

Supplement Formulas of the Fine-structure Constant α , New Formulas of Euler Number e and Their Relationships with Nuclides

Gang Chen[†], Tianman Chen, Tianyi Chen

Guangzhou Huifu Research Institute Co., Ltd., Guangzhou, P. R. China

7-20-4, Greenwich Village, Wangjianglu 1, Chengdu, P. R. China

[†]Correspondence to: gang137.chen@connect.polyu.hk

Dedicated to Prof. Albert Sun-Chi Chan on the occasion of his 70th birthday

Abstract

This paper is a subsequent paper to the paper “Chen’s Formulas of the Fine-structure Constant” (viXra:2002.0203) for giving some supplements. In the previous paper, many formulas of the fine-structure constant α based on the most important key number 112 had been given. In this paper, some new formulas of α based on a subsequent key number 173 were deduced, some formulas of α were expressed with large integers, some new formulas of Euler number e and their relationships with nuclides were given, the relationships of some formulas of 2π with nuclides were revealed, the relationships between some constants (e , γ , γ_c , γ_g and γ_{cg}) and nuclides were disclosed, a picture showing the unification of mathematics and physics through α was depicted, the most important formula of the speed of light in atomic unites c_{au} was revised to be more reasonable, a Fibonacci sequence containing 173 and its relationships with nuclides were proposed, the meanings of the numerical values of α and c_{au} were discovered, some formulas of α based on 137, 83, 83^2 , 112×173 , 163×173 , $36/112$, 100×112 and so on were presented, two continued fractions of α were gained, and an overall picture of the set of formulas of α was designed.

Keywords: formulas of the fine-structure constant α , formulas of Euler number e ; the speed of light; the unification of mathematics and physics; relationships; nuclides.

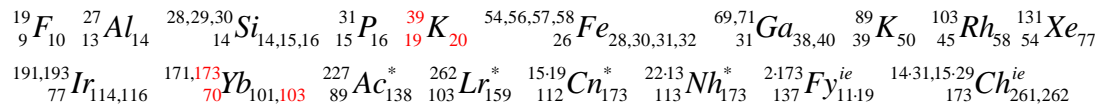
1. Construct formulas of α with 173 instead of 112

Referring to ${}_{112}\text{Cn}_{173}$, ${}_{137}\text{Fy}_{209}$ and ${}_{173}\text{Ch}_{261,262}$ presented in the previous paper “Chen’s Formulas of the Fine-structure Constant” (viXra:2002.0203)¹, the natural end

of elements is ${}_{112}\text{Cn}$, the Fynmann end of elements is ${}_{137}\text{Fy}$, and the end of ideal extended elements is ${}_{173}\text{Ch}$. We had already constructed formulas of α with 112 which is double of the most stable number 56 in the world of nuclides according to our Chirality and Poetry Model of Atomic Nuclei², and 173 seems to be a subsequent stable number connected to 112, so it should be possible to construct some reasonable (but may less important) formulas of α with 173 instead of 112 as follows. And the factors in these formulas are supposed to relate to nuclides as in the previous paper¹.

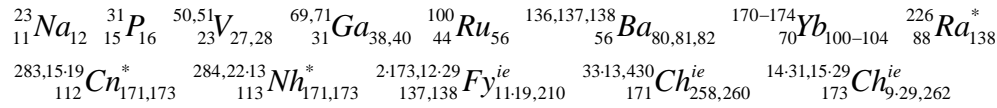
$$\alpha_{1-(173)} = \frac{8}{e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{20}{19}\right)^{39}}} \frac{1}{173 + \frac{1}{31} - \frac{1}{10 \cdot 227 - \frac{103}{7 \cdot 193} \text{ or } \frac{60}{6 \cdot 131 + 1} \text{ or } \frac{112}{13 \cdot 113}}}$$

$$= 1/137.035999037435$$



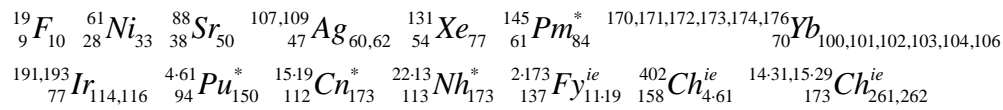
$$\alpha_{1-(173)-\text{Wallis}} = \frac{2}{\left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{58}{59} \frac{4 \cdot 3 \cdot 5}{2 \cdot 29 + 1}\right)} \frac{1}{173 + \frac{1}{31} - \frac{1}{8 \cdot 11 \cdot 23 + \frac{171}{2 \cdot 112}}}$$

$$= 1/137.035999037435$$



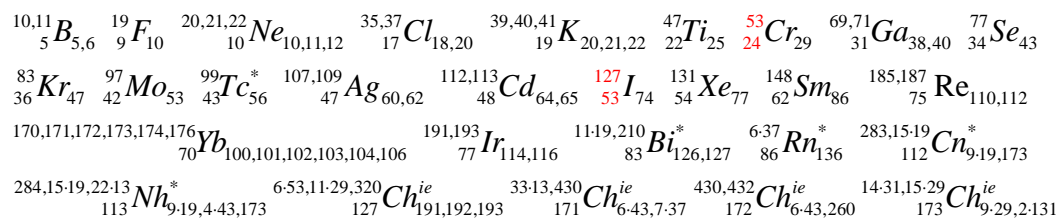
$$\alpha_{1-(173)-\text{GL}} = \frac{1}{1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 19 + 1}} \frac{1}{173 + \frac{1}{14} - \frac{1}{4 \cdot 61} + \frac{1}{2 \cdot 173 \cdot 191 + \frac{1}{50}}}$$

$$= 1/137.035999037435$$



$$\alpha_{1-(173)-\text{NC}} = \frac{8}{(2\pi)_{\text{NC}-1}} \frac{1}{173 + \frac{1}{10} - \frac{1}{17 \cdot 31} + \frac{1}{2 \cdot 47 \cdot 53 \cdot 127}} = 1/137.035999037435$$

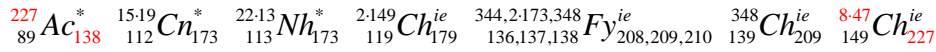
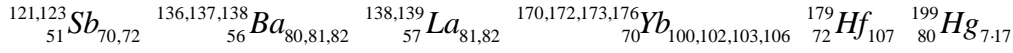
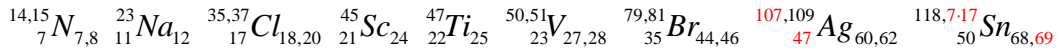
$$= \frac{8}{6 + \frac{1}{3}} \frac{1}{173 + \frac{1}{10} - \frac{9 \cdot 19 \cdot (3 \cdot 11 \cdot 112 + 1)}{2 \cdot 17 \cdot 31 \cdot 47 \cdot 53 \cdot 127}} = \frac{24}{19} \frac{1}{173 + \frac{1}{10} - \frac{171 \cdot (2 \cdot 43^2 - 1)}{2 \cdot 17 \cdot 31 \cdot 47 \cdot 53 \cdot 127}}$$



Note: $(2\pi)_{NC-k} = 6 + \sum_{n=1}^k \frac{(-1)^{n+1}}{n(n+1/2)(n+1)}$

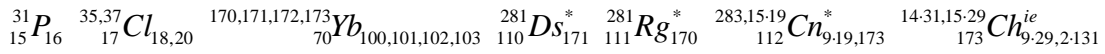
$$\alpha_{1-15(173)} = \frac{7 \cdot 17}{3 \cdot 5 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot (6 \cdot 47 - 1)}{6 \cdot 11 \cdot 17 + 1}\right)^{3 \cdot 7 \cdot 107}}} \frac{1}{173 + \frac{1}{20 \cdot (138 + 1) \cdot 227}}$$

$$= 1/137.035999037435$$



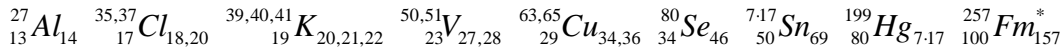
$$\alpha_{1-15(173)-Wallis} = \frac{7 \cdot 17}{4 \cdot 3 \cdot 5 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{3370}{3371} \frac{4 \cdot 3 \cdot 281}{2 \cdot 5 \cdot 337 + 1}\right)} \frac{1}{173 + \frac{1}{11 \cdot 103 \cdot (18 \cdot 31 - 1) + \frac{7}{10}}}$$

$$= 1/137.035999037435$$



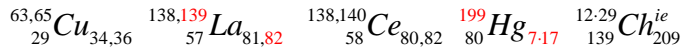
$$\alpha_{1-15(173)-GL} = \frac{7 \cdot 17}{8 \cdot 3 \cdot 5 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 29 \cdot 37 + 1}\right)} \frac{1}{173 + \frac{1}{2 \cdot 19 \cdot 23 \cdot 157 - \frac{10}{13}}}$$

$$= 1/137.035999037435$$



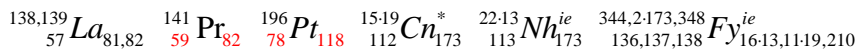
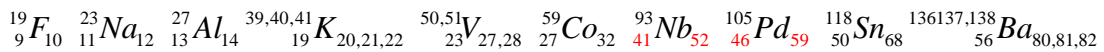
$$\alpha_{1-15(173)-NC} = \frac{7 \cdot 17}{3 \cdot 5 \cdot (2\pi)_{NC-9}} \frac{1}{173 + \frac{1}{2 \cdot 41} - \frac{1}{2 \cdot 9 \cdot (2 \cdot 199 - 1) - \frac{29}{139}}}$$

$$= 1/137.035999037435$$



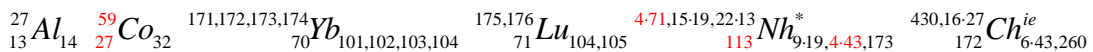
$$\alpha_{1-59(173)} = \frac{4 \cdot 9 \cdot 13}{59 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot (8 \cdot 13 \cdot 137 + 1)}{3 \cdot 7 \cdot 23 \cdot 59}\right)^{5 \cdot (2 \cdot 41 \cdot 139 + 1)}}} \frac{1}{173 + \frac{2 \cdot 9 \cdot 11 \cdot 59}{5 \cdot 10^{11}}}$$

$$= 1/137.035999037435$$



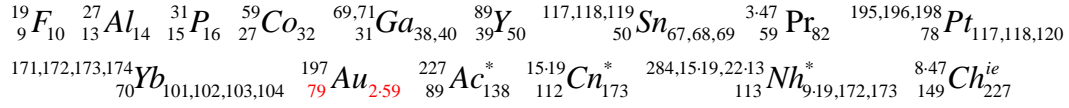
$$\alpha_{1-59(173)-Wallis} = \frac{9 \cdot 13}{59 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{85482}{85483} \frac{4 \cdot 7 \cdot 43 \cdot 71}{2 \cdot 27 \cdot (2 \cdot 7 \cdot 113 + 1) + 1}\right)} \frac{1}{173 + \frac{9 \cdot 7 \cdot 71}{4 \cdot 10^{11}}}$$

$$= 1/137.035999037435$$

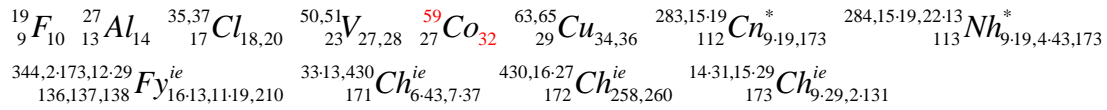


$$\alpha_{1-59(173)-GL} = \frac{9 \cdot 13}{2 \cdot 59 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 4 \cdot 3 \cdot 5 \cdot (4 \cdot 227 - 1) + 1})} \frac{1}{173 + \frac{9 \cdot 79}{4 \cdot 10^{10}}}$$

$$= 1/137.035999037435$$

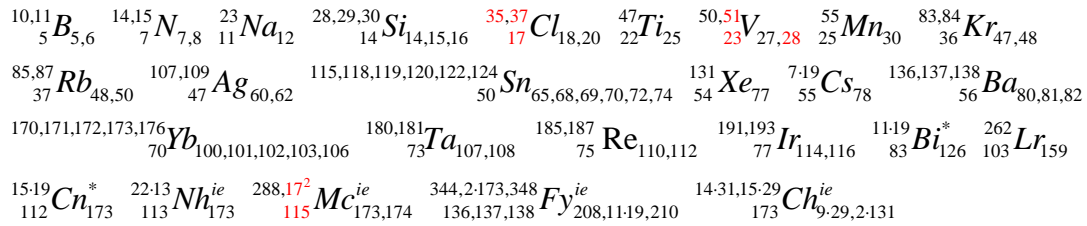


$$\alpha_{1-59(173)-NC} = \frac{4 \cdot 9 \cdot 13}{59 \cdot (2\pi)_{NC-23}} \frac{1}{173 + \frac{1}{32 \cdot 9 \cdot (10 \cdot 19 + 1) - \frac{29}{2 \cdot 17}}} = 1/137.035999037435$$



