Supplement Formulas of the Fine-structure Constant α, New Formulas of Euler Number e and Their Relationships with Nuclides

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Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70^{th} birthday

Abstract

This paper is a subsequent paper to the paper "Chen's Formulas of the Fine-structure Constant" (viXra:2002.0203) for giving some supplements. In the previous paper, many formulas of the fine-structure constant α based on the most important key number 112 had been given. In this paper, some new formulas of α based on a subsequent key number 173 were deduced, some formulas of α were expressed with large integers, some new formulas of Euler number e and their relationships with nuclides were given, the relationships of some formulas of 2π with nuclides were revealed, the relationships between some constants (e, γ, γ_c, γ_g and γ_{cg}) and nuclides were disclosed, a picture showing the unification of mathematics and physics through α was depicted, the most important formula of the speed of light in atomic unites c_{au} was revised to be more reasonable, a Fibonacci sequence containing 173 and its relationships with nuclides were proposed, the meanings of the numerical values of α and c_{au} were discovered, some formulas of α based on 137, 83, 83^2, 112×173, 163×173, 36/112, 100×112 and so on were presented, continued fractions of α and c_{au} were gained, an overall picture of the set of formulas of α was designed, and comparison of the calculated and measured values of α was exhibited.

Keywords: formulas of the fine-structure constant α, formulas of Euler number e; the speed of light; the unification of mathematics and physics; relationships; nuclides.

1. Construct formulas of α with 173 instead of 112

Referring to $_{112}Cn_{173}$, $_{137}Fy_{209}$ and $_{173}Ch_{261,262}$ presented in the previous paper

"Chen's Formulas of the Fine-structure Constant" (viXra:2002.0203)[1], the natural end of elements is $_{112}$Cn, the Fynmann end of elements is $_{137}$Fy, and the end of ideal extended elements is $_{173}$Ch. We had already constructed formulas of α with 112 which is double of the most stable number 56 in the world of nuclides according to our Chirality and Poetry Model of Atomic Neclei[2], and 173 seems to be a subsequent stable number connected to 112, so it should be possible to construct some reasonable (but may less important) formulas of α with 173 instead of 112 as follows. And the factors in these formulas are supposed to relate to nuclides as in the previous paper[1].

$$\alpha_{1-1(173)}=\frac{8}{e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{20}{19})^{39}}}\frac{1}{173+\frac{1}{31}-\frac{1}{10\cdot 227-\frac{103}{7\cdot 193}\text{ or }\frac{60}{6\cdot 131+1}\text{ or }\frac{112}{13\cdot 113}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{39}_{19}K_{20}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{89}_{39}K_{50}$ ${}^{103}_{45}Rh_{58}$ ${}^{131}_{54}Xe_{77}$
${}^{191,193}_{77}Ir_{114,116}$ ${}^{171,173}_{70}Yb_{101,103}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{*}_{173}$ ${}^{2\cdot173}_{137}Fy^{ie}_{11\cdot19}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-1(173)-Wallis}=\frac{2}{(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{58}{59}\frac{4\cdot 3\cdot 5}{2\cdot 29+1})}\frac{1}{173+\frac{1}{31}-\frac{1}{8\cdot 11\cdot 23+\frac{171}{2\cdot 112}}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{100}_{44}Ru_{56}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{170-174}_{70}Yb_{100-104}$ ${}^{226}_{88}Ra^{*}_{138}$
${}^{283,15\cdot19}_{112}Cn^{*}_{171,173}$ ${}^{284,22\cdot13}_{113}Nh^{*}_{171,173}$ ${}^{2\cdot173,12\cdot29}_{137,138}Fy^{ie}_{11\cdot19,210}$ ${}^{33\cdot13,430}_{171}Ch^{ie}_{258,260}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,262}$

$$\alpha_{1-1(173)-GL}=\frac{1}{1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 19+1}}\frac{1}{173+\frac{1}{14}-\frac{1}{4\cdot 61}+\frac{1}{2\cdot 173\cdot 191+\frac{1}{50}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{61}_{28}Ni_{33}$ ${}^{88}_{38}Sr_{50}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{131}_{54}Xe_{77}$ ${}^{145}_{61}Pm^{*}_{84}$ ${}^{170,171,172,173,174,176}_{70}Yb_{100,101,102,103,104,106}$
${}^{191,193}_{77}Ir_{114,116}$ ${}^{4\cdot61}_{94}Pu^{*}_{150}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{*}_{173}$ ${}^{2\cdot173}_{137}Fy^{ie}_{11\cdot19}$ ${}^{402}_{158}Ch^{ie}_{4\cdot61}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-1(173)-NC}=\frac{8}{(2\pi)_{NC-1}}\frac{1}{173+\frac{1}{10}-\frac{1}{17\cdot 31}+\frac{1}{2\cdot 47\cdot 53\cdot 127}}=1/137.035999037435$$

$$=\frac{8}{6+\frac{1}{3}}\frac{1}{173+\frac{1}{10}-\frac{9\cdot 19\cdot(3\cdot 11\cdot 112+1)}{2\cdot 17\cdot 31\cdot 47\cdot 53\cdot 127}}=\frac{24}{19}\frac{1}{173+\frac{1}{10}-\frac{171\cdot(2\cdot 43^2-1)}{2\cdot 17\cdot 31\cdot 47\cdot 53\cdot 127}}$$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{47}_{22}Ti_{25}$ ${}^{53}_{24}Cr_{29}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{77}_{34}Se_{43}$
${}^{83}_{36}Kr_{47}$ ${}^{97}_{42}Mo_{53}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{127}_{53}I_{74}$ ${}^{131}_{54}Xe_{77}$ ${}^{148}_{62}Sm_{86}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$
${}^{170,171,172,173,174,176}_{70}Yb_{100,101,102,103,104,106}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{11\cdot19,210}_{83}Bi^{*}_{126,127}$ ${}^{6\cdot37}_{86}Rn^{*}_{136}$ ${}^{283,15\cdot19}_{112}Cn^{*}_{9\cdot19,173}$
${}^{284,15\cdot19,22\cdot13}_{113}Nh^{*}_{9\cdot19,4\cdot43,173}$ ${}^{6\cdot53,11\cdot29,320}_{127}Ch^{ie}_{191,192,193}$ ${}^{33\cdot13,430}_{171}Ch^{ie}_{6\cdot43,7\cdot37}$ ${}^{430,432}_{172}Ch^{ie}_{6\cdot43,260}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

Note: $(2\pi)_{NC-k} = 6+\sum_{n=1}^{k}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}}$

$$\alpha_{1-15(173)} = \frac{7\cdot 17}{3\cdot 5\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot(6\cdot 47-1)}{6\cdot 11\cdot 17+1})^{3\cdot 7\cdot 107}}}\frac{1}{173+\frac{1}{20\cdot(138+1)\cdot 227}}$$

$=1/137.035999037435$

${}^{14,15}_{7}N_{7,8}$ ${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{45}_{21}Sc_{24}$ ${}^{47}_{22}Ti_{25}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{118,7\cdot 17}_{50}Sn_{68,69}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{170,172,173,176}_{70}Yb_{100,102,103,106}$ ${}^{179}_{72}Hf_{107}$ ${}^{199}_{80}Hg_{7\cdot 17}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{*}_{173}$ ${}^{2\cdot 149}_{119}Ch^{ie}_{179}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{348}_{139}Ch^{ie}_{209}$ ${}^{8\cdot 47}_{149}Ch^{ie}_{227}$

$$\alpha_{1-15(173)-Wallis} = \frac{7\cdot 17}{4\cdot 3\cdot 5\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{3370}{3371}\frac{4\cdot 3\cdot 281}{2\cdot 5\cdot 337+1})}\frac{1}{173+\frac{1}{11\cdot 103\cdot(18\cdot 31-1)+\frac{7}{10}}}$$

$=1/137.035999037435$

${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{170,171,172,173}_{70}Yb_{100,101,102,103}$ ${}^{281}_{110}Ds^{*}_{171}$ ${}^{281}_{111}Rg^{*}_{170}$ ${}^{283,15\cdot 19}_{112}Cn^{*}_{9\cdot 19,173}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,2\cdot 131}$

$$\alpha_{1-15(173)-GL} = \frac{7\cdot 17}{8\cdot 3\cdot 5\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 29\cdot 37+1})}\frac{1}{173+\frac{1}{2\cdot 19\cdot 23\cdot 157-\frac{10}{13}}}$$

$=1/137.035999037435$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{80}_{34}Se_{46}$ ${}^{7\cdot 17}_{50}Sn_{69}$ ${}^{199}_{80}Hg_{7\cdot 17}$ ${}^{257}_{100}Fm^{*}_{157}$

$$\alpha_{1-15(173)-NC} = \frac{7\cdot 17}{3\cdot 5\cdot(2\pi)_{NC-9}}\frac{1}{173+\frac{1}{2\cdot 41}-\frac{1}{2\cdot 9\cdot(2\cdot 199-1)-\frac{29}{139}}}$$

$=1/137.035999037435$

${}^{63,65}_{29}Cu_{34,36}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{138,140}_{58}Ce_{80,82}$ ${}^{199}_{80}Hg_{7\cdot 17}$ ${}^{12\cdot 29}_{139}Ch^{ie}_{209}$

$$\alpha_{1-59(173)} = \frac{4\cdot 9\cdot 13}{59\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot(8\cdot 13\cdot 137+1)}{3\cdot 7\cdot 23\cdot 59})^{5\cdot(2\cdot 41\cdot 139+1)}}}\frac{1}{173+\frac{2\cdot 9\cdot 11\cdot 59}{5\cdot 10^{11}}}$$

$=1/137.035999037435$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{93}_{41}Nb_{52}$ ${}^{105}_{46}Pd_{59}$ ${}^{118}_{50}Sn_{68}$ ${}^{136137,138}_{56}Ba_{80,81,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{196}_{78}Pt_{118}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{16\cdot 13,11\cdot 19,210}$

$$\alpha_{1-59(173)-Wallis} = \frac{9\cdot 13}{59\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{85482}{85483}\frac{4\cdot 7\cdot 43\cdot 71}{2\cdot 27\cdot(2\cdot 7\cdot 113+1)+1})}\frac{1}{173+\frac{9\cdot 7\cdot 71}{4\cdot 10^{11}}}$$

$=1/137.035999037435$

${}^{27}_{13}Al_{14}$ ${}^{59}_{27}Co_{32}$ ${}^{171,172,173,174}_{70}Yb_{101,102,103,104}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{4\cdot 71,15\cdot 19,22\cdot 13}_{113}Nh^{*}_{9\cdot 19,4\cdot 43,173}$ ${}^{430,16\cdot 27}_{172}Ch^{ie}_{6\cdot 43,260}$

$$\alpha_{1-59(173)-GL} = \frac{9\cdot 13}{2\cdot 59\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot4\cdot3\cdot5\cdot(4\cdot227-1)+1})}\frac{1}{173+\frac{9\cdot79}{4\cdot10^{10}}}$$

$=1/137.035999037435$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{59}_{27}Co_{32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{89}_{39}Y_{50}$ ${}^{117,118,119}_{50}Sn_{67,68,69}$ ${}^{3\cdot47}_{59}\mathrm{Pr}_{82}$ ${}^{195,196,198}_{78}Pt_{117,118,120}$

${}^{171,172,173,174}_{70}Yb_{101,102,103,104}$ ${}^{197}_{79}Au_{2\cdot59}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{284,15\cdot19,22\cdot13}_{113}Nh^{*}_{9\cdot19,172,173}$ ${}^{8\cdot47}_{149}Ch^{ie}_{227}$

$$\alpha_{1-59(173)-NC} = \frac{4\cdot9\cdot13}{59\cdot(2\pi)_{NC-23}}\frac{1}{173+\frac{1}{32\cdot9\cdot(10\cdot19+1)-\frac{29}{2\cdot17}}} = 1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{283,15\cdot19}_{112}Cn^{*}_{9\cdot19,173}$ ${}^{284,15\cdot19,22\cdot13}_{113}Nh^{*}_{9\cdot19,4\cdot43,173}$

${}^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$ ${}^{33\cdot13,430}_{171}Ch^{ie}_{6\cdot43,7\cdot37}$ ${}^{430,16\cdot27}_{172}Ch^{ie}_{258,260}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-1(173)} = \frac{e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{37}{36})^{73}}}{5}\frac{1}{173-\frac{1}{5\cdot11}+\frac{1}{2\cdot7\cdot(2\cdot17^2-1)+\frac{2\cdot47}{3\cdot103}\text{ or }\frac{5\cdot23}{14\cdot27}}}$$

$=1/137.035999111818$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{47}_{22}Ti_{25}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{83,84}_{36}Kr_{47,48}$

${}^{85,87}_{37}Rb_{48,50}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{115,118,119,120,122,124}_{50}Sn_{65,68,69,70,72,74}$ ${}^{131}_{54}Xe_{77}$ ${}^{7\cdot19}_{55}Cs_{78}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$

${}^{170,171,172,173,176}_{70}Yb_{100,101,102,103,106}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{11\cdot19}_{83}Bi^{*}_{126}$ ${}^{262}_{103}Lr_{159}$

${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{288,17^2}_{115}Mc^{ie}_{173,174}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,11\cdot19,210}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-1(173)-Wallis} = \frac{4\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{108}{109}\frac{2\cdot5\cdot11}{2\cdot2\cdot27+1})}{5}\frac{1}{173-\frac{1}{7\cdot13}+\frac{1}{4\cdot25\cdot19\cdot23-\frac{10}{71}}}$$

$=1/137.035999111818$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{91}_{40}Zr_{51}$

${}^{99,100}_{44}Ru_{55,56}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{7\cdot19}_{55}Cs^{ie}_{78}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$ ${}^{169}_{69}Tm_{100}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{21\cdot11}_{91}Pa^{*}_{140}$

${}^{257}_{100}Fm^{*}_{157}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{4\cdot71,15\cdot19,22\cdot13}_{113}Nh^{ie}_{9\cdot19,4\cdot43,173}$ ${}^{6\cdot71}_{169}Ch^{ie}_{257}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-1(173)-GL} = \frac{8\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot4\cdot17+1})}{5}\frac{1}{173-\frac{1}{128\cdot7}+\frac{1}{5\cdot11^2\cdot19\cdot73}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{60,62}_{28}Ni_{32,34}$ ${}^{73}_{32}Ge_{41}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{125}_{52}Te_{73}$ ${}^{166,167,168}_{68}Er_{98,99,100}$

${}^{180,181}_{73}Ta_{107,108}$ ${}^{11\cdot19}_{83}Bi^{*}_{126}$ ${}^{11\cdot19}_{84}Po^{*}_{125}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$

$$\alpha_{2-1(173)-NC}=\frac{(2\pi)_{NC-3}}{5}\frac{1}{173-1+\frac{1}{2}-\frac{1}{10}+\frac{1}{2\cdot5\cdot23}-\frac{1}{4\cdot9\cdot7\cdot(400+1)+\frac{1}{10}}}$$

$=1/137.035999111818$

${}^{19}_{9}F_{10}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{82,93,84,86}_{36}Kr_{46,47,48,50}$ ${}^{125,126}_{52}Te_{73,74}$ ${}^{169}_{69}Tm_{100}$ ${}^{208}_{82}Pt_{126}$ ${}^{11\cdot19}_{83}Bi^{*}_{126}$ ${}^{11\cdot19}_{84}Po^{*}_{125}$
${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{288,289}_{115}Cn^{*}_{173,174}$

$$\alpha_{2-44(173)}=\frac{4\cdot11\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\frac{e^{2}}{(\frac{4}{3})^{7}}\cdots\frac{e^{2}}{(\frac{3\cdot(8\cdot131+1)}{2\cdot11^{2}\cdot13})^{7\cdot29\cdot31}}}{3\cdot73}\frac{1}{173-\frac{1}{11\cdot29\cdot(62\cdot59+1)+\frac{5}{6}}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{100}_{44}Ru_{56}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{125}_{52}Te_{73}$ ${}^{131}_{54}Xe_{77}$ ${}^{171,173,174}_{70}Yb_{101,103,104}$ ${}^{180,181}_{73}Ta_{107,108}$
${}^{201}_{80}Hg_{121}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{302}_{121}Ch^{ie}_{181}$ ${}^{332}_{131}Ch^{ie}_{201}$ ${}^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$ ${}^{33\cdot13}_{171}Ch^{ie}_{258}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-44(173)-Wallis}=\frac{16\cdot11\cdot(2\frac{2}{3}\frac{4}{3}\cdots\frac{9440}{9441}\frac{9442}{2\cdot16\cdot5\cdot59+1})}{3\cdot73}\frac{1}{173-\frac{1}{59\cdot(2\cdot3\cdot7\cdot13\cdot17+1)-\frac{4}{11}}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{73}_{32}Ge_{41}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{105}_{46}Pd_{59}$ ${}^{107,109}_{47}Ag_{60,62}$
${}^{118,119}_{50}Sn_{68,69}$ ${}^{125}_{52}Te_{73}$ ${}^{3\cdot47}_{59}\mathrm{Pr}_{82}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{9\cdot22,199}_{80}Hg_{2\cdot59,7\cdot17}$

$$\alpha_{2-44(173)-GL}=\frac{32\cdot11\cdot(1-\frac{1}{3}+\frac{1}{5}-\cdots+\frac{1}{2\cdot10\cdot(14\cdot43-1)+1})}{3\cdot73}\frac{1}{173-\frac{1}{(8\cdot61-1)\cdot(42\cdot23-1)}}$$

$=1/137.035999111818$

${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{46,47,50}_{22}Ti_{24,25,28}$ ${}^{61}_{28}Ni_{33}$ ${}^{73}_{32}Ge_{41}$ ${}^{92,96,97,98,100}_{42}Mo_{50,54,55,56,58}$ ${}^{98,99}_{43}Tc^{*}_{55,56}$
${}^{99,100}_{44}Ru_{55,56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{125}_{52}Te_{73}$ ${}^{138}_{56}Ba_{82}$ ${}^{2\cdot73}_{61}Pm^{*}_{85}$ ${}^{169}_{69}Tm_{100}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{238}_{92}U^{*}_{2\cdot73}$

$$\alpha_{2-44(173)-NC}=\frac{4\cdot11\cdot(2\pi)_{11}}{3\cdot73}\frac{1}{173-\frac{1}{7\cdot113}+\frac{2\cdot7\cdot29\cdot31\cdot47}{25\cdot10^{11}}}=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{47,49,50}_{22}Ti_{25,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{73}_{32}Ge_{41}$
${}^{99,100}_{44}Ru_{55,56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{65,66}$ ${}^{125}_{52}Te_{73}$ ${}^{173}_{70}Yb_{103}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$
${}^{2\cdot173,12\cdot29}_{137,138}Fy^{ie}_{11\cdot19,210}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-45(173)} = \frac{9\cdot5\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{8\cdot5\cdot27}{13\cdot83})^{17\cdot127}}}{2\cdot112}\frac{1}{173-\frac{1}{2\cdot151\cdot173-\frac{9}{8\cdot5}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{77}_{34}Se_{43}$ ${}^{83}_{36}Kr_{47}$ ${}^{90,91,92,94}_{40}Zr_{50,51,52,54}$
${}^{103}_{45}Rh_{58}$ ${}^{126,128}_{52}Te_{74,76}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{131}_{54}Xe_{77}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{173}_{70}Yb_{103}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{13\cdot19}_{96}Cm^{*}_{151}$
${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{2\cdot151}_{121}Ch^{ie}_{181}$ ${}^{318,319,320}_{127}Ch^{ie}_{191,192,193}$ ${}^{344,2\cdot173}_{136,137}Fy^{ie}_{16\cdot13,11\cdot19}$ ${}^{380}_{151}Ch^{ie}_{229}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-45(173)-Wallis} = \frac{9\cdot5\cdot(2\frac{2}{3}\frac{4}{3}\cdots\frac{3236}{3237}\frac{2\cdot(20\cdot81-1)}{2\cdot2\cdot(8\cdot101+1)+1})}{8\cdot7}\frac{1}{173-\frac{1}{8\cdot101\cdot(4\cdot9\cdot13-1)-\frac{2}{15}}}$$

$$=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{45}_{21}Sc_{24}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{99}_{43}Tc^{*}_{56}$
${}^{100,101}_{44}Ru_{56,57}$ ${}^{103}_{45}Rh_{58}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{171,172,173}_{70}Yb_{101,102,103}$ ${}^{6\cdot43}_{101}Md^{*}_{157}$ ${}^{283,15\cdot19}_{112}Cn^{*}_{171,173}$
${}^{284,15\cdot19,22\cdot13}_{113}Nh^{ie}_{171,172,173}$ ${}^{8\cdot43,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$ ${}^{33\cdot13,430}_{171}Ch^{ie}_{6\cdot43,7\cdot37}$ ${}^{430,16\cdot27}_{172}Ch^{ie}_{6\cdot43,260}$
${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-45(173)-GL} = \frac{9\cdot5\cdot(1-\frac{1}{3}+\frac{1}{5}-\cdots+\frac{1}{2\cdot2\cdot(10\cdot103+1)+1})}{4\cdot7}\frac{1}{173-\frac{1}{4\cdot17\cdot(44\cdot13-1)-\frac{32}{3\cdot19}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{100,101}_{44}Ru_{56,57}$
${}^{103}_{45}Rh_{58}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{168,170}_{68}Er_{100,102}$ ${}^{170,171,172,173,174,176}_{70}Yb_{100,101,102,103,104,106}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$
${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{8\cdot43,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,262}$

$$\alpha_{2-45(173)-NC} = \frac{9\cdot5\cdot(2\pi)_7}{2\cdot112}\frac{1}{173-\frac{1}{3\cdot17\cdot59+\frac{121}{320}}} = \frac{9\cdot5\cdot(2\pi)_7}{2\cdot112}\frac{1}{173-\frac{1}{2\cdot5\cdot7\cdot43-\frac{199}{320}}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{85,87}_{37}Rb_{48,59}$ ${}^{91}_{40}Zr_{51}$ ${}^{99}_{43}Tc^{*}_{56}$
${}^{105}_{46}Pd_{59}$ ${}^{121,123}_{51}Sb_{70,71}$ ${}^{112}_{48}Cd_{64}$ ${}^{185,11\cdot17}_{75}\mathrm{Re}_{110,112}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{173}_{70}Yb_{103}$ ${}^{199,200,201}_{80}Hg_{7\cdot19,120,121}$
${}^{222}_{86}Rn^{*}_{136}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{8\cdot43,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,262}$

$$\alpha_{2-87(173)}=\frac{3\cdot 29\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\dfrac{e^2}{(\frac{4}{3})^7}\cdots\dfrac{e^2}{(\frac{341846}{341845})^{683691}}}{14\cdot 31-1}\frac{1}{173-\dfrac{14560}{10^{13}}}$$

$$=\frac{3\cdot 29\cdot e^2e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\dfrac{e^2}{(\frac{4}{3})^7}\cdots\dfrac{e^2}{(\frac{2\cdot 59\cdot(2\cdot 9\cdot 7\cdot 23-1)}{5\cdot 7\cdot(24\cdot 11\cdot 37-1)})^{3\cdot(4\cdot 163+1)\cdot(12\cdot 29+1)}}}{2\cdot 7\cdot 31-1}\frac{1}{173-\dfrac{112\cdot 13}{10^{12}}}$$

