \frac{\int\int_0^\infty \varepsilon_\lambda \cdot I(\lambda)\cdot \mathrm{d}\lambda \cdot \mathrm{d}\Omega}{\int\int_0^\infty I(\lambda)\cdot \mathrm{d}\lambda \cdot \mathrm{d}\Omega} \qquad \text{from [4] «} \qquad (50)$$

But we don't want to make it as quite as complicated. Therefore we assume, that the root of the area ratio should equal the average of ε_v, i.e. be equal to ε_T. It applies: $\varepsilon_T=\varepsilon_v\varepsilon_K$, with $\varepsilon_v=⅔\sqrt{2}=0.942809$ and $\varepsilon_K=0.987440402$. Multiplying the calculated $T_k=2.79837$K with ε_K, we obtain a value of 2.76322K, which is about +0.0377K above the measured one. But is it correct, to apply ε_K resp. ε_T simply as a factor to WIEN's displacement law? The answer is no. It's about a factor from PLANCK's radiation-rule. Applying ε_T to (1)…(7), it cancels out at the end. Herewith the inclination 2 at WIEN's displacement rule ($\tilde{x}$ is the ratio slope/peak-line) also applies to the *gray body*. But even a constant of integration would be possible here. There are influences on the displacement indeed. But these depend on the shape of the envelope-curve and, with it, on the function $\varepsilon_v(\omega)$, which we do not know. Therefore we must improvise, contriving a function, which well-complies the requirements. Then, at least, we can see, which influence a frequency-dependent ε_v has onto the shape of the curve and with it even onto the displacement itself.

As a start the function before the in-coupling must have the value $\varepsilon_{vmax}=⅔\sqrt{2}=0.942809$. Furthermore it must vary somehow. We choose a simple change from one to another value. As inflection point we choose the moment of in-coupling with Q=1/2 resp. $2\omega_1$. Then $y=\Omega$ applies. The 0.930967739 from the area ratio of div$\mathbf{S_0}$ are our $\overline{\varepsilon}_T$. We use the function as per (51). Therefrom a lower limit of $\varepsilon_{vmin}=0.920464$ arises. With it $\overline{\varepsilon}_T$ is a little bit smaller than the average, due to the function used. All that appears plausible on the whole, because the metric wave field mostly picks up energy before the in-coupling. Thus, it has a higher absorption coefficient as thereafter, when a share of energy is re-emitted. Even the offset of the zero-transition of div$\mathbf{S_0}$ of Q=0.52549 is mapped very well. If you don't like it, it's only a model and an optimized example function. Whether it really happens in that manner, is another matter.

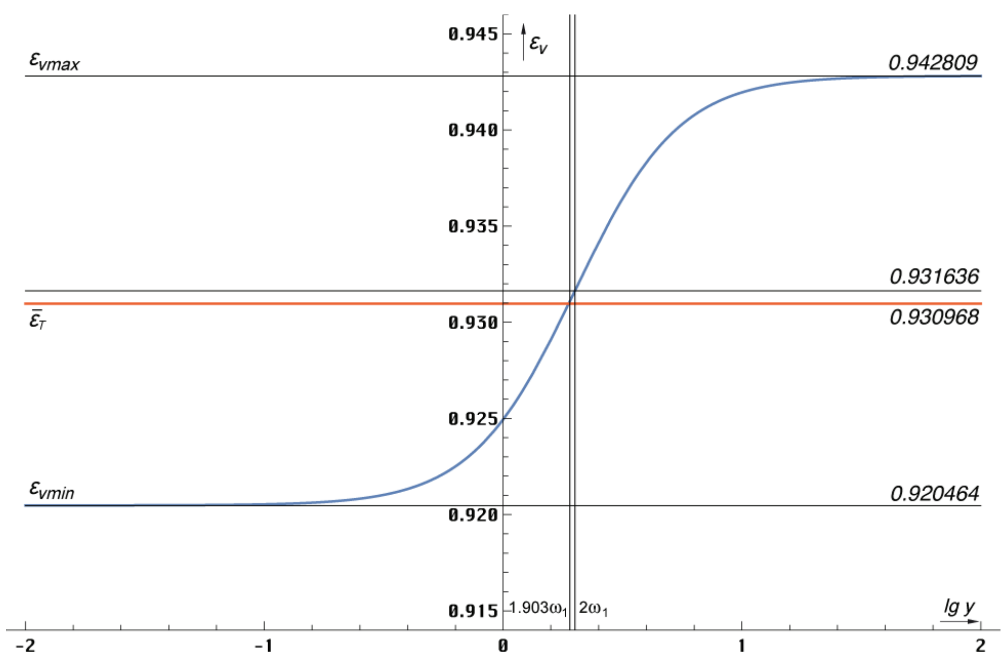

Figure 13
Vacuum coefficient of absorption ε_v as a function of ω

$$\varepsilon_\nu = \varepsilon_{\nu\max}\left(1 - 2\frac{\varepsilon_{\nu\max} - \overline{\varepsilon}_T}{1+\Omega^2}\right) \qquad \varepsilon_{\nu\min} = \varepsilon_{\nu\max}\left(1 - 2\left(\varepsilon_{\nu\max} - \overline{\varepsilon}_T\right)\right) \tag{51}$$

$$\varepsilon_T = \frac{2}{3}\sqrt{2}\left(1 - \frac{0.02368}{1+\Omega^2}\right) \qquad \varepsilon_K = 1 - \frac{0.02368}{1+\Omega^2} \qquad \begin{matrix}\varepsilon_{K\max} = 1.00000\\ \varepsilon_{K\min} = 0.97630\end{matrix} \tag{52}$$

Now we want to analyze the effect of ε_K on the envelope-curve. We believe in the „*self-healing powers*“ of the solution of figure 9 using a clean PLANCK-curve. Since the effect on (51) is hardly to be seen in the graphics, we use an additional, *exaggerated function* ε_{T5} to the better presentation.

$$\varepsilon_{T5} = \frac{2}{3}\sqrt{2}\left(1 - \frac{0.5}{1+\Omega^2}\right) \qquad \varepsilon_{K5} = 1 - \frac{0.5}{1+\Omega^2} \tag{53}$$

That corresponds to an $\overline{\varepsilon}_{T5}$=0.69281. We obtain the following course with it:

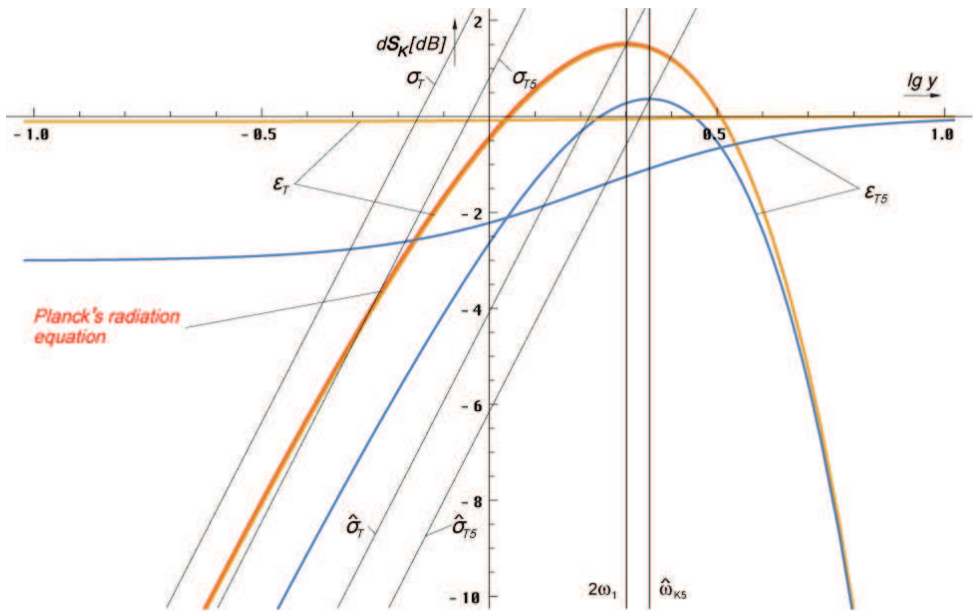

Figure 14
Effect of the absorption coefficient ε_ν
onto the envelope-curve, high resolution

One sees, the function (52) mostly affects the lower-frequent part of the envelope-curve. The maximum is up-shifted in frequency. But the inclination in the left part remains constant. That applies as I said to the example function only. Natural materials may distort the envelope-curve significantly even in this region. Then the regression line applies as a function of $\overline{\varepsilon}_T$ according to (50). Then it has the same inclination and even only, it's more or less amplitude-shifted (constant of integration!). B.t.w. the regression line σ_T resp. the lower-frequent slope is also the line, the WIEN displacement happens at. Here we can see the benefit of the duplicate logarithmic presentation, the curve becomes a line then.

The regression line σ_T can be determined by trying out most suitably. It applies $y=\Omega$ too. In the duplicate logarithmic presentation the following functions arise:

$$\sigma_T(\Omega) = 10\left(2\Omega + \lg\left(2\varepsilon_{K\min}\right)\right) \quad [\mathrm{dB}] \qquad \text{Slope} \tag{54}$$

$$\hat{\sigma}_T(\Omega) = 10\left(2\Omega - \lg\tilde{x} + \lg\varepsilon_{K\min}\right) \quad [\mathrm{dB}] \qquad \text{Maximum} \tag{55}$$

$$\sigma_T(\Omega) = 2\varepsilon_{\mathrm{K\,min}}10^{2\Omega} = 2\varepsilon_{\mathrm{K\,min}}e^{2\ln 10\,\Omega} = 2\varepsilon_{\mathrm{K\,min}}e^{4.60517\,\Omega} \qquad \text{Slope linearly} \qquad (56)$$

That only applies to the example function used here. The 2 on the right side stems from the definition of Ω according to (9). To the *black body* and with it, even to the PLANCK-curve applies $\varepsilon_{Kmin}=\varepsilon_T=\varepsilon_{Kmax}=1$. With natural materials we must replace ε_{Kmin} by $\overline{\varepsilon}_T$ from (50). The course is shown in figure 15. Of course even a regression line for the maximum can be defined. With it ($\tilde{x}$), the circle closes to WIEN's displacement law. However expression (55) isn't very accurate and the line may miss the maximum with smaller ε_{vmin}. But it applies exactly to the *black body* and to our example function. With natural materials even more than one maximum may occur. The more the envelope-curve differs from the ideal, the less reasonable is it, to speak of a radiation temperature.

From (55) arises, that we, nevertheless can define a WIEN's displacement law for the *gray body*, at least for the example function and when the curve-shape do not differ too far from that of a *black body*:

$$T \approx \frac{1}{\tilde{x}\,\varepsilon_{\mathrm{K\,min}}}\frac{\hbar\omega_{\max}}{k} \qquad \text{WIEN's displacement law for the } \textit{gray body} \qquad (57)$$

With natural materials ε_{Kmin} must be replaced by $\overline{\varepsilon}_T$ again.

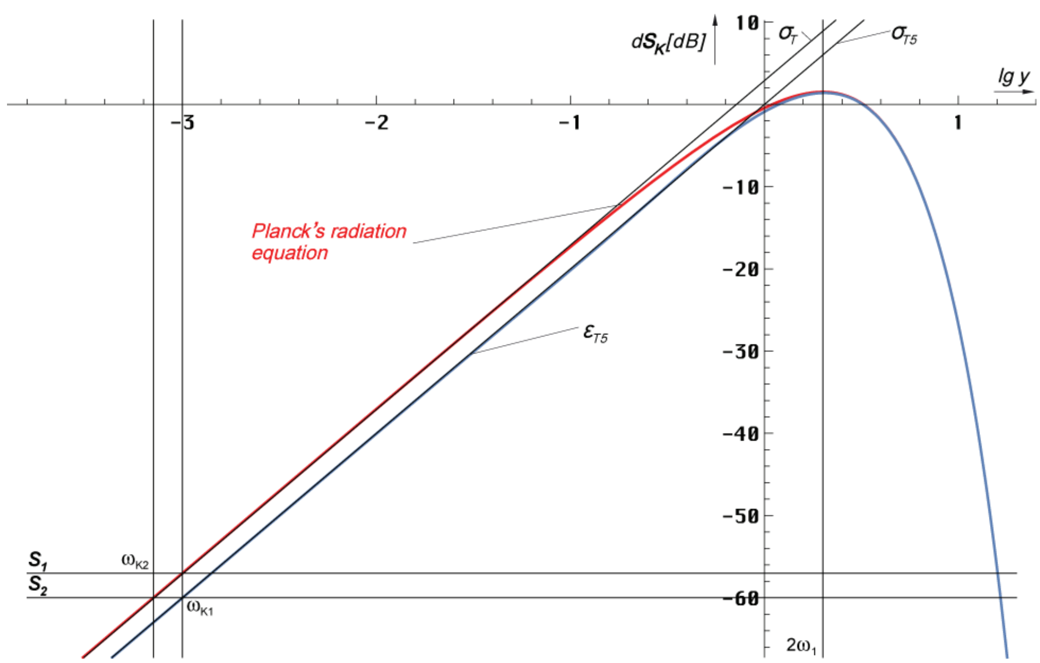

Figure 15
Displacement lines σ_T and σ_{T5}
as well as envelope-curves, low resolution