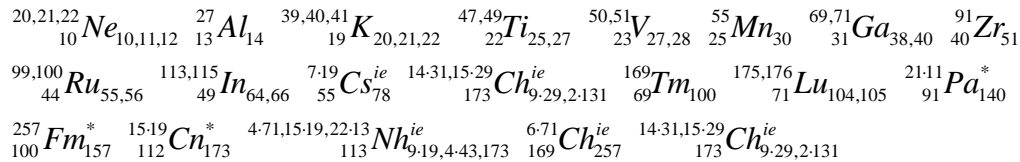
$$\alpha_{2-1(173)} = \frac{e^2 \frac{e^2}{(\frac{2}{-})^3} \frac{e^2}{(\frac{3}{-})^5} \frac{e^2}{(\frac{4}{-})^7} \dots \frac{e^2}{(\frac{37}{-})^{73}}}{5} \frac{1}{173 - \frac{1}{5 \cdot 11} + \frac{1}{2 \cdot 7 \cdot (2 \cdot 17^2 - 1) + \frac{2 \cdot 47}{3 \cdot 103} \text{ or } \frac{5 \cdot 23}{14 \cdot 27}}}$$

$$= 1/137.035999111818$$



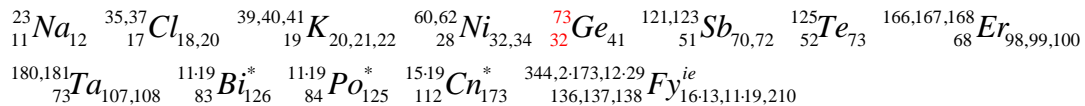
$$\alpha_{2-1(173)-Wallis} = \frac{4 \cdot (2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{108}{109} \frac{2 \cdot 5 \cdot 11}{2 \cdot 2 \cdot 27 + 1})}{5} \frac{1}{173 - \frac{1}{7 \cdot 13} + \frac{1}{4 \cdot 25 \cdot 19 \cdot 23 - \frac{10}{71}}}$$

$$= 1/137.035999111818$$



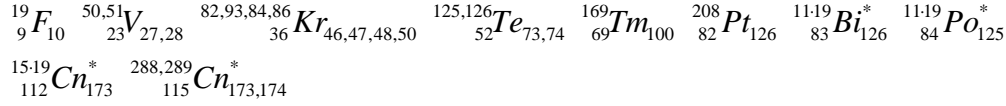
$$\alpha_{2-1(173)-GL} = \frac{8 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 4 \cdot 17 + 1})}{5} \frac{1}{173 - \frac{1}{128 \cdot 7} + \frac{1}{5 \cdot 11^2 \cdot 19 \cdot 73}}$$

$$= 1/137.035999111818$$



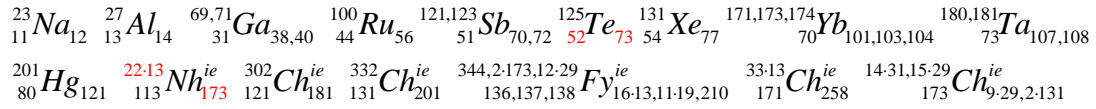
$$\alpha_{2-1(173)-NC} = \frac{(2\pi)_{NC-3}}{5} \frac{1}{173-1+\frac{1}{2}-\frac{1}{10}+\frac{1}{2\cdot 5\cdot 23}-\frac{1}{4\cdot 9\cdot 7\cdot (400+1)}+\frac{1}{10}}$$

$$= 1/137.035999111818$$



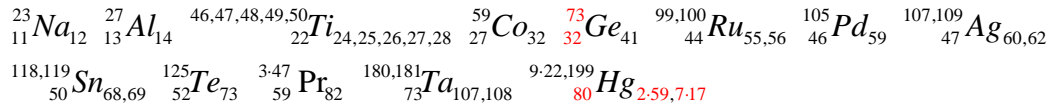
$$\alpha_{2-44(173)} = \frac{4\cdot 11\cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{3\cdot (8\cdot 131+1)}{2\cdot 11^2\cdot 13}\right)^{7\cdot 29\cdot 31}}}{3\cdot 7\cdot 3} \frac{1}{173-\frac{1}{11\cdot 29\cdot (62\cdot 59+1)}+\frac{5}{6}}$$

$$= 1/137.035999111818$$



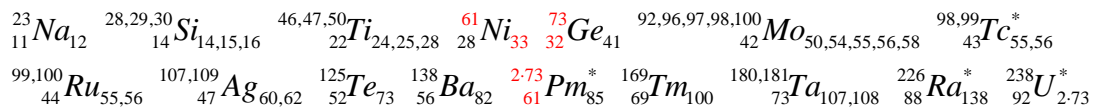
$$\alpha_{2-44(173)-Wallis} = \frac{16\cdot 11\cdot \left(2\frac{2}{3}\frac{4}{3}\dots\frac{9440}{9441}\frac{9442}{2\cdot 16\cdot 5\cdot 59+1}\right)}{3\cdot 7\cdot 3} \frac{1}{173-\frac{1}{59\cdot (2\cdot 3\cdot 7\cdot 13\cdot 17+1)}-\frac{4}{11}}$$

$$= 1/137.035999111818$$

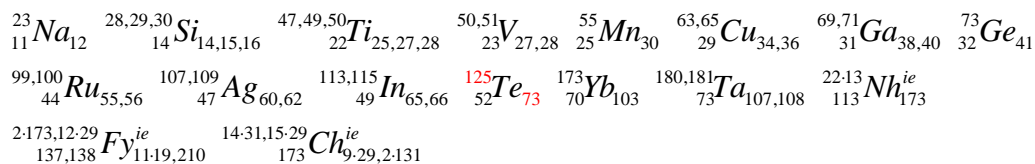


$$\alpha_{2-44(173)-GL} = \frac{32\cdot 11\cdot \left(1-\frac{1}{3}+\frac{1}{5}-\dots+\frac{1}{2\cdot 10\cdot (14\cdot 43-1)+1}\right)}{3\cdot 7\cdot 3} \frac{1}{173-\frac{1}{(8\cdot 61-1)\cdot (42\cdot 23-1)}}$$

$$= 1/137.035999111818$$

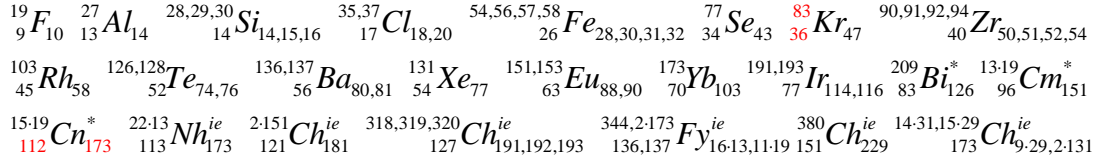


$$\alpha_{2-44(173)-NC} = \frac{4\cdot 11\cdot (2\pi)_{11}}{3\cdot 7\cdot 3} \frac{1}{173-\frac{1}{7\cdot 113}+\frac{2\cdot 7\cdot 29\cdot 31\cdot 47}{25\cdot 10^{11}}} = 1/137.035999111818$$



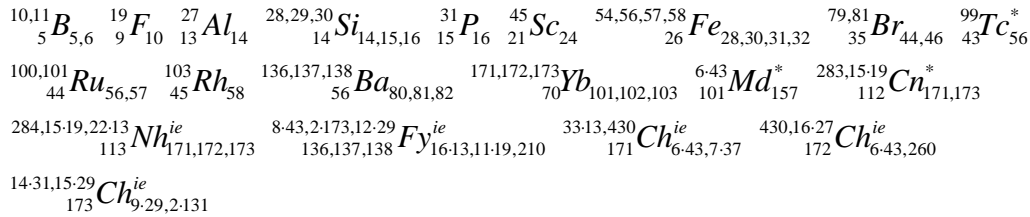
$$\alpha_{2-45(173)} = \frac{9 \cdot 5 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{8 \cdot 5 \cdot 27}{13 \cdot 83}\right)^{17 \cdot 127}}}{2 \cdot 112} \frac{1}{173 - \frac{1}{2 \cdot 151 \cdot 173 - \frac{9}{8 \cdot 5}}}$$

$$= 1/137.035999111818$$



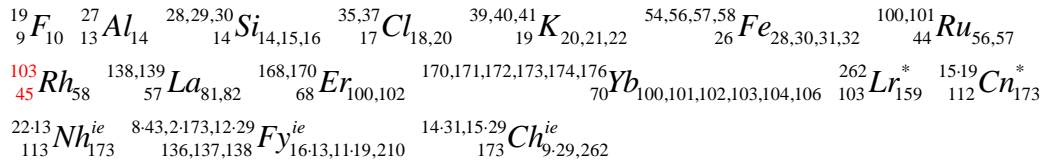
$$\alpha_{2-45(173)-Wallis} = \frac{9 \cdot 5 \cdot \left(2 \frac{2}{3} \frac{4}{3} \dots \frac{3236}{3237} \frac{2 \cdot (20 \cdot 81 - 1)}{2 \cdot 2 \cdot (8 \cdot 101 + 1) + 1}\right)}{8 \cdot 7} \frac{1}{173 - \frac{1}{8 \cdot 101 \cdot (4 \cdot 9 \cdot 13 - 1) - \frac{2}{15}}}$$

$$= 1/137.035999111818$$



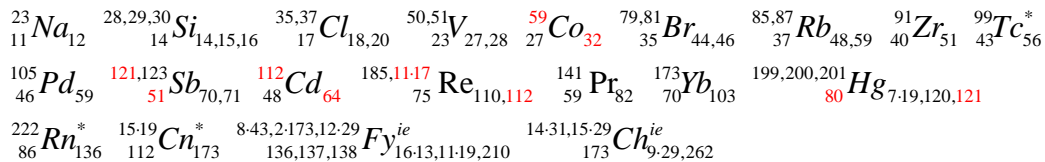
$$\alpha_{2-45(173)-GL} = \frac{9 \cdot 5 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \dots + \frac{1}{2 \cdot 2 \cdot (10 \cdot 103 + 1) + 1}\right)}{4 \cdot 7} \frac{1}{173 - \frac{1}{4 \cdot 17 \cdot (44 \cdot 13 - 1) - \frac{32}{3 \cdot 19}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-45(173)-NC} = \frac{9 \cdot 5 \cdot (2\pi)_7}{2 \cdot 112} \frac{1}{173 - \frac{1}{3 \cdot 17 \cdot 59 + \frac{121}{320}}} = \frac{9 \cdot 5 \cdot (2\pi)_7}{2 \cdot 112} \frac{1}{173 - \frac{1}{2 \cdot 5 \cdot 7 \cdot 43 - \frac{199}{320}}}$$