$=1/137.035999111818$

${}^{11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Cr_{32}$
${}^{50,52,,53,54}_{24}Cr_{26,28,29,30}$ ${}^{54,56,57,58}_{26}Pd_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{67}_{30}Zn_{37}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{85,87}_{37}Rb_{48,50}$
${}^{99,100}_{44}Ru_{55,56}$ ${}^{105}_{46}Pd_{59}$ ${}^{131}_{54}Xe_{77}$ ${}^{149}_{62}Sm_{87}$ ${}^{163}_{66}Dy_{97}$ ${}^{4\cdot 67}_{105}Db^{*}_{163}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{410}_{163}Ch^{ie}_{13\cdot 19}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$

$$\alpha_{2-87(173)-Wallis}=\frac{4\cdot 87\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{1044088}{1044089}\frac{1044090}{1044089+1})}{433}\frac{1}{173-\dfrac{434\text{ or }435}{10^{13}}}$$

$$=\frac{4\cdot 3\cdot 29\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{1044088}{1044089}\frac{2\cdot 81\cdot 5\cdot(23\cdot 56+1)}{2\cdot 4\cdot 19\cdot(4\cdot 17\cdot 101+1)})}{2\cdot 7\cdot 31-1}\frac{1}{173-\dfrac{7\cdot 31}{5\cdot 10^{12}}\text{ or }\dfrac{3\cdot 29}{2\cdot 10^{12}}}$$

$=1/137.035999111818$

${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,57,58}_{26}Pd_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{97,99}_{43}Tc^{*}_{54,56}$ ${}^{79,81}_{45}Br_{44,46}$
${}^{100,101,102}_{44}Ru_{56,57,58}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{149}_{62}Sm_{87}$ ${}^{157}_{64}Gd_{93}$ ${}^{168,170}_{68}Er_{100,102}$ ${}^{169}_{69}Tm_{100}$ ${}^{9\cdot 19,172,173}_{70}Yb_{101,102,103}$
${}^{7\cdot 29,205}_{81}Tl_{122,124}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{258}_{101}Md^{*}_{157}$ ${}^{7\cdot 37}_{102}No^{*}_{157}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{284,286}_{113}Nh^{ie}_{9\cdot 19,173}$
${}^{288,17^2}_{115}Mc^{ie}_{173,2\cdot 87}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,11\cdot 19,210}$ ${}^{429,430}_{171}Ch^{ie}_{6\cdot 43,7\cdot 37}$ ${}^{430,16\cdot 27}_{172}Ch^{ie}_{6\cdot 43,260}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$

$$\alpha_{2-87(173)-GL}=\frac{8\cdot 3\cdot 29\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 16\cdot(72\cdot(2\cdot 17^2-1)+1})}{2\cdot 7\cdot 31-1}\frac{1}{173-\dfrac{137}{125\cdot 10^{11}}}$$

$=1/137.035999111818$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{149}_{62}Sm_{87}$ ${}^{223}_{87}Fr^{*}_{136}$
${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{288,17^2}_{115}Mc^{ie}_{173,2\cdot 87}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$

$$\alpha_{2-87(173)-NC}=\frac{3\cdot 29\cdot(2\pi)_{NC-55}}{2\cdot 7\cdot 31-1}\frac{1}{173-\dfrac{1}{2\cdot 3\cdot 7\cdot 29\cdot 61+\dfrac{7}{4\cdot 3}}}=1/137.035999111818$$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{61}_{28}Ni_{33}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{87}_{38}Sr_{49}$ ${}^{107.109}_{47}Ag_{60,62}$ ${}^{97}_{42}Mo_{55}$ ${}^{133}_{55}Cs_{78}$
${}^{5\cdot 29}_{61}Pm^{*}_{84}$ ${}^{149}_{62}Sm_{87}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{2-89(173)} = \frac{89\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{3\cdot13\cdot41}{2\cdot17\cdot47})^{23\cdot(138+1)}}}{2\cdot13\cdot17+1}\frac{1}{173-\frac{1}{7\cdot17\cdot(4\cdot11\cdot29+1)+\frac{2}{25}}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{46,47,48,50}_{22}Ti_{24,25,26,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{80,82}_{34}Se_{46,48}$ ${}^{82,83,84,86}_{36}Kr_{46,47,48,50}$

${}^{89}_{39}Y_{50}$ ${}^{93}_{41}Nb_{52}$ ${}^{100}_{44}Ru_{56}$ ${}^{116-120}_{50}Sn_{66-70}$ ${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{151,153}_{63}Eu_{88,90}$

${}^{170-174,176}_{70}Yb_{100-104,106}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{8\cdot43,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{16\cdot13,11\cdot19,210}$ ${}^{12\cdot29}_{139}Ch^{ie}_{209}$

${}^{33\cdot13}_{171}Ch^{ie}_{258}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,2\cdot131}$

$$\alpha_{2-89(173)-Wallis} = \frac{4\cdot89\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\cdots\frac{4794}{4795}\frac{4\cdot11\cdot109}{2\cdot3\cdot17\cdot47+1})}{4\cdot3\cdot37-1}\frac{1}{173-\frac{1}{2\cdot49\cdot(32\cdot113+1)+\frac{7}{8}}}$$

$$=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{74}_{32}Ge_{42}$ ${}^{86,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{100}_{44}Ru_{56}$ ${}^{107,109}_{47}Ag_{60,62}$

${}^{113,115}_{49}In_{65,66}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{183}_{74}W_{109}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{2\cdot173}_{137}Fy^{ie}_{11\cdot19}$ ${}^{8\cdot47}_{149}Ch^{ie}_{227}$

$$\alpha_{2-89(173)-GL} = \frac{8\cdot89\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot4\cdot7\cdot109+1})}{4\cdot3\cdot37-1}\frac{1}{173-\frac{1}{9\cdot(4\cdot5\cdot49\cdot47+1)+\frac{3}{8}}}$$

$$=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{58,60,64}_{28}Ni_{30,32,36}$ ${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{103}_{45}Rh_{58}$

${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{65,66}$ ${}^{137}_{56}Ba_{81}$ ${}^{173}_{70}Yb_{103}$ ${}^{183}_{74}W_{109}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{6\cdot37}_{86}Rn^{*}_{136}$ ${}^{227}_{89}Ac^{*}_{138}$

${}^{262}_{103}Lr^{*}_{159}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{288,289}_{115}Mc^{ie}_{173,174}$ ${}^{2\cdot173}_{137}Fy^{ie}_{209}$ ${}^{8\cdot47}_{149}Ch^{ie}_{227}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,262}$

$$\alpha_{2-89(173)-NC} = \frac{89\cdot(2\pi)_{NC-9}}{4\cdot3\cdot37-1}\frac{1}{173-\frac{1}{9\cdot25}+\frac{1}{13\cdot(2\cdot3\cdot7\cdot19^2-1)+\frac{6}{17}}}$$

$$=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{40,41,42}_{19}K_{20,21,22}$ ${}^{45}_{21}Sc_{24}$ ${}^{55}_{25}Mn_{30}$

${}^{85,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{103}_{45}Rh_{58}$ ${}^{173}_{70}Yb_{103}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{288,289}_{115}Mc^{ie}_{173,174}$

${}^{2\cdot173}_{137}Fy^{ie}_{209}$ ${}^{8\cdot47}_{149}Ch^{ie}_{227}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{9\cdot29,262}$

$$\alpha_{2-131(173)} = \frac{131\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{4\cdot 3\cdot 13\cdot 59}{2\cdot 43\cdot 107+1})^{79\cdot(8\cdot 29+1)}}}{4\cdot 163}\frac{1}{173-\frac{4\cdot 59\cdot(2\cdot 11\cdot 19+1)}{25\cdot 10^{11}}}$$

$$=1/137.035999111818$$

$^{23}_{11}Na_{12}$ $^{47}_{22}Ti_{25}$ $^{79,81}_{35}Br_{44,46}$ $^{97,98,99}_{43}Tc^{*}_{54,55,56}$ $^{105}_{46}Pd_{59}$ $^{107,109}_{47}Ag_{60,62}$ $^{116,117,118}_{50}Sn_{66,67,68}$ $^{131}_{54}Xe_{77}$
$^{141}_{59}Pr_{82}$ $^{163}_{66}Dy_{97}$ $^{2\cdot 97,195,196}_{78}Pt_{116,117,118}$ $^{197}_{79}Au_{118}$ $^{209}_{83}Bi^{*}_{126}$ $^{270}_{107}Ac^{*}_{163}$ $^{15\cdot 19}_{112}Cn^{*}_{173}$ $^{22\cdot 13}_{113}Nh^{ie}_{173}$ $^{293}_{116}Lv^{ie}_{3\cdot 59}$
$^{294}_{118}Og^{ie}_{16\cdot 11}$ $^{4\cdot 83}_{131}Ch^{ie}_{201}$ $^{8\cdot 43,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{16\cdot 13,11\cdot 19,210}$ $^{410}_{163}Ch^{ie}_{13\cdot 19}$ $^{33\cdot 13,430}_{171}Ch^{ie}_{6\cdot 43,260}$ $^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,2\cdot 131}$

$$\alpha_{2-131(173)-Wallis} = \frac{131\cdot(2\frac{2}{3}\frac{4}{3}\cdots\frac{27610}{27611}\frac{4\cdot 9\cdot 13\cdot 59}{2\cdot 2\cdot 5\cdot 11\cdot(4\cdot 9\cdot 7-1)+1})}{163}\frac{1}{173-\frac{8\cdot 67\cdot 157}{25\cdot 10^{11}}}$$

$$=1/137.035999111818$$

$^{19}_{9}F_{10}$ $^{23}_{11}Na_{12}$ $^{27}_{13}Al_{14}$ $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{59}_{27}Cr_{32}$ $^{83,84}_{36}Kr_{47,48}$ $^{105}_{46}Pd_{59}$ $^{9\cdot 13,118}_{50}Sn_{67,68}$
$^{131}_{54}Xe_{77}$ $^{141}_{59}Pr_{82}$ $^{157}_{64}Gd_{93}$ $^{163}_{66}Dy_{97}$ $^{165}_{67}Ho_{98}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{257}_{100}Fm^{*}_{157}$ $^{4\cdot 67}_{105}Db^{*}_{163}$ $^{15\cdot 19}_{112}Cn^{*}_{173}$
$^{22\cdot 13}_{113}Nh^{ie}_{173}$ $^{294}_{118}Og^{ie}_{16\cdot 11}$ $^{4\cdot 83}_{131}Ch^{ie}_{3\cdot 67}$ $^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{16\cdot 13,11\cdot 19,210}$ $^{410}_{163}Ch^{ie}_{13\cdot 19}$ $^{435}_{173}Ch^{ie}_{2\cdot 131}$

$$\alpha_{2-131(173)-GL} = \frac{2\cdot 131\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 11\cdot 17\cdot 47+1})}{163}\frac{1}{173-\frac{11\cdot 67\cdot 97}{4\cdot 10^{11}}}$$

$$=1/137.035999111818$$

$^{23}_{11}Na_{12}$ $^{35,37}_{17}Cl_{18,20}$ $^{47}_{22}Ti_{25}$ $^{67}_{30}Zn_{37}$ $^{97}_{42}Mo_{55}$ $^{107,109}_{47}Ag_{60,62}$ $^{131}_{54}Xe_{77}$ $^{163}_{66}Dy_{97}$ $^{15\cdot 11}_{67}Ho_{98}$ $^{247}_{97}Bk^{*}_{150}$
$^{4\cdot 67}_{105}Db^{*}_{163}$ $^{270}_{107}Bh^{*}_{163}$ $^{15\cdot 19}_{112}Cn^{*}_{173}$ $^{22\cdot 13}_{113}Nh^{ie}_{173}$ $^{4\cdot 83}_{131}Ch^{ie}_{3\cdot 67}$ $^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{16\cdot 13,11\cdot 19,210}$ $^{410}_{163}Ch^{ie}_{247}$ $^{435}_{173}Ch^{ie}_{2\cdot 131}$

$$\alpha_{2-131(173)-NC} = \frac{131\cdot(2\pi)_{NC-17}}{4\cdot 163}\frac{1}{173-\frac{1}{20\cdot 64}+\frac{49\cdot(12\cdot 269+1)}{25\cdot 10^{11}}}$$

$$=1/137.035999111818$$

$^{35,37}_{17}Cl_{18,20}$ $^{85,87}_{37}Rb_{48,50}$ $^{113,115}_{49}In_{65,66}$ $^{131}_{54}Xe_{77}$ $^{163}_{66}Dy_{97}$ $^{268}_{105}Db^{*}_{163}$ $^{269}_{106}Sg^{*}_{163}$ $^{270}_{107}Bh^{*}_{163}$ $^{435}_{173}Ch^{ie}_{2\cdot 131}$

$\alpha_{2-175(173)}$

$$= \frac{7\cdot 25\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{4\cdot 3\cdot 11\cdot 47-1}{14\cdot(2\cdot 13\cdot 17+1)})^{15\cdot(14\cdot 59+1)}}}{13\cdot 67}\frac{1}{173-\frac{1}{2\cdot 167\cdot(32\cdot 3\cdot 49-1)}}$$

$$=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$
${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{59}_{27}Co_{32}$ ${}^{64,66,67,68,70}_{30}Zn_{34,36,37,38,40}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{77,78}_{34}Se_{43,44}$ ${}^{79,81}_{35}Br_{44,46}$
${}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{100}_{44}Ru_{56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{131,132,134}_{54}Xe_{77,78,80}$
${}^{114,115,116,117,118,119,120}_{50}Sn_{64,65,66,67,68,69,70}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{165}_{67}Ho_{98}$ ${}^{167,168}_{68}Er_{99,100}$ ${}^{169}_{69}Tm_{100}$ ${}^{175,176}_{71}Lu_{104,105}$
${}^{172,173,174,176}_{70}Yb_{102,103,104,106}$ ${}^{4\cdot 49}_{78}Pt_{118}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{6\cdot 49}_{118}Og^{ie}_{176}$
${}^{4\cdot 83}_{131}Ch^{ie}_{3\cdot 67}$ ${}^{2\cdot 167}_{132}Ch^{ie}_{202}$ ${}^{340}_{134}Ch^{ie}_{206}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$ ${}^{2\cdot 209}_{167}Ch^{ie}_{251}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,262}$

$\alpha_{2-175(173)-Wallix}$

$$= \frac{4\cdot 7\cdot 25\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{18606}{19607}\frac{16\cdot(2\cdot 7\cdot 83+1)}{2\cdot 3\cdot 7\cdot(2\cdot 13\cdot 17+1)+1})}{13\cdot 67}\frac{1}{173-\frac{1}{5\cdot(16\cdot 9\cdot 5\cdot 17\cdot 41+1)}}$$

$$=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{40,42,44,46,48}_{20}Ca_{20,22,24,26,28}$
${}^{45}_{21}Sc_{24}$ ${}^{48}_{22}Ti_{26}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{68,60,62,64}_{28}Ni_{30,32,34,36}$ ${}^{64,66,67,68,70}_{30}Zn_{34,36,37,38,40}$
${}^{74,76,77,78,80,82}_{34}Se_{40,42,43,44,46,48}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{78,80,82,83,84,86}_{36}Kr_{42,44,46,47,48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{90,91,92}_{40}Zn_{50,51,52}$ ${}^{93}_{41}Nb_{52}$
${}^{103}_{45}Rh_{58}$ ${}^{115,117,118,119,120,122}_{50}Sn_{65,67,68,69,70,72}$ ${}^{129,130,131,132,134,136}_{54}Xe_{75,76,77,78,80,82}$ ${}^{134,136,137,138}_{56}Ba_{78,80,81,82}$
${}^{166,167,168}_{68}Er_{98,99,100}$ ${}^{170,172,173,174}_{70}Yb_{100,102,103,104}$ ${}^{204,206,207,208}_{82}Pb_{122,124,125,126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{262}_{103}Lr^{*}_{159}$
${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{4\cdot 83}_{131}Ch^{ie}_{3\cdot 67}$ ${}^{340}_{134}Ch^{ie}_{206}$ ${}^{344,2\cdot 173}_{136,137}Fy^{ie}_{208,209}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,262}$

$\alpha_{2-175(173)-GL}$

$$= \frac{8\cdot 7\cdot 25\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot(2\cdot 9\cdot 7\cdot 47+1)+1})}{13\cdot 67}\frac{1}{173-\frac{1}{5\cdot 59\cdot(4\cdot 5\cdot 13\cdot 17+1)}}$$

$$=1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{47}_{22}Ti_{25}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{40,42,44,46,48}_{20}Ca_{20,22,24,26,28}$ ${}^{59}_{27}Co_{32}$ ${}^{115,117,118,120}_{50}Sn_{65,67,68,70}$
${}^{134,136,137,138}_{56}Ba_{78,80,81,82}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$

$$\alpha_{2-175(173)-NC} = \frac{7\cdot 25\cdot(2\pi)_{NC-15}}{13\cdot 67}\frac{1}{173-\frac{1}{2\cdot(3\cdot 128-1)}+\frac{1}{125\cdot 7\cdot 29^2}} = 1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{125,128}_{52}Te_{73,76}$ ${}^{145}_{60}Nd_{85}$
${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{7\cdot 29,205}_{81}Tl_{122,124}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{24\cdot 13}_{125}Ch^{ie}_{187}$ ${}^{326}_{128}Ch^{ie}_{198}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,262}$

2. Formulas of $\alpha_{1\text{-}7\text{-NC}}$ and $\alpha_{2\text{-}13\text{-NC}}$ expressed with large integers

$$\alpha_{1-7-NC}=\frac{36}{7\cdot(2\pi)_{NC-3}}\frac{1}{112+\frac{1}{4\cdot7}-\cfrac{1}{2\cdot5\cdot(3\cdot7)^2-\frac{113}{7\cdot17}}}$$

$$=\frac{36}{7\cdot(6+\sum_{n=1}^{3}\frac{(-1)^{n+1}}{n(n+1/2)(n+1)})}\frac{1}{112+\frac{1}{4\cdot7}-\cfrac{1}{2\cdot5\cdot(3\cdot7)^2-\frac{113}{7\cdot17}}}$$

$$=\frac{36}{7\cdot(6+\cfrac{1}{1\cdot\frac{3}{2}\cdot2}-\cfrac{1}{2\cdot\frac{5}{2}\cdot3}+\cfrac{1}{3\cdot\frac{7}{2}\cdot4})}\frac{1}{112+\frac{1}{4\cdot7}-\cfrac{1}{2\cdot5\cdot(3\cdot7)^2-\frac{113}{7\cdot17}}}$$

$$=\frac{36}{7\cdot(6+\frac{1}{3}-\frac{1}{15}+\frac{1}{42})}\frac{1}{112+\frac{1}{4\cdot7}-\cfrac{1}{2\cdot5\cdot(3\cdot7)^2-\frac{113}{7\cdot17}}}$$

$$=\frac{36}{7\cdot(6+\frac{61}{210})}\frac{1}{112+\frac{1}{4\cdot7}-\cfrac{1}{2\cdot5\cdot(3\cdot7)^2-\frac{113}{7\cdot17}}}$$

$$=\frac{36}{7\cdot(\frac{44}{7}+\frac{1}{210})}\frac{1}{112+\frac{1}{4\cdot7}-\cfrac{1}{2\cdot5\cdot(3\cdot7)^2-\frac{113}{7\cdot17}}}$$

$$=\frac{36}{44+\frac{1}{30}}\frac{1}{112+\frac{1}{4\cdot7}-\frac{7\cdot17}{41\cdot67\cdot191}}$$

$$=\frac{1080}{1321}\frac{1}{112+\frac{5\cdot11\cdot9479}{4\cdot7\cdot41\cdot67\cdot191}}=\frac{1080}{1321}\frac{28\cdot41\cdot67\cdot191}{81\cdot89\cdot229\cdot997}$$

$$=\frac{2^5\cdot5\cdot7\cdot(2^3\cdot5+1)(2\cdot3\cdot11+1)(2\cdot5\cdot19+1)}{3\cdot(2^3\cdot11+1)(2^3\cdot3\cdot5\cdot11+1)(4\cdot3\cdot19+1)(4\cdot3\cdot83+1)}$$

$$=1/137.035999037435$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$

${}^{45}_{21}Sc_{24}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{64,66,67,68,70}_{30}Zn_{34,36,37,38,40}$ ${}^{73}_{32}Ge_{41}$

${}^{79,81}_{35}Br_{44,46}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{89}_{39}Y_{50}$ ${}^{96,98,99,100,101,102,104}_{44}Ru_{52,54,55,56,57,58,60}$ ${}^{112,113}_{48}Cd_{64,65}$

${}^{113,115}_{49}In_{64,66}$ ${}^{112-120,122,124}_{50}Sn_{62-70,72,74}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{165}_{67}Ho_{98}$

${}^{164,166,167,168,170}_{68}Er_{96,98,99,100,102}$ ${}^{169}_{69}Tm_{100}$ ${}^{185,11\cdot17}_{75}\mathrm{Re}_{110,112}$ ${}^{188,9\cdot21}_{76}Os_{112,113}$ ${}^{191,193}_{77}Ir_{114,116}$

${}^{199,200,201}_{80}Hg_{119,120,121}$ ${}^{208}_{82}Pt_{126}$ ${}^{11\cdot19}_{83}Bi^{*}_{126}$ ${}^{11\cdot19}_{84}Po^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$