As next we want to determine the frequency-shift ω_{K2}/ω_{K1}. We choose the *exaggerated function* (53), since we cannot see anything otherwise. We want to navigate in the lower-frequent range, namely at $\omega_{K1}=0.5\cdot10^{-3}\omega_{max}$. Therefore we can employ WIEN's radiation-rule:

$$d\mathbf{S_1} \approx \frac{1}{4\pi^2}\frac{\hbar\omega_{K1}^3}{c^2}\,e^{\frac{\hbar\omega_{K1}}{kT}}\,\mathbf{e_s}\,d\omega \qquad \text{WIEN's radiation rule} \qquad (58)$$

To the amplitude of $d\mathbf{S_2}$ applies ($T_1=T_2=T$) :

$$\mathrm{d}\mathbf{S}_2 \approx \frac{\varepsilon_{\mathrm{K\,min}}}{4\pi^2}\,\frac{\hbar\omega_{\mathrm{K1}}^3}{\mathrm{c}^2}\,\mathrm{e}^{\frac{\hbar\omega_{\mathrm{K1}}}{\mathrm{k}T}}\,\mathbf{e}_s\,\mathrm{d}\omega \qquad = \qquad \frac{1}{4\pi^2}\,\frac{\hbar\omega_{\mathrm{K2}}^3}{\mathrm{c}^2}\,\mathrm{e}^{\frac{\hbar\omega_{\mathrm{K2}}}{\mathrm{k}T}}\,\mathbf{e}_s\,\mathrm{d}\omega \tag{59}$$

By equating we obtain the following expression:

$$\omega_{\mathrm{K2}}^3 = \varepsilon_{\mathrm{K\,min}}\,\omega_{\mathrm{K1}}^3\,\mathrm{e}^{\frac{\hbar\omega_{\mathrm{K1}}}{\mathrm{k}T}-\frac{\hbar\omega_{\mathrm{K2}}}{\mathrm{k}T}} = \varepsilon_{\mathrm{K\,min}}\,\omega_{\mathrm{K1}}^3\,\mathrm{e}^{\frac{\hbar}{\mathrm{k}T}(\omega_{\mathrm{K1}}-\omega_{\mathrm{K2}})} \tag{60}$$

$$\frac{\hbar}{\mathrm{k}T} = \frac{2.821439372}{\omega_{\max}} = \frac{2.821439}{2\cdot 10^3\,\omega_{\mathrm{K1}}} = \frac{1.41072\cdot 10^{-3}}{\omega_{\mathrm{K1}}} \tag{61}$$

$$\omega_{\mathrm{K2}}^3 = \varepsilon_{\mathrm{K\,min}}\,\omega_{\mathrm{K1}}^3\,\mathrm{e}^{1.41072\,10^{3}\left(1-\frac{\omega_{\mathrm{K2}}}{\omega_{\mathrm{K1}}}\right)} \approx \varepsilon_{\mathrm{K\,min}}\,\omega_{\mathrm{K1}}^3\,\mathrm{e}^0 = \varepsilon_{\mathrm{K\,min}}\,\omega_{\mathrm{K1}}^3 \tag{62}$$

$$\omega_{\mathrm{K2}} = \sqrt[3]{\varepsilon_{\mathrm{K\,min}}}\,\omega_{\mathrm{K1}} = \sqrt[3]{0.97630}\,\omega_{\mathrm{K1}} = 0.992037\,\omega_{\mathrm{K1}} \tag{63}$$

With it the frequency of our example function shifts downward by +0,8027% at the base. The offset of the maximum is +0,4860% (Function FindMaximum[#]). Just for information, with the *exaggerated function* ε_{T5} the base-shift is at +25,99%, at the maximum at +12,64%. Thus, in both cases a narrowing of the envelope-curve occurs, at which point the frequency shift at the base is nearly twice as large, as at the maximum. Because with the real values only fractions of a percent come into effect, it looks like the curve is *black*.

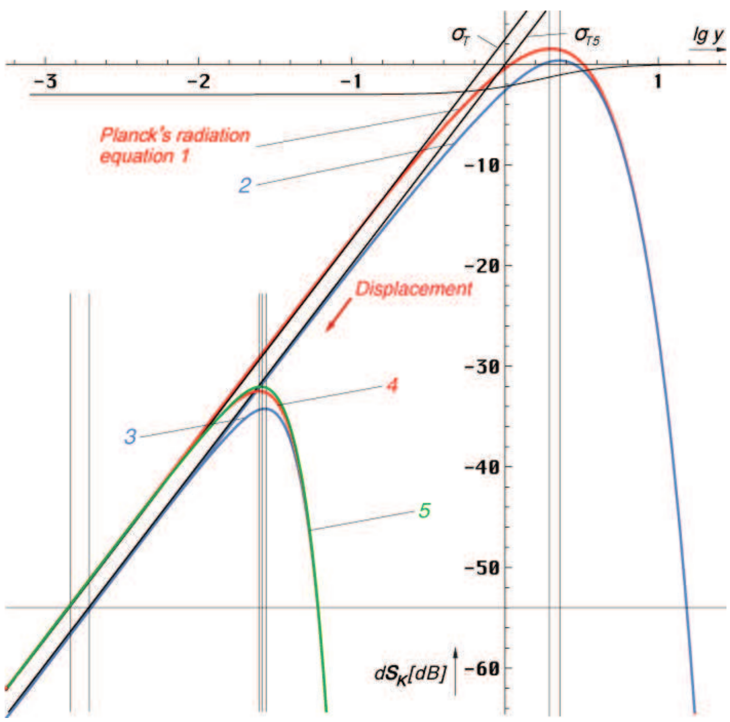

Figure 16
Possible error sources by misinterpretation of the curve-characteristic

Subsequently it's about errors in the interpretation of actual measured data only. The model itself is no issue and it's irrelevant, whether any universal natural constants change over time or not and how. Figure 16 shows what may happen, if we misinterprete the curve-characteristic, by a mistaken application of the *black body* mathematics to a *gray curve*. Curve #1 is the curve of a *black body* at the moment of in-coupling, curve #2 is the *gray curve*. The redshift z (displacement) takes place in the direction of arrow along the displacement line σ_T and σ_{T5}. You can perform it in a graphics program even manually in the following manner: At first duplicate the graph. Then scale it equably by shifting the corner point right above to the bottom left with pressed shift key, maintaining the contact with the displacement line left.