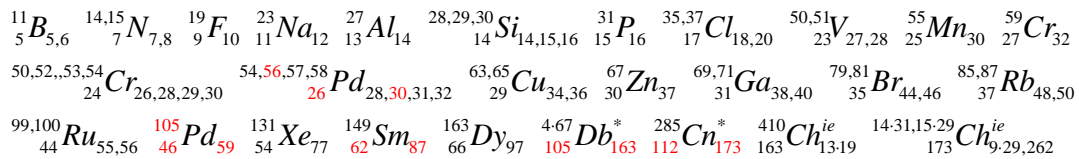
$$= 1/137.035999111818$$



$$\alpha_{2-87(173)} = \frac{3 \cdot 29 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{341846}{341845}\right)^{683691}}}{14 \cdot 31 - 1} \frac{1}{173 - \frac{14560}{10^{13}}}$$

$$= \frac{3 \cdot 29 \cdot e^2 e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot 59 \cdot (2 \cdot 9 \cdot 7 \cdot 23 - 1)}{5 \cdot 7 \cdot (24 \cdot 11 \cdot 37 - 1)}\right)^{3 \cdot (4 \cdot 163 + 1) \cdot (12 \cdot 29 + 1)}}}{2 \cdot 7 \cdot 31 - 1} \frac{1}{173 - \frac{112 \cdot 13}{10^{12}}}$$

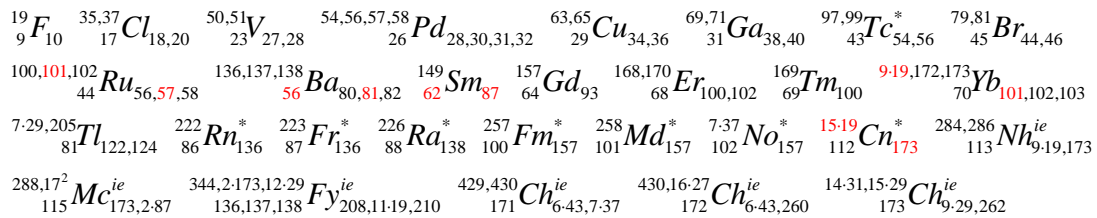
$$= 1/137.035999111818$$



$$\alpha_{2-87(173)-Wallis} = \frac{4 \cdot 87 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{1044088}{1044089} \frac{1044090}{1044089+1}\right)}{433} \frac{1}{173 - \frac{434 \text{ or } 435}{10^{13}}}$$

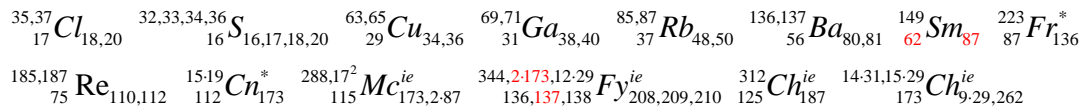
$$= \frac{4 \cdot 3 \cdot 29 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{1044088}{1044089} \frac{2 \cdot 81 \cdot 5 \cdot (23 \cdot 56 + 1)}{2 \cdot 4 \cdot 19 \cdot (4 \cdot 17 \cdot 101 + 1)}\right)}{2 \cdot 7 \cdot 31 - 1} \frac{1}{173 - \frac{7 \cdot 31}{5 \cdot 10^{12}} \text{ or } \frac{3 \cdot 29}{2 \cdot 10^{12}}}$$

$$= 1/137.035999111818$$

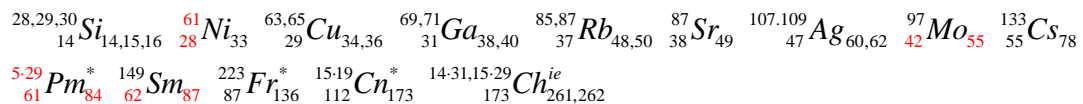


$$\alpha_{2-87(173)-GL} = \frac{8 \cdot 3 \cdot 29 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 16 \cdot (72 \cdot (2 \cdot 17^2 - 1) + 1)}\right)}{2 \cdot 7 \cdot 31 - 1} \frac{1}{173 - \frac{137}{125 \cdot 10^{11}}}$$

$$= 1/137.035999111818$$

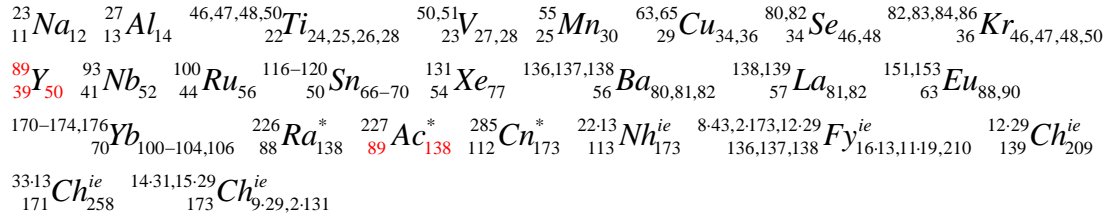


$$\alpha_{2-87(173)-NC} = \frac{3 \cdot 29 \cdot (2\pi)_{NC-55}}{2 \cdot 7 \cdot 31 - 1} \frac{1}{173 - \frac{1}{2 \cdot 3 \cdot 7 \cdot 29 \cdot 61 + \frac{7}{4 \cdot 3}}} = 1/137.035999111818$$



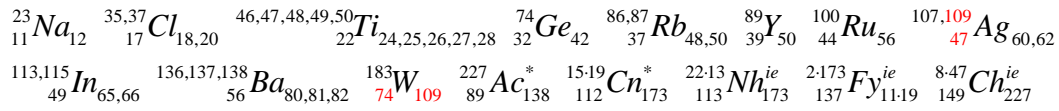
$$\alpha_{2-89(173)} = \frac{89 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{3 \cdot 13 \cdot 41}{2 \cdot 17 \cdot 47}\right)^{23 \cdot (138+1)}}}{2 \cdot 13 \cdot 17 + 1} \frac{1}{173 - \frac{1}{7 \cdot 17 \cdot (4 \cdot 11 \cdot 29 + 1) + \frac{2}{25}}}$$

$$= 1/137.035999111818$$



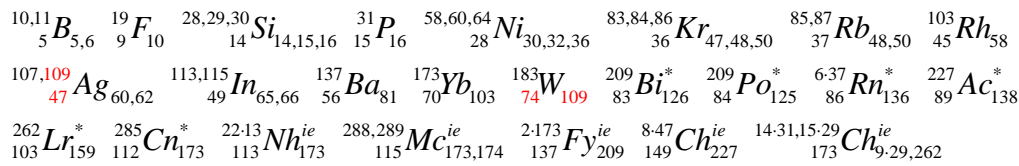
$$\alpha_{2-89(173)\text{-Wallis}} = \frac{4 \cdot 89 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \dots \frac{4794}{4795} \frac{4 \cdot 11 \cdot 109}{2 \cdot 3 \cdot 17 \cdot 47 + 1}\right)}{4 \cdot 3 \cdot 37 - 1} \frac{1}{173 - \frac{1}{2 \cdot 49 \cdot (32 \cdot 113 + 1) + \frac{7}{8}}}$$

$$= 1/137.035999111818$$



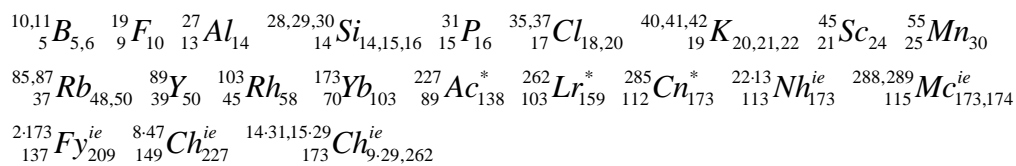
$$\alpha_{2-89(173)\text{-GL}} = \frac{8 \cdot 89 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 4 \cdot 7 \cdot 109 + 1}\right)}{4 \cdot 3 \cdot 37 - 1} \frac{1}{173 - \frac{1}{9 \cdot (4 \cdot 5 \cdot 49 \cdot 47 + 1) + \frac{3}{8}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-89(173)\text{-NC}} = \frac{89 \cdot (2\pi)_{\text{NC-9}}}{4 \cdot 3 \cdot 37 - 1} \frac{1}{173 - \frac{1}{9 \cdot 25} + \frac{1}{13 \cdot (2 \cdot 3 \cdot 7 \cdot 19^2 - 1) + \frac{6}{17}}}$$

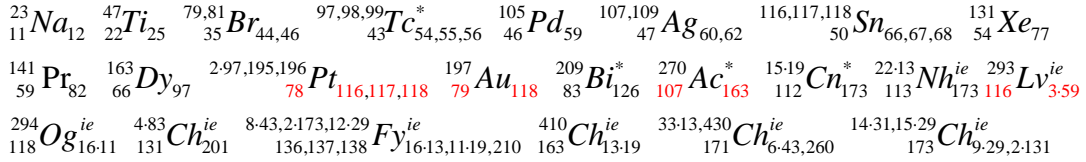
$$= 1/137.035999111818$$



$$\alpha_{2-131(173)}$$

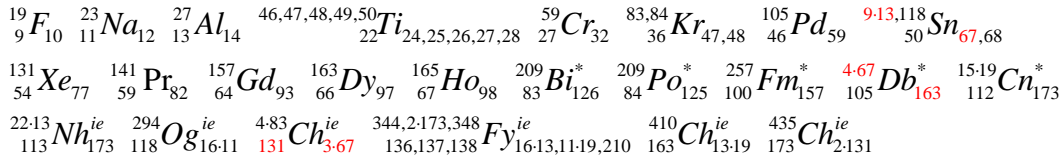
$$= \frac{131 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{4 \cdot 3 \cdot 13 \cdot 59}{2 \cdot 43 \cdot 107 + 1}\right)^{79 \cdot (8 \cdot 29 + 1)}}}{4 \cdot 163} \frac{1}{173 - \frac{4 \cdot 59 \cdot (2 \cdot 11 \cdot 19 + 1)}{25 \cdot 10^{11}}}$$

$$= 1/137.035999111818$$



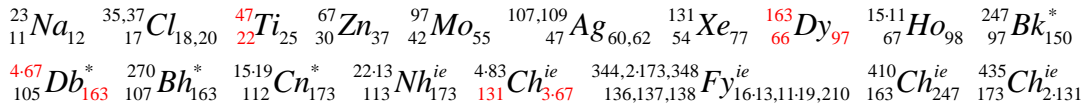
$$\alpha_{2-131(173)\text{-Wallis}} = \frac{131 \cdot \left(2 \frac{2}{3} \frac{4}{3} \dots \frac{27610}{27611} \frac{4 \cdot 9 \cdot 13 \cdot 59}{2 \cdot 2 \cdot 5 \cdot 11 \cdot (4 \cdot 9 \cdot 7 - 1) + 1}\right)}{163} \frac{1}{173 - \frac{8 \cdot 67 \cdot 157}{25 \cdot 10^{11}}}$$

$$= 1/137.035999111818$$



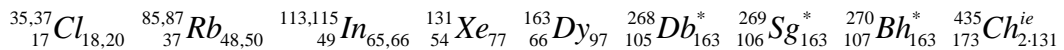
$$\alpha_{2-131(173)\text{-GL}} = \frac{2 \cdot 131 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 11 \cdot 17 \cdot 47 + 1}\right)}{163} \frac{1}{173 - \frac{11 \cdot 67 \cdot 97}{4 \cdot 10^{11}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-131(173)\text{-NC}} = \frac{131 \cdot (2\pi)_{\text{NC}-17}}{4 \cdot 163} \frac{1}{173 - \frac{1}{20 \cdot 64} + \frac{49 \cdot (12 \cdot 269 + 1)}{25 \cdot 10^{11}}}$$