${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{318,11\cdot29,320}_{127}Ch^{*}_{191,192,193}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,11\cdot19,210}$ ${}^{22\cdot17}_{148}Ch^{*}_{226}$ ${}^{380}_{151}Ch^{*}_{229}$

$$\alpha_{2-13-NC}=\frac{13\cdot(2\pi)_{NC-5}}{100}\frac{1}{112-\frac{1}{4\cdot 9}+\frac{1}{1600}-\frac{1}{2(2\cdot 7\cdot 199\cdot 257+1)}}$$

$$=1/137.035999111818$$

$${}^{27}_{13}Al_{14}\quad {}^{28,29,30}_{14}Si_{14,15,16}\quad {}^{36,38,40}_{18}Ar_{18,20,22}\quad {}^{40,42,43,44,46,48}_{20}Ca_{20,22,23,24,26,28}\quad {}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$$

$${}^{54,56,57,58}_{26}Fe_{28,30,31,32}\quad {}^{69,71}_{31}Ga_{38,40}\quad {}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}\quad {}^{90}_{40}Zr_{50}\quad {}^{100}_{44}Ru_{56}\quad {}^{119}_{50}Sn_{69}\quad {}^{169}_{69}Tm_{100}$$

$${}^{199}_{80}Hg_{119}\quad {}^{257}_{100}Fm^{*}_{157}\quad {}^{328}_{129}Ch^{ie}_{199}\quad {}^{6\cdot 71}_{169}Ch^{ie}_{257}$$

$$(2\pi)_{NC-5}=6+\sum_{n=1}^{5}\frac{(-1)^{n+1}}{\mathrm{n(n+1/2)(n+1)}}=6+\frac{1}{1\cdot 3}-\frac{1}{3\cdot 5}+\frac{1}{2\cdot 3\cdot 7}-\frac{1}{2\cdot 3^2\cdot 5}+\frac{1}{3\cdot 5\cdot 11}$$

$$=6+\frac{2\cdot 3\cdot 5\cdot 7\cdot 11}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}-\frac{2\cdot 3\cdot 7\cdot 11}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}+\frac{3\cdot 5\cdot 11}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}-\frac{7\cdot 11}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}+\frac{2\cdot 3\cdot 7}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}$$

$$=6+\frac{2310-462+165-77+42}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}=6+\frac{1978}{2\cdot 3^2\cdot 5\cdot 7\cdot 11}=6+\frac{989}{5\cdot 7\cdot 9\cdot 11}=\frac{29\cdot 751}{5\cdot 7\cdot 9\cdot 11}$$

$$\alpha_{2-13-NC}=\frac{13\cdot 29\cdot(2\cdot 3\cdot 5^3+1)}{2^2\cdot 5^3\cdot 7\cdot 9\cdot 11}\frac{1}{112-\frac{1}{4\cdot 9}+\frac{1}{1600}-\frac{1}{2(2\cdot 7\cdot 199\cdot 257+1)}}$$

$$=\frac{13\cdot 29\cdot(2\cdot 3\cdot 5^3+1)}{2^2\cdot 5^3\cdot 7\cdot 9\cdot 11}\frac{1}{112-\frac{1}{2}(\frac{1}{2\cdot 9}-\frac{1}{800}+\frac{1}{(2\cdot 7\cdot 199\cdot 257+1)})}$$

$$=\frac{13\cdot 29\cdot(2\cdot 3\cdot 5^3+1)}{2^2\cdot 3^2\cdot 5^3\cdot 7\cdot 11}\frac{1}{112-\frac{13\cdot 19\cdot(2\cdot 17^2\cdot 37\cdot 53+1)}{2^6\cdot 3^2\cdot 5^2\cdot(2\cdot 7\cdot 199\cdot 257+1)}}$$

$$=1/137.035999111818$$

$${}^{27}_{13}Al_{14}\quad {}^{28,29,30}_{14}Si_{14,15,16}\quad {}^{35,37}_{17}Cl_{18,20}\quad {}^{39,40,41}_{19}K_{20,21,22}\quad {}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}\quad {}^{53}_{24}Cr_{29}\quad {}^{55}_{25}Mn_{30}$$

$${}^{63,65}_{29}Cu_{34,36}\quad {}^{67}_{30}Zn_{37}\quad {}^{75}_{33}As_{42}\quad {}^{85,87}_{37}Rb_{48,50}\quad {}^{5\cdot 19}_{42}Mo_{53}\quad {}^{99,100}_{44}Ru_{55,56}\quad {}^{127}_{53}I_{2\cdot 37}\quad {}^{7\cdot 19}_{55}Cs_{78}\quad {}^{163}_{66}Dy_{97}$$

$${}^{185,11\cdot 17}_{75}\mathrm{Re}_{110,112}\quad {}^{191,193}_{77}Ir_{114,116}\quad {}^{199}_{80}Hg_{7\cdot 17}\quad {}^{11\cdot 19}_{83}Bi^{*}_{126}\quad {}^{11\cdot 19}_{84}Po^{*}_{125}\quad {}^{13\cdot 19}_{96}Cm^{*}_{151}\quad {}^{13\cdot 19}_{97}Bk^{*}_{50}\quad {}^{257}_{100}Fm^{*}_{157}$$

$${}^{269,271}_{106}Sg^{*}_{163,165}\quad {}^{17^2}_{115}Mc^{*}_{6\cdot 29}\quad {}^{24\cdot 13}_{125}Ch^{ie}_{11\cdot 17}\quad {}^{6\cdot 53,11\cdot 29,320}_{127}Ch^{ie}_{191,3\cdot 64,193}\quad {}^{328}_{129}Ch^{ie}_{199}\quad {}^{410}_{163}Ch^{ie}_{13\cdot 19}\quad {}^{426}_{169}Ch^{ie}_{257}$$

3. Deduction of new formulas of e

Define $\infty=\lim\limits_{n\to\infty}n$

$$e=(1+\frac{1}{\infty})^{\infty}=1+\frac{\infty}{1!}\frac{1}{\infty}+\frac{\infty(\infty-1)}{2!}\frac{1}{\infty^2}+\frac{\infty(\infty-1)(\infty-2)}{3!}\frac{1}{\infty^3}+\cdots$$

$$=(1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots)+(\frac{1}{2!}+\frac{1+2}{3!}+\frac{1+2+3}{4!}+\cdots)\frac{1}{\infty}+$$

$$[\frac{1\cdot 2}{3!}+\frac{1\cdot 2+(1+2)3}{4!}+\frac{1\cdot 2+(1+2)3+(1+2+3)4}{5!}+\cdots]\frac{1}{\infty^2}+$$

$$\{\frac{1\cdot 2\cdot 3}{4!}+\frac{1\cdot 2\cdot 3+[1\cdot 2+(1+2)3]4}{5!}+\cdots\}\frac{1}{\infty^3}+\cdots$$

3-1. There are or should be some formulas as follows.

$$e=1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots=1+\sum_{k=1}^{\infty}\frac{1}{k!}=2.718281828459045\cdots$$

$$e=2(\frac{1}{2!}+\frac{1+2}{3!}+\frac{1+2+3}{4!}+\cdots)=2\sum_{k=2}^{\infty}\frac{\sum_{l=1}^{k-1}l}{k!}=1+\sum_{k=1}^{\infty}\frac{1}{k!}$$

$$e=\frac{24}{11}[\frac{1\cdot2}{3!}+\frac{1\cdot2+(1+2)3}{4!}+\frac{1\cdot2+(1+2)3+(1+2+3)4}{5!}+\cdots]=\frac{24}{11}\sum_{k=3}^{\infty}\frac{\sum_{l=2}^{k-1}l\sum_{m=1}^{l-1}m}{k!}$$

$$e=\frac{16}{7}\{\frac{1\cdot2\cdot3}{4!}+\frac{1\cdot2\cdot3+[1\cdot2+(1+2)3)]4}{5!}+\cdots\}=\frac{16}{7}\sum_{k=4}^{\infty}\frac{\sum_{l_1=3}^{k-1}l_1\sum_{l_2=2}^{l_1-1}l_2\sum_{m=1}^{l_2-1}m}{k!}$$

$$\frac{1}{e}=1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots=1+\sum_{k=1}^{\infty}\frac{(-1)^k}{k!}=1/2.718281828459045\cdots$$

$$\frac{1}{e}=2(\frac{1}{2!}-\frac{1+2}{3!}+\frac{1+2+3}{4!}-\cdots)=2\sum_{k=2}^{\infty}\frac{(-1)^{k-2}\sum_{l=1}^{k-1}l}{k!}=1+\sum_{k=1}^{\infty}\frac{(-1)^k}{k!}$$

$$\frac{1}{e}=\frac{24}{5}[\frac{1\cdot2}{3!}-\frac{1\cdot2+(1+2)3}{4!}+\frac{1\cdot2+(1+2)3+(1+2+3)4}{5!}-\cdots]=\frac{24}{5}\sum_{k=3}^{\infty}\frac{(-1)^{k-3}\sum_{l=2}^{k-1}l\sum_{m=1}^{l-1}m}{k!}$$

$$\frac{1}{e}=\frac{48}{5}\{\frac{1\cdot2\cdot3}{4!}-\frac{1\cdot2\cdot3+[1\cdot2+(1+2)3)]4}{5!}+\cdots\}=\frac{48}{5}\sum_{k=4}^{\infty}\frac{(-1)^{k-4}\sum_{l_1=3}^{k-1}l_1\sum_{l_2=2}^{l_1-1}l_2\sum_{m=1}^{l_2-1}m}{k!}$$

3-2. There are or should be some general formulas as follows.

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots=1+\sum_{k=1}^{\infty}\frac{x^k}{k!}=1+x(\frac{1}{1!}+\frac{x}{2!}+\frac{x^2}{3!}+\cdots)=1+x\sum_{k=1}^{\infty}\frac{x^{k-1}}{k!}$$

$$e^x=2[\frac{1}{2!}+\frac{1+2}{3!}x+\frac{1+2+3}{4!}x^2+\cdots]=2(\frac{1}{2!}+\sum_{k=3}^{\infty}\frac{x^{k-2}\sum_{m=1}^{k-1}m}{k!})=2\sum_{k=2}^{\infty}\frac{x^{k-2}\sum_{m=1}^{k-1}m}{k!}$$

$$e^x=\frac{8}{x+8/3}[\frac{1\cdot2}{3!}+\frac{1\cdot2+(1+2)3}{4!}x+\cdots]=\frac{8}{x+8/3}\sum_{k=3}^{\infty}\frac{x^{k-3}\sum_{l=2}^{k-1}l\sum_{m=1}^{l-1}m}{k!}$$

$$e^x=\frac{6\cdot8}{(x+2)(x+6)}\{\frac{1\cdot2\cdot3}{4!}+\frac{1\cdot2\cdot3+[1\cdot2+(1+2)3)]4}{5!}x+\cdots\}$$

$$=\frac{6\cdot8}{(x+2)(x+6)}\sum_{k=4}^{\infty}\frac{x^{k-4}\sum_{l_1=3}^{k-1}l_1\sum_{l_2=2}^{l_1-1}l_2\sum_{m=1}^{l_2-1}m}{k!}\qquad(x\in\mathrm{R},\ 0^0=1)$$

3-3. There should be some special zero points for the above formulas as follows.

$$\sum_{k=3}^{\infty}\frac{(-\frac{8}{3})^{k-3}\sum_{l=2}^{k-1}l\sum_{m=1}^{l-1}m}{k!}=\frac{1\cdot 2}{3!}+\frac{1\cdot 2+(1+2)3}{4!}(-\frac{8}{3})+\frac{1\cdot 2+(1+2)3+(1+2+3)4}{5!}(-\frac{8}{3})^2+\cdots=0$$

$$\sum_{k=4}^{\infty}\frac{(-2)^{k-4}\sum_{l_1=3}^{k-1}l_1\sum_{l_2=2}^{l_1-1}l_2\sum_{m=1}^{l_2-1}m}{k!}=\frac{1\cdot 2\cdot 3}{4!}+\frac{1\cdot 2\cdot 3+[1\cdot 2+(1+2)3]4}{5!}(-2)+\frac{1\cdot 2\cdot 3+[1\cdot 2+(1+2)3)]4+[1\cdot 2+(1+2)3+(1+2+3)4]5}{6!}(-2)^2+\cdots=0$$

$$\sum_{k=4}^{\infty}\frac{(-6)^{k-3}\sum_{l_1=3}^{k-1}l_1\sum_{l_2=2}^{l_1-1}l_2\sum_{m=1}^{l_2-1}m}{k!}=\frac{1\cdot 2\cdot 3}{4!}+\frac{1\cdot 2\cdot 3+[1\cdot 2+(1+2)3)]4}{5!}(-6)+\frac{1\cdot 2\cdot 3+[1\cdot 2+(1+2)3)]4+[1\cdot 2+(1+2)3+(1+2+3)4]5}{6!}(-6)^2+\cdots=0$$

3-4. There should be some special extended formulas as follows.

$$e=\frac{5760}{2447}\sum_{k=5}^{\infty}\frac{\sum_{l_1=4}^{k-1}l_1\sum_{l_2=3}^{l_1-1}l_2\sum_{l_3=2}^{l_2-1}l_3\sum_{m=1}^{l_3-1}m}{k!}=\frac{2^7\cdot 3^2\cdot 5}{2\cdot 9\cdot 136-1}\sum_{k=5}^{\infty}\frac{\sum_{l_1=4}^{k-1}l_1\sum_{l_2=3}^{l_1-1}l_2\sum_{l_3=2}^{l_2-1}l_3\sum_{m=1}^{l_3-1}m}{k!}$$

$$\frac{1}{e}=\frac{5760}{337}\sum_{k=5}^{\infty}\frac{(-1)^{k-5}\sum_{l_1=4}^{k-1}l_1\sum_{l_2=3}^{l_1-1}l_2\sum_{l_3=2}^{l_2-1}l_3\sum_{m=1}^{l_3-1}m}{k!}=\frac{2^7\cdot 3^2\cdot 5}{3\cdot 112+1}\sum_{k=5}^{\infty}\frac{(-1)^{k-5}\sum_{l_1=4}^{k-1}l_1\sum_{l_2=3}^{l_1-1}l_2\sum_{l_3=2}^{l_2-1}l_3\sum_{m=1}^{l_3-1}m}{k!}$$

$$e=\frac{2304}{959}\sum_{k=6}^{\infty}\frac{\sum_{l_1=5}^{k-1}l_1\sum_{l_2=4}^{l_1-1}l_2\sum_{l_3=3}^{l_2-1}l_3\sum_{l_4=2}^{l_3-1}l_4\sum_{m=1}^{l_4-1}m}{k!}=\frac{2^8\cdot 3^2}{7\cdot 137}\sum_{k=6}^{\infty}\frac{\sum_{l_1=5}^{k-1}l_1\sum_{l_2=4}^{l_1-1}l_2\sum_{l_3=3}^{l_2-1}l_3\sum_{l_4=2}^{l_3-1}l_4\sum_{m=1}^{l_4-1}m}{k!}$$

$$\frac{1}{e}=\frac{3840}{137}\sum_{k=6}^{\infty}\frac{(-1)^{k-6}\sum_{l_1=5}^{k-1}l_1\sum_{l_2=4}^{l_1-1}l_2\sum_{l_3=3}^{l_2-1}l_3\sum_{l_4=2}^{l_3-1}l_4\sum_{m=1}^{l_4-1}m}{k!}=\frac{2^8\cdot 3\cdot 5}{137}\sum_{k=6}^{\infty}\frac{(-1)^{k-6}\sum_{l_1=5}^{k-1}l_1\sum_{l_2=4}^{l_1-1}l_2\sum_{l_3=3}^{l_2-1}l_3\sum_{l_4=2}^{l_3-1}l_4\sum_{m=1}^{l_4-1}m}{k!}$$

4. Relationships between the new formulas of e and nuclides

$$e=\frac{8\cdot 3}{11}\sum_{k=3}^{\infty}\frac{\sum_{l=2}^{k-1}l\sum_{m=1}^{l-1}m}{k!}\qquad \frac{1}{e}=\frac{8\cdot 3}{5}\sum_{k=3}^{\infty}\frac{(-1)^{k-3}\sum_{l=2}^{k-1}l\sum_{m=1}^{l-1}m}{k!}$$

$${}^{7}_{3}Li_4\quad {}^{9}_{4}Be_5\quad {}^{10,11}_{5}B_{5,6}\quad {}^{12,13}_{6}C_{6,7}\quad {}^{14,15}_{7}N_{7,8}\quad {}^{20,21,22}_{10}Ne_{10,11,12}\quad {}^{23}_{11}Na_{12}\quad {}^{24,25,26}_{12}Mg_{12,13,14}\quad {}^{48}_{22}Ti_{26}$$

$$e=\frac{16}{7}\sum_{k=4}^{\infty}\frac{\sum_{l_1=3}^{k-1}l_1\sum_{l_2=2}^{l_1-1}l_2\sum_{m=1}^{l_2-1}m}{k!}$$

$$\frac{1}{e}=\frac{16\cdot 3}{5}\sum_{k=4}^{\infty}\frac{(-1)^{k-4}\sum_{l_1=3}^{k-1}l_1\sum_{l_2=2}^{l_1-1}l_2\sum_{m=1}^{l_2-1}m}{k!}$$

$$\frac{1}{e^4}=-12\sum_{k=4}^{\infty}\frac{(-4)^{k-4}\sum_{l_1=3}^{k-1}l_1\sum_{l_2=2}^{l_1-1}l_2\sum_{m=1}^{l_2-1}m}{k!}$$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{32,34,36}_{16}S_{16,18,20}$ ${}^{40,42,44,48}_{20}Ca_{20,22,24,28}$ ${}^{45}_{21}Sc_{24}$ ${}^{50,52,54}_{24}Cr_{26,28,30}$

${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{58,60,64}_{28}Ni_{30,32,36}$ ${}^{64,36,70}_{30}Zn_{34,36,40}$ ${}^{66,68,72}_{32}Ge_{34,36,40}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{80,84}_{36}Kr_{44,48}$

${}^{112}_{48}Cd_{64}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{200}_{80}Hg_{120}$ ${}^{285}_{112}Cn^{*}_{173}$

$$e=\frac{5760}{2447}\sum_{k=5}^{\infty}\frac{\sum_{l_1=4}^{k-1}l_1\sum_{l_2=3}^{l_1-1}l_2\sum_{l_3=2}^{l_2-1}l_3\sum_{m=1}^{l_3-1}m}{k!}=\frac{2^7\cdot 3^2\cdot 5}{2\cdot 9\cdot 136-1}\sum_{k=5}^{\infty}\frac{\sum_{l_1=4}^{k-1}l_1\sum_{l_2=3}^{l_1-1}l_2\sum_{l_3=2}^{l_2-1}l_3\sum_{m=1}^{l_3-1}m}{k!}$$

$$\frac{1}{e}=\frac{5760}{337}\sum_{k=5}^{\infty}\frac{(-1)^{k-5}\sum_{l_1=4}^{k-1}l_1\sum_{l_2=3}^{l_1-1}l_2\sum_{l_3=2}^{l_2-1}l_3\sum_{m=1}^{l_3-1}m}{k!}=\frac{2^7\cdot 3^2\cdot 5}{3\cdot 112+1}\sum_{k=5}^{\infty}\frac{(-1)^{k-5}\sum_{l_1=4}^{k-1}l_1\sum_{l_2=3}^{l_1-1}l_2\sum_{l_3=2}^{l_2-1}l_3\sum_{m=1}^{l_3-1}m}{k!}$$

${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{64,68,70}_{30}Zn_{34,38,40}$ ${}^{74,80,82}_{34}Se_{40,46,48}$

${}^{112,114,118,119,120,122}_{50}Sn_{62,64,68,69,70,72}$ ${}^{168}_{68}Er_{100}$ ${}^{112}_{48}Cd_{64}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{285}_{112}Cn^{*}_{173}$

$$e=\frac{2304}{959}\sum_{k=6}^{\infty}\frac{\sum_{l_1=5}^{k-1}l_1\sum_{l_2=4}^{l_1-1}l_2\sum_{l_3=3}^{l_2-1}l_3\sum_{l_4=2}^{l_3-1}l_4\sum_{m=1}^{l_4-1}m}{k!}$$

$$=\frac{2^8\cdot 3^2}{7\cdot 137}\sum_{k=6}^{\infty}\frac{\sum_{l_1=5}^{k-1}l_1\sum_{l_2=4}^{l_1-1}l_2\sum_{l_3=3}^{l_2-1}l_3\sum_{l_4=2}^{l_3-1}l_4\sum_{m=1}^{l_4-1}m}{k!}$$

$$\frac{1}{e}=\frac{3840}{137}\sum_{k=6}^{\infty}\frac{(-1)^{k-6}\sum_{l_1=5}^{k-1}l_1\sum_{l_2=4}^{l_1-1}l_2\sum_{l_3=3}^{l_2-1}l_3\sum_{l_4=2}^{l_3-1}l_4\sum_{m=1}^{l_4-1}m}{k!}$$

$$=\frac{2^8\cdot 3\cdot 5}{137}\sum_{k=6}^{\infty}\frac{(-1)^{k-6}\sum_{l_1=5}^{k-1}l_1\sum_{l_2=4}^{l_1-1}l_2\sum_{l_3=3}^{l_2-1}l_3\sum_{l_4=2}^{l_3-1}l_4\sum_{m=1}^{l_4-1}m}{k!}$$

${}^{83,84}_{36}Kr_{47,48}$ ${}^{112}_{48}Cd_{64}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{138}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$

${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

5. Relationships between formulas of 2π and nuclides

$$\pi = \frac{22}{7} - \int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx \qquad 2\pi = \frac{4\cdot 11}{7} - 2\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$$

${}^{14,15}_{7}N_{7,8}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{46,47,48,49,50}_{22}Sc_{24,25,26,27,28}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{77}_{34}Se_{43}$

${}^{88}_{38}Sr_{50}$ ${}^{96,98,99,100,102,104}_{44}Ru_{52,54,55,56,58,60}$ ${}^{150}_{62}Sm_{88}$ ${}^{226}_{88}Ra^{*}_{138}$

$$\pi = \frac{355}{113} - \frac{1}{3164}\int_0^1 \frac{x^8(1-x)^8(25+816x^2)}{1+x^2}dx$$

$$2\pi = \frac{2\cdot 5\cdot 71}{113} - \frac{1}{2\cdot 7\cdot 113}\int_0^1 \frac{x^8(1-x)^8(25+16\cdot 3\cdot 17x^2)}{1+x^2}dx$$

${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{31}_{15}P_{16}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{55}_{25}Mn_{30}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{112,113}_{48}Cd_{64,65}$

${}^{77,78,80,82}_{34}Se_{43,44,46,48}$ ${}^{118,119}_{50}Sn_{68,69}$ ${}^{121,122}_{51}Sb_{70,72}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{175,11\cdot 16}_{71}Lu_{104,105}$

${}^{170,171,172,173,176}_{70}Yb_{100,101,102,103,106}$ ${}^{189}_{76}Os_{113}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{259}_{102}Fm^{*}_{157}$ ${}^{4\cdot 71,2\cdot 11\cdot 13}_{113}Nh^{ie}_{171,173}$ ${}^{2\cdot 11\cdot 17}_{148}Ch^{ie}_{226}$

6. Relationships between γ, e, γ_c, γ_g and γ_{cg} and nuclides

Euler-Mascheroni constant $\gamma = \lim_{N\to\infty}(\sum_{n=1}^{N}\frac{1}{n} - \ln n) = 0.5772156649\cdots$

$$\gamma = 0.5772156649\cdots \approx \frac{3\cdot 5}{2\cdot 13} \approx \frac{3\cdot 41}{71}$$

${}^{11}_{5}B_{6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{31}_{15}P_{16}$ ${}^{39}_{19}K_{20}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$

${}^{64,66}_{30}Zn_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{89}_{39}Y_{50}$ ${}^{93}_{41}Nb_{52}$ ${}^{121,3\cdot 41}_{51}Sb_{70,72}$ ${}^{3\cdot 41}_{52}Te_{71}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{18\cdot 17,4\cdot 77}_{123}Ch^{ie}_{183,185}$

$$e = 2.718281828\cdots \approx \frac{3\cdot 29}{32} \approx \frac{193}{71}$$

${}^{48}_{22}Ti_{26}$ ${}^{53}_{24}Cr_{29}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{87,84}_{36}Kr_{47,48}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{106,112,113,116}_{48}Cd_{58,64,65,68}$