The result are the curves 3 and 4. Now, however the *gray curve* 3 can be „inflated“ in such a manner, that it almost fits the *black curve* 4, that's curve 5 (green). This happens, when a too small redshift z is being assumed, a value, which we actually wanted to determine. One sees, it's possible to wangle a nearly perfect covering of the maxima. The difference is, in practice, nearly undetectable with ε_T-values near 1. The result is, that a too small z and a too small radiation temperature T_k is calculated, and that by half the offset at the base.

Presuming the calculated T_k-value in the amount of 2.79837K to be the *gray temperature,* under consideration of the interpretation error at the measured value of 2.72548K, the application of (57) a measured *gray temperature* of 2,79164K turns out. Then the calculated temperature is only +0.0067K above (+0.25%). Thus, in contrast to the hitherto +7.29%, the improvement wouldn't be insignificant. Of course, I could have configured the example function even such, that I hit the measured value exactly. But that would not have been very meaningful.

In any case, the effects of a possible *gray radiation-characteristic* should be considered, especially then, when we want to measure extremely accurate. But then we can forget the declared accuracy of ±0.00057K for the measured value resp. it applies only relatively and not absolutely.

6. Summary

In the course of this article, according to the model in [1], we succeeded in approximating the envelope-curve of PLANCK's radiation-rule as a function of a dynamic frequency response under application of a phase- and group-delay-correction with a residual deviation of ±1dB. Furthermore was shown, that the temperature calculated in [1] is in the proximity of the value measured by the COBE-satellite. By consideration of the gray characteristic of the CMBR, predicted by the model, could be shown, that and how the measured value is determined too low under misapplication of the black-body-mathematics to a gray radiation source. Under consideration of this issue the calculated CMBR-temperature would be only +0,0067K above the corrected, gray temperature. Whether the self-made gray radiation characteristic coincides with reality, remains unsettled. It's about an example here, just showing the conditions with the gray body. Altogether no contradictions have been found between the model and reality. Furthermore was shown, why the BOLTZMANN-constant has the known value and not another one. The reason is the curve inclination at an oscillating circuit with the Q-factor ½.

The results of the work on hand don't exclude the possibility, that the course of the PLANCK's radiation-rule could be the result of the existence of an upper cut-off frequency of the vacuum. Whether it's so or not, in both cases the classic deduction [2] would not be overruled. Both deductions are compatible and complement each other.

THE END

5. References

[1] **Dipl. Ing. Gerd Pommerenke**
E-Mail-Adresse: GerdPommerenke@arcor.de
The Shape of the Universe, Augsburg 2000-2013, 2020 *viXra:1310.0189*
4. strongly reworked edition, please update older versions

[2] **Ottmar Marti**
Institut für Experimentelle Physik, Universität Ulm
Strahlungsgesetze
http://wwwex.physik.uni-ulm.de/lehre/gk4-2005/node13.html#1474
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[3] **Prof. Dr. sc. techn. Dr. techn. h.c. Eugen Philippow**, TH Ilmenau
Taschenbuch der Elektrotechnik, Band 2, Grundlagen der Informationstechnik
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[4] **Page „Grauer Körper"**
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https://de.wikipedia.org/w/index.php?title=Grauer_K%C3%B6rper&oldid=176666036
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(Last visit: 5. August 2020, 06:58 UTC)