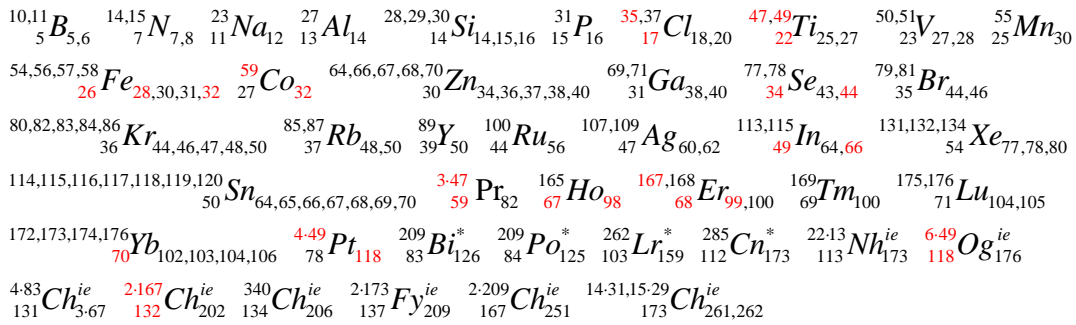
$$= 1/137.035999111818$$



$\alpha_{2-175(173)}$

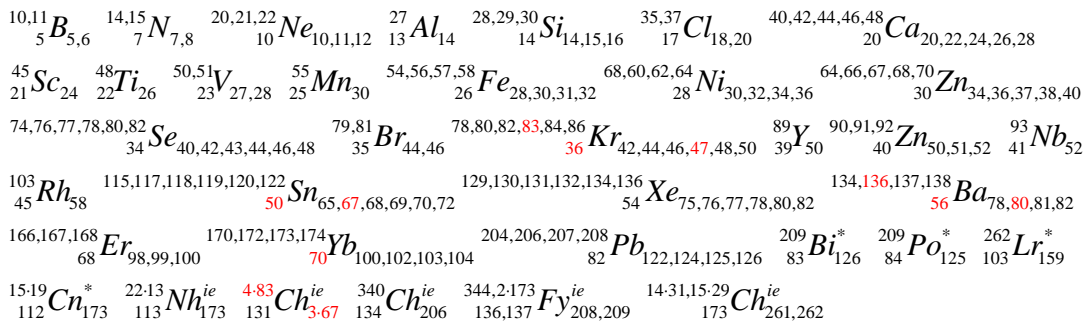
$$= \frac{7 \cdot 25 \cdot e^2 \cdot \frac{e^2}{\left(\frac{2}{1}\right)^3} \cdot \frac{e^2}{\left(\frac{3}{2}\right)^5} \cdot \frac{e^2}{\left(\frac{4}{3}\right)^7} \cdots \frac{e^2}{\left(\frac{4 \cdot 3 \cdot 11 \cdot 47 - 1}{14 \cdot (2 \cdot 13 \cdot 17 + 1)}\right)^{15 \cdot (14 \cdot 59 + 1)}}}{13 \cdot 67} \cdot \frac{1}{173 - \frac{1}{2 \cdot 167 \cdot (32 \cdot 3 \cdot 49 - 1)}}$$

$$= 1/137.035999111818$$

 $\alpha_{2-175(173)-Wallix}$

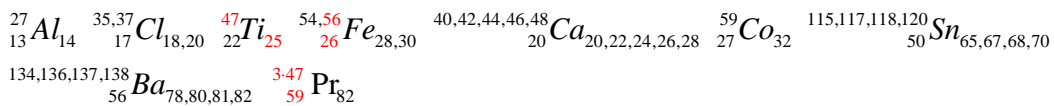
$$= \frac{4 \cdot 7 \cdot 25 \cdot \left(2 \frac{2}{3} \frac{4}{5} \frac{4}{5} \frac{6}{7} \frac{6}{7} \frac{8}{7} \cdots \frac{18606}{19607} \frac{16 \cdot (2 \cdot 7 \cdot 83 + 1)}{2 \cdot 3 \cdot 7 \cdot (2 \cdot 13 \cdot 17 + 1) + 1}\right)}{13 \cdot 67} \cdot \frac{1}{173 - \frac{1}{5 \cdot (16 \cdot 9 \cdot 5 \cdot 17 \cdot 41 + 1)}}$$

$$= 1/137.035999111818$$

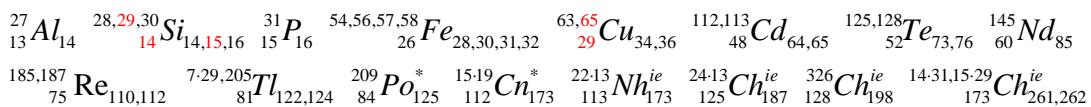
 $\alpha_{2-175(173)-GL}$

$$= \frac{8 \cdot 7 \cdot 25 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{1}{2 \cdot 2 \cdot (2 \cdot 9 \cdot 7 \cdot 47 + 1) + 1}\right)}{13 \cdot 67} \cdot \frac{1}{173 - \frac{1}{5 \cdot 59 \cdot (4 \cdot 5 \cdot 13 \cdot 17 + 1)}}$$

$$= 1/137.035999111818$$

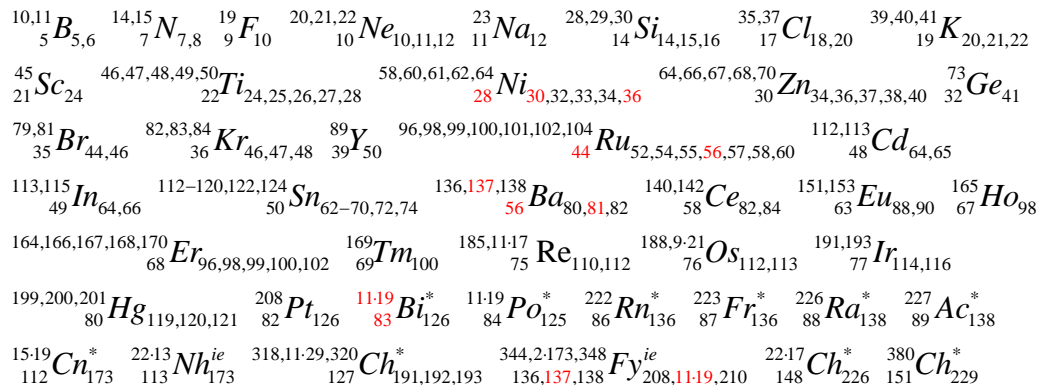


$$\alpha_{2-175(173)-NC} = \frac{7 \cdot 25 \cdot (2\pi)_{NC-15}}{13 \cdot 67} \cdot \frac{1}{173 - \frac{1}{2 \cdot (3 \cdot 128 - 1)} + \frac{1}{125 \cdot 7 \cdot 29^2}} = 1/137.035999111818$$



2. Formulas of α_{1-7-NC} and $\alpha_{2-13-NC}$ expressed with large integers

$$\begin{aligned}
 \alpha_{1-7-NC} &= \frac{36}{7 \cdot (2\pi)_{NC-3}} \frac{1}{112 + \frac{1}{4 \cdot 7} - \frac{1}{2 \cdot 5 \cdot (3 \cdot 7)^2 - \frac{113}{7 \cdot 17}}} \\
 &= \frac{36}{7 \cdot (6 + \sum_{n=1}^3 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)})} \frac{1}{112 + \frac{1}{4 \cdot 7} - \frac{1}{2 \cdot 5 \cdot (3 \cdot 7)^2 - \frac{113}{7 \cdot 17}}} \\
 &= \frac{36}{7 \cdot (6 + \frac{1}{1 \cdot \frac{3}{2} \cdot 2} - \frac{1}{2 \cdot \frac{5}{2} \cdot 3} + \frac{1}{3 \cdot \frac{7}{2} \cdot 4})} \frac{1}{112 + \frac{1}{4 \cdot 7} - \frac{1}{2 \cdot 5 \cdot (3 \cdot 7)^2 - \frac{113}{7 \cdot 17}}} \\
 &= \frac{36}{7 \cdot (6 + \frac{1}{3} - \frac{1}{15} + \frac{1}{42})} \frac{1}{112 + \frac{1}{4 \cdot 7} - \frac{1}{2 \cdot 5 \cdot (3 \cdot 7)^2 - \frac{113}{7 \cdot 17}}} \\
 &= \frac{36}{7 \cdot (6 + \frac{61}{210})} \frac{1}{112 + \frac{1}{4 \cdot 7} - \frac{1}{2 \cdot 5 \cdot (3 \cdot 7)^2 - \frac{113}{7 \cdot 17}}} \\
 &= \frac{36}{7 \cdot (\frac{44}{7} + \frac{1}{210})} \frac{1}{112 + \frac{1}{4 \cdot 7} - \frac{1}{2 \cdot 5 \cdot (3 \cdot 7)^2 - \frac{113}{7 \cdot 17}}} \\
 &= \frac{36}{44 + \frac{1}{30}} \frac{1}{112 + \frac{1}{4 \cdot 7} - \frac{1}{41 \cdot 67 \cdot 191}} \\
 &= \frac{1080}{1321} \frac{1}{112 + \frac{5 \cdot 11 \cdot 9479}{4 \cdot 7 \cdot 41 \cdot 67 \cdot 191}} = \frac{1080}{1321} \frac{28 \cdot 41 \cdot 67 \cdot 191}{81 \cdot 89 \cdot 229 \cdot 997} \\
 &= \frac{2^5 \cdot 5 \cdot 7 \cdot (2^3 \cdot 5 + 1)(2 \cdot 3 \cdot 11 + 1)(2 \cdot 5 \cdot 19 + 1)}{3 \cdot (2^3 \cdot 11 + 1)(2^3 \cdot 3 \cdot 5 \cdot 11 + 1)(4 \cdot 3 \cdot 19 + 1)(4 \cdot 3 \cdot 83 + 1)} \\
 &= 1/137.035999037435
 \end{aligned}$$



$$\alpha_{2-13-NC} = \frac{13 \cdot (2\pi)_{NC-5}}{100} \frac{1}{112 - \frac{1}{4 \cdot 9} + \frac{1}{1600} - \frac{1}{2(2 \cdot 7 \cdot 199 \cdot 257 + 1)}}$$

$$= 1/137.035999111818$$

²⁷₁₃Al₁₄ ^{28,29,30}₁₄Si_{14,15,16} ^{36,38,40}₁₈Ar_{18,20,22} ^{40,42,43,44,46,48}₂₀Ca_{20,22,23,24,26,28} ^{46,47,48,49,50}₂₂Ti_{24,25,26,27,28}
^{54,56,57,58}₂₆Fe_{28,30,31,32} ^{69,71}₃₁Ga_{38,40} ^{80,82,83,84,86}₃₆Kr_{44,46,47,48,50} ⁹⁰₄₀Zr₅₀ ¹⁰⁰₄₄Ru₅₆ ¹¹⁹₅₀Sn₆₉ ¹⁶⁹₆₉Tm₁₀₀
¹⁹⁹₈₀Hg₁₁₉ ²⁵⁷₁₀₀Fm₁₅₇ ³²⁸₁₂₉Ch₁₉₉^{ie} ⁶⁷¹₁₆₉Ch₂₅₇^{ie}