${}^{113,115}_{49}In_{64,66}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{4\cdot 71,286}_{113}Nh^{*}_{171,173}$ ${}^{11\cdot 29,320}_{127}Ch^{ie}_{3\cdot 64,193}$ ${}^{434}_{173}Ch^{ie}_{9\cdot 29}$

$$e^2 \approx \frac{37}{5} \approx \frac{7\cdot 19}{2\cdot 9}$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{38}_{18}Ar_{20}$ ${}^{67,68}_{30}Zn_{37,38}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{84,87}_{38}Sr_{46,49}$

${}^{7\cdot 19}_{55}Cs_{78}$ ${}^{9\cdot 19}_{70}Yb_{101}$ ${}^{185,187}_{75}Tm_{110,112}$ ${}^{284,286}_{113}Nh^{ie}_{9\cdot 19,173}$ ${}^{4\cdot 77}_{123}Ch^{ie}_{185}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{3\cdot 112}_{133}Ch^{ie}_{7\cdot 29}$ ${}^{429,430}_{171}Ch^{ie}_{258,7\cdot 37}$

$$e^{\gamma} = 1.7810724\cdots \approx \frac{3\cdot 19}{32} \approx \frac{7\cdot 43}{167}$$

${}^{7}_{3}Li_{4}$ ${}^{48}_{22}Ti_{26}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{70}_{32}Ge_{38}$ ${}^{77}_{34}Se_{43}$ ${}^{84}_{36}Kr_{48}$ ${}^{86,87}_{38}Sr_{48,49}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{112,113,114}_{48}Cd_{64,65,66}$

${}^{167,168}_{68}Er_{99,100}$ ${}^{171}_{70}Yb_{101}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{284,22\cdot 13}_{113}Nh^{ie}_{9\cdot 19,173}$ ${}^{33\cdot 13,430}_{171}Ch^{ie}_{258,259}$ ${}^{22\cdot 19}_{167}Ch^{ie}_{251}$

$$e^{\gamma_c} = e^{0.0810614668\cdots} = 1.08443755\cdots \approx \frac{7\cdot 11}{71} \approx \frac{5\cdot 131}{4\cdot 151}$$

${}^{69,71}_{31}Ga_{38,40}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{77}_{34}Se_{43}$ ${}^{131}_{54}Xe_{77}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{247}_{96}Cm^{*}_{151}$

$$e^{2\gamma_c} = e^{2\times 0.0810614668\cdots} = 1.17600480\cdots \approx \frac{4\cdot 5}{17} \approx \frac{3\cdot 49}{125} \approx \frac{167}{2\cdot 71}$$

$${}^{35,37}_{17}Cl_{18,20} \quad {}^{49}_{22}Ti_{27} \quad {}^{68}_{30}Ga_{38} \quad {}^{69,71}_{31}Ga_{38,40} \quad {}^{85,87}_{37}Rb_{48,50} \quad {}^{87}_{38}Sr_{49} \quad {}^{113,115}_{49}In_{64,66} \quad {}^{118}_{50}Sn_{68} \quad {}^{121,123}_{51}Sb_{70,72}$$

$${}^{125}_{52}Te_{73} \quad {}^{147}_{62}Sm_{85} \quad {}^{167,168}_{68}Er_{99,100} \quad {}^{175,176}_{71}Lu_{104,105} \quad {}^{209}_{84}Po^{*}_{125} \quad {}^{210}_{85}At^{*}_{125} \quad {}^{312}_{125}Ch^{ie}_{11\cdot 17} \quad {}^{12\cdot 31}_{147}Ch^{ie}_{225} \quad {}^{418}_{167}Ch^{ie}_{251}$$

$$2(1-\gamma_c) = 2(1-0.0810614668\cdots) \approx \frac{4\cdot 17}{37} \qquad \text{Note: } 2\pi = (\frac{e}{e^{\gamma_c}})^2$$

$${}^{35,37}_{17}Cl_{18,20} \quad {}^{64,67,68}_{30}Zn_{34,37,38} \quad {}^{74}_{34}Se_{40} \quad {}^{85,87}_{37}Rb_{38,40} \quad {}^{118}_{50}Sn_{68} \quad {}^{145}_{60}Nd_{85} \quad {}^{168}_{68}Er_{100} \quad {}^{185,187}_{75}Tm_{110,112}$$

$${}^{210}_{85}At^{*}_{125} \quad {}^{4\cdot 77}_{123}Ch^{ie}_{185} \quad {}^{312}_{125}Ch^{ie}_{187}$$

$$2(1-\gamma_g) = 2(1-0.7742086474\cdots) \approx \frac{2\cdot 7}{31} \qquad \text{Note: } \frac{\pi}{2} = (\frac{e}{e^{\gamma_g}})^2$$

$${}^{14,15}_{7}N_{7,8} \quad {}^{28,29,30}_{14}Si_{14,15,16} \quad {}^{31}_{15}P_{16} \quad {}^{54,56,57,58}_{26}Fe_{28,30,31,32} \quad {}^{62}_{28}Ni_{34} \quad {}^{69,71}_{31}Ga_{38,40} \quad {}^{147,148,150,152}_{62}Sm_{85,86,88,90}$$

$$e^{2\gamma_{cg}} \approx \frac{17}{3\cdot 5} \approx \frac{127}{112} \approx \frac{144}{127}, \quad 2\gamma_{cg} = 2\times 0.0628164798\cdots \approx \frac{23}{3\cdot 61} \approx \frac{8\cdot 3}{191} \approx \frac{25}{199} \qquad \frac{\pi}{2} = (\frac{e^{\gamma/2}}{e^{\gamma_{cg}}})^2$$

$${}^{50,51}_{23}V_{27,28} \quad {}^{53}_{24}Cr_{29} \quad {}^{64}_{30}Zn_{34} \quad {}^{83,84}_{36}Kr_{47,48} \quad {}^{112}_{48}Cd_{64} \quad {}^{127}_{53}I_{74} \quad {}^{144}_{60}Nd_{84} \quad {}^{3\cdot 61,186}_{74}W_{109,112} \quad {}^{185,187}_{75}Tm_{110,112} \quad {}^{199}_{80}Hg_{7\cdot 17}$$

$${}^{191,193}_{77}Ir_{114,116} \quad {}^{209,210}_{83}Bi^{*}_{126,127} \quad {}^{237}_{93}Np^{*}_{144} \quad {}^{285}_{112}Cn^{*}_{173} \quad {}^{18\cdot 17}_{123}Ch^{ie}_{3\cdot 31} \quad {}^{6\cdot 53,11\cdot 29,320}_{127}Ch^{ie}_{191,192,193} \quad {}^{328}_{129}Ch^{ie}_{199} \quad {}^{6\cdot 61}_{144}Ch^{ie}_{222}$$

7. Integrated Relationships between 2π, γ, e, γ_c, γ_g and γ_{cg} and nuclides

As we had demonstrated that 2π is directly and indirectly related to nuclides in the previous paper[1], it should be reasonable for some constants comparable to 2π such as γ, e, e^{γ}, $2(1-\gamma_c)$, $2(1-\gamma_g)$ and $2\gamma_{cg}$ to directly relate to nuclides. The above examples exhibit not only the relationships between the above constants and nuclides respectively, but also the integrated relationships between them and nuclides. For example, it seems they cooperatively define all the 4 stable isotopes of Fe as follows.

$$\gamma \approx \frac{3\cdot 5}{2\cdot 13} \quad e \approx \frac{3\cdot 29}{32} \quad e^{\gamma} \approx \frac{3\cdot 19}{32} \quad 2(1-\gamma_g) \approx \frac{2\cdot 7}{31}$$

$${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$$

The following is another example of this kind of integrated relationships.

$$2\pi = \frac{4\cdot 11}{7} - 2\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$$

$$2\pi = \frac{2\cdot 5\cdot 71}{113} - \frac{1}{2\cdot 7\cdot 113}\int_0^1 \frac{x^8(1-x)^8(25+16\cdot 3\cdot 17x^2)}{1+x^2}dx$$

$$2\pi \approx \frac{4\cdot 157}{100}, \quad \gamma \approx \frac{3\cdot 5}{2\cdot 13} \approx \frac{3\cdot 41}{71}, \quad e \approx \frac{3\cdot 29}{32} \approx \frac{193}{71}, \quad e^2 \approx \frac{37}{5} \approx \frac{7\cdot 19}{2\cdot 9}, \quad e^{\gamma} \approx \frac{3\cdot 19}{32} \approx \frac{7\cdot 43}{167}$$

$$e^{\gamma_c} \approx \frac{7\cdot 11}{71} \approx \frac{5\cdot 131}{4\cdot 151},\quad e^{2\gamma_c} \approx \frac{4\cdot 5}{17} \approx \frac{3\cdot 49}{125} \approx \frac{167}{2\cdot 71},\quad e^{2\gamma_{cg}} \approx \frac{17}{3\cdot 5} \approx \frac{127}{112} \approx \frac{127}{16\cdot 9}$$

$$2(1-\gamma_c) \approx \frac{4\cdot 17}{37},\quad 2(1-\gamma_g) \approx \frac{2\cdot 7}{31},\quad 2\gamma_{cg} \approx \frac{23}{3\cdot 61} \approx \frac{8\cdot 3}{191} \approx \frac{25}{199}$$

$${}^{23}_{11}Na_{12}\ \ {}^{35,37}_{17}Cl_{18,20}\ \ {}^{48,49}_{22}Ti_{26,27}\ \ {}^{50,51}_{23}V_{27,28}\ \ {}^{53}_{24}Cr_{29}\ \ {}^{61}_{28}Ni_{33}\ \ {}^{64}_{30}Zn_{34}\ \ {}^{69,71}_{31}Ga_{38,40}\ \ {}^{77}_{34}Se_{43}\ \ {}^{85,87}_{37}Rb_{48,50}$$

$${}^{87}_{38}Sr_{49}\ \ {}^{93}_{41}Nb_{52}\ \ {}^{95}_{42}Mo_{53}\ \ {}^{98,99}_{43}Tc^{*}_{55,56}\ \ {}^{100}_{44}Ru_{56}\ \ {}^{112-114}_{48}Cd_{64-66}\ \ {}^{113,115}_{49}In_{64,66}\ \ {}^{121,123}_{51}Sb_{70,72}\ \ {}^{3\cdot 41,125}_{52}Te_{71,73}$$

$${}^{127}_{53}I_{74}\ \ {}^{131}_{54}Xe_{77}\ \ {}^{136-138}_{56}Ba_{80-82}\ \ {}^{144}_{60}Nd_{84}\ \ {}^{5\cdot 29,146,3\cdot 49}_{61}Pm^{*}_{84,85,86}\ \ {}^{144,147-150,152,154}_{62}Sm_{82,85-88,90,92}\ \ {}^{157}_{64}Gd_{93}$$

$${}^{162}_{66}Dy_{96}\ \ {}^{166,167,168,170}_{68}Er_{98,99,100,102}\ \ {}^{169}_{69}Tm_{100}\ \ {}^{168,170-174,176}_{70}Yb_{98,100-104,106}\ \ {}^{175,176}_{71}Lu_{104,105}\ \ {}^{3\cdot 61,6\cdot 31}_{74}W_{109,112}$$

$${}^{185,187}_{75}Tm_{110,112}\ \ {}^{188,189}_{76}Os_{112,113}\ \ {}^{191,193}_{77}Ir_{114,116}\ \ {}^{199}_{80}Hg_{7\cdot 17}\ \ {}^{209,210}_{83}Bi^{*}_{126,127}\ \ {}^{209}_{84}Po^{*}_{125}\ \ {}^{210}_{85}At^{*}_{125}\ \ {}^{6\cdot 37}_{86}Rn^{*}_{136}$$

$${}^{223}_{87}Fr^{*}_{136}\ \ {}^{2\cdot 116}_{88}Ra^{*}_{138}\ \ {}^{237}_{93}Np^{*}_{144}\ \ {}^{243}_{95}Am^{*}_{148}\ \ {}^{13\cdot 19}_{96}Cm^{*}_{151}\ \ {}^{257}_{100}Fm^{*}_{157}\ \ {}^{6\cdot 43}_{101}Md^{*}_{157}\ \ {}^{7\cdot 37}_{102}No^{*}_{157}\ \ {}^{9\cdot 29,262}_{103}Lr^{*}_{158,159}$$

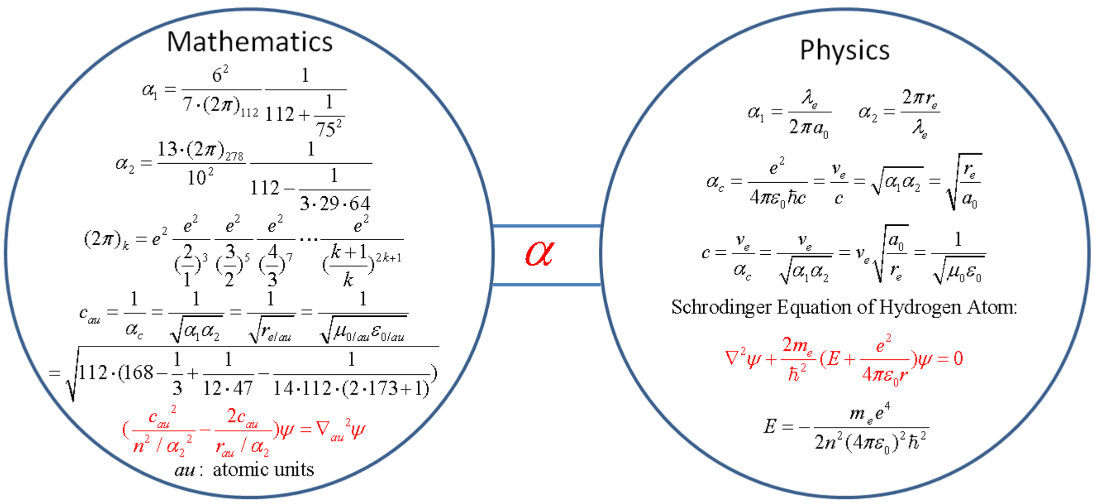
$${}^{281}_{110}Ds^{*}_{171}\ \ {}^{15\cdot 19}_{112}Cn^{*}_{173}\ \ {}^{4\cdot 71,22\cdot 13}_{113}Nh^{ie}_{9\cdot 19,173}\ \ {}^{18\cdot 17,4\cdot 77}_{123}Ch^{ie}_{3\cdot 61,185}\ \ {}^{312}_{125}Ch^{ie}_{187}\ \ {}^{6\cdot 53,11\cdot 29,320}_{127}Ch^{ie}_{191,3\cdot 64,193}\ \ {}^{328}_{129}Ch^{ie}_{199}$$

$${}^{4\cdot 83}_{131}Ch^{ie}_{201}\ \ {}^{8\cdot 43,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{16\cdot 13,11\cdot 19,210}\ \ {}^{6\cdot 61}_{144}Ch^{ie}_{6\cdot 37}\ \ {}^{12\cdot 31}_{147}Ch^{ie}_{225}\ \ {}^{22\cdot 17}_{148}Ch^{ie}_{226}\ \ {}^{380}_{151}Ch^{ie}_{229}\ \ {}^{400}_{157}Ch^{ie}_{243}\ \ {}^{22\cdot 19}_{167}Ch^{ie}_{251}$$

$${}^{33\cdot 13,430}_{171}Ch^{ie}_{6\cdot 43,7\cdot 37}\ \ {}^{430,432}_{172}Ch^{ie}_{6\cdot 43,260}\ \ {}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$$

These relationships are marvelous, they should not be just coincidences, and they should be science. This mechanism is analogous to that between DNA and protein.

8. Unification of Mathematics and Physics



Unification of Mathematics and Physics through α

Gang Chen, Tianman Chen, Tianyi Chen. 2018/4/12-2020/7/20-21

Fig. 1

Mathematics and science are usually regarded to be independent to each other, although mathematics is used as tool or language of science. It seems science has mathematic properties, so on the other hand, does mathematics has science features? We found the fine-structure constant α could act as a bridge between mathematics and physics (**Fig. 1**). For example, Schrödinger equation of hydrogen atom could be simplified to a reasonable and pure mathematic equation through α (red color parts).

9. Formula of the Speed of Light in Atomic Unites

In the previous paper[1], many formulas of the speed of light in atomic unites c_{au} had been deduced. Among them the most important should be the following formula.

$$c_{au}{}^2 = \frac{1}{\alpha_c{}^2} = \frac{1}{\alpha_1\alpha_2} = \frac{2\pi a_0}{\lambda_e}\frac{\lambda_e}{2\pi r_e} = \frac{a_0}{r_e} = (\frac{c}{v_e})^2$$
$$= 137.035999037435 \times 137.035999111818 = 137.035999074627^2$$
$$= 112 \times (168 - \frac{1}{3} + \frac{1}{12\cdot 47} - \frac{1}{6\cdot 29\cdot 53\cdot 59 - 79/47})$$

Here, we try to revise it to the following more reasonable form.

$$c_{au}{}^2 = \frac{1}{\alpha_c{}^2} = \frac{1}{\alpha_1\alpha_2} = \frac{2\pi a_0}{\lambda_e}\frac{\lambda_e}{2\pi r_e} = \frac{a_0}{r_e} = (\frac{c}{v_e})^2$$
$$= 137.035999037435 \times 137.035999111818 = 137.035999074626^2$$
$$= 112 \times (168 - \frac{1}{3} + \frac{1}{12\cdot 47} - \frac{1}{14\cdot 112\cdot (2\cdot 173+1)})$$
$$= 112 \times 167.668437878402 = 18778.865042381$$

$${}^{12,13}_{6}C_{6,7}\quad {}^{14,15}_{7}N_{7,8}\quad {}^{24,25,26}_{12}Mg_{12,13,14}\quad {}^{28,29,30}_{14}Si_{14,15,16}\quad {}^{32,36}_{16}S_{16,20}\quad {}^{47,49}_{22}Ti_{25,27}\quad {}^{63,65}_{29}Cu_{34,36}$$
$${}^{54,56,57,58}_{26}Fe_{28,30,31,32}\quad {}^{58,60,64}_{28}Ni_{30,32,36}\quad {}^{69,71}_{31}Ga_{38,40}\quad {}^{70,72}_{32}Ge_{38,40}\quad {}^{83,84}_{36}Kr_{47,48}\quad {}^{98,99}_{43}Tc^{*}_{55,56}$$
$${}^{98,99,100}_{44}Ru_{54,55,56}\quad {}^{107,109}_{47}Ag_{60,62}\quad {}^{112,113}_{48}Cd_{64,65}\quad {}^{113,115}_{49}In_{64,66}\quad {}^{131}_{54}Xe_{77}\quad {}^{136,137,138}_{56}Ba_{80,81,82}$$
$${}^{159}_{65}Tb_{94}\quad {}^{158}_{64}Gd_{94}\quad {}^{168}_{68}Er_{100}\quad {}^{169}_{69}Tm_{100}\quad {}^{173}_{70}Yb_{103}\quad {}^{185,187}_{75}\mathrm{Re}_{110,112}\quad {}^{4\cdot 47,189}_{76}Os_{112,113}\quad {}^{209}_{83}Bi^{*}_{126}$$
$${}^{209}_{84}Po^{*}_{125}\quad {}^{244}_{94}Pa^{*}_{150}\quad {}^{2\cdot 131}_{103}Lr^{*}_{159}\quad {}^{285}_{112}Cn^{*}_{173}\quad {}^{286}_{113}Nh^{ie}_{173}\quad {}^{288,289}_{115}Mc^{ie}_{173,174}\quad {}^{312}_{125}Ch^{ie}_{187}\quad {}^{314}_{126}Ch^{ie}_{4\cdot 47}$$
$${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}\quad {}^{418,420}_{168}Ch^{ie}_{250,252}\quad {}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,2\cdot 131}$$

10. Fibonacci Sequence *He* and its relationships with nuclides

We noticed 0.618×(112+168)=0.618×280≈173, so we construct the following Fibonacci sequence and present its relationships with nuclides.