Enveloppe Curve Approximation

Copy Friendly Version

Definitions

```
Off[General::"spell"]
Off[General::"spell1"]
Off[NIntegrate::inumr]
Off[NIntegrate::precw]
Off[NIntegrate::ncvb]

Hankel1=Function[BesselJ[0,#]+I*BesselY[0,#]];
Hankel21=Function[BesselJ[2,#]+I*BesselY[2,#]];

A=Function[(BesselJ[0,#]*BesselJ[2,#]+BesselY[0,#]*BesselY[2,#])/
(BesselJ[0,#]^2+BesselY[0,#]^2)];
B=Function[(BesselY[0,#]*BesselJ[2,#]-
BesselJ[0,#]*BesselY[2,#])/(BesselJ[0,#]^2+BesselY[0,#]^2)];

RhoQ=Function[If[#<30,
Abs[-2*I/#/Sqrt[1-(Hankel21[#]/Hankel1[#])^2]],#^(-1/2)]];
Rho=Function[
  Abs[-2*I/Sqrt[#]/Sqrt[1-(Hankel21[Sqrt[#]]/Hankel1[Sqrt[#]])^2]]];
 (* Rho = Rho(t) c_ *)
InvRhoQ=Function[If[(Abs[#]>.851661),Infinity,If[Abs[#]<=.1,1/#^2,0.346365+0.998383
/#^2-2.50962*#+5.63857*#^2-4.39788*#^3]]];

PhiQ=Function[If[#>20,-Pi/4-3/4/#,N[Arg[-2*I/#/Sqrt[1-
(Hankel21[#]/Hankel1[#])^2]]]]];
InvPhiQ=Function[If[(((-#)>Pi)||((-#)<Pi/4)),Infinity,(*4% Error*)If[((-
3/4/(#+Pi/4))>6),-3/4/(#+Pi/4),3/4/(1/(#-0.5493137)+Pi/4)-
1.4578336150663990315 6]]];

RhoQQ=Function[If[#<30,Sqrt[Sqrt[(1-A[#]^2+B[#]^2)^2+(2*A[#]*B[#])^2]],2/Sqrt[#]]];
(* Arc length unequal RhoQ!!!*)
RhoQQQ=Function[Sqrt[Sqrt[(1-A[#]^2+B[#]^2)^2+(2*A[#]*B[#])^2]]];

rq={{0,0}};
For[x=(-8); i=0,x<4,(++i),x+=.01; AppendTo[rq,{10^x,N[1/RhoQQQ[10^x]]}]];
RhoQ1=Interpolation[rq];

RhoQQ1=Function[If[#<10^4,RhoQ1[#],.5*Sqrt[#]]];
AlphaQ=Function[N[Pi/4-PhiQ[#]]];
BetaQ=Function[Sqrt[#1]*((#2)^2+#1^2*(1-(#2)^2)^2)^(-.25)];
DeltaQ=Function[ArcSin[RhoQ[#]*Sin[AlphaQ[#]]]];

GammaPQ=Function[N[PhiQ[#]+ArcCos[RhoQ[#]*Sin[AlphaQ[#]]]+Pi/4]];
GammaPQa=Function[N[-PhiQ[#]-ArcSin[RhoQ[#]*Sin[AlphaQ[#]]]+Pi/4]];
GammaNQ=Function[N[-PhiQ[#]+ArcSin[RhoQ[#]*Cos[AlphaQ[#]]]-Pi/4]];
GammaNQa=Function[N[PhiQ[#]-ArcCos[RhoQ[#]*Cos[AlphaQ[#]]]-Pi/4]];

Rk=If[#1<1000,3Sqrt[#1] NIntegrate[RhoQQ1[x1],{x1,0,#1}],#1^2]&;
(* Exact world radius/r1 *)

BRQP=Function[Rk[#] Sqrt[(Sin[AlphaQ[#]]/Sin[GammaPQ[#]])^4-1]];
BRQPa=Function[Rk[#] Sqrt[(Sin[AlphaQ[#]]/Sin[GammaPQa[#]])^4-1]];
BRQN=Function[Rk[#] Sqrt[(-Cos[AlphaQ[#]]/Sin[GammaNQ[#]])^4-1]];
BRQNa=Function[Rk[#] Sqrt[(Cos[AlphaQ[#]]/Sin[GammaNQa[#]])^4-1]];
BNQP=Function[0.4073456 #^(-6/4)];
BNQN=Function[0.282048 #^(-7/4)];
BGN=1/3 Sqrt[2]*BRQP[.5];
```

```
brq={{0,0}};
For[x=(-8); i=0,x<25,(++i),x+=.1;
AppendTo[brq,{10^x,N[BRQP[10^x]/BGN/(2.5070314770581117*10^x) ]}]]
BRQ0=Interpolation[brq];
BRQ1=Function[If[#<10^15,BRQ0[#],Sqrt[#]]];

M1=Function[Abs[Hankel1[#]]];
SGenau=Function[Pi/2*Rho[#]^2*Abs[Hankel1[Sqrt[#]]^2]];
kk=Function[Expp[Sqrt[2]*Log10[E]*#/(1+#^2)]];
AnU=Function[.5*1/Sqrt[1+#^2]*(1+1/Sqrt[1+#^2])];
FG=Function[.5/(1+I*#)*(1+1/(1+I*#))];

Xline=Function[10^33*(#1-#2(*Wert_x*))];
Xlline=Function[33+(10^#1-Log10[#2](*Wert_x*))];

Pom=Function[Print[StringJoin["x = ",ToString[10^Chop[First[xx/.Rest[%]],10^-7]],
" Om1","  (",ToString[.5*10^Chop[First[xx/.Rest[#]],10^-7]],
" OmU)"]]];
Pol=Function[Print["y = "<>ToString[First[#]]<>" dB   ("<>If[First[#]-
zzz>0,"+",""]<>
ToString[First[#]-zzz]<>" dB)"]];
Expp=Function[If[#<0,1/Exp[-#],Exp[#]]];
(* Strictly needed to avoid calculation errors *)

xtilde = N[3+ProductLog[-3 E^-3],16];
c = xtilde^2;
b=xtilde;
S1 = 8*(#1/(2*((#1/2)^2 + 1)))^2 & ;
S2 = (b*(#1/2))^3/(Expp[b*(#1/2)] - 1) & ;
Psi0 = (1/2)*Log[1 + (#1/(c*Sqrt[#2]))^2] -
      (#1/(c*Sqrt[#2]))^2/(1 + (#1/(c*Sqrt[#2]))^2) +
      Log[Cos[ArcTan[#1/(c*Sqrt[#2])]] -
         #1/((c*Sqrt[#2])*(1 + (#1/(c*Sqrt[#2]))^
             2))]] & ;
Psi1 = NIntegrate[(1/2)*Log[1 + (#1/(c*Sqrt[Q]))^2] -
       (#1/(c*Sqrt[Q]))^2/(1 + (#1/(c*Sqrt[Q]))^2) +
       Log[Cos[ArcTan[#1/(c*Sqrt[Q])]] -
          #1/((c*Sqrt[Q])*(1 + (#1/(c*Sqrt[Q]))^2))]],
      {Q, 0.5, 3000}] & ;
Psi2 = NIntegrate[(1/2)*Log[1 + (#1/(c*BRQ1[Q]))^2] -
       (#1/(c*BRQ1[Q]))^2/(1 + (#1/(c*BRQ1[Q]))^2) +
       Log[Cos[ArcTan[#1/(c*BRQ1[Q])]] -
          #1/((c*BRQ1[Q])*(1 + (#1/(c*BRQ1[Q]))^2))]],
      {Q, 0.5, 3000}] & ;

G=6.6732*10^-11;     (*Bruker*)
qe=1.60217733*10^-19;
me=9.1093897*10^-31;
mp=1.6726231*10^-27;
mn=1.6749286*10^-27;
ma=1.66057*10^-27;
anull=5.29177*10^-11 (* Bohr's hydrogen radius *);
re=2.81792*10^-15;
km=1000;
Mpc=3.08572*10^19 km;
my0=4 Pi 10^-7;
ep0=8.854187817*10^-12;
ka0=c^3/(my0 G hg H) (*1.23879 10^93*);
k=1.380658 10^-23;
hg=1.05457266*10^-34;
h=2 Pi hg;
hi=4.99697*10^27;
h1=hg*Q0;
hb1=7.95297*10^26;
hiSp=4.99697*10^27;
Z0=Sqrt[my0/ep0];    (*2 Pi 60*)
```

```
Phi0=1.99383*10^-16;
Phi1=6.8626*10^14;
Q884=Function[3/2*(qe^2/ep0/#/me/mp)^(3/2)];        (* #=G *)
Q892=Function[3/8/Pi*qe^4/(ep0^2*me^2*mp*Sqrt[#^3*hg*c])];
Q890=Function[3/2*(1/4/Pi*qe^2*Z0/me*Sqrt[c/#/hg])^3];
c=1/Sqrt[my0 ep0];                             (*2.99792458 10^8*)
Om0=Sqrt[c^5/G/hg];
Om1=Om0 Q0;
Q0=Q890[G];
(*3/2*(qe^2/ep0/G/me/mp)^(3/2)                "884"*)
(*3/2*(1/4/Pi*qe^2*Z0/me*Sqrt[c/G/hg])^3      "890"*)
(*3/8/Pi*qe^4/(ep0^2*me^2*mp*Sqrt[G^3*hg*c])"892"*)
(*7.5419 10^60 "Arbeit"*)
QTAB=7.5419 10^60;
Qrel=Function[Q0*(Sqrt[1+#1]-(2*#2)^(2/3))];
Qabs=Function[(Sqrt[2*ka0*#1/ep0]-Q0*(2*#2)^(2/3))];
H=Om0/Q0;
(*8/3*Pi*G/my0/Z0*me^2*mp/qe^4 2.45972*10^-18*)
r1=1/(ka0 Z0);
r0=Q0 r1;                                         (*1.596 10^-35*)
R=Q0^2 r1;
T=1/(2 H);                                        (*2.03275 10^17*)
t1=T/Q0^2;                                      (*3.57372 10^-105*)
qn=Sqrt[hg/Z0];
```