$$(2\pi)_{NC-5} = 6 + \sum_{n=1}^5 \frac{(-1)^{n+1}}{n(n+1/2)(n+1)} = 6 + \frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{2 \cdot 3 \cdot 7} - \frac{1}{2 \cdot 3^2 \cdot 5} + \frac{1}{3 \cdot 5 \cdot 11}$$

$$= 6 + \frac{2 \cdot 3 \cdot 5 \cdot 7 \cdot 11}{2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11} - \frac{2 \cdot 3 \cdot 7 \cdot 11}{2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11} + \frac{3 \cdot 5 \cdot 11}{2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11} - \frac{7 \cdot 11}{2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11} + \frac{2 \cdot 3 \cdot 7}{2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11}$$

$$= 6 + \frac{2310 - 462 + 165 - 77 + 42}{2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11} = 6 + \frac{1978}{2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11} = 6 + \frac{989}{5 \cdot 7 \cdot 9 \cdot 11} = \frac{29 \cdot 751}{5 \cdot 7 \cdot 9 \cdot 11}$$

$$\alpha_{2-13-NC} = \frac{13 \cdot 29 \cdot (2 \cdot 3 \cdot 5^3 + 1)}{2^2 \cdot 5^3 \cdot 7 \cdot 9 \cdot 11} \frac{1}{112 - \frac{1}{4 \cdot 9} + \frac{1}{1600} - \frac{1}{2(2 \cdot 7 \cdot 199 \cdot 257 + 1)}}$$

$$= \frac{13 \cdot 29 \cdot (2 \cdot 3 \cdot 5^3 + 1)}{2^2 \cdot 5^3 \cdot 7 \cdot 9 \cdot 11} \frac{1}{112 - \frac{1}{2} \left(\frac{1}{2 \cdot 9} - \frac{1}{800} + \frac{1}{(2 \cdot 7 \cdot 199 \cdot 257 + 1)} \right)}$$

$$= \frac{13 \cdot 29 \cdot (2 \cdot 3 \cdot 5^3 + 1)}{2^2 \cdot 3^2 \cdot 5^3 \cdot 7 \cdot 11} \frac{1}{112 - \frac{13 \cdot 19 \cdot (2 \cdot 17^2 \cdot 37 \cdot 53 + 1)}{2^6 \cdot 3^2 \cdot 5^2 \cdot (2 \cdot 7 \cdot 199 \cdot 257 + 1)}}$$

$$= 1/137.035999111818$$

²⁷₁₃Al₁₄ ^{28,29,30}₁₄Si_{14,15,16} ^{35,37}₁₇Cl_{18,20} ^{39,40,41}₁₉K_{20,21,22} ^{46,47,48,49,50}₂₂Ti_{24,25,26,27,28} ⁵³₂₄Cr₂₉ ⁵⁵₂₅Mn₃₀
^{63,65}₂₉Cu_{34,36} ⁶⁷₃₀Zn₃₇ ⁷⁵₃₃As₄₂ ^{85,87}₃₇Rb_{48,50} ⁵⁻¹⁹₄₂Mo₅₃ ^{99,100}₄₄Ru_{55,56} ¹²⁷₅₃I₂₋₃₇ ⁷⁻¹⁹₅₅Cs₇₈ ¹⁶³₆₆Dy₉₇
^{185,11-17}₇₅Re_{110,112} ^{191,193}₇₇Ir_{114,116} ¹⁹⁹₈₀Hg₇₋₁₇ ¹¹⁻¹⁹₈₃Bi₁₂₆ ¹¹⁻¹⁹₈₄Po₁₂₅ ¹³⁻¹⁹₉₆Cm₁₅₁ ¹³⁻¹⁹₉₇Bk₅₀ ²⁵⁷₁₀₀Fm₁₅₇
^{269,271}₁₀₆Sg_{163,165} ¹⁷²₁₁₅Mc₆₋₂₉ ²⁴⁻¹³₁₂₅Ch₁₁₇^{ie} ^{6-53,11-29,320}₁₂₇Ch_{191,3-64,193}^{ie} ³²⁸₁₂₉Ch₁₉₉^{ie} ⁴¹⁰₁₆₃Ch₁₃₋₁₉^{ie} ⁴²⁶₁₆₉Ch₂₅₇^{ie}

3. Deduction of new formulas of e

Define $\infty = \lim_{n \rightarrow \infty} n$

$$e = \left(1 + \frac{1}{\infty}\right)^\infty = 1 + \frac{\infty}{1! \infty} + \frac{\infty(\infty-1)}{2! \infty^2} + \frac{\infty(\infty-1)(\infty-2)}{3! \infty^3} + \dots$$

$$= \left(1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots\right) + \left(\frac{1}{2!} + \frac{1+2}{3!} + \frac{1+2+3}{4!} + \dots\right) \frac{1}{\infty} +$$

$$\left[\frac{1 \cdot 2}{3!} + \frac{1 \cdot 2 + (1+2)3}{4!} + \frac{1 \cdot 2 + (1+2)3 + (1+2+3)4}{5!} + \dots\right] \frac{1}{\infty^2} +$$

$$\left\{\frac{1 \cdot 2 \cdot 3}{4!} + \frac{1 \cdot 2 \cdot 3 + [1 \cdot 2 + (1+2)3]4}{5!} + \dots\right\} \frac{1}{\infty^3} + \dots$$

3-1. There are or should be some formulas as follows.

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} = 2.718281828459045 \dots$$

$$e = 2 \left(\frac{1}{2!} + \frac{1+2}{3!} + \frac{1+2+3}{4!} + \dots \right) = 2 \sum_{k=2}^{\infty} \frac{\sum_{l=1}^{k-1} l}{k!} = 1 + \sum_{k=1}^{\infty} \frac{1}{k!}$$

$$e = \frac{24}{11} \left[\frac{1 \cdot 2}{3!} + \frac{1 \cdot 2 + (1+2)3}{4!} + \frac{1 \cdot 2 + (1+2)3 + (1+2+3)4}{5!} + \dots \right] = \frac{24}{11} \sum_{k=3}^{\infty} \frac{\sum_{l=2}^{k-1} l \sum_{m=1}^{l-1} m}{k!}$$

$$e = \frac{16}{7} \left\{ \frac{1 \cdot 2 \cdot 3}{4!} + \frac{1 \cdot 2 \cdot 3 + [1 \cdot 2 + (1+2)3]4}{5!} + \dots \right\} = \frac{16}{7} \sum_{k=4}^{\infty} \frac{\sum_{l_1=3}^{k-1} l_1 \sum_{l_2=2}^{l_1-1} l_2 \sum_{m=1}^{l_2-1} m}{k!}$$

$$\frac{1}{e} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots = 1 + \sum_{k=1}^{\infty} \frac{(-1)^k}{k!} = 1/2.718281828459045 \dots$$

$$\frac{1}{e} = 2 \left(\frac{1}{2!} - \frac{1+2}{3!} + \frac{1+2+3}{4!} - \dots \right) = 2 \sum_{k=2}^{\infty} \frac{(-1)^{k-2} \sum_{l=1}^{k-1} l}{k!} = 1 + \sum_{k=1}^{\infty} \frac{(-1)^k}{k!}$$

$$\frac{1}{e} = \frac{24}{5} \left[\frac{1 \cdot 2}{3!} - \frac{1 \cdot 2 + (1+2)3}{4!} + \frac{1 \cdot 2 + (1+2)3 + (1+2+3)4}{5!} - \dots \right] = \frac{24}{5} \sum_{k=3}^{\infty} \frac{(-1)^{k-3} \sum_{l=2}^{k-1} l \sum_{m=1}^{l-1} m}{k!}$$

$$\frac{1}{e} = \frac{48}{5} \left\{ \frac{1 \cdot 2 \cdot 3}{4!} - \frac{1 \cdot 2 \cdot 3 + [1 \cdot 2 + (1+2)3]4}{5!} + \dots \right\} = \frac{48}{5} \sum_{k=4}^{\infty} \frac{(-1)^{k-4} \sum_{l_1=3}^{k-1} l_1 \sum_{l_2=2}^{l_1-1} l_2 \sum_{m=1}^{l_2-1} m}{k!}$$

3-2. There are or should be some general formulas as follows.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots = 1 + \sum_{k=1}^{\infty} \frac{x^k}{k!} = 1 + x \left(\frac{1}{1!} + \frac{x}{2!} + \frac{x^2}{3!} + \dots \right) = 1 + x \sum_{k=1}^{\infty} \frac{x^{k-1}}{k!}$$

$$e^x = 2 \left[\frac{1}{2!} + \frac{1+2}{3!} x + \frac{1+2+3}{4!} x^2 + \dots \right] = 2 \left(\frac{1}{2!} + \sum_{k=3}^{\infty} \frac{x^{k-2} \sum_{m=1}^{k-1} m}{k!} \right) = 2 \sum_{k=2}^{\infty} \frac{x^{k-2} \sum_{m=1}^{k-1} m}{k!}$$

$$e^x = \frac{8}{x+8/3} \left[\frac{1 \cdot 2}{3!} + \frac{1 \cdot 2 + (1+2)3}{4!} x + \dots \right] = \frac{8}{x+8/3} \sum_{k=3}^{\infty} \frac{x^{k-3} \sum_{l=2}^{k-1} l \sum_{m=1}^{l-1} m}{k!}$$

$$e^x = \frac{6 \cdot 8}{(x+2)(x+6)} \left\{ \frac{1 \cdot 2 \cdot 3}{4!} + \frac{1 \cdot 2 \cdot 3 + [1 \cdot 2 + (1+2)3]4}{5!} x + \dots \right\}$$

$$= \frac{6 \cdot 8}{(x+2)(x+6)} \sum_{k=4}^{\infty} \frac{x^{k-4} \sum_{l_1=3}^{k-1} l_1 \sum_{l_2=2}^{l_1-1} l_2 \sum_{m=1}^{l_2-1} m}{k!} \quad (x \in \mathbb{R}, 0^0 = 1)$$

3-3. There should be some special zero points for the above formulas as follows.

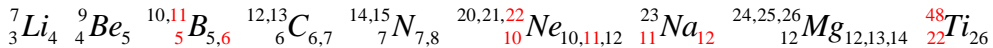
$$\begin{aligned} \sum_{k=3}^{\infty} \frac{\left(-\frac{8}{3}\right)^{k-3} \sum_{l=2}^{k-1} l \sum_{m=1}^{l-1} m}{k!} &= \frac{1 \cdot 2}{3!} + \frac{1 \cdot 2 + (1+2)3}{4!} \left(-\frac{8}{3}\right) + \\ &\frac{1 \cdot 2 + (1+2)3 + (1+2+3)4}{5!} \left(-\frac{8}{3}\right)^2 + \dots = 0 \\ \sum_{k=4}^{\infty} \frac{(-2)^{k-4} \sum_{l_1=3}^{k-1} l_1 \sum_{l_2=2}^{l_1-1} l_2 \sum_{m=1}^{l_2-1} m}{k!} &= \frac{1 \cdot 2 \cdot 3}{4!} + \frac{1 \cdot 2 \cdot 3 + [1 \cdot 2 + (1+2)3]4}{5!} (-2) + \\ &\frac{1 \cdot 2 \cdot 3 + [1 \cdot 2 + (1+2)3]4 + [1 \cdot 2 + (1+2)3 + (1+2+3)4]5}{6!} (-2)^2 + \dots = 0 \\ \sum_{k=4}^{\infty} \frac{(-6)^{k-3} \sum_{l_1=3}^{k-1} l_1 \sum_{l_2=2}^{l_1-1} l_2 \sum_{m=1}^{l_2-1} m}{k!} &= \frac{1 \cdot 2 \cdot 3}{4!} + \frac{1 \cdot 2 \cdot 3 + [1 \cdot 2 + (1+2)3]4}{5!} (-6) + \\ &\frac{1 \cdot 2 \cdot 3 + [1 \cdot 2 + (1+2)3]4 + [1 \cdot 2 + (1+2)3 + (1+2+3)4]5}{6!} (-6)^2 + \dots = 0 \end{aligned}$$