Fibonacci Sequence *He*: 2 7 9 16 25 41 66 107 173 280

$${}^{1,2}_{1}H_{0,1}\quad {}^{4}_{2}He_{2}\quad {}^{7}_{3}Li_{4}\quad {}^{9}_{4}Be_{5}\quad {}^{14,15}_{7}N_{7,8}\quad {}^{16,17,18}_{8}O_{8,9,10}\quad {}^{19}_{9}F_{10}\quad {}^{25}_{12}Mg_{13}\quad {}^{28,29,30}_{14}Si_{14,15,16}\quad {}^{31}_{15}P_{16}$$
$${}^{32}_{16}S_{16}\quad {}^{41}_{19}K_{22}\quad {}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}\quad {}^{55}_{25}Mn_{30}\quad {}^{66}_{30}Zn_{36}\quad {}^{73}_{32}Ge_{41}\quad {}^{93}_{41}Nb_{52}\quad {}^{106,108}_{46}Pd_{60,62}$$
$${}^{107,109}_{47}Ag_{60,62}\quad {}^{106,108,112,113,114,116}_{48}Cd_{58,60,64,65,66,68}\quad {}^{113,115}_{49}In_{64,66}\quad {}^{116}_{50}Sn_{66}\quad {}^{163}_{66}Dy_{97}\quad {}^{173}_{70}Yb_{103}$$
$${}^{179}_{72}Hf_{107}\quad {}^{180}_{73}Ta_{107}\quad {}^{262}_{103}Lr^{*}_{159}\quad {}^{270}_{107}Bh^{*}_{163}\quad {}^{285}_{112}Cn^{*}_{173}\quad {}^{286}_{113}Nh^{ie}_{173}\quad {}^{288}_{115}Nh^{ie}_{173}\quad {}^{434,435}_{173}Ch^{ie}_{261,262}$$

11. The Meanings of the Numerical Values of the Fine-structure Constant

$$\alpha_1=\frac{1}{137.035999037435}=\frac{1}{137+\frac{1}{27}-\frac{1}{9\cdot 107}+\frac{1}{4\cdot 79\cdot(2\cdot 23\cdot 163+1)}}$$

$${}^{7}_{3}Li_{4}\ {}^{9}_{4}Be_{5}\ {}^{19}_{9}F_{10}\ {}^{23}_{11}Na_{12}\ {}^{27}_{13}Al_{14}\ {}^{46,47,49}_{22}Ti_{24,25,27}\ {}^{50,51}_{23}V_{27,28}\ {}^{59}_{27}Co_{32}\ {}^{64,66}_{30}Zn_{34,36}\ {}^{79,81}_{35}Br_{44,46}\ {}^{92}_{40}Zr_{52}$$

$${}^{105,106,108}_{46}Pd_{59,60,62}\ {}^{107,109}_{47}In_{60,62}\ {}^{5\cdot 27,136,137,6\cdot 23}_{56}Ba_{79,80,81,82}\ {}^{3\cdot 47}_{59}\mathrm{Pr}_{82}\ {}^{156,157,158}_{64}Gd_{92,93,94}\ {}^{162,163}_{66}Dy_{96,97}$$

$${}^{168}_{68}Er_{100}\ {}^{169}_{69}Tm_{100}\ {}^{179,180}_{72}Hf_{107,108}\ {}^{180,181}_{73}Ta_{107,108}\ {}^{197}_{79}Au_{2\cdot 59}\ {}^{226}_{88}Ra^{*}_{138}\ {}^{227}_{89}Ac^{*}_{138}\ {}^{5\cdot 47,238}_{92}U^{*}_{143,146}$$

$${}^{247}_{97}Bk^{*}_{150}\ {}^{243}_{98}Am^{*}_{145}\ {}^{257}_{100}Fm^{*}_{157}\ {}^{270}_{107}Bh^{*}_{163}\ {}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,11\cdot 19,210}\ {}^{400}_{157}Ch^{ie}_{243}\ {}^{410}_{163}Ch^{ie}_{247}$$

$$\alpha_2=\frac{1}{137.035999111818}=\frac{1}{137+\frac{1}{27}-\frac{1}{9\cdot 107}+\frac{1}{5\cdot 17\cdot 137\cdot 173}}$$

$${}^{10,11}_{5}B_{5,6}\ {}^{19}_{9}F_{10}\ {}^{27}_{13}Al_{14}\ {}^{31}_{15}P_{16}\ {}^{35,37}_{17}Cl_{18,20}\ {}^{49}_{22}Ti_{27}\ {}^{50,3\cdot 17}_{23}V_{27,28}\ {}^{85,87}_{37}Rb_{48,50}\ {}^{107,109}_{47}In_{60,62}\ {}^{131}_{54}Xe_{77}\ {}^{173}_{70}Yb_{103}$$

$${}^{5\cdot 27,8\cdot 17,137,138}_{56}Ba_{79,80,81,82}\ {}^{179,180}_{72}Hf_{107,108}\ {}^{180,181}_{73}Ta_{107,108}\ {}^{210}_{85}At^{*}_{125}\ {}^{222}_{86}Rn^{*}_{136}\ {}^{223}_{87}Fr^{*}_{136}\ {}^{2\cdot 131}_{103}Lr^{*}_{159}$$

$${}^{270}_{107}Bh^{*}_{163}\ {}^{15\cdot 19}_{112}Cn^{*}_{173}\ {}^{284}_{113}Nh^{ie}_{173}\ {}^{288,17^2}_{115}Mc^{ie}_{173,174}\ {}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}\ {}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 19,2\cdot 131}$$

12. Construct Formulas of the Fine-structure Constant with 137 instead of 112 or 173

$$\alpha_{1-7(137)}=\frac{4\cdot 11}{7\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\,\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot 126-1}{2\cdot 125})^{3\cdot 167}}}\;\frac{1}{137+\frac{1}{37\cdot 173-\frac{5}{32}}}$$

$$=1/137.035999037435$$

$${}^{14,15}_{7}N_{7,8}\ {}^{23}_{11}Na_{12}\ {}^{28,29,30}_{14}Si_{14,15,16}\ {}^{35,37}_{17}Na_{18,20}\ {}^{47,48,49,50}_{22}Ti_{25,26,27,28}\ {}^{55}_{25}Mn_{30}\ {}^{58,60}_{28}Ni_{30,32}\ {}^{83,84}_{36}Kr_{47,48}$$

$${}^{70,72,73,74,76}_{32}Ge_{38,40,41,42,44}\ {}^{85,87}_{37}Rb_{48,50}\ {}^{99,100}_{44}Ru_{55,56}\ {}^{112,113}_{48}Cd_{64,65}\ {}^{113,115}_{49}In_{64,66}\ {}^{125,126}_{52}Te_{73,74}$$

$${}^{136,137}_{56}Ba_{80,81}\ {}^{167,168}_{68}Er_{99,100}\ {}^{187,188}_{75}\mathrm{Re}_{110,112}\ {}^{209}_{83}Bi^{*}_{126}\ {}^{209}_{84}Po^{*}_{125}\ {}^{6\cdot 37}_{86}Rn^{*}_{136}\ {}^{15\cdot 19}_{112}Cn^{*}_{173}\ {}^{286}_{113}Nh^{ie}_{173}$$

$${}^{288,289}_{115}Mc^{ie}_{173,174}\ {}^{312}_{125}Ch^{ie}_{187}\ {}^{314}_{126}Ch^{ie}_{188}\ {}^{2\cdot 173}_{137}Fy^{ie}_{209}\ {}^{2\cdot 209}_{167}Ch^{ie}_{251}\ {}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,262}$$

$$\alpha_{1-39(137)}=\frac{5\cdot 7^2}{39\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\,\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{4\cdot 3\cdot 13^2-1}{2\cdot(4\cdot 11\cdot 23+1)})^{3\cdot 7\cdot 193}}}\;\frac{1}{137+\frac{1}{13\cdot 29\cdot(4\cdot 3\cdot 5\cdot 47-1)}}$$

$$=1/137.035999037435$$

$${}^{23}_{11}Na_{12}\ {}^{27}_{13}Al_{14}\ {}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}\ {}^{50,51}_{23}V_{27,28}\ {}^{54,56,58}_{26}Fe_{28,30,32}\ {}^{63,65}_{29}Cu_{34,36}\ {}^{79,81}_{35}Br_{44,46}\ {}^{89}_{39}Y_{50}$$

$${}^{94,98}_{42}Mo_{52,56}\ {}^{99,100,102,104}_{44}Ru_{55,56,58,60}\ {}^{104,106,110}_{46}Pd_{58,60,64}\ {}^{107,109}_{47}Ag_{60,62}\ {}^{113,115}_{49}In_{64,66}\ {}^{136,137}_{56}Ba_{80,81}$$

$${}^{140,142}_{58}Ce_{82,84}\ {}^{141}_{59}\mathrm{Pr}_{82}\ {}^{160,164}_{66}Dy_{94,98}\ {}^{169}_{69}Tm_{100}\ {}^{4\cdot 47,189,192}_{76}Os_{112,113,116}\ {}^{191,193}_{77}Ir_{114,116}\ {}^{227}_{89}Ac^{*}_{138}$$

$${}^{5\cdot 47,238}_{92}U^{*}_{143,146}\ {}^{244}_{94}Pu^{*}_{150}\ {}^{15\cdot 19}_{112}Cn^{*}_{173}\ {}^{286}_{113}Nh^{ie}_{173}\ {}^{288,289}_{115}Mc^{ie}_{173,174}\ {}^{2\cdot 173}_{137}Fy^{ie}_{209}\ {}^{314}_{126}Ch^{ie}_{4\cdot 47}$$

$${}^{318,11\cdot 29,320}_{127}Ch^{ie}_{191,192,193}\ {}^{4\cdot 89,21\cdot 17}_{141}Ch^{ie}_{215,216}\ {}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{261,262}$$

$$\alpha_{1-46(137)} = \frac{17^2}{2\cdot 23\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{3\cdot 13\cdot 25}{2\cdot(2\cdot 3^5+1)})^{1949}}}\frac{1}{137+\frac{1}{2\cdot 5\cdot 13\cdot(4\cdot 5\cdot 163-1)-\frac{7}{8}}}$$

$=1/137.035999037435$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Na_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{89}_{39}Y_{50}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{163}_{66}Dy_{97}$ ${}^{227}_{89}Ac^{*}_{138}$

${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{288,17^2}_{115}Mc^{ie}_{173,174}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{16\cdot13,209,210}$ ${}^{400}_{157}Ch^{ie}_{243}$ ${}^{410}_{163}Ch^{ie}_{13\cdot19}$

$\alpha_{1-71(137)}$

$$= \frac{2\cdot(2\cdot 112-1)}{71\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{6\cdot(6\cdot 11\cdot 17+1)-1}{16\cdot(4\cdot 3\cdot 5\cdot 7+1)})^{27\cdot(6\cdot 83+1)}}}\frac{1}{137+\frac{1}{256\cdot 11\cdot(8\cdot 27\cdot 7-1)}}$$

$=1/137.035999037435$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Na_{18,20}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Cr_{32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{105}_{46}Pd_{59}$

${}^{112,113}_{48}Cd_{64,65}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{128}_{54}Xe_{74}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{143,144}_{60}Nd_{83,84}$ ${}^{171,173}_{70}Yb_{101,103}$

${}^{175,11\cdot16}_{71}Lu_{104,105}$ ${}^{185,11\cdot17}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{4\cdot71,286}_{113}Nh^{ie}_{171,173}$ ${}^{288,289}_{115}Mc^{ie}_{173,174}$

${}^{2\cdot163}_{128}Ch^{ie}_{3\cdot66}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-103(137)} = \frac{8\cdot 81-1}{103\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot 29\cdot(6\cdot 7\cdot 23+1)}{15\cdot(6\cdot 7\cdot 89+1)})^{23\cdot(4\cdot 23\cdot 53+1)}}}\frac{1}{137+\frac{2\cdot 41\cdot 163+1}{125\cdot 10^{11}}}$$

$=1/137.035999037435$

${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{78,81}_{35}Br_{44,46}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{89}_{39}Y_{50}$ ${}^{103}_{45}Rh_{58}$

${}^{92,95,96,97,98,100}_{42}Mo_{50,53,54,55,56,58}$ ${}^{104,105,106,108}_{46}Pd_{58,59,60,62}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112,113,116}_{48}Cd_{64,65,68}$ ${}^{127}_{53}I_{74}$

${}^{129,132,134,136}_{54}Xe_{75,78,80,82}$ ${}^{137,138}_{56}Ba_{81,82}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{163}_{66}Dy_{97}$ ${}^{169}_{69}Tm_{100}$

${}^{173,174,176}_{70}Yb_{103,104,106}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{7\cdot29,5\cdot41}_{81}Tl_{122,124}$ ${}^{206,9\cdot23,208}_{82}Pb_{124,125,126}$ ${}^{209}_{83}Bi^{*}_{126}$

${}^{209}_{84}Bi^{*}_{125}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{268,271}_{106}Sg^{*}_{163,165}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot173,12\cdot29}_{137,138}Fy^{ie}_{209,210}$ ${}^{410}_{163}Ch^{ie}_{261,262}$

$$\alpha_{1-110(137)} = \frac{4\cdot 173-1}{110\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot(168\cdot 11-1)}{3\cdot(112\cdot 11-1)})^{83\cdot 89}}}\frac{1}{137+\frac{1}{3\cdot 23\cdot 71\cdot(270-1)}}$$

$=1/137.035999037435$

${}^{23}_{11}Na_{12}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{99,100}_{44}Ru_{55,56}$

${}^{105,106,108,110}_{46}Pd_{59,60,62,64}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{110,112,113,114}_{48}Cd_{62,64,65,66}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{131}_{54}Xe_{77}$

${}^{116,118,119,120,122}_{50}Sn_{66,68,69,70,72}$ ${}^{133}_{55}Cs_{78}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{143}_{60}Nd_{83}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{163}_{66}Dy_{97}$ ${}^{168}_{68}Er_{100}$

${}^{169}_{69}Tm_{100}$ ${}^{171,172,173,174,176}_{70}Yb_{101,102,103,104,106}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{191,193}_{77}Ir_{114,116}$

${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{281}_{110}Ds^{*}_{171}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{4\cdot71,22\cdot13}_{113}Nh^{ie}_{171,173}$

${}^{288,289}_{115}Nh^{ie}_{173,174}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{418,420}_{168}Ch^{ie}_{250,252}$ ${}^{434,435}_{173}Ch^{ie}_{261,262}$

$$\alpha_{2-7(137)}=\frac{7\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\,\dfrac{e^2}{(\frac{4}{3})^7}\cdots\dfrac{e^2}{(\frac{6\cdot 199}{8\cdot 149+1})^{7\cdot 11\cdot 31}}}{4\cdot 11}\,\frac{1}{137-\dfrac{1}{17\cdot(4\cdot 5\cdot 7\cdot 31+1)-\dfrac{13}{3\cdot 19}}}$$

$$=\frac{7\cdot e^2\,\dfrac{e^2}{(\frac{2}{1})^3}\,\dfrac{e^2}{(\frac{3}{2})^5}\,\dfrac{e^2}{(\frac{4}{3})^7}\cdots\dfrac{e^2}{(\frac{6\cdot 199}{8\cdot 149+1})^{7\cdot 11\cdot 31}}}{4\cdot 11}\,\frac{1}{137-\dfrac{1}{17\cdot(2\cdot 13\cdot 167-1)-\dfrac{13}{3\cdot 19}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{48}_{22}Ti_{26}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$

${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{76,77,78,80}_{34}Se_{42,43,44,46}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{89}_{39}Y_{50}$

${}^{99,100,101,102,104}_{44}Ru_{55,56,57,58,60}$ ${}^{110,112,113,114,116}_{48}Cd_{62,64,65,66,68}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{121,123}_{51}Sb_{70,72}$

${}^{122,124,128,130}_{52}Te_{70,72,76,78}$ ${}^{130,131,132}_{54}Xe_{76,77,78}$ ${}^{136,137}_{56}Ba_{80,81}$ ${}^{149}_{62}Sm_{87}$ ${}^{167,168}_{68}Er_{99,100}$

${}^{171,172,173,174,176}_{70}Yb_{101,102,103,104,106}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{186,187,188,189}_{76}Os_{110,111,112,113,114}$ ${}^{191,193}_{77}Ir_{114,116}$

${}^{192,195,196}_{78}Pt_{114,117,118}$ ${}^{199,204}_{80}Hg_{7\cdot 17,124}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Bi^{*}_{125}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{284,22\cdot 13}_{113}Nh^{ie}_{9\cdot 19,173}$

${}^{310}_{124}Ch^{ie}_{6\cdot 31}$ ${}^{326,328}_{129}Ch^{ie}_{197,199}$ ${}^{344,2\cdot 173}_{136,137}Fy^{ie}_{208,209}$ ${}^{376}_{149}Ch^{ie}_{227}$ ${}^{22\cdot 19}_{167}Ch^{ie}_{251}$ ${}^{14\cdot 31,435}_{173}Ch^{ie}_{9\cdot 19,262}$

13. The Meanings of the Numerical Value of the Speed of Light in Atomic Unites

$$c_{au}=137.035999074627=137+\frac{1}{27}-\frac{1}{9\cdot 107}+\frac{1}{8\cdot 137\cdot(4\cdot 7\cdot 71-1)}$$

$$c_{au}=137.035999074627=137+\frac{1}{27}-\frac{1}{9\cdot 107}+\frac{1}{3\cdot 17\cdot(4\cdot 7\cdot 25\cdot 61+1)}$$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$

${}^{31}_{15}P_{16}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{45}_{21}Sc_{24}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Co_{32}$

${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64,66,68}_{30}Zn_{34,36,38}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{70,72,74}_{32}Ge_{38,40,42}$ ${}^{79,81}_{35}Br_{44,46}$

${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{96,97,98}_{42}Mo_{54,55,56}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{103}_{45}Rh_{58}$ ${}^{105,106,108}_{46}Pd_{59,60,62}$

${}^{96,98,99,100,102,104}_{44}Ru_{52,54,55,56,58,60}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{118,119,122}_{50}Sn_{68,69,72}$ ${}^{131}_{54}Xe_{77}$

${}^{121,123}_{51}Sb_{70,72}$ ${}^{5\cdot 27,8\cdot 17,137,138}_{56}Ba_{79,80,81,82}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{145,146,147}_{61}Pm^{*}_{84,85,86}$ ${}^{157}_{64}Gd_{93}$ ${}^{162,163}_{66}Dy_{96,97}$

${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{170,171,172,173.174}_{70}Yb_{100,101,102,103,104}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{3\cdot 59,179,180}_{72}Hf_{105,107,108}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$

${}^{188,189}_{76}Os_{112,113}$ ${}^{203,205}_{81}Tl_{122,124}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$

${}^{232}_{90}Th^{*}_{142}$ ${}^{247}_{97}Bk^{*}_{150}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{258}_{101}Md^{*}_{157}$ ${}^{259}_{102}No^{*}_{157}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269,271}_{106}Sg^{*}_{163,165}$ ${}^{270}_{107}Bh^{*}_{163}$

${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{4\cdot 71,286}_{113}Nh^{ie}_{171,173}$ ${}^{288,289}_{115}Mc^{ie}_{173,174}$ ${}^{304}_{122}Ch^{ie}_{182}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{2\cdot 179,360}_{142}Ch^{ie}_{216,218}$ ${}^{14\cdot 27}_{150}Ch^{ie}_{228}$

${}^{400}_{157}Ch^{ie}_{243}$ ${}^{410}_{163}Ch^{ie}_{247}$ ${}^{6\cdot 71}_{169}Ch^{ie}_{257}$ ${}^{33\cdot 13,430}_{171}Ch^{ie}_{6\cdot 43,7\cdot 37}$ ${}^{430,16\cdot 27}_{172}Ch^{ie}_{6\cdot 43,260}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,262}$

14. Other Formulas of the Fine-structure Constant with 137

$$\alpha_{1-7(137)\text{-}Wallis}=\frac{11}{7\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{750}{751}\frac{16\cdot 47}{2\cdot 3\cdot 125+1})}\frac{1}{137+\frac{1}{13\cdot 37\cdot 59-\frac{4\cdot 11}{3\cdot 25}}}$$

$$=1/137.035999037435$$

${}^{7}_{3}Li_{4}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{47}_{22}Ti_{25}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Co_{32}$ ${}^{64,66,67,68}_{30}Zn_{34,36,37,38}$
${}^{75}_{33}As_{42}$ ${}^{82,83,84,86}_{36}Kr_{46,47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{105}_{46}Pd_{59}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$
${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{187,188}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{6\cdot 37}_{86}Rn^{*}_{136}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{2\cdot 173}_{137}Fy^{ie}_{209}$

$$\alpha_{1-7(137)-GL}=\frac{11}{2\cdot 7\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot(2\cdot 7\cdot 17+1)+1})}\frac{1}{137+\frac{1}{4\cdot 11\cdot(4\cdot 7^{2}+1)+\frac{19}{27}}}$$

$$=\frac{11}{2\cdot 7\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot(2\cdot 7\cdot 17+1)+1})}\frac{1}{137+\frac{1}{4\cdot 11\cdot(2\cdot 9\cdot 11-1)+\frac{19}{27}}}$$

$$=1/137.035999037435$$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{49,50}_{22}Ti_{27,28}$ ${}^{50,51}_{23}V_{27,28}$
${}^{59}_{27}Co_{32}$ ${}^{76,77,78,80,82}_{34}Se_{42,43,44,46,48}$ ${}^{98,99,100,101,102}_{44}Ru_{54,55,56,57,58}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$
${}^{162,164,166,167,168,170}_{68}Er_{94,96,98,99,100,102}$ ${}^{344,2\cdot 173}_{136,137}Fy^{ie}_{208,11\cdot 19}$

$$\alpha_{1-7(137)-NC}=\frac{4\cdot 11}{7\cdot(2\pi)_{NC-5}}\frac{1}{137+\frac{1}{23}-\frac{1}{2\cdot(4\cdot 3\cdot 5\cdot 7+1)}+\frac{1}{2\cdot 3\cdot 7\cdot(2\cdot 5\cdot 53\cdot 83+1)}}$$

$$=\frac{4\cdot 11}{7\cdot(2\pi)_{NC-5}}\frac{1}{137+\frac{1}{23}-\frac{1}{2\cdot(4\cdot 3\cdot 5\cdot 7+1)}+\frac{1}{2\cdot 3\cdot 7\cdot(8\cdot 9\cdot 13\cdot 47+1)}}$$

$$=1/137.035999037435$$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{45}_{21}Sc_{24}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{53}_{24}Cr_{29}$
${}^{95}_{42}Mo_{53}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{127}_{53}I_{74}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$
${}^{210}_{85}At^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{6\cdot 53,11\cdot 29,320}_{127}Ch^{ie}_{191,192,13}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{16\cdot 13,,209,210}$

$$\alpha_{1-39(137)-Wallis}=\frac{5\cdot 7^{2}}{4\cdot 39\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{6080}{6081}\frac{2\cdot(2\cdot 9\cdot 13^{2}-1)}{2\cdot 32\cdot 5\cdot 19+1})}\frac{1}{137+\frac{1}{2\cdot 3\cdot 5^{2}\cdot 7\cdot 11-\frac{1}{10}}}$$

$$=1/137.035999037435$$

${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$
${}^{31}_{15}P_{16}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{40,41,42,43,44,46,48}_{20}Ca_{20,22,23,24,26,28}$ ${}^{45}_{21}Sc_{24}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$
${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{58,60,61,64}_{28}Ni_{30,32,33,36}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64,66,68,70}_{30}Zn_{34,36,38,40}$ ${}^{69,71}_{31}Ga_{38,40}$

$^{79,81}_{35}Br_{44,46}$ $^{87,88}_{38}Sr_{49,50}$ $^{89}_{39}Y_{50}$ $^{90,91,92}_{40}Zr_{50,51,52}$ $^{92,94,95,96,97,98,100}_{42}Mo_{50,52,53,54,55,56,58}$ $^{103}_{45}Rh_{58}$
$^{96,98,99,100,101,102,104}_{44}Ru_{52,54,55,56,57,58,60}$ $^{112,114,115,116,117,118,119,120,122,124}_{50}Sn_{62,64,65,66,67,68,69,70,72,74}$
$^{113,115}_{49}In_{64,66}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{173}_{70}Yb_{103}$ $^{191,193}_{77}Ir_{104,105}$ $^{15\cdot 19}_{112}Cn^{*}_{173}$ $^{286}_{113}Nh^{ie}_{173}$ $^{344,2\cdot 173,348}_{137}Fy^{ie}_{208,209,210}$

$$\alpha_{1-39(137)-GL}=\frac{5\cdot 7^2}{8\cdot 39\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 9\cdot 5\cdot 43+1})}\frac{1}{137+\frac{1}{8\cdot 5\cdot 7\cdot(2\cdot 5\cdot 23\cdot 27-1)}}$$

$$=\frac{5\cdot 7^2}{8\cdot 39\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 9\cdot 5\cdot 43+1})}\frac{1}{137+\frac{1}{8\cdot 5\cdot 7\cdot(64\cdot 97+1)}}$$

$$=1/137.035999037435$$

$^{19}_{9}F_{10}$ $^{20,21,22}_{10}Ne_{10,11,12}$ $^{27}_{13}Al_{14}$ $^{40,42,43,44,46,50}_{20}Ca_{20,22,23,24,26,28}$ $^{50,51}_{23}V_{27,28}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{77}_{34}Se_{43}$
$^{79,81}_{35}Br_{44,46}$ $^{90,91,92,94,96}_{40}Zr_{50,51,52,54,56}$ $^{97,98,99}_{43}Tc^{*}_{54,55,56}$ $^{112,113}_{48}Cd_{64,65}$ $^{113,115}_{49}In_{64,66}$ $^{163}_{66}Dy_{97}$ $^{173}_{70}Yb_{103}$
$^{136,137,138}_{56}Ba_{80,81,82}$ $^{13\cdot 19}_{97}Bk^{*}_{150}$ $^{15\cdot 19}_{112}Cn^{*}_{173}$ $^{8\cdot 43,2\cdot 173,348}_{137}Fy^{ie}_{208,209,210}$ $^{410}_{163}Ch^{ie}_{13\cdot 19}$

$$\alpha_{1-39(137)-NC}=\frac{5\cdot 7^2}{39\cdot(2\pi)_9}\frac{1}{137+\frac{109\cdot(12\cdot 29+1)}{4\cdot 10^{11}}}=1/137.035999037435$$