Source Function

```
(*b = xtilde; Figure 1 *)
Plot[{
Log10[(b*.5*10^y)^3/(Expp[b*.5*10^y]-1)],
Log10[ 8*(.5*10^y/((.5*10^y)^2+1))^2],
Xline[y,Log10[2]]},{y, -5, 3},PlotRange->{-10.1,.45}]
```

Expansion

```
Plot[{(*Log10[BRQP[10^qqq]/BGN/(2.5070314770581117*10^qqq)], Figure 2a *)
Log10[BRQ1[10^qqq]], Log10[Sqrt[10^qqq]]}, {qqq, -1, 10}]
Plot[{(*BRQP[qqq]/BGN/(2.5070314770581117*qqq), Figure 2b *)
BRQ1[qqq], Sqrt[qqq]}, {qqq, 0, 10}, PlotRange -> {-0.3, 9.6}]
Integral

 c=8; (*Factor 8 approx ξ Figure 3 *)
Plot[{Psi1[y],Psi2[y]},{y,0,10},
(*PlotRange->{-5.8,0.2},*)PlotStyle->RGBColor[0.91,0.15,0.25],
PlotLabel->None,LabelStyle->{FontFamily->"Chicago",10,GrayLevel[0]}]

 c=8; (*Factor 8 approx ξ Skipped *)
Plot[{Expp[Psi1[y]],Expp[Psi2[y]]},{y,-4,4}(*,
PlotRange->{0,2.35}*),PlotLabel->None,LabelStyle->{FontFamily-
>"Chicago",10,GrayLevel[0]}]

 c=8; (*Factor 8 approx ξ Figure 4 *)
Plot[{10Log10[Expp[Psi1[10^y]]],10 Log10[Expp[Psi2[10^y]]]},{y,-3,2},
PlotRange->{-88,2},LabelStyle->{FontFamily->"Chicago",12,GrayLevel[0]}];

 Plot[{10 Log10[Abs[FG[10^y]]]},{y,-3,2},PlotRange->{-88,2},
PlotLabel->None,PlotStyle->RGBColor[0,0,0],
LabelStyle->{FontFamily->"Chicago",10,GrayLevel[0]}];

 Show[%%,%]
```

Approximation 1

```
 c=8; (* Factor 8 approximated BGN exact Figure 5 *)
Plot[{10 Log10[S2[10^y]],10
(Log10[S1[10^y]*Expp[Psi1[10^y]]]),Xline[y,Log10[2]]},{y,-3,3},
```

```
PlotRange->{-51,10.5},ImageSize->Full,LabelStyle->{FontFamily->"Chicago",
10,GrayLevel[0]}]

 c=7.519884824; (* Sqrt[π] exact ξ Figure 6 *)
Plot[{10 Log10[S2[10^y]],10 (Log10[S1[10^y]]+Log10[E]*Psi2[10^y]),
Xline[y,Log10[2]]},{y,-3,3},PlotRange->{-51,4.5},ImageSize->Full,
LabelStyle->{FontFamily->"Chicago",10,GrayLevel[0]}]
```

Extreme Values 1

```
FindMaximum[10 Log10[S2[10^xx]],{xx, 0}];
(* Planck's curve *)
Print[StringJoin["x = ",ToString[(10^First[xx/.Rest[%%]])],
" Om1        (1.000000 OmU)"]]
Print[StringJoin["y = ",ToString[zzz = First[%%%]]," dB    (±0.000000 dB)"]]

FindMaximum[10 (Log10[S1[10^xx]*Expp[Psi1[10^xx]]])-10Log10[S2[10^xx]],{xx,1}] ;
(* Maximum deviation Psi1 *)
Pom[%]
Pol[%%]

FindMaximum[10 (Log10[S1[10^xx]*Expp[Psi2[10^xx]]])-10Log10[S2[10^xx]],{xx,0}];
(* Maximum deviation 1 Psi2 *)
Pom[%]
Pol[%%]

FindMaximum[10 (Log10[S1[10^xx]*Expp[Psi2[10^xx]]])-10Log10[S2[10^xx]],{xx,1}];
(* Maximum deviation 2 Psi2 *)
Pom[%]
Pol[%%]

FindMaximum[10 (Log10[S1[10^xx]]+Log10[E]*Psi1[10^xx]),{xx,-1}];
(* Deviation between maxima Psi1*)
Pom[%]
Pol[%%]

FindMaximum[10 (Log10[S1[10^xx]]+Log10[E]*Psi2[10^xx]),{xx,-3,2}];
(* Deviation between maxima Psi2 *)
Pom[%]
Pol[%%]
```

Deviation 1

```
 c=8; (*Factor 8 approx ξ Figure 7 *)
Plot[{10 Log10[S1[10^y]*Expp[Psi1[10^y]]/S2[10^y]],Xline[y,Log10[2]]},
{y,-3,2},PlotRange->{-3.1,1.35},ImageSize->Full,LabelStyle->
{FontFamily->"Chicago",10,GrayLevel[0]}];

 c=7.519884824; (* Sqrt[π] exact ξ *)
Plot[{10 Log10[S1[10^y]*Expp[Psi2[10^y]]/S2[10^y]]},{y,-3,2},
ImageSize->Full,LabelStyle->{FontFamily->"Chicago",10,GrayLevel[0]}];

 Show[%,%%%,PlotRange->{-3.1,1.35}]

 Plot[{10Log10[kk[10^x]]},{x,-3,2.2},PlotRange->{-0.6,3},
PlotStyle->RGBColor[0.06,0.52,0.]];

Show[%%,%,ImageSize->Full,LabelStyle->{FontFamily->"Chicago",12,GrayLevel[0]}]
```