3-4. There should be some special extended formulas as follows.

$$\begin{aligned} e &= \frac{5760}{2447} \sum_{k=5}^{\infty} \frac{\sum_{l_1=4}^{k-1} l_1 \sum_{l_2=3}^{l_1-1} l_2 \sum_{l_3=2}^{l_2-1} l_3 \sum_{m=1}^{l_3-1} m}{k!} = \frac{2^7 \cdot 3^2 \cdot 5}{2 \cdot 9 \cdot 136 - 1} \sum_{k=5}^{\infty} \frac{\sum_{l_1=4}^{k-1} l_1 \sum_{l_2=3}^{l_1-1} l_2 \sum_{l_3=2}^{l_2-1} l_3 \sum_{m=1}^{l_3-1} m}{k!} \\ \frac{1}{e} &= \frac{5760}{337} \sum_{k=5}^{\infty} \frac{(-1)^{k-5} \sum_{l_1=4}^{k-1} l_1 \sum_{l_2=3}^{l_1-1} l_2 \sum_{l_3=2}^{l_2-1} l_3 \sum_{m=1}^{l_3-1} m}{k!} = \frac{2^7 \cdot 3^2 \cdot 5}{3 \cdot 112 + 1} \sum_{k=5}^{\infty} \frac{(-1)^{k-5} \sum_{l_1=4}^{k-1} l_1 \sum_{l_2=3}^{l_1-1} l_2 \sum_{l_3=2}^{l_2-1} l_3 \sum_{m=1}^{l_3-1} m}{k!} \\ e &= \frac{2304}{959} \sum_{k=6}^{\infty} \frac{\sum_{l_1=5}^{k-1} l_1 \sum_{l_2=4}^{l_1-1} l_2 \sum_{l_3=3}^{l_2-1} l_3 \sum_{l_4=2}^{l_3-1} l_4 \sum_{m=1}^{l_4-1} m}{k!} = \frac{2^8 \cdot 3^2}{7 \cdot 137} \sum_{k=6}^{\infty} \frac{\sum_{l_1=5}^{k-1} l_1 \sum_{l_2=4}^{l_1-1} l_2 \sum_{l_3=3}^{l_2-1} l_3 \sum_{l_4=2}^{l_3-1} l_4 \sum_{m=1}^{l_4-1} m}{k!} \\ \frac{1}{e} &= \frac{3840}{137} \sum_{k=6}^{\infty} \frac{(-1)^{k-6} \sum_{l_1=5}^{k-1} l_1 \sum_{l_2=4}^{l_1-1} l_2 \sum_{l_3=3}^{l_2-1} l_3 \sum_{l_4=2}^{l_3-1} l_4 \sum_{m=1}^{l_4-1} m}{k!} = \frac{2^8 \cdot 3 \cdot 5}{137} \sum_{k=6}^{\infty} \frac{(-1)^{k-6} \sum_{l_1=5}^{k-1} l_1 \sum_{l_2=4}^{l_1-1} l_2 \sum_{l_3=3}^{l_2-1} l_3 \sum_{l_4=2}^{l_3-1} l_4 \sum_{m=1}^{l_4-1} m}{k!} \end{aligned}$$

4. Relationships between the new formulas of e and nuclides

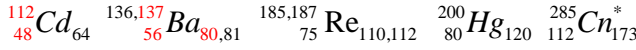
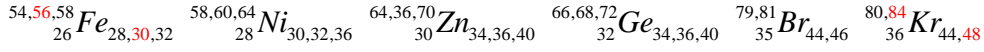
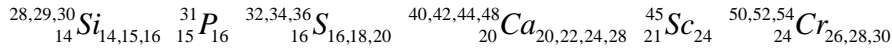
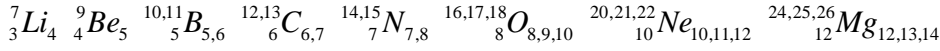
$$e = \frac{8 \cdot 3}{11} \sum_{k=3}^{\infty} \frac{\sum_{l=2}^{k-1} l \sum_{m=1}^{l-1} m}{k!} \quad \frac{1}{e} = \frac{8 \cdot 3}{5} \sum_{k=3}^{\infty} \frac{(-1)^{k-3} \sum_{l=2}^{k-1} l \sum_{m=1}^{l-1} m}{k!}$$



$$e = \frac{16}{7} \sum_{k=4}^{\infty} \frac{\sum_{l_1=3}^{k-1} \sum_{l_2=2}^{l_1-1} \sum_{m=1}^{l_2-1} m}{k!}$$

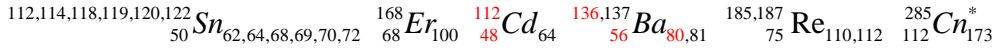
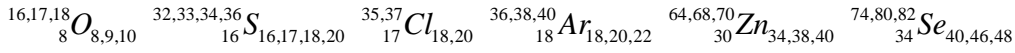
$$\frac{1}{e} = \frac{16 \cdot 3}{5} \sum_{k=4}^{\infty} \frac{(-1)^{k-4} \sum_{l_1=3}^{k-1} \sum_{l_2=2}^{l_1-1} \sum_{m=1}^{l_2-1} m}{k!}$$

$$\frac{1}{e^4} = -12 \sum_{k=4}^{\infty} \frac{(-4)^{k-4} \sum_{l_1=3}^{k-1} \sum_{l_2=2}^{l_1-1} \sum_{m=1}^{l_2-1} m}{k!}$$



$$e = \frac{5760}{2447} \sum_{k=5}^{\infty} \frac{\sum_{l_1=4}^{k-1} \sum_{l_2=3}^{l_1-1} \sum_{l_3=2}^{l_2-1} \sum_{m=1}^{l_3-1} m}{k!} = \frac{2^7 \cdot 3^2 \cdot 5}{2 \cdot 9 \cdot 136 - 1} \sum_{k=5}^{\infty} \frac{\sum_{l_1=4}^{k-1} \sum_{l_2=3}^{l_1-1} \sum_{l_3=2}^{l_2-1} \sum_{m=1}^{l_3-1} m}{k!}$$

$$\frac{1}{e} = \frac{5760}{337} \sum_{k=5}^{\infty} \frac{(-1)^{k-5} \sum_{l_1=4}^{k-1} \sum_{l_2=3}^{l_1-1} \sum_{l_3=2}^{l_2-1} \sum_{m=1}^{l_3-1} m}{k!} = \frac{2^7 \cdot 3^2 \cdot 5}{3 \cdot 112 + 1} \sum_{k=5}^{\infty} \frac{(-1)^{k-5} \sum_{l_1=4}^{k-1} \sum_{l_2=3}^{l_1-1} \sum_{l_3=2}^{l_2-1} \sum_{m=1}^{l_3-1} m}{k!}$$

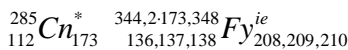
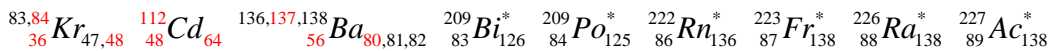


$$e = \frac{2304}{959} \sum_{k=6}^{\infty} \frac{\sum_{l_1=5}^{k-1} \sum_{l_2=4}^{l_1-1} \sum_{l_3=3}^{l_2-1} \sum_{l_4=2}^{l_3-1} \sum_{m=1}^{l_4-1} m}{k!}$$

$$= \frac{2^8 \cdot 3^2}{7 \cdot 137} \sum_{k=6}^{\infty} \frac{\sum_{l_1=5}^{k-1} \sum_{l_2=4}^{l_1-1} \sum_{l_3=3}^{l_2-1} \sum_{l_4=2}^{l_3-1} \sum_{m=1}^{l_4-1} m}{k!}$$

$$\frac{1}{e} = \frac{3840}{137} \sum_{k=6}^{\infty} \frac{(-1)^{k-6} \sum_{l_1=5}^{k-1} \sum_{l_2=4}^{l_1-1} \sum_{l_3=3}^{l_2-1} \sum_{l_4=2}^{l_3-1} \sum_{m=1}^{l_4-1} m}{k!}$$

$$= \frac{2^8 \cdot 3 \cdot 5}{137} \sum_{k=6}^{\infty} \frac{(-1)^{k-6} \sum_{l_1=5}^{k-1} \sum_{l_2=4}^{l_1-1} \sum_{l_3=3}^{l_2-1} \sum_{l_4=2}^{l_3-1} \sum_{m=1}^{l_4-1} m}{k!}$$



5. Relationships between formulas of 2π and nuclides

$$\pi = \frac{22}{7} - \int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx \quad 2\pi = \frac{4 \cdot 11}{7} - 2 \int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$$

$^{14,15}_7N_{7,8}$ $^{23}_{11}Na_{12}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{46,47,48,49,50}_{22}Sc_{24,25,26,27,28}$ $^{58,60,61,62,64}_{28}Ni_{30,32,33,34,36}$ $^{77}_{34}Se_{43}$
 $^{88}_{38}Sr_{50}$ $^{96,98,99,100,102,104}_{44}Ru_{52,54,55,56,58,60}$ $^{150}_{62}Sm_{88}$ $^{226}_{88}Ra_{138}^*$

$$\pi = \frac{355}{113} - \frac{1}{3164} \int_0^1 \frac{x^8(1-x)^8(25+816x^2)}{1+x^2} dx$$

$$2\pi = \frac{2 \cdot 5 \cdot 71}{113} - \frac{1}{2 \cdot 7 \cdot 113} \int_0^1 \frac{x^8(1-x)^8(25+16 \cdot 3 \cdot 17x^2)}{1+x^2} dx$$

$^{14,15}_7N_{7,8}$ $^{16,17,18}_8O_{8,9,10}$ $^{31}_{15}P_{16}$ $^{32,33,34,36}_{16}S_{16,17,18,20}$ $^{35,37}_{17}Cl_{18,20}$ $^{55}_{25}Mn_{30}$ $^{69,71}_{31}Ga_{38,40}$ $^{112,113}_{48}Cd_{64,65}$
 $^{77,78,80,82}_{34}Se_{43,44,46,48}$ $^{118,119}_{50}Sn_{68,69}$ $^{121,122}_{51}Sb_{70,72}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{168}_{68}Er_{100}$ $^{169}_{69}Tm_{100}$ $^{175,116}_{71}Lu_{104,105}$
 $^{170,171,172,173,176}_{70}Yb_{100,101,102,103,106}$ $^{189}_{76}Os_{113}$ $^{226}_{88}Ra_{138}^*$ $^{257}_{100}Fm_{157}$ $^{259}_{102}Fm_{157}^*$ $^{4 \cdot 71, 2 \cdot 11 \cdot 13}_{113}Nh_{171,173}^{ie}$ $^{2 \cdot 11 \cdot 17}_{148}Ch_{226}^{ie}$

6. Relationships between γ , e , γ_c , γ_g and γ_{cg} and nuclides

$$\text{Euler-Mascheroni constant } \gamma = \lim_{N \rightarrow \infty} \left(\sum_{n=1}^N \frac{1}{n} - \ln n \right) = 0.5772156649 \dots$$

$$\gamma = 0.5772156649 \dots \approx \frac{3 \cdot 5}{2 \cdot 13} \approx \frac{3 \cdot 41}{71}$$