$^{19}_{9}F_{10}$ $^{63,65}_{29}Cu_{34,36}$ $^{79,81}_{35}Br_{44,46}$ $^{107,109}_{47}Ag_{60,62}$ $^{113,115}_{49}In_{64,66}$ $^{183}_{74}W_{109}$ $^{278}_{109}Mt^{*}_{169}$ $^{344,346,12\cdot 29}_{137}Fy^{ie}_{208,209,210}$

$$\alpha_{1-46(137)-Wallis}$$

$$=\frac{17^2}{8\cdot 23\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{2924}{9\cdot 25\cdot 13}\frac{2\cdot 7\cdot 11\cdot 19}{2\cdot 2\cdot 17\cdot 43+1})}\frac{1}{137+\frac{1}{4\cdot 3\cdot 11\cdot 17\cdot 43-\frac{13}{17}}}$$

$$=1/137.035999037435$$

$^{14,15}_{7}N_{7,8}$ $^{19}_{9}F_{10}$ $^{23}_{11}Na_{12}$ $^{27}_{13}Al_{14}$ $^{35,37}_{17}Na_{18,20}$ $^{39,40,41}_{19}K_{20,21,22}$ $^{43}_{20}Ca_{23}$ $^{46,47,48,48,50}_{22}Ti_{24,25,26,27,28}$
$^{50,51}_{23}V_{27,28}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{77}_{34}Se_{43}$ $^{79,81}_{35}Br_{44,46}$ $^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ $^{86,87,88}_{38}Sr_{48,49,50}$
$^{97,98,99}_{43}Tc^{*}_{54,55,56}$ $^{112,113}_{48}Cd_{64,65}$ $^{113,115}_{49}In_{64,66}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$
$^{222}_{86}Rn^{*}_{136}$ $^{223}_{87}Fr^{*}_{136}$ $^{226}_{88}Ra^{*}_{138}$ $^{227}_{89}Ac^{*}_{138}$ $^{285}_{112}Cn^{*}_{173}$ $^{22\cdot 13}_{113}Nh^{ie}_{173}$ $^{17^2}_{115}Mc^{ie}_{174}$ $^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-46(137)-GL}=\frac{17^2}{16\cdot 23\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 49\cdot 19+1})}\frac{1}{137+\frac{1}{16\cdot 3\cdot 17\cdot 79-\frac{3\cdot 13}{4\cdot 11}}}$$

$$=1/137.035999037435$$

$^{23}_{11}Na_{12}$ $^{27}_{13}Al_{14}$ $^{35,37}_{17}Na_{18,20}$ $^{39,40,41}_{19}K_{20,21,22}$ $^{46,48,49,50}_{22}Ti_{24,26,27,28}$ $^{50,51}_{23}V_{27,28}$ $^{79,81}_{35}Br_{44,46}$ $^{85,87}_{37}Rb_{48,50}$
$^{84,86,87,88}_{38}Sr_{46,48,49,50}$ $^{89}_{39}Y_{50}$ $^{96,98,99,100,101,102,104}_{44}Ru_{52,54,55,56,57,58,60}$ $^{112,113}_{48}Cd_{64,65}$ $^{113,115}_{49}In_{64,66}$
$^{125,126,128,130}_{52}Te_{73,74,76,78}$ $^{135,136,137,138}_{56}Ba_{79,80,81,82}$ $^{197}_{79}Au_{118}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{222}_{86}Rn^{*}_{136}$ $^{223}_{87}Fr^{*}_{136}$
$^{226}_{88}Ra^{*}_{138}$ $^{227}_{89}Ac^{*}_{138}$ $^{285}_{112}Cn^{*}_{173}$ $^{22\cdot 13}_{113}Nh^{ie}_{173}$ $^{17^2}_{115}Mc^{ie}_{174}$ $^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-46(137)-NC} = \frac{17^2}{2\cdot 23\cdot (2\pi)_7}\frac{1}{137+\frac{1}{5\cdot 79}-\frac{1}{3\cdot 19\cdot (4\cdot 3\cdot 7\cdot 47-1)+\frac{4}{13}}}$$

$$=1/137.035999037435$$

${}^{7}_{3}Li_{4}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Na_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,71}_{23}V_{27,28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$

${}^{79,81}_{35}Br_{44,46}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{101}_{44}Ru_{57}$ ${}^{125,126,128,130}_{52}Te_{73,74,76,78}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{138,139}_{57}La_{81,82}$

${}^{197}_{79}Au_{118}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{17^2}_{115}Mc^{ie}_{174}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-71(137)-Wallis}$$

$$=\frac{2\cdot 112-1}{2\cdot 71\cdot (2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{20208}{20209}\frac{2\cdot 5\cdot 43\cdot 47}{16\cdot 3\cdot (4\cdot 3\cdot 5\cdot 7+1)+1})}\frac{1}{137+\frac{2\cdot 9\cdot 7\cdot 13\cdot 109}{25\cdot 10^{11}}}$$

$$=1/137.035999037435$$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{69,71}_{31}Ga_{38,40}$

${}^{83,84}_{36}Kr_{47,48}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{94}_{42}Mo_{52}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112}_{48}Cd_{64}$ ${}^{159}_{65}Tb_{94}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{4\cdot 47}_{76}Os_{112}$

${}^{183}_{74}W_{109}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{244}_{94}Pu^{*}_{150}$ ${}^{278}_{109}Mt^{*}_{169}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-71(137)-GL}$$

$$=\frac{2\cdot 112-1}{4\cdot 71\cdot (1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot 2\cdot 7\cdot (2\cdot 27\cdot 17+1)+1})}\frac{1}{137+\frac{1}{2\cdot 5\cdot 7\cdot 31\cdot (2\cdot 3\cdot 7\cdot 31-1)}}$$

$$=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{58,60,61,62}_{28}Ni_{30,32,33,34}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{107,109}_{47}Ag_{60,62}$

${}^{96}_{42}Mo_{54}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{173}_{70}Yb_{103}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-71(137)-NC} = \frac{2\cdot (2\cdot 112-1)}{71\cdot (2\pi)_{NC-15}}\frac{1}{137+\frac{1}{2\cdot 27\cdot 25+\frac{3\cdot 11}{4\cdot 131}}} = 1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{47,49}_{22}Ti_{25,27}$ ${}^{31}_{15}P_{16}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Co_{32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{75}_{33}As_{42}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{120,122}_{50}Sn_{70,72}$

${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$

${}^{4\cdot 83}_{131}Ch^{ie}_{201}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{14\cdot 31,435}_{173}Ch^{ie}_{261,2\cdot 131}$

$$\alpha_{1-103(137)-Wallis}$$

$$=\frac{8\cdot 81-1}{4\cdot 103\cdot (2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{168234}{168235}\frac{4\cdot 137\cdot (4\cdot 7\cdot 11-1)}{2\cdot 3\cdot 11\cdot (4\cdot 49\cdot 13+1)+1})}\frac{1}{137+\frac{113\cdot 157}{5\cdot 10^{12}}}$$

$$=\frac{8\cdot 81-1}{4\cdot 103\cdot (2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{168234}{168235}\frac{4\cdot 137\cdot (4\cdot 7\cdot 11-1)}{2\cdot 3\cdot 11\cdot (2\cdot 3\cdot 25\cdot 17-1)+1})}\frac{1}{137+\frac{7\cdot 37\cdot 137}{10^{13}}}$$

$$=1/137.035999037435$$

$^{23}_{11}Na_{12}$ $^{27}_{13}Al_{14}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{35,37}_{17}Cl_{18,20}$ $^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$
$^{67}_{30}Zn_{37}$ $^{75}_{33}As_{42}$ $^{74,76,77,78,80,82}_{34}Se_{40,42,43,44,46,48}$ $^{85,87}_{37}Rb_{48,50}$ $^{96,98,99,100,101,102,104}_{44}Ru_{52,54,55,56,57,58,60}$
$^{103}_{45}Ru_{58}$ $^{106,108,110,111,112,113,114,116}_{48}Cd_{58,60,62,63,64,65,66,68}$ $^{113,115}_{49}In_{64,66}$ $^{116,118,119,120,124}_{50}Sn_{66,68,69,70,74}$
$^{121,123}_{51}Sb_{70,72}$ $^{131}_{54}Xe_{77}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{157}_{64}Gd_{93}$ $^{162,163}_{66}Dy_{96,97}$ $^{168}_{68}Er_{100}$ $^{173}_{70}Yb_{103}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$
$^{257}_{100}Fm^{*}_{157}$ $^{262}_{103}Lr^{*}_{159}$ $^{285}_{112}Cn^{*}_{173}$ $^{286}_{113}Nh^{ie}_{173}$ $^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{400}_{157}Ch^{ie}_{243}$ $^{434,435}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-103(137)-GL}=\frac{8\cdot81-1}{8\cdot103\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot(2\cdot9\cdot25\cdot7\cdot17+1)+1})}\frac{1}{137+\frac{19\cdot173}{5\cdot10^{11}}}$$

$$=1/137.035999037435$$

$^{10,11}_{5}B_{5,6}$ $^{14,15}_{7}N_{7,8}$ $^{16,17,18}_{8}O_{8,9,10}$ $^{19}_{9}F_{10}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{35,37}_{17}Cl_{18,20}$ $^{39,40,41}_{19}K_{20,21,22}$ $^{45}_{21}Sc_{24}$ $^{55}_{25}Mn_{30}$
$^{79,81}_{35}Br_{44,46}$ $^{103}_{45}Ru_{58}$ $^{118,7\cdot17}_{50}Sn_{68,69}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{173}_{70}Yb_{103}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{7\cdot29,205}_{81}Tl_{122,124}$
$^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{210}_{85}At^{*}_{125}$ $^{262}_{103}Lr^{*}_{159}$ $^{285}_{112}Cn^{*}_{173}$ $^{2\cdot157}_{126}Ch^{ie}_{188}$ $^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{434,435}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-103(137)-NC}=\frac{8\cdot81-1}{103\cdot(2\pi)_{NC-29}}\frac{1}{137+\frac{1}{32\cdot3\cdot7\cdot13\cdot29+\frac{2}{15}}}=1/137.035999037435$$

$^{14,15}_{7}N_{7,8}$ $^{27}_{13}Al_{14}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{31}_{15}P_{16}$ $^{45}_{21}Sc_{24}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{63,65}_{29}Cu_{34,36}$ $^{64,70}_{30}Zn_{34,40}$
$^{79,81}_{35}Br_{44,46}$ $^{103}_{45}Ru_{58}$ $^{112,113}_{48}Cd_{64,65}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{173}_{70}Yb_{103}$ $^{7\cdot29,205}_{81}Tl_{122,124}$ $^{262}_{103}Lr^{*}_{159}$
$^{285}_{112}Cn^{*}_{173}$ $^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{434,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-110(137)-Wallis}$$

$$=\frac{4\cdot173-1}{4\cdot110\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{11080}{11081}\frac{2\cdot3\cdot(168\cdot11-1)}{2\cdot8\cdot5\cdot(12\cdot23+1)+1})}\frac{1}{137+\frac{1}{4\cdot27\cdot29\cdot(420+1)}}$$

$$=1/137.035999037435$$

$^{19}_{9}F_{10}$ $^{20,21,22}_{10}Ne_{10,11,12}$ $^{23}_{11}Na_{12}$ $^{27}_{13}Al_{14}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{36,38,40}_{18}Ar_{18,20,22}$ $^{40,42,44,46,48}_{20}Ca_{20,22,23,24,26,28}$
$^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ $^{50,51}_{23}V_{27,28}$ $^{50,52,53,54}_{24}Cr_{26,28,29,30}$ $^{59}_{27}Co_{32}$ $^{63,65}_{29}Cu_{34,36}$ $^{80,82,83,84,86}_{36}Kr_{44,46,47,48}$
$^{90,92,94}_{40}Zr_{50,52,54}$ $^{98,99,100,102,104}_{44}Ru_{54,55,56,58,60}$ $^{103}_{45}Rh_{58}$ $^{102,104,105,106,108,110}_{46}Pd_{56,58,59,60,62,64}$ $^{131}_{54}Xe_{77}$ $^{133}_{55}Cs_{78}$
$^{106,108,110,111,112,113,114,116}_{48}Cd_{58,60,62,63,64,65,66,68}$ $^{112,115,116,118,120,122}_{50}Sn_{62,65,66,68,70,72}$ $^{136,137,138}_{56}Ba_{80,81,82}$
$^{136,138,140,142}_{58}Ce_{78,80,82,84}$ $^{144,145,148}_{60}Ce_{84,85,88}$ $^{168}_{68}Er_{100}$ $^{173}_{70}Yb_{103}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{191,193}_{77}Ir_{114,116}$
$^{200}_{80}Hg_{120}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$ $^{210}_{85}At^{*}_{125}$ $^{223}_{87}Fr^{*}_{136}$ $^{226}_{88}Ra^{*}_{138}$ $^{262}_{103}Lr^{*}_{159}$ $^{270}_{108}Hs^{*}_{162}$ $^{293}_{116}Lv^{*}_{177}$ $^{285}_{112}Cn^{*}_{173}$
$^{22\cdot13}_{113}Nh^{ie}_{173}$ $^{288,289}_{115}Mc^{ie}_{173,174}$ $^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{418,420}_{168}Ch^{ie}_{250,252}$ $^{434,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-110(137)-GL}$$

$$=\frac{4\cdot173-1}{8\cdot110\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot(8\cdot9\cdot49-1)+1})}\frac{1}{137+\frac{1}{3\cdot7\cdot11\cdot(8\cdot3\cdot5\cdot41-1)}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{70,72,73,74,76}_{32}Ge_{38,40,41,42,44}$ ${}^{75}_{33}As_{42}$ ${}^{93}_{41}Nb_{52}$
${}^{82,84}_{36}Kr_{46,48}$ ${}^{96,98,99,100,101,102,103,104}_{44}Ru_{52,54,55,56,57,58,60}$ ${}^{110,112,113,114,116}_{48}Cd_{62,64,65,66,68}$ ${}^{113,115}_{49}In_{64,66}$
${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{173}_{70}Yb_{103}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{262}_{103}Lr^{*}_{159}$
${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{434,435}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-110(137)-NC}=\frac{4\cdot173-1}{110\cdot(2\pi)_{NC-13}}\frac{1}{137+\frac{1}{3\cdot149}-\frac{1}{29\cdot(2\cdot3\cdot13\cdot137+1)+\frac{4}{7}}}$$

$$=\frac{4\cdot173-1}{110\cdot(2\pi)_{NC-13}}\frac{1}{137+\frac{1}{3\cdot149}-\frac{1}{29\cdot(64\cdot167-1)+\frac{4}{7}}}=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{89}_{39}Y_{50}$ ${}^{99,100,101,102,104}_{44}Ru_{55,56,58,60}$ ${}^{149}_{62}Sm_{87}$
${}^{136,137,138}_{56}Ba_{8,81,82}$ ${}^{167,168}_{68}Er_{99,100}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{376}_{149}Ch^{ie}_{227}$ ${}^{22\cdot19}_{167}Ch^{ie}_{251}$
${}^{434,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{2-7(137)-wallis}=\frac{7\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{3580}{3579}\frac{4\cdot5\cdot179}{2\cdot(4\cdot3\cdot149+1)+1})}{11}\frac{1}{137-\frac{1}{32\cdot9\cdot7\cdot173+1}}$$

$$=1/137.035999111818$$

${}^{7}_{3}Li_{4}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{149}_{62}Sm_{87}$ ${}^{173}_{70}Yb_{103}$ ${}^{179}_{72}Hf_{107}$
${}^{262}_{103}Lr^{*}_{159}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{2\cdot149}_{119}Ch^{ie}_{179}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{376}_{149}Ch^{ie}_{227}$ ${}^{434,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{2-7(137)-GL}=\frac{2\cdot7\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot2\cdot17\cdot67+1})}{11}\frac{1}{137-\frac{1}{4\cdot7\cdot19\cdot(2\cdot9\cdot31-1)-\frac{10}{37}}}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$
${}^{64,66,67,68}_{30}Zn_{34,36,37,38}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{74,76,77,78}_{34}Se_{40,42,43,44}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{84,86,87,88}_{38}Sr_{46,48,49,50}$ ${}^{117,118}_{50}Sn_{67,68}$
${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{165}_{67}Ho_{98}$ ${}^{173}_{70}Yb_{103}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{434,435}_{173}Ch^{ie}_{261,262}$

$$\alpha_{2-7(137)-NC}=\frac{7\cdot(2\pi)_{NC-9}}{4\cdot11}\frac{1}{137-\frac{1}{7\cdot17}+\frac{1}{3\cdot19\cdot(8\cdot61-1)-\frac{61}{100}}}=1/137.035999111818$$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{61,62}_{28}Ni_{33,34}$ ${}^{77}_{34}Se_{43}$ ${}^{118,7\cdot17}_{50}Sn_{68,69}$
${}^{99,100,101,102,104}_{44}Ru_{55,56,57,58,60}$ ${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{145,146,147}_{61}Pm^{*}_{84,85,86}$ ${}^{168}_{68}Er_{100}$
${}^{173}_{70}Yb_{103}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{199,200}_{80}Hg_{119,120}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{434,435}_{173}Ch^{ie}_{261,262}$

15. Formulas of the Fine-structure Constant with 83

As the 83th element $_{83}$Bi should be the end of stable elements and the start of radioactive elements (except $_{43}$Tc and $_{61}$Pm) in the periodic table of elements, some formulas of the fine-structure constant could be constructed with the factor 83 instead of 112, 173 and 137 as follows.

$$\alpha_{1-31(83)}=\frac{2\cdot 59}{31\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\dfrac{e^2}{(\frac{4}{3})^7}\cdots\dfrac{e^2}{(\frac{9\cdot 83}{2\cdot(2\cdot 11\cdot 17-1)})^{1493}}}\frac{1}{83+\dfrac{1}{3\cdot 29\cdot(6\cdot 83-1)-\dfrac{19}{2\cdot 37}}}$$

$$=\frac{2\cdot 59}{31\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\dfrac{e^2}{(\frac{4}{3})^7}\cdots\dfrac{e^2}{(\frac{9\cdot 83}{2\cdot(12\cdot 31+1)+1})^{1493}}}\frac{1}{83+\dfrac{1}{3\cdot 29\cdot(16\cdot 31+1)-\dfrac{19}{2\cdot 37}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{59}_{27}Co_{32}$
${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{63,65}_{29}Co_{34,36}$ ${}^{64,66,67,68}_{30}Zn_{34,36,37,38}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{74,76,77,78,80,82}_{34}Se_{40,42,43,44,46,48}$
${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{84.86,87,88}_{38}Sr_{46,48,49,50}$ ${}^{99,100,102}_{44}Ru_{55,56,58}$ ${}^{105,107}_{46}Pd_{59,62}$ ${}^{107,109}_{47}Ag_{60,62}$
${}^{110,111,112,113,114,116}_{48}Cd_{62,63,64,65,66,68}$ ${}^{2\cdot 59}_{50}Sn_{68}$ ${}^{126}_{52}Te_{74}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$
${}^{143}_{60}Nd_{83}$ ${}^{149}_{62}Sm_{87}$ ${}^{166,167,168,170}_{68}Er_{98,99,100,102}$ ${}^{173}_{70}Yb_{103}$ ${}^{184,6\cdot 31}_{74}W_{110,112}$ ${}^{185,11\cdot 17}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$
${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{286}_{113}Nh^{ie}_{173}$ ${}^{294}_{118}Og^{ie}_{176}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137.138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-31(83)-Wallis}$$

$$=\frac{59}{2\cdot 31\cdot(2\dfrac{2}{3}\dfrac{4}{3}\dfrac{4}{5}\dfrac{6}{5}\dfrac{6}{7}\dfrac{8}{7}\cdots\dfrac{2238}{2239}\dfrac{4\cdot 5\cdot 112}{2\cdot 3\cdot(2\cdot 11\cdot 17-1)+1})}\frac{1}{83+\dfrac{1}{2\cdot 83\cdot(3\cdot 128-1)-\dfrac{5}{11}}}$$

$$=1/137.035999037435$$

${}^{10,11}_{5}B_{5,6}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{112}_{48}Cd_{64}$ ${}^{185,11\cdot 17}_{75}\mathrm{Re}_{110,112}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137.138}Fy^{ie}_{208,209,210}$

$$\alpha_{1\text{-}31(83)\text{-}GL}=\frac{59}{4\cdot 31\cdot(1-\dfrac{1}{3}+\dfrac{1}{5}-\dfrac{1}{7}+\cdots+\dfrac{1}{2\cdot 16\cdot 89+1})}\frac{1}{83+\dfrac{1}{13\cdot 17\cdot(14\cdot 73-1)-\dfrac{13}{4\cdot 7}}}$$

$$=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{59}_{27}Co_{32}$ ${}^{58,60,62,64}_{28}Ni_{30,32,34,36}$ ${}^{73}_{32}Ge_{41}$ ${}^{89}_{39}Y_{50}$ ${}^{125}_{52}Te_{73}$
${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{24\cdot 13}_{125}Ch^{ie}_{11\cdot 17}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137.138}Fy^{ie}_{16\cdot 13,209,210}$

$$\alpha_{1\text{-}31(83)\text{-}NC}=\frac{2\cdot 59}{31\cdot(2\pi)_{NC-7}}\frac{1}{83+\frac{1}{13^2}-\frac{1}{2\cdot(4\cdot81\cdot47-1)-\frac{47}{61}}}=1/137.035999037435$$

${}^{14,15}_{7}N_{7,8}$ ${}^{27}_{13}Al_{14}$ ${}^{59}_{27}Co_{32}$ ${}^{61}_{28}Ni_{33}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{83}_{36}Kr_{47}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$
${}^{145}_{61}Pm^{*}_{84}$ ${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{203,205}_{81}Tl_{2\cdot61,124}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,12\cdot29}_{136,137.138}Fy^{ie}_{16\cdot13,209,210}$ ${}^{426}_{169}Ch^{ie}_{257}$

$$\alpha_{2-8(83)}=\frac{8\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots\frac{e^2}{(\frac{2\cdot(4\cdot25\cdot7+1)}{3\cdot(4\cdot9\cdot13-1)})^{2803}}}{83}\frac{1}{83-\frac{1}{2\cdot31\cdot(2\cdot3\cdot5\cdot7\cdot13+1)-\frac{1}{10}}}$$