Approximation 2

```
 c=8; (* Factor 8 approximated BGN exact Figure 8 *)
Plot[{10 Log10[S2[10^y]],10 (Log10[S1[10^y]*Expp[Psi1[10^y]]])-10Log10[kk[10^y]],
Xline[y,Log10[2]]},{y,-3,3},PlotRange->{-51,4.5},ImageSize->Full,LabelStyle-
>{FontFamily->"Chicago",10,GrayLevel[0]}]
```

```
 c=7.519884824; (* Sqrt[π] exact ξ Figure 9 *)
Plot[{10 Log10[S2[10^y]],10 (Log10[S1[10^y]]+Log10[E]*Psi2[10^y])-
10 Log10[kk[10^y]],Xline[y,Log10[2]]},{y,-3,3},PlotRange->{-51,4.5},
ImageSize->Full,LabelStyle->{FontFamily->"Chicago",10,GrayLevel[0]}]
```

Extreme Values 2

```
FindMaximum[10 Log10[S2[10^xx]],{xx, 0}];
(* Planck's curve *)
Print[StringJoin["x = ",ToString[(10^First[xx/.Rest[%%]])],
" Om1        (1.000000 OmU)"]]
Print[StringJoin["y = ",ToString[zzz = First[%%%]]," dB    (±0.000000 dB)"]]

FindMaximum[10 Log10[(S1[10^xx]*Expp[Psi1[10^xx]]/kk[10^xx])/S2[10^xx]],{xx,0}];
(* Maximum deviation Psi1 *)
Pom[%]
Pol[%%]

FindMaximum[10 Log10[(S1[10^xx]*Expp[Psi2[10^xx]]/kk[10^xx])/S2[10^xx]],{xx,0}];
(* Maximum deviation 1 Psi2 *)
Pom[%]
Pol[%%]

FindMinimum[10 Log10[(S1[10^xx]*Expp[Psi2[10^xx]]/kk[10^xx])/S2[10^xx]],{xx,.5}];
(* Maximum deviation 2 Psi2 *)
Pom[%]
Pol[%%]

FindMaximum[10 Log10[(S1[10^xx]*Expp[Psi2[10^xx]]/kk[10^xx])/S2[10^xx]],{xx,1}];
(* Maximum deviation 3 Psi2 *)
Pom[%]
Pol[%%]

FindMaximum[10 Log10[S1[10^xx]*Expp[Psi1[10^xx]]/kk[10^xx]],{xx,0}];
(* Deviation between maxima Psi1 *)
Pom[%]
Pol[%%]

FindMaximum[10 Log10[S1[10^xx]*Expp[Psi2[10^xx]]/kk[10^xx]],{xx,0}];
(* Deviation between maxima Psi2 *)
Pom[%]
Pol[%%]

 Plot[{(* Figure 10 *)
10 Log10[S1[10^y]],
10 Log10[S2[10^y]],
10(Log10[S1[10^y]]+Log10[E]*Psi2[10^y]),
10 (Log10[S1[10^y]]+Log10[E]*Psi2[10^y]-Log10[kk[10^y]]),
Xline[y,Log10[2]]
},{y,-0.8,1.4},PlotRange->{-11,4.5},PlotLabel->None,ImageSize->Full,LabelStyle-
>{FontFamily->"Chicago",10,GrayLevel[0]}]
```

Deviation 2

```
 c=7.519884824; (* Sqrt[π] exact ξ Figure 11 *)
Plot[{10 Log10[S1[10^y]*Expp[Psi1[10^y]]/S2[10^y]]-10Log10[kk[10^y]],
10 Log10[S1[10^y]*Expp[Psi2[10^y]]/S2[10^y]]-10Log10[kk[10^y]]},{y,-3,2},
ImageSize->Full,LabelStyle->{FontFamily->"Chicago",10,GrayLevel[0]}];

 Show[%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%,%,PlotRange->{-3.1,1.35}]
```

Roots

```
 FindRoot[10 (Log10[S1[10^y]]+Log10[E]*Psi2[10^y])-10Log10[kk[10^y]]-
10Log10[S2[10^y]]==0,{y,.5}]

 FindRoot[10 (Log10[S1[10^y]]+Log10[E]*Psi2[10^y])-10Log10[kk[10^y]]-
10Log10[S2[10^y]]==0,{y,1}]

 FindRoot[10 (Log10[S1[10^y]]+Log10[E]*Psi2[10^y])-10Log10[kk[10^y]]-
10Log10[S2[10^y]]==0,{y,2}]

 N[10^0.846931]  (* Level at 2nd null *)
ToString[10 Log10[S2[%]]]<>" dB"

 N[10^1.1612]    (* Level at 3rd null *)
ToString[10 Log10[S2[%]]]<>" dB"

 N[10^1.4142]    (* Level after 3rd null *)
ToString[10 Log10[S2[%]]]<>" dB"

 Plot[{(* Skipped *)
10 Log10[S1[10^y]],
10 Log10[S2[10^y]],
10(Log10[S1[10^y]]+Log10[E]*Psi2[10^y]),
10 (Log10[S1[10^y]]+Log10[E]*Psi2[10^y]-Log10[kk[10^y]]),
Xline[y,Log10[2]]},{y,-3,3},PlotRange->{-51,4.5},PlotLabel->None,
ImageSize->Full,LabelStyle->{FontFamily->"Chicago",10,GrayLevel[0]}]
```

Energy Flux Density Vector

```
w0g=Function[Sqrt[Pi^3/8]*M1[Sqrt[#]]^3*Rho[#]^3];
w0n=Function[#^-(3/2)];
w0nPunkt2Int=Function[-(w0n[#])^2+.897659];
w0gPunkt=Function[(w0g[#+.00001]-w0g[#])/.00001];
w0gPunkt2=Function[(w0g[#+.00001]^2-w0g[#]^2)/.00001];
w0gPunkt2Int=Function[-(w0g[#])^2+.897659];
ka0g=Function[Pi/4*M1[Sqrt[#]]^2*Rho[#]^2];
ka0g2=Function[Pi^2/12*M1[Sqrt[#]]^4*Rho[#]^4];
ka0g2n=Function[1/3*#^(-2)];
ka0g2Int=Function[NIntegrate[ka0g2[tt],{tt,0,#}]];
ka0g2nInt=Function[-1/(6*#1^(3/2))+1/(6*10^(3/2))+0.345818];

Plot[{-w0gPunkt2[t^2]-ka0g2[t^2]},{t,0,3},PlotRange->{-0.22,0.88}, (* Figure 12 *)
PlotLabel->None,ImageSize->Full,LabelStyle->{FontFamily-
>"Chicago",10,GrayLevel[0]}]
```