$^{11}_5B_6$ $^{12,13}_6C_{6,7}$ $^{31}_{15}P_{16}$ $^{39}_{19}K_{20}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$
 $^{64,66}_{30}Zn_{34,36}$ $^{69,71}_{31}Ga_{38,40}$ $^{89}_{39}Y_{50}$ $^{93}_{41}Nb_{52}$ $^{121,3 \cdot 41}_{51}Sb_{70,72}$ $^{3 \cdot 41}_{52}Te_{71}$ $^{175,176}_{71}Lu_{104,105}$ $^{18 \cdot 17, 4 \cdot 77}_{123}Ch_{183,185}^{ie}$

$$e = 2.718281828 \dots \approx \frac{3 \cdot 29}{32} \approx \frac{193}{71}$$

$^{48}_{22}Ti_{26}$ $^{53}_{24}Cr_{29}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{63,65}_{29}Cu_{34,36}$ $^{87,84}_{36}Kr_{47,48}$ $^{85,87}_{37}Rb_{48,50}$ $^{106,112,113,116}_{48}Cd_{58,64,65,68}$
 $^{113,115}_{49}In_{64,66}$ $^{175,176}_{71}Lu_{104,105}$ $^{191,193}_{77}Ir_{114,116}$ $^{4 \cdot 71, 286}_{113}Nh_{171,173}^*$ $^{11 \cdot 29, 320}_{127}Ch_{3 \cdot 64, 193}^{ie}$ $^{434}_{173}Ch_{9 \cdot 29}^{ie}$

$$e^2 \approx \frac{37}{5} \approx \frac{7 \cdot 19}{2 \cdot 9}$$

$^{10,11}_5B_{5,6}$ $^{14,15}_7N_{7,8}$ $^{19}_9F_{10}$ $^{28,29,30}_{14}Si_{14,15,16}$ $^{35,37}_{17}Cl_{18,20}$ $^{38}_{18}Ar_{20}$ $^{67,68}_{30}Zn_{37,38}$ $^{85,87}_{37}Rb_{48,50}$ $^{84,87}_{38}Sr_{46,49}$
 $^{7 \cdot 19}_{55}Cs_{78}$ $^{9 \cdot 19}_{70}Yb_{101}$ $^{185,187}_{75}Tm_{110,112}$ $^{284,286}_{113}Nh_{9 \cdot 19, 173}^{ie}$ $^{4 \cdot 77}_{123}Ch_{185}^{ie}$ $^{312}_{125}Ch_{187}^{ie}$ $^{3 \cdot 112}_{133}Ch_{7 \cdot 29}^{ie}$ $^{429,430}_{171}Ch_{258,7 \cdot 37}^{ie}$

$$e^\gamma = 1.7810724 \dots \approx \frac{3 \cdot 19}{32} \approx \frac{7 \cdot 43}{167}$$

7_3Li_4 $^{48}_{22}Ti_{26}$ $^{54,56,57,58}_{26}Fe_{28,30,31,32}$ $^{70}_{32}Ge_{38}$ $^{77}_{34}Se_{43}$ $^{84}_{36}Kr_{48}$ $^{86,87}_{38}Sr_{48,49}$ $^{99}_{43}Tc_{56}^*$ $^{112,113,114}_{48}Cd_{64,65,66}$
 $^{167,168}_{68}Er_{99,100}$ $^{171}_{70}Yb_{101}$ $^{15 \cdot 19}_{112}Cn_{173}^*$ $^{284,22 \cdot 13}_{113}Nh_{9 \cdot 19, 173}^{ie}$ $^{33 \cdot 13, 430}_{171}Ch_{258,259}^{ie}$ $^{22 \cdot 19}_{167}Ch_{251}^{ie}$

$$e^{\gamma_c} = e^{0.0810614668 \dots} = 1.08443755 \dots \approx \frac{7 \cdot 11}{71} \approx \frac{5 \cdot 131}{4 \cdot 151}$$

$^{69,71}_{31}Ga_{38,40}$ $^{121,123}_{51}Sb_{70,72}$ $^{175,176}_{71}Lu_{104,105}$ $^{77}_{34}Se_{43}$ $^{131}_{54}Xe_{77}$ $^{151,153}_{63}Eu_{88,90}$ $^{191,193}_{77}Ir_{114,116}$ $^{247}_{96}Cm_{151}^*$

$$e^{2\gamma_c} = e^{2 \times 0.0810614668 \dots} = 1.17600480 \dots \approx \frac{4 \cdot 5}{17} \approx \frac{3 \cdot 49}{125} \approx \frac{167}{2 \cdot 71}$$

$${}^{35,37}_{17}\text{Cl}_{18,20} \quad {}^{49}_{22}\text{Ti}_{27} \quad {}^{68}_{30}\text{Ga}_{38} \quad {}^{69,71}_{31}\text{Ga}_{38,40} \quad {}^{85,87}_{37}\text{Rb}_{48,50} \quad {}^{87}_{38}\text{Sr}_{49} \quad {}^{113,115}_{49}\text{In}_{64,66} \quad {}^{118}_{50}\text{Sn}_{68} \quad {}^{121,123}_{51}\text{Sb}_{70,72}$$

$${}^{125}_{52}\text{Te}_{73} \quad {}^{147}_{62}\text{Sm}_{85} \quad {}^{167,168}_{68}\text{Er}_{99,100} \quad {}^{175,176}_{71}\text{Lu}_{104,105} \quad {}^{209}_{84}\text{Po}_{125}^* \quad {}^{210}_{85}\text{At}_{125}^* \quad {}^{312}_{125}\text{Ch}_{11-17}^{ie} \quad {}^{12-31}_{147}\text{Ch}_{225}^{ie} \quad {}^{418}_{167}\text{Ch}_{251}^{ie}$$

$$2(1 - \gamma_c) = 2(1 - 0.0810614668 \dots) \approx \frac{4 \cdot 17}{37} \quad \text{Note: } 2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2$$

$${}^{35,37}_{17}\text{Cl}_{18,20} \quad {}^{64,67,68}_{30}\text{Zn}_{34,37,38} \quad {}^{74}_{34}\text{Se}_{40} \quad {}^{85,87}_{37}\text{Rb}_{38,40} \quad {}^{118}_{50}\text{Sn}_{68} \quad {}^{145}_{60}\text{Nd}_{85} \quad {}^{168}_{68}\text{Er}_{100} \quad {}^{185,187}_{75}\text{Tm}_{110,112}$$

$${}^{210}_{85}\text{At}_{125}^* \quad {}^{4-77}_{123}\text{Ch}_{185}^{ie} \quad {}^{312}_{125}\text{Ch}_{187}^{ie}$$

$$2(1 - \gamma_g) = 2(1 - 0.7742086474 \dots) \approx \frac{2 \cdot 7}{31} \quad \text{Note: } \frac{\pi}{2} = \left(\frac{e}{e^{\gamma_g}}\right)^2$$

$${}^{14,15}_{7}\text{N}_{7,8} \quad {}^{28,29,30}_{14}\text{Si}_{14,15,16} \quad {}^{31}_{15}\text{P}_{16} \quad {}^{54,56,57,58}_{26}\text{Fe}_{28,30,31,32} \quad {}^{62}_{28}\text{Ni}_{34} \quad {}^{69,71}_{31}\text{Ga}_{38,40} \quad {}^{147,148,150,152}_{62}\text{Sm}_{85,86,88,90}$$

$$e^{2\gamma_{cg}} \approx \frac{17}{3 \cdot 5} \approx \frac{127}{112} \approx \frac{144}{127}, \quad 2\gamma_{cg} = 2 \times 0.0628164798 \dots \approx \frac{23}{3 \cdot 61} \approx \frac{8 \cdot 3}{191} \approx \frac{25}{199} \quad \frac{\pi}{2} = \left(\frac{e^{\gamma/2}}{e^{\gamma_{cg}}}\right)^2$$

$${}^{50,51}_{23}\text{V}_{27,28} \quad {}^{53}_{24}\text{Cr}_{29} \quad {}^{64}_{30}\text{Zn}_{34} \quad {}^{83,84}_{36}\text{Kr}_{47,48} \quad {}^{112}_{48}\text{Cd}_{64} \quad {}^{127}_{53}\text{I}_{74} \quad {}^{144}_{60}\text{Nd}_{84} \quad {}^{3-61,186}_{74}\text{W}_{109,112} \quad {}^{185,187}_{75}\text{Tm}_{110,112} \quad {}^{199}_{80}\text{Hg}_{7-17}$$

$${}^{191,193}_{77}\text{Ir}_{114,116} \quad {}^{209,210}_{83}\text{Bi}_{126,127}^* \quad {}^{237}_{93}\text{Np}_{144}^* \quad {}^{285}_{112}\text{Cn}_{173}^* \quad {}^{18-17}_{123}\text{Ch}_{3-31}^{ie} \quad {}^{6-53,11-29,320}_{127}\text{Ch}_{191,192,193}^{ie} \quad {}^{328}_{129}\text{Ch}_{199}^{ie} \quad {}^{6-61}_{144}\text{Ch}_{222}^{ie}$$

7. Integrated Relationships between 2π , γ , e , γ_c , γ_g and γ_{cg} and nuclides

As we had demonstrated that 2π is directly and indirectly related to nuclides in the previous paper¹, it should be reasonable for some constants comparable to 2π such as γ , e , e^γ , $2(1-\gamma_c)$, $2(1-\gamma_g)$ and $2\gamma_{cg}$ to directly relate to nuclides. The above examples exhibit not only the relationships between the above constants and nuclides respectively, but also the integrated relationships between them and nuclides. For example, it seems they cooperatively define all the 4 stable isotopes of Fe as follows.

$$\gamma \approx \frac{3 \cdot 5}{2 \cdot 13} \quad e \approx \frac{3 \cdot 29}{32} \quad e^\gamma \approx \frac{3 \cdot 19}{32} \quad 2(1 - \gamma_g) \approx \frac{2 \cdot 7}{31}$$

$${}^{54,56,57,58}_{26}\text{Fe}_{28,30,31,32}$$

The following is another example of this kind of integrated relationships.

$$2\pi = \frac{4 \cdot 11}{7} - 2 \int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx$$

$$2\pi = \frac{2 \cdot 5 \cdot 71}{113} - \frac{1}{2 \cdot 7 \cdot 113} \int_0^1 \frac{x^8(1-x)^8(25+16 \cdot 3 \cdot 17x^2)}{1+x^2} dx$$

$$2\pi \approx \frac{4 \cdot 157}{100}, \quad \gamma \approx \frac{3 \cdot 5}{2 \cdot 13} \approx \frac{3 \cdot 41}{71}, \quad e \approx \frac{3 \cdot 29}{32} \approx \frac{193}{71}, \quad e^2 \approx \frac{37}{5} \approx \frac{7 \cdot 19}{2 \cdot 9}, \quad e^\gamma \approx \frac{3 \cdot 19}{32} \approx \frac{7 \cdot 43}{167}$$