$=1/137.035999111818$

${}^{7}_{3}Li_{4}$ ${}^{9}_{4}Be_{5}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$
${}^{31}_{15}P_{16}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{58,60,62,64}_{28}Ni_{30,32,34,36}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{89}_{39}Y_{50}$
${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{90,7\cdot13,92,94}_{40}Zr_{50,51,4\cdot13,54}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{110,112,113}_{48}Cd_{62,64,65}$ ${}^{115,9\cdot13,120,124}_{50}Sn_{65,67,70,74}$
${}^{124,125,126,130}_{52}Te_{72,73,74,78}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{157}_{64}Gd_{93}$ ${}^{159}_{65}Tb_{94}$ ${}^{173}_{70}Yb_{103}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{200}_{80}Hg_{120}$ ${}^{209}_{83}Bi^{*}_{126}$
${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{286}_{113}Nh^{ie}_{173}$ ${}^{300}_{120}Ch^{*}_{180}$ ${}^{330}_{130}Ch^{*}_{200}$ ${}^{344,2\cdot173,348}_{136,137.138}Fy^{ie}_{16\cdot13,209,210}$

$$\alpha_{2-8(83)-Wallis}=\frac{32\cdot(2\frac{2}{3}\frac{4}{3}\frac{4}{5}\frac{6}{5}\frac{6}{7}\frac{8}{7}\cdots\frac{4202}{4203}\frac{4\cdot(2\cdot3\cdot25\cdot7+1)}{2\cdot11\cdot(2\cdot5\cdot19+1)+1})}{83}\frac{1}{83-\frac{1}{4\cdot7\cdot89\cdot(4\cdot83-1)}}$$

$=1/137.035999111818$

${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{47}_{22}Ti_{25}$ ${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{143,144}_{60}Nd_{83,84}$ ${}^{227}_{89}Ac^{*}_{138}$

$$\alpha_{2-8(83)-GL}=\frac{64\cdot(1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots+\frac{1}{2\cdot4\cdot3\cdot223+1})}{83}\frac{1}{83-\frac{1}{4\cdot9\cdot13\cdot(4\cdot127+1)-\frac{7}{11}}}$$

$=1/137.035999111818$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{53}_{24}Cr_{29}$ ${}^{83}_{36}Kr_{47}$ ${}^{127}_{53}I_{74}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{6\cdot53}_{127}Ch^{ie}_{191}$

$$\alpha_{2-8(83)-NC}=\frac{8\cdot(2\pi)_{NC-9}}{83}\frac{1}{83-\frac{1}{2\cdot149}+\frac{1}{2\cdot3\cdot79\cdot(2\cdot3\cdot47-1)-\frac{7}{8}}}$$

$=1/137.035999111818$

${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{83}_{36}Kr_{47}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{3\cdot47}_{59}\mathrm{Pr}_{82}$ ${}^{149}_{62}Sm_{87}$
${}^{2\cdot79}_{64}Gd_{2\cdot47}$ ${}^{197}_{79}Au_{118}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot149}_{119}Ch^{ie}_{179}$ ${}^{344,2\cdot173,348}_{136,137.138}Fy^{ie}_{16\cdot13,209,210}$ ${}^{4\cdot89}_{141}Ch^{ie}_{215}$ ${}^{8\cdot47}_{149}Ch^{ie}_{227}$

16. Formulas of the Fine-structure Constant with 83^2

$$\alpha_{1-(50)/83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37}-\frac{4\cdot 3\cdot 5\cdot 41-1}{125\cdot 10^{10}}}{83^2}=1/137.035999037435$$

Note: $\frac{1}{3}-\frac{1}{16}=\frac{13}{16\cdot 3}$

${}^{55}_{25}Mn_{30}$ ${}^{67}_{30}Zn_{37}$ ${}^{82,83,84,86}_{36}Kr_{46,47,48,50}$ ${}^{85,87}_{37}Nb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{93}_{41}Nb_{52}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{112,113}_{48}Cd_{64,65}$
${}^{117,122,124}_{50}Sn_{67,72,74}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{143}_{60}Nd_{83}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{207,16\cdot 13}_{82}Pb_{125,126}$ ${}^{16\cdot 13,209}_{83}Bi^{*}_{125,126}$ ${}^{209}_{84}Po^{*}_{125}$
${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot 143}_{113}Nh^{ie}_{173}$ ${}^{24\cdot 13}_{125}Ch^{ie}_{187}$ ${}^{8\cdot 43,2\cdot 173,348}_{136,137,138}Fy^{ie}_{16\cdot 13,209,210}$

$$\alpha_{1-(50)/83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37+\frac{59\cdot(4\cdot 53-1)}{25\cdot 10^{5}}}}{83^2}=1/137.035999037435$$

${}^{53}_{24}Cr_{29}$ ${}^{59}_{27}Co_{32}$ ${}^{95}_{42}Mo_{53}$ ${}^{105,106}_{46}Pd_{59,60}$ ${}^{2\cdot 59,124}_{50}Sn_{68,74}$ ${}^{127}_{53}I_{74}$ ${}^{141}_{59}\mathrm{Pr}_{82}$

$$\alpha_{1-(50)/83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37+\frac{1}{200+\frac{2\cdot 7}{17}}}}{83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37+\frac{17}{2\cdot 3\cdot(8\cdot 71+1)}}}{83^2}$$
$$=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{77}_{34}Se_{43}$ ${}^{118,7\cdot 17,120,122,124}_{50}Sn_{68,69,70,72,74}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{143}_{60}Nd_{83}$
${}^{175,176}_{71}Lu_{104,105}$ ${}^{200}_{80}Hg_{120}$ ${}^{4\cdot 71,2\cdot 143}_{113}Nh^{ie}_{171,173}$

$$\alpha_{2-(50)/83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37}-\frac{27\cdot 11\cdot(2\cdot 9\cdot 11-1)}{2\cdot 10^{12}}}{83^2}=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{82,83,84,86}_{36}Kr_{46,47,48,50}$ ${}^{85,87}_{37}Nb_{48,50}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$
${}^{117,124}_{50}Sn_{67,74}$ ${}^{143}_{60}Nd_{83}$ ${}^{16\cdot 13,209}_{83}Bi^{*}_{125,126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot 143}_{113}Nh^{ie}_{173}$ ${}^{8\cdot 43,2\cdot 173,348}_{136,137,138}Fy^{ie}_{16\cdot 13,209,210}$

$$\alpha_{2-(50)/83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37+\frac{43\cdot 79\cdot 109}{5\cdot 10^{6}}}}{83^2}=1/137.035999111818$$

${}^{79,81}_{35}Br_{44,46}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{118}_{50}Sn_{68}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{183}_{74}W_{109}$ ${}^{197}_{79}Au_{118}$ ${}^{278}_{109}Mt^{*}_{169}$

$$\alpha_{2-(50)/83^2}=\frac{50+\frac{1}{3}-\frac{1}{16}+\frac{1}{43\cdot 37+\frac{2}{27+\frac{7}{1000}}}}{83^2}=\frac{50+\frac{13}{16\cdot 3}+\frac{1}{43\cdot 37+\frac{16\cdot 125}{113\cdot(2\cdot 7\cdot 17+1)}}}{83^2}$$
$$=1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{85,87}_{37}Nb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{97,99}_{43}Tc^{*}_{54,56}$ ${}^{112,113}_{48}Cd_{64,65}$
${}^{143}_{60}Nd_{83}$ ${}^{188,27\cdot 7}_{76}Os_{112,113}$ ${}^{16\cdot 13,209}_{83}Bi^{*}_{125,126}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot 143}_{113}Nh^{ie}_{173}$

17. Formulas of the Fine-structure Constant with 112×173

$$\alpha_{1-(141)/112\cdot 173}=\frac{3\cdot 47+\frac{1}{2}-\frac{1}{9}+\frac{1}{8\cdot 27}-\frac{1}{2\cdot 31\cdot(2\cdot 5\cdot 107-1)-\frac{5}{17}}}{112\cdot 173}=1/137.035999037435$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$

${}^{50,51}_{23}Cl_{27,28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{59}_{27}Co_{32}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$

${}^{96}_{42}Mo_{54}$ ${}^{105,108}_{46}Pd_{59,62}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{131}_{54}Xe_{77}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$

${}^{147}_{62}Sm_{85}$ ${}^{173}_{70}Yb_{103}$ ${}^{179,180}_{72}Hf_{107,108}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{4\cdot 47,27\cdot 7}_{76}Os_{112,113}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$

${}^{2\cdot 131}_{103}Lr^{*}_{159}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,11\cdot 19,210}$ ${}^{4\cdot 71,22\cdot 13}_{113}Nh^{ie}_{9\cdot 19,173}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 29,2\cdot 131}$

$$\alpha_{2-(141)/112\cdot 173}=\frac{3\cdot 47+\frac{1}{2}-\frac{1}{9}+\frac{1}{8\cdot 27}-\frac{1}{9\cdot 17\cdot(16\cdot 27-1)-\frac{2\cdot 17}{47}}}{112\cdot 173}=1/137.035999111818$$

${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{50,51}_{23}Cl_{27,28}$ ${}^{60,62,64}_{28}Ni_{32,34,36}$ ${}^{83,84}_{36}Kr_{47,48}$

${}^{91,94}_{40}Zr_{51,54}$ ${}^{94,96,98}_{42}Mo_{52,54,56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{121,123}_{51}Sb_{70,72}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{152,154,158,160}_{64}Gd_{88,90,94,96}$

${}^{159}_{65}Tb_{94}$ ${}^{160}_{66}Dy_{94}$ ${}^{244}_{94}Pu^{*}_{150}$ ${}^{251}_{98}Cf^{*}_{9\cdot 17}$ ${}^{36\cdot 7}_{99}Es^{*}_{9\cdot 17}$ ${}^{128\cdot 3}_{153}Ch^{ie}_{3\cdot 77}$

18. Formulas of the Fine-structure Constant with 163×173

163×173≈167.9^2≈168^2, so in the world of nuclides, 163 should also be a sub-stable number, or 163 and 173 could be regarded as twin numbers split from 168, and hence some formulas of the fine-structure constant could be constructed with 163×173 as follows.

$$\alpha_{1-(206)/163\cdot 173}=\frac{2\cdot 103+\frac{1}{4}-\frac{1}{5\cdot 7}+\frac{1}{4\cdot 25\cdot 19+\frac{2\cdot 7\cdot 17-1}{2\cdot 81\cdot 7}}}{163\cdot 173}=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{118,119}_{50}Sn_{68,69}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$

${}^{173}_{70}Yb_{103}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{2\cdot 131}_{103}Lr^{*}_{159}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{410}_{163}Ch^{ie}_{13\cdot 19}$ ${}^{434,435}_{173}Ch^{ie}_{261,2\cdot 131}$

$$\alpha_{2-(206)/163\cdot 173}=\frac{2\cdot 103+\frac{1}{4}-\frac{1}{5\cdot 7}+\frac{1}{4\cdot 25\cdot 19-\frac{2\cdot 23}{2\cdot 7\cdot 17-1}}}{163\cdot 173}=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{76,80}_{34}Se_{42,46}$ ${}^{79,81}_{35}Br_{44,46}$

${}^{99,100,102,104}_{44}Ru_{55,56,58,60}$ ${}^{103}_{45}Rh_{58}$ ${}^{118,119}_{50}Sn_{68,69}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$

${}^{188,189}_{76}Os_{112,113}$ ${}^{2\cdot 131}_{103}Lr^{*}_{159}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{410}_{163}Ch^{ie}_{13\cdot 19}$ ${}^{434,435}_{173}Ch^{ie}_{261,2\cdot 131}$

$$\alpha_{1-(206)/163\cdot173}=\frac{2\cdot103+\frac{1}{4}-\frac{1}{5\cdot7}+\frac{1}{9\cdot(2\cdot3\cdot5\cdot7+1)}-\frac{1}{4\cdot47\cdot83\cdot191}}{163\cdot173}$$

$$=1/137.035999037435$$

${}^{10,11}_{5}B_{5,6}$ ${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{83}_{36}Kr_{47}$ ${}^{103}_{45}Rh_{58}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{131}_{54}Xe_{77}$ ${}^{143}_{60}Nd_{83}$ ${}^{163}_{66}Dy_{97}$
${}^{173}_{70}Yb_{103}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{2\cdot131}_{103}Lr^{*}_{159}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269,271}_{106}Sg^{*}_{163,165}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{2\cdot143}_{113}Nh^{ie}_{173}$
${}^{318,319,320}_{127}Ch^{ie}_{191,192,193}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{410}_{163}Ch^{ie}_{247}$ ${}^{434,435}_{173}Ch^{ie}_{261,2\cdot131}$

$$\alpha_{2-(206)/163\cdot173}=\frac{2\cdot103+\frac{1}{4}-\frac{1}{5\cdot7}+\frac{1}{2\cdot13\cdot73}-\frac{1}{27\cdot5\cdot7\cdot(64\cdot3\cdot11-1)}}{163\cdot173}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{59}_{27}Co_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{73}_{32}Ge_{41}$
${}^{75}_{33}As_{42}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{97}_{42}Mo_{55}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{103}_{45}Rh_{58}$ ${}^{112,113,115}_{48}Cd_{64,65,66}$ ${}^{125}_{52}Te_{73}$
${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{143,146}_{60}Nd_{83,86}$ ${}^{146}_{61}Pm^{*}_{84}$ ${}^{163}_{66}Dy_{97}$ ${}^{173}_{70}Yb_{103}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{210}_{84}Po^{*}_{125}$
${}^{235,238}_{92}U^{*}_{143,146}$ ${}^{2\cdot131}_{103}Lr^{*}_{159}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269,271}_{106}Sg^{*}_{163,5\cdot33}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{340}_{135}Ch^{ie}_{205}$
${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{3\cdot121,28\cdot13}_{143}Ch^{ie}_{220,221}$ ${}^{370}_{146}Ch^{ie}_{224}$ ${}^{410}_{163}Ch^{ie}_{13\cdot19}$ ${}^{434,435}_{173}Ch^{ie}_{261,2\cdot131}$

19. Formulas of the Fine-structure Constant with 36/112 and 100×112

$$\alpha_{1-36/(44)\cdot112}=\frac{36}{(44+\frac{1}{3\cdot7}-\frac{1}{4\cdot7\cdot107}+\frac{1}{2\cdot27\cdot25\cdot43\cdot127})\cdot112}=1/137.035999037435$$

${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{45}_{21}Sc_{24}$ ${}^{55}_{25}Mn_{30}$
${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{58,60,61,64}_{28}Ni_{30,32,33,36}$ ${}^{66}_{30}Zn_{36}$ ${}^{75}_{33}As_{42}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$
${}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{103}_{45}Rh_{58}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{112,113,114}_{48}Cd_{64,65,66}$ ${}^{112,116,122}_{50}Sn_{62,66,72}$
${}^{127}_{53}I_{74}$ ${}^{126,129,131,132}_{54}Xe_{72,75,77,78}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{143,144,146,148}_{60}Nd_{83,84,86,88}$ ${}^{163}_{66}Dy_{97}$ ${}^{173}_{70}Yb_{103}$
${}^{179}_{72}Hf_{107}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{191,193}_{77}Ir_{104,106}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209,210,211}_{84}Po^{*}_{125,126,127}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$
${}^{270}_{107}Bh^{*}_{163}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{286}_{113}Nh^{ie}_{173}$ ${}^{6\cdot53,11\cdot29,320}_{127}Ch^{ie}_{191,192,193}$ ${}^{8\cdot43,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{410}_{163}Ch^{ie}_{247}$ ${}^{434,435}_{173}Ch^{ie}_{261,2\cdot131}$

$$\alpha_{2-36/(44)\cdot112}=\frac{36}{(44+\frac{1}{3\cdot7}-\frac{1}{4\cdot7\cdot107}+\frac{1}{3\cdot7\cdot157\cdot(4\cdot25\cdot19+1)})\cdot112}$$

$$=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{45}_{21}Sc_{24}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{86,88}_{38}Sr_{48,50}$
${}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ ${}^{99,100,101,104}_{44}Ru_{55,56,57,60}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{163}_{66}Dy_{97}$ ${}^{179}_{72}Hf_{107}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$
${}^{186,4\cdot47}_{76}Os_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{270}_{107}Bh^{*}_{163}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{400}_{157}Ch^{ie}_{243}$

$$\alpha_{1-36/(44)\cdot 112}=\frac{36}{\left(44+\frac{1}{3\cdot 7}-\frac{1}{81\cdot 37+\frac{11^2}{3\cdot 5\cdot 37}}\right)\cdot 112}=1/137.035999037435$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$

${}^{45}_{21}Sc_{24}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{66,67}_{30}Zn_{36,37}$ ${}^{75}_{33}As_{42}$ ${}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$

${}^{85,87}_{37}Rb_{48,50}$ ${}^{96,97,98}_{42}Mo_{54,55,56}$ ${}^{98,99,100}_{44}Ru_{54,55,56}$ ${}^{111,112,114}_{48}Cd_{63,64,66}$ ${}^{128,131,132,134}_{54}Xe_{74,77,78,80}$

${}^{133}_{55}Cs_{78}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{182,184,186}_{74}W_{108,110,112}$ ${}^{5\cdot 37,187}_{75}\mathrm{Re}_{110,112}$ ${}^{201}_{80}Hg_{121}$ ${}^{203,205}_{81}Tl_{122,124}$

${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{6\cdot 37}_{86}Rn^{*}_{136}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{2-36/(44)\cdot 112}=\frac{36}{\left(44+\frac{1}{3\cdot 7}-\frac{1}{81\cdot 37+\frac{2\cdot 9\cdot 17}{7\cdot 101}}\right)\cdot 112}=1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{46,48,49,50}_{22}Ti_{24,26,27,28}$ ${}^{76,77,78}_{34}Se_{42,43,44}$ ${}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$

${}^{85,87}_{37}Rb_{48,50}$ ${}^{99,100,101}_{44}Ru_{55,56,57}$ ${}^{112,114}_{48}Cd_{64,66}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{170,171,172,173,176}_{70}Yb_{100,101,102,103,106}$

${}^{167,168,170}_{68}Er_{99,100,102}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{6\cdot 37}_{86}Rn^{*}_{136}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-(82)/100\cdot 112}=\frac{82-\frac{1}{3}+\frac{1}{3\cdot 5}-\frac{1}{5\cdot 67}+\frac{1}{2\cdot 7\cdot 29\cdot(8\cdot 5\cdot 113-1)}}{100\cdot 112}=1/137.035999037435$$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{67}_{30}Zn_{37}$ ${}^{100}_{44}Ru_{56}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{117}_{50}Sn_{67}$ ${}^{165}_{67}Ho_{98}$

${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{200,3\cdot 67}_{80}Hg_{120,121}$

${}^{207,208}_{82}Pb_{125,126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{286}_{113}Nh^{ie}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{2-(82)/100\cdot 112}=\frac{82-\frac{1}{3}+\frac{1}{3\cdot 5}-\frac{1}{5\cdot 67}+\frac{1}{32\cdot 9\cdot 5\cdot 19\cdot 73}}{100\cdot 112}=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{70,72,73,74,76}_{32}Ge_{38,40,41,42,44}$ ${}^{100}_{44}Ru_{56}$ ${}^{103}_{45}Rh_{58}$ ${}^{112}_{48}Cd_{64}$

${}^{117}_{50}Sn_{67}$ ${}^{125}_{52}Te_{73}$ ${}^{165}_{67}Ho_{98}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{142,143,144,146}_{60}Nd_{82,83,84,86}$ ${}^{168}_{68}Er_{100}$

${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{200,3\cdot 67}_{80}Hg_{120,121}$ ${}^{207,208}_{82}Pb_{125,126}$ ${}^{209}_{83}Bi^{*}_{126}$

${}^{209}_{84}Po^{*}_{125}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

20. Other Formulas of the Fine-structure Constant with some square numbers

$$\alpha_{1-(29)/36\cdot 112}=\frac{29+\frac{1}{2}-\frac{1}{12}+\frac{1}{3\cdot 53}-\frac{1}{3\cdot 19\cdot(2\cdot 17^2-1)-\frac{2\cdot 19}{5\cdot 37}}}{36\cdot 112}=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{53}_{24}Cr_{29}$ ${}^{58,62,64}_{28}Ni_{30,34,36}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{83,84}_{36}Kr_{47,48}$

${}^{85,87}_{37}Rb_{48,50}$ ${}^{95}_{42}Mo_{53}$ ${}^{84,86,87}_{38}Sr_{46,48,49}$ ${}^{100,101,102}_{44}Ru_{56,57,58}$ ${}^{122,123}_{51}Sb_{70,72}$ ${}^{127}_{53}I_{74}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$

${}^{180,186}_{74}W_{106,112}$ ${}^{188}_{76}Os_{112}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{6\cdot 37}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{17^2}_{115}Mc^{ie}_{6\cdot 29}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{2-(29)/36\cdot112} = \frac{29+\frac{1}{2}-\frac{1}{12}+\frac{1}{3\cdot53}-\frac{1}{8\cdot7\cdot(4\cdot3\cdot49-1)-\frac{3\cdot11}{2\cdot5\cdot7}}}{36\cdot112} = 1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{46,48,49}_{22}Ti_{24,26,27}$ ${}^{53}_{24}Cr_{29}$

${}^{58,60}_{28}Ni_{30,32,33,36}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{75}_{33}As_{42}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{84}_{36}Kr_{48}$ ${}^{95,97,98,100}_{42}Mo_{53,55,56,58}$ ${}^{98,99,100,102}_{44}Ru_{54,55,56,58}$

${}^{106,110,111,112,114,116}_{48}Cd_{58,62,63,64,66,68}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{127}_{53}I_{74}$ ${}^{131}_{54}Xe_{77}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{140,142}_{58}Ce_{82,84}$

${}^{173}_{70}Yb_{103}$ ${}^{185,185}_{75}\mathrm{Re}_{110,112}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{191,193}_{77}Ir_{114,116}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{262}_{103}Lv^{*}_{3\cdot53}$

${}^{15\cdot19}_{112}Cn^{*}_{173}$ ${}^{286}_{113}Nh^{ie}_{173}$ ${}^{6\cdot53,11\cdot29,320}_{127}Ch^{ie}_{191,192,193}$ ${}^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{14\cdot31,15\cdot29}_{173}Ch^{ie}_{261,262}$

$$\alpha_{1-81/(66)\cdot112} = \frac{66+\frac{1}{4}-\frac{1}{4\cdot5}+\frac{1}{2\cdot9\cdot5\cdot7+1}-\frac{1}{64\cdot13^2\cdot41-\frac{1}{7}}}{81\cdot112} = 1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{70,72,73,74,76}_{32}Ge_{38,40,41,42,44}$ ${}^{64,66,70}_{30}Zn_{34,36,40}$ ${}^{75}_{33}As_{42}$