Displacement Line

```
b = xtilde;
Plot[{(* Skipped *)
Log10[S2[10^y]], Log10[S1[10^y]],Xline[y,Log10[2]],
   2*y + Log10[2], 2*y - Log10[xtilde]}, {y, -3.05, 3.05},
  PlotRange -> {0.55, -5.05}, ImageSize -> Full,
  LabelStyle -> {FontFamily -> "Chicago", 10,
    GrayLevel[0]}]

b = 2.821439;
Plot[{(* Skipped *)
N[(b*y)^3/(E^(b*y) - 1)], 10^N[2*Log10[y] + Sin[2]]},
{y, 0, 0.15}, PlotRange -> {0, 0.2}]
```

Gray body

Definitions

```
x=2.972456 10^-63;
y=8.6556 10^-64;
z=y 2^(1/6)/3^(2/3) Q0^-.5;fff=Function[1/(1+(#1/#2)^2)];
fff=Function[1/(1+(#1/#2)^2)];
ggg=Function[1/(1+((#1/#2)-(#2/#1))^2)];
hhh=Function[2*(#1/#2)/(1+(#1/#2)^2)];
Ek3=Function[1-0.0236820832fff[#1,#2]];
Ek5=Function[1-0.5fff[#1,#2]];              (* Ek5 over-scaled !!! *);
```

Absorbing Coefficient

```
 Plot[{
2/3Sqrt[2]Ek3[10^xxx,2Om0],0.942807,.920464,.930967739,
Xline[xxx,Log10[2Om0]]},
(* Epsilon T *)
{xxx,-2+ Log10[Om0],2+ Log10[Om0]},PlotRange->{0.91,0.95}]

 Plot[{(* Figure 13 *)
2/3Sqrt[2]Ek3[10^xxx,2],0.942807,.920464, 0.930967739,(0.942807+.920464)/2,
Xline[xxx,Log10[2]],Xline[xxx,Log10[1.903]]},
{xxx,-2,2},PlotRange->{0.914,0.946},ImageSize->Full,PlotLabel->None,
LabelStyle->{FontFamily->"Chicago",11,GrayLevel[0]}]
(* Epsilon T *)


aaa = Log10[2];
bbb = xtilde (*2*Sqrt[2]*);
ccc = 1;
Plot[{(* Figure 14 *)
  10*Log10[(bbb*10^(zzz - aaa))^3/(E^(bbb*10^(zzz - aaa)) - 1)],
  10*Log10[Ek3[10^(zzz - aaa), ccc]*((bbb*10^(zzz - aaa))^3/
       (E^(bbb*10^(zzz - aaa)) - 1))],
  10*Log10[Ek5[10^(zzz - aaa), ccc]*((bbb*10^(zzz - aaa))^3/
       (E^(bbb*10^(zzz - aaa)) - 1))], 10*Log10[Ek3[10^(zzz - aaa), ccc]],
  10*Log10[Ek5[10^(zzz - aaa), ccc]],
  Xline[zzz, Log10[2]],Xline[zzz,0.35271201428301324],
  10*(2*zzz + Log10[2]), 10*(2*zzz - Log10[xtilde]),
  10*(2*zzz + Log10[2*0.69281]),
  10*(2*zzz - Log10[xtilde] + Log10[(0.69281+.5)/2])
  }, {zzz, -1.02, 1.02}, PlotRange -> {-10.25, 3.25}, ImageSize -> Full,
  PlotLabel -> None, LabelStyle -> {FontFamily -> "Chicago", 12, GrayLevel[0]}]
```

Extreme Values 3

```
 FindMaximum[10*Log10[S2[10^zzz]],{zzz,-1.02,1.02}]

 FindMaximum[10*Log10[(bbb*10^(zzz-aaa))^3/(Exp[(bbb*10^(zzz-aaa))]-1)],
{zzz,-1.02,1.02}]

 FindMaximum[10*Log10[Ek3[10^(zzz-aaa),ccc]*((bbb*10^(zzz-aaa))^3/
(E^(bbb*10^(zzz-aaa))-1))],{zzz,-1.02,1.02}]

 FindMaximum[10*Log10[Ek5[10^(zzz - aaa), ccc]*((bbb*10^(zzz - aaa))^3/
       (E^(bbb*10^(zzz - aaa)) - 1))],{zzz,-1.02,1.02}]

aaa = 0*Log10[2];
bbb = xtilde (*2*Sqrt[2]*);
ccc = 0.5 (* Q(max) *);
```

```
Plot[{(* Figure 15 *)
  10*Log10[S2[10^zzz]],
  10*Log10[Ek5[10^zzz, ccc]*S2[10^zzz]],
  Xline[zzz, Log10[2]], Xline[zzz,-3], 10*Log10[S2[10^-3]],
  10*(2*zzz + Log10[2(1-0.0268)]),
  10*(2*zzz + Log10[2(1-0.5)])(* 2 εKmin *)},
  {zzz, -3.8, 1.3}, PlotRange -> {-67.25, 10.25}, ImageSize -> Full,
  PlotLabel -> None, LabelStyle -> {FontFamily -> "Chicago", 12, GrayLevel[0]}]

aaa = 1*Log10[2];
bbb = xtilde;
ccc = 0.5;
Plot[{(* Figure 16 *)
      10*Log10[(bbb*10^(zzz - aaa))^3/(Expp[bbb*10^(zzz - aaa)] - 1)],
      10*Log10[Ek5[10^(zzz - aaa), ccc]*((bbb*10^(zzz - aaa))^3/
      (E^(bbb*10^(zzz - aaa)) - 1))], 10*Log10[Ek5[10^(zzz - aaa), ccc]],
      Xline[zzz, Log10[2]],Xline[zzz,0.35271201428301324]},
      {zzz, -3.8, 3.4}, PlotRange -> {-67.25, 5.25}, ImageSize -> Full,
      LabelStyle -> {FontFamily -> "Chicago", 10, GrayLevel[0]}]
Beep[]
Beep[]
Beep[]
```