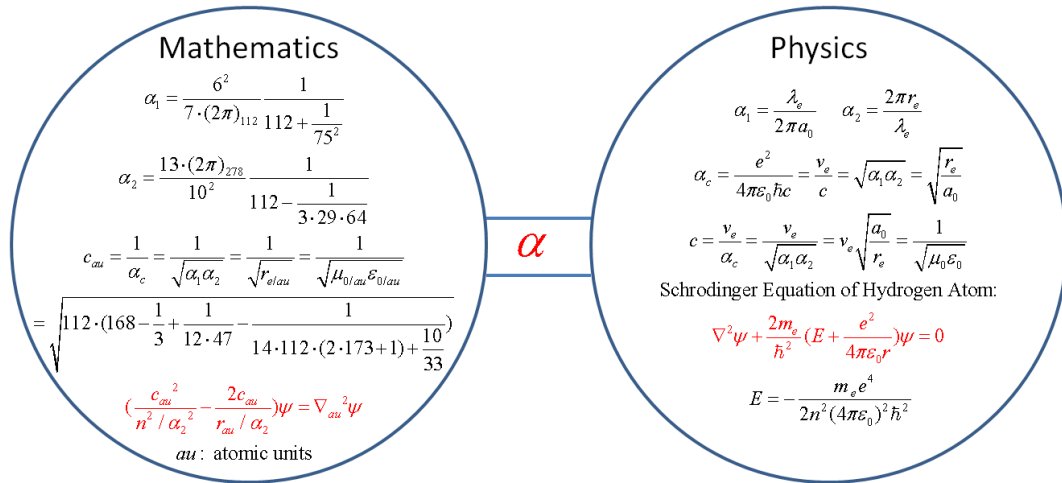
$$e^{\gamma_c} \approx \frac{7 \cdot 11}{71} \approx \frac{5 \cdot 131}{4 \cdot 151}, \quad e^{2\gamma_c} \approx \frac{4 \cdot 5}{17} \approx \frac{3 \cdot 49}{125} \approx \frac{167}{2 \cdot 71}, \quad e^{2\gamma_{cg}} \approx \frac{17}{3 \cdot 5} \approx \frac{127}{112} \approx \frac{127}{16 \cdot 9}$$

$$2(1-\gamma_c) \approx \frac{4 \cdot 17}{37}, \quad 2(1-\gamma_g) \approx \frac{2 \cdot 7}{31}, \quad 2\gamma_{cg} \approx \frac{23}{3 \cdot 61} \approx \frac{8 \cdot 3}{191} \approx \frac{25}{199}$$

²³Na₁₂ ^{35,37}Cl_{18,20} ^{48,49}Ti_{26,27} ^{50,51}V_{27,28} ⁵³Cr₂₉ ⁶¹Ni₃₃ ⁶⁴Zn₃₄ ^{69,71}Ga_{38,40} ⁷⁷Se₄₃ ^{85,87}Rb_{48,50}
⁸⁷Sr₃₈ ⁹³Nb₄₁ ⁹⁵Mo₄₂ ^{98,99}Tc*₄₃ ¹⁰⁰Ru₄₄ ¹¹²⁻¹¹⁴Cd₆₄₋₆₆ ^{113,115}In₄₉ ^{121,123}Sb_{70,72} ^{3-41,125}Te₅₂ ^{71,73}
¹²⁷I₇₄ ¹³¹Xe₇₇ ¹³⁶⁻¹³⁸Ba₈₀₋₈₂ ¹⁴⁴Nd₆₀ ^{5-29,146,3-49}Pm*₆₁ ^{84,85,86} ^{144,147-150,152,154}Sm₆₂ ^{82,85-88,90,92} ¹⁵⁷Gd₆₄
¹⁶²Dy₆₆ ^{166,167,168,170}Er₆₈ ¹⁶⁹Tm₆₉ ^{168,170-174,176}Yb₇₀ ^{175,176}Lu₇₁ ^{104,105} ^{3-61,6-31}W₇₄ ^{109,112}
^{185,187}Tm₇₅ ^{188,189}Os₇₆ ^{191,193}Ir₇₇ ¹⁹⁹Hg₈₀ ^{209,210}Po*₈₃ ²⁰⁹Po*₈₄ ²¹⁰At*₈₅ ⁶⁻³⁷Rn*₈₆ ¹³⁶
²²³Fr*₁₃₆ ²⁻¹¹⁶Ra*₈₈ ²³⁷Np*₉₃ ²⁴³Am*₉₅ ¹³⁻¹⁹Cm*₉₆ ²⁵⁷Fm*₁₀₀ ⁶⁻⁴³Md*₁₀₁ ⁷⁻³⁷No*₁₀₂ ^{9-29,262}Lr*₁₀₃ ^{158,159}
²⁸¹Ds*₁₁₀ ¹⁵⁻¹⁹Cn*₁₁₂ ^{4-71,22-13}Nh^{ie}₁₁₃ ^{9-19,173} ^{18-17,4-77}Ch^{ie}₁₂₃ ^{3-61,185} ³¹²Ch^{ie}₁₂₅ ^{6-53,11-29,320}Ch^{ie}₁₂₇ ^{191,3-64,193} ³²⁸Ch^{ie}₁₂₉ ¹⁹⁹
⁴⁻⁸³Ch^{ie}₁₃₁ ^{8-43,2-173,12-29}Fy^{ie}₁₃₆ ^{16-13,11-19,210} ⁶⁻⁶¹Ch^{ie}₁₄₄ ¹²⁻³¹Ch^{ie}₁₄₇ ⁶⁻³⁷ ²²⁻¹⁷Ch^{ie}₁₄₈ ²²⁶ ³⁸⁰Ch^{ie}₁₅₁ ²²⁹ ⁴⁰⁰Ch^{ie}₁₅₇ ²²⁻¹⁹Ch^{ie}₁₆₇ ²⁵¹
^{33-13,430}Ch^{ie}₁₇₁ ^{6-43,7-37} ^{430,432}Ch^{ie}₁₇₂ ^{6-43,260} ^{14-31,15-29}Ch^{ie}₁₇₃ ^{9-29,262}

These relationships are marvelous, they should not be just coincidences, and they should be science. This mechanism is analogous to that between DNA and protein.

8. Unification of Mathematics and Physics



Unification of Mathematics and Physics through α

Gang Chen, Tianman Chen, Tianyi Chen. 2020/7/20-21

Fig. 1

Mathematics and science are usually regarded to be independent to each other, although mathematics is used as tool or language of science. It seems science has mathematic properties, so on the other hand, does mathematics has science features? We found the fine-structure constant α could act as a bridge between mathematics and

physics (**Fig. 1**). For example, Schrödinger equation of hydrogen atom could be simplified to a reasonable and pure mathematic equation through α (red color parts).

9. Formula of the Speed of Light in Atomic Unites

In the previous paper¹, many formulas of the speed of light in atomic unites c_{au} had been deduced. Among them the most important should be the following formula.

$$c_{au}^2 = \frac{1}{\alpha_c^2} = \frac{1}{\alpha_1 \alpha_2} = \frac{2\pi a_0}{\lambda_e} \frac{\lambda_e}{2\pi r_e} = \frac{a_0}{r_e} = \left(\frac{c}{v_e}\right)^2$$

$$= 137.035999037435 \times 137.035999111818 = 137.035999074627^2$$

$$= 112 \times \left(168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{6 \cdot 29 \cdot 53 \cdot 59 - 79 / 47}\right)$$

Here, we try to revise it to the following more reasonable form.

$$c_{au}^2 = \frac{1}{\alpha_c^2} = \frac{1}{\alpha_1 \alpha_2} = \frac{2\pi a_0}{\lambda_e} \frac{\lambda_e}{2\pi r_e} = \frac{a_0}{r_e} = \left(\frac{c}{v_e}\right)^2$$

$$= 137.035999037435 \times 137.035999111818 = 137.035999074627^2$$

$$= 112 \times \left(168 - \frac{1}{3} + \frac{1}{12 \cdot 47} - \frac{1}{14 \cdot 112 \cdot (2 \cdot 173 + 1) + \frac{2 \cdot 5}{3 \cdot 11}}\right)$$

$$= 112 \times 167.668437878402 = 18778.865042381$$

⁷ ₃ Li ₄	^{10,11} ₅ B _{5,6}	^{12,13} ₆ C _{6,7}	^{14,15} ₇ N _{7,8}	^{20,21,22} ₁₀ Ne _{10,11,12}	²³ ₁₁ Na ₁₂	^{24,25,26} ₁₂ Mg _{12,13,14}	^{28,29,30} ₁₄ Si _{14,15,16}
³¹ ₁₅ P ₁₆	^{32,33} ₁₆ S _{16,17}	^{47,49} ₂₂ Ti _{25,27}	⁵⁵ ₂₅ Mn ₃₀	^{54,56,57,58} ₂₆ Fe _{28,30,31,32}	^{63,65} ₂₉ Cu _{34,36}	^{58,60,61,64} ₂₈ Ni _{30,32,33,36}	
^{64,66,68} ₃₀ Zn _{34,36,38}	^{69,71} ₃₁ Ga _{38,40}	^{70,72} ₃₂ Ge _{38,40}	⁷⁵ ₃₃ As ₄₂	^{83,84} ₃₆ Kr _{47,48}	^{98,99} ₄₃ Tc _{55,56} *	^{98,99,100} ₄₄ Ru _{54,55,56}	
^{107,109} ₄₇ Ag _{60,62}	^{110,112-114} ₄₈ Cd _{62,64-66}	^{113,115} ₄₉ In _{64,66}	¹³¹ ₅₄ Xe ₇₇	¹³³ ₅₅ Cs ₇₈	¹³⁷ ₅₆ Ba ₈₁	¹⁵⁸ ₆₄ Gd ₉₄	¹⁶⁸ ₆₈ Er ₁₀₀
¹⁶⁹ ₆₉ Tm ₁₀₀	¹⁷³ ₇₀ Yb ₁₀₃	^{185,187} ₇₅ Re _{110,112}	^{4-47,189} ₇₆ Os _{112,113}	²⁰⁹ ₈₃ Bi ₁₂₆ *	²⁰⁹ ₈₄ Po ₁₂₅ *	²⁴⁴ ₉₄ Pa ₁₅₀ *	²⁸⁵ ₁₁₂ Cn ₁₇₃ *
²⁸⁶ ₁₁₃ Nh ₁₇₃ ^{ie}	^{288,289} ₁₁₅ Mc _{173,174} ^{ie}	³¹² ₁₂₅ Ch ₁₈₇ ^{ie}	³¹⁴ ₁₂₆ Ch ₄₋₄₇ ^{ie}	²⁻¹⁷³ ₁₃₇ Fy ₂₀₉ ^{ie}	^{418,420} ₁₆₈ Ch _{250,252} ^{ie}	^{14-31,15-29} ₁₇₃ Ch _{261,2-131} ^{ie}	

10. Fibonacci Sequence He and its relationships with nuclides

We noticed $0.618 \times (112 + 168) = 0.618 \times 280 \approx 173$, so we construct the following

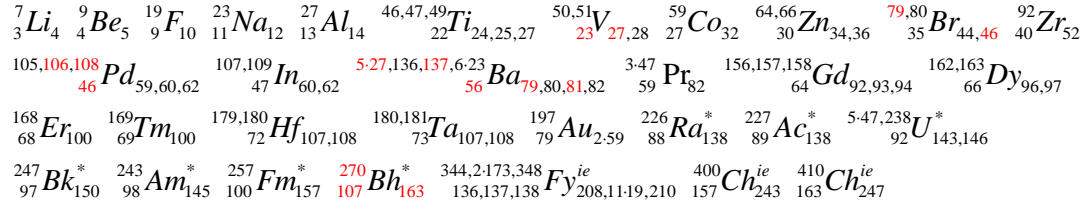
Fibonacci sequence and present its relationships with nuclides.

Fibonacci Sequence He: 2 7 9 16 25 41 66 107 173 280

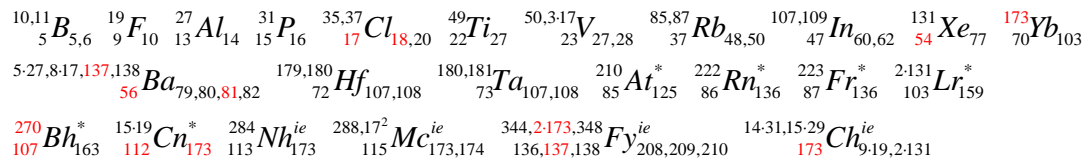
^{1,2} ₁ H _{0,1}	⁴ ₂ He ₂	⁷ ₃ Li ₄	⁹ ₄ Be ₅	^{14,15} ₇ N _{7,8}	^{16,17,18} ₈ O _{8,9,10}	¹⁹ ₉ F ₁₀	²⁵ ₁