${}^{79,81}_{35}Br_{44,46}$ ${}^{93}_{41}Nb_{52}$ ${}^{103}_{45}Rh_{58}$ ${}^{112,114}_{48}Cd_{64,66}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$

${}^{156,162,163,164}_{66}Dy_{90,96,97,98}$ ${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{203,5\cdot41}_{81}Tl_{122,124}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$

${}^{268}_{105}Db^{*}_{163}$ ${}^{278}_{109}Mt^{*}_{169}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot13}_{113}Nh^{ie}_{173}$ ${}^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{2-81/(66)\cdot112} = \frac{66+\frac{1}{4}-\frac{1}{4\cdot5}+\frac{1}{2\cdot9\cdot5\cdot7+1}-\frac{1}{4\cdot9\cdot125\cdot97+\frac{1}{8}}}{81\cdot112} = 1/137.035999111818$$

${}^{55}_{25}Mn_{30}$ ${}^{75}_{33}As_{42}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{97}_{42}Mo_{55}$ ${}^{103}_{45}Rh_{58}$ ${}^{125}_{52}Te_{73}$ ${}^{129}_{54}Xe_{75}$ ${}^{163}_{66}Dy_{97}$ ${}^{173}_{70}Yb_{103}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$

${}^{210}_{85}At^{*}_{125}$ ${}^{247}_{97}Bk^{*}_{150}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{312}_{125}Ch^{ie}_{187}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-23^2/(8\cdot81)\cdot112} = \frac{23^2}{(8\cdot81-1+\frac{1}{3}-\frac{1}{12}+\frac{1}{23\cdot112-\frac{5\cdot89}{2\cdot13\cdot23+1}})\cdot112} = 1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{46,48,49,50}_{22}Ti_{24,26,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{89}_{39}Y_{50}$

${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{169}_{69}Tm_{100}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{2-23^2/(8\cdot81)\cdot112} = \frac{23^2}{(8\cdot81-1+\frac{1}{3}-\frac{1}{4\cdot3}+\frac{1}{31\cdot83-\frac{3\cdot5}{4\cdot53}})\cdot112} = 1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{52,53,54}_{24}Cr_{28,29,30}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{83}_{36}Kr_{47}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$

${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-49/(60)\cdot 112} = \frac{49}{\left(60 - \frac{1}{3\cdot 7} + \frac{1}{2\cdot 25\cdot 23} - \frac{1}{64\cdot 5\cdot (256\cdot 13+1)}\right)\cdot 112}$$

$$= 1/137.035999037435$$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{27}_{13}Al_{14}$ ${}^{45}_{21}Sc_{24}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{84,86,87,88}_{38}Sr_{46,48,49,50}$ ${}^{102,108,110}_{46}Pd_{56,60,64}$ ${}^{112}_{48}Cd_{64}$

${}^{113,115}_{49}In_{64,66}$ ${}^{119}_{50}Sn_{69}$ ${}^{144}_{60}Nd_{84}$ ${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{22\cdot 13}_{113}Nh^{ie}_{173}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{2-49/(60)\cdot 112} = \frac{49}{\left(60 - \frac{1}{3\cdot 7} + \frac{1}{2\cdot 25\cdot 23} - \frac{1}{16\cdot 7\cdot 59\cdot 167}\right)\cdot 112} = 1/137.035999111818$$

${}^{45}_{21}Sc_{24}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{84,86,87,88}_{38}Sr_{46,48,49,50}$ ${}^{102,105,108}_{46}Pd_{56,59,60}$ ${}^{112}_{48}Cd_{64}$ ${}^{113,115}_{49}In_{64,66}$

${}^{118,119}_{50}Sn_{68,69}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{144}_{60}Nd_{84}$ ${}^{167,168}_{68}Er_{99,100}$ ${}^{169}_{69}Tm_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$

${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-(5\cdot 157)/31^2\cdot 112} = \frac{5\cdot 157 + \frac{1}{2} - \frac{1}{2\cdot 7} + \frac{1}{2\cdot 5\cdot 17\cdot 73 + \frac{50}{3\cdot 23}}}{31^2\cdot 112} = 1/137.035999037435$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{125}_{52}Te_{73}$ ${}^{112,115,118,119,120,124}_{50}Sn_{62,65,68,69,70,74}$ ${}^{157}_{64}Gd_{93}$ ${}^{169}_{69}Tm_{100}$

${}^{173}_{70}Yb_{103}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{238}_{92}U^{*}_{146}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{400}_{157}Ch^{ie}_{243}$

$$\alpha_{2-(5\cdot 157)/31^2\cdot 112} = \frac{5\cdot 157 + \frac{1}{2} - \frac{1}{2\cdot 7} + \frac{1}{4\cdot(16\cdot 3\cdot 5\cdot 13-1) + \frac{37}{50}}}{31^2\cdot 112} = 1/137.035999111818$$

${}^{12,13}_{6}C_{6,7}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$

${}^{67}_{30}Zn_{37}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{112,115,117,120,124}_{50}Sn_{62,65,67,70,74}$ ${}^{157}_{64}Gd_{93}$ ${}^{173}_{70}Yb_{103}$

${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{400}_{157}Ch^{ie}_{243}$

21. Formulas of the Fine-structure Constant in Continued Fraction Form

Some natural constant such as φ, e and 2π can be expressed as continued fraction form as follows.

$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{\cdots}}}}$$

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\cdots}}}} = 1.618\cdots$$

$$e = 1 + 1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{4 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{6 + \cfrac{1}{\cdots}}}}}}}}} = 2.71828\cdots$$

$$2\pi = 6 + \cfrac{2}{6 + \cfrac{9}{6 + \cfrac{25}{6 + \cfrac{49}{\cdots}}}} = 6.283185\cdots$$

$$\frac{\pi}{2} = 1 + \cfrac{1}{1 + \cfrac{1}{1/2 + \cfrac{1}{1/3 + \cfrac{1}{1/4 + \cfrac{1}{\cdots}}}}} = 1.570796\cdots$$

$$\left(\sqrt{\frac{\sqrt{5}+5}{2}} - \frac{\sqrt{5}+1}{2}\right)e^{\frac{2\pi}{5}} = 1 + \cfrac{1}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \cfrac{e^{-6\pi}}{\cdots}}}}$$

As we have demonstrated the formulas of the fine-structure constant α are related to e, 2π and nuclides (φ is related to nuclides too), it should be reasonable to express α in continued fraction form as follows. When these continued fractions are calculated, they miraculously give some specific integers which should correspond to some typical nuclides.

$$\alpha_1 = \cfrac{1}{137+\cfrac{1}{28-\cfrac{1}{4+\cfrac{1}{2-\cfrac{1}{17+\cfrac{1}{16-\cfrac{1}{1+\cfrac{1}{10-\cfrac{1}{3}}}}}}}}} = \cfrac{1}{56+81+\cfrac{1}{28-\cfrac{13\cdot(112\cdot 11-1)}{3\cdot 5\cdot(112\cdot 43+1)}}}$$

$$=1/137.035999037435$$

${}^{7}_{3}Li_{4}$ ${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$

${}^{31}_{15}P_{16}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$

${}^{74,76,77,78,80,82}_{34}Se_{40,42,43,44,46,48}$ ${}^{80,82,83,84,86}_{36}Kr_{44,46,47,48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{94,96,97,98}_{42}Mo_{52,54,55,56}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$

${}^{99,100,104}_{44}Ru_{55,56,60}$ ${}^{110,112,113,114}_{48}Cd_{62,64,65,66}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{112,115,116,117,118,120}_{50}Sn_{62,65,66,67,68,70}$ ${}^{131}_{54}Xe_{77}$

${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{159}_{65}Tb_{94}$ ${}^{168,170}_{68}Er_{100,102}$ ${}^{171,4\cdot 43,173}_{70}Yb_{101,6\cdot 17,103}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{188,189}_{76}Os_{112,113}$

${}^{195,198}_{78}Pt_{117,120}$ ${}^{200,201}_{80}Hg_{120,121}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$

${}^{227}_{89}Ac^{*}_{138}$ ${}^{2\cdot 131}_{103}Lr^{*}_{159}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{284,22\cdot 13}_{113}Nh^{ie}_{171,173}$ ${}^{288,289}_{115}Mc^{ie}_{173,174}$ ${}^{4\cdot 83}_{131}Ch^{ie}_{201}$ ${}^{8\cdot 43,2\cdot 173,348}_{136,137,138}Fy^{ie}_{16\cdot 13,209,210}$

$$\alpha_2 = \cfrac{1}{137+\cfrac{1}{28-\cfrac{1}{4+\cfrac{1}{2-\cfrac{1}{18+\cfrac{1}{3-\cfrac{1}{1+\cfrac{1}{5-\cfrac{1}{2}}}}}}}}} = \cfrac{1}{56+81+\cfrac{1}{28-\cfrac{2\cdot(16\cdot 27-1)}{3\cdot(16\cdot 81+1)}}}$$

$$=1/137.035999111818$$

${}^{10,11}_{5}B_{5,6}$ ${}^{14,15}_{7}N_{7,8}$ ${}^{16,17,18}_{8}O_{8,9,10}$ ${}^{19}_{9}F_{10}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$

${}^{32,34,36}_{16}S_{16,18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{59}_{27}Co_{32}$ ${}^{58,60,62,64}_{28}Ni_{30,32,34,36}$ ${}^{80,82}_{34}Se_{46,48}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{112,113}_{48}Cd_{64,65}$

${}^{131}_{54}Xe_{77}$ ${}^{143,144}_{60}Nd_{83,84}$ ${}^{135,136,137,138}_{56}Ba_{79,80,81,82}$ ${}^{168}_{68}Er_{100}$ ${}^{173}_{70}Yb_{103}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{188,7\cdot 27}_{76}Os_{112,113}$

${}^{203,205}_{81}Tl_{122,124}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{286}_{113}Nh^{ie}_{173}$ ${}^{343,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{16\cdot 13,209,210}$ ${}^{14\cdot 31,15\cdot 29}_{173}Ch^{ie}_{9\cdot 19,2\cdot 131}$

22. Set of Formulas of the Fine-structure Constant

We had found many formulas of the fine-structure constant with hard efforts for more than two years. In the end of this paper, we realized that all of these formulas should constitute a mathematical set in which some formulas exhibit much more importance than others. The set and the four most important formulas are depicted as follows (**Fig. 2**). The set is designed as a circle, the two originally found formulas should be close to the center of the circle, and the two last found continued fraction formulas should be on the circumference of the circle.

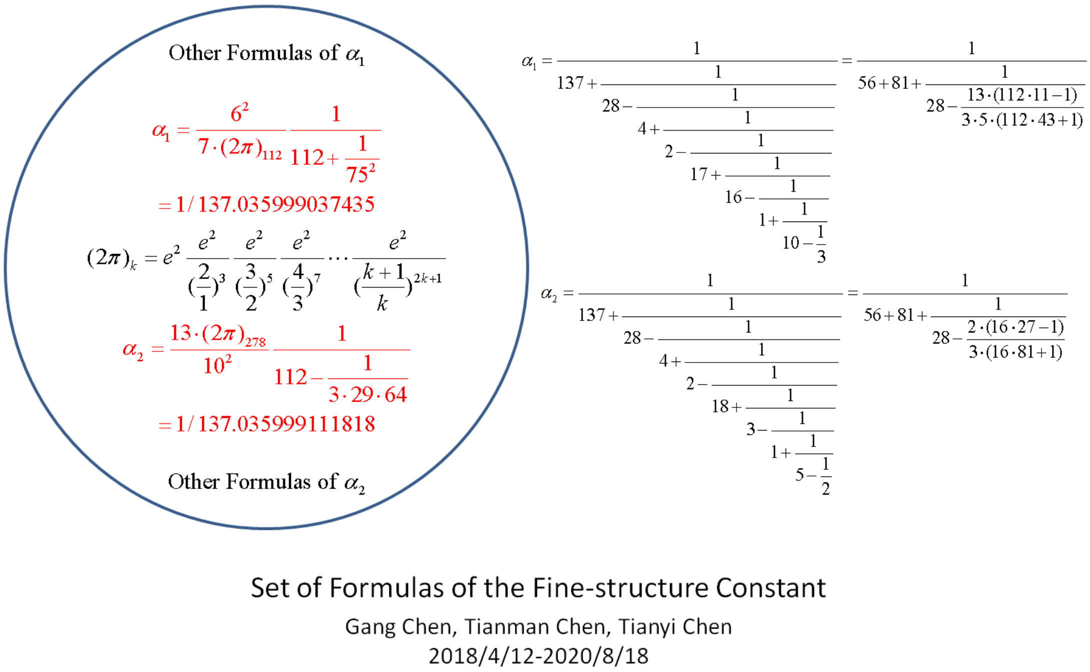


Fig. 2

23. Formulas of the speed of light in atomic units in Continued Fraction Form

$$c_{au} = \frac{1}{\alpha_c} = 137 + \cfrac{1}{28 - \cfrac{1}{4 + \cfrac{1}{2 - \cfrac{1}{17 + \cfrac{1}{2 - \cfrac{1}{1 + \cfrac{1}{2 - \cfrac{1}{4 + \cfrac{1}{5 - \cfrac{1}{1 + \cfrac{1}{2}}}}}}}}}}}$$

$$= 56 + 81 + \cfrac{1}{28 - \cfrac{5\cdot(4\cdot 3\cdot 7\cdot 17 - 1)}{2\cdot 5\cdot(2\cdot 5\cdot(2\cdot 7\cdot 23)+1)+1}} = 137.035999074626$$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{58,60,62}_{28}Ni_{30,32,34}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{76,80}_{34}Se_{42,46}$ ${}^{79,81}_{35}Br_{44,46}$

${}^{90,91,92}_{40}Zr_{50,51,52}$ ${}^{100,102}_{44}Ru_{56,58}$ ${}^{102,105,106}_{46}Pd_{56,59,60}$ ${}^{112,115,118,119,120}_{50}Sn_{62,65,68,69,70}$ ${}^{121,123}_{51}Sb_{70,72}$

${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{156,157}_{64}Gd_{92,93}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{172,173}_{70}Yb_{102,103}$ ${}^{199,200,204}_{80}Hg_{119,120,124}$

${}^{203,205}_{81}Tl_{122,124}$ ${}^{14\cdot 17}_{92}U^{*}_{146}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{7\cdot 37}_{102}No^{*}_{157}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{17^2}_{115}Mc^{ie}_{174}$ ${}^{344,2\cdot 173,12\cdot 29}_{136,137,138}Fy^{ie}_{208,209,210}$

$$c_{au}=\frac{1}{\alpha_c}=137+\cfrac{1}{28-\cfrac{1}{4+\cfrac{1}{2-\cfrac{1}{17+\cfrac{1}{2-\cfrac{1}{1+\cfrac{1}{2-\cfrac{1}{4+\cfrac{1}{5-\cfrac{1}{1+\cfrac{1}{2}}}}}}}}}}}$$

$$=56+81+\cfrac{1}{28-\cfrac{1}{4+\cfrac{1}{2-\cfrac{1}{17+\cfrac{8\cdot 19}{9\cdot 23}}}}}=137.035999074626$$

$${}^{16,17,18}_{8}O_{8,9,10}\quad {}^{19}_{9}F_{10}\quad {}^{23}_{11}Na_{12}\quad {}^{28,29,30}_{14}Si_{14,15,16}\quad {}^{35,37}_{17}Cl_{18,20}\quad {}^{36,38,40}_{18}Ar_{18,20,22}\quad {}^{39,40,41}_{19}K_{20,21,22}$$

$${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}\quad {}^{50,51}_{23}V_{27,28}\quad {}^{58,60,62,64}_{28}Ni_{30,32,34,36}\quad {}^{64,68}_{30}Zn_{34,38}\quad {}^{68,70}_{31}Ga_{38,40}\quad {}^{76,80}_{34}Se_{42,46}$$

$${}^{79,81}_{35}Br_{44,46}\quad {}^{83,84,86}_{36}Kr_{47,48,50}\quad {}^{85,87}_{37}Rb_{48,50}\quad {}^{84,86,87,88}_{38}Sr_{46,48,49,50}\quad {}^{89}_{39}Y_{50}\quad {}^{100,102}_{44}Ru_{56,58}\quad {}^{102}_{46}Pd_{56}$$

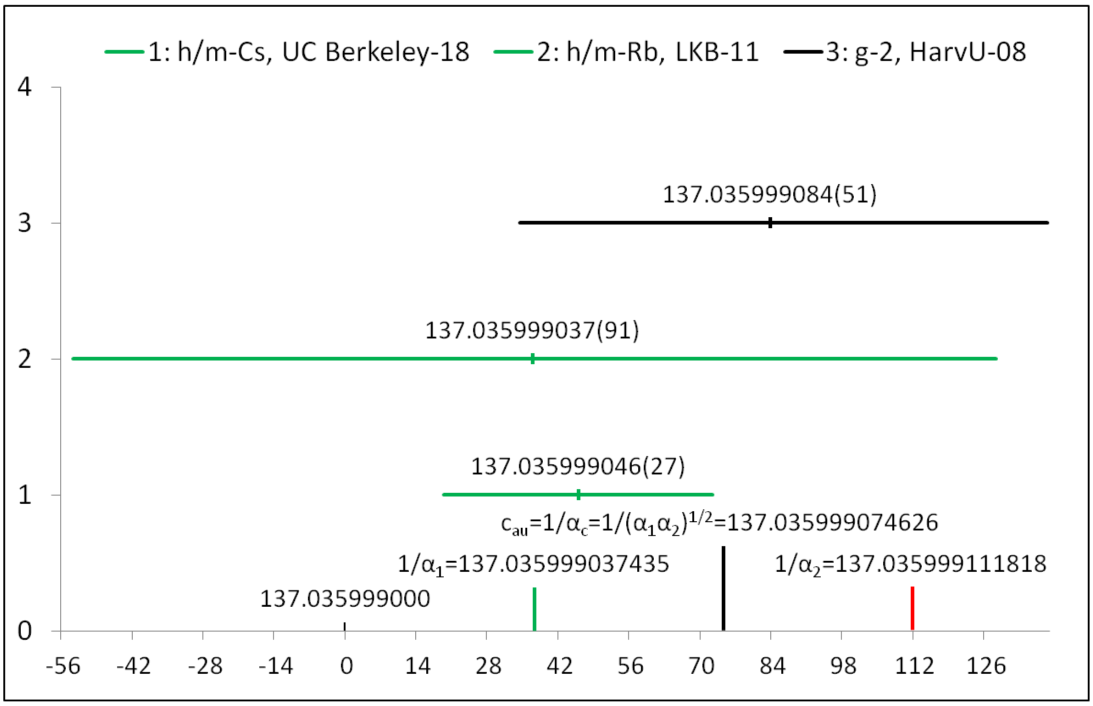
$${}^{112,114,115,118,119}_{50}Sn_{62,64,65,68,69}\quad {}^{136,137,138}_{56}Ba_{80,81,82}\quad {}^{138,139}_{57}La_{81,82}\quad {}^{151,153}_{63}Eu_{88,90}\quad {}^{168}_{68}Er_{100}\quad {}^{169}_{69}Tm_{100}$$

$${}^{195,187}_{75}\mathrm{Re}_{110,112}\quad {}^{188}_{76}Os_{112}\quad {}^{203,205}_{81}Tl_{122,124}\quad {}^{204,207,208}_{82}Pb_{122,125,126}\quad {}^{209}_{83}Bi^{*}_{126}\quad {}^{209}_{84}Po^{*}_{125}\quad {}^{210}_{85}At^{*}_{125}$$

$${}^{222}_{86}Rn^{*}_{136}\quad {}^{223}_{87}Fr^{*}_{136}\quad {}^{226}_{88}Ra^{*}_{138}\quad {}^{227}_{89}Ac^{*}_{138}\quad {}^{15\cdot 19}_{112}Cn^{*}_{173}\quad {}^{289}_{115}Mc^{ie}_{174}\quad {}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}\quad {}^{382}_{152}Ch^{ie}_{230}$$

24. Comparison of Calculated Values of α to Measured Values of α

In recent years there were some much accurate measurements of the fine-structure constant α. For example, in 2008 D. Hanneke *et al* reported α value of 1/137.035999084(51) by g-2 method[3], in 2011 R. Bouchendira *et al* reported α value of 1/137.035999037(91) by h/m_{Rb} method[4], and R. H. Parker *et al* reported α value of 1/137.035999046(27) by h/m_{Cs} method[5]. Comparison of our calculated values of α to these relatively accurate measured values of α was exhibited in **Fig. 3**. It seems that α_1 corresponds to the values measured by h/m method and α_c (or α_2) corresponds to the values measured by g-2 method. These measurements should be good evidences of our points of view that there are two values of α, and in the future more accurate measurements of α by these two methods or other methods may give better or even perfect proofs to our theories, formulas and values of the fine-structure constant.



Comparison of Calculated and Measured Values of 1/α

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Fig. 3

In **Fig. 3**, it seems that the green line 2 and 1 have a trend to focus on the calculated value of $1/\alpha_1$, and there is a lack of a more accurate black line to point to the calculated value of c_{au}. Based on these analyses of **Fig. 3**, we suggest experimental physicists should develop new methods corresponding to α_2 and improve the measurement accuracy of all methods (which could be called green, red and black methods), so someday some measurement results which can't overlap each other and embrace the calculated values of α_1, α_2 and α_c (especially almost as centers) could be achieved, and hence confirm the calculated values of α_1, α_2 and α_c.

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Acknowledgements

Yichang Huifu Silicon Material Co., Ltd., Guangzhou Huifu Research Institute Co., Ltd. and Yichang Huifu Nanometer Material Co., Ltd. have been giving Dr. Gang Chen a part-time employment since Dec. 2018. Thank these companies for their financial support. Specially thank Dr. Yuelin Wang and other colleagues of these companies for their appreciation, support and help.

Appendix I: Research History

Section	Page	Date	Remarks
1	1-10	2020/7/8-23	
2	11-12	2020/7/11-12	
3	12-14	2018/8/21-9/6	
3-1	13	2018/8/21-25	
3-2	13	2018/9/2-6	
3-3	14	2018/9/2-6	
3-4	14	2018/9/5-6	
4	14-15	2020/6/28-30	
5	16	2020/6/30-7/1	
6	16-17	2020/6/30-7/1, 7/5	
7	17-18	2020/7/3-5	
8	18-19	2020/7/20-21	
9	19	2020/7/21-25	Revise: 2020/8/23
10	19-20	2020/7/27-28	
11	20	2020/7/28	
12	20-22	2020/7/28-30	
13	22	2020/8/2	
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17	31	2020/8/14	
18	31-32	2020/8/15-17	
19	32-33	2020/8/15-17	
20	33-35	2020/8/15-16	
21	35-37	2020/8/17	
22	37-38	2020/8/18-21	
23	38	2020/8/23, 9/2-3	
24	39-40	2020/9/4-5	
Preparing this paper	1-42	2020/7/8-2020/9/6	

Notes: Dates were recorded according to Beijing Time; *ie* means ideal extended elements; *GL* means Gregory-Leibniz formula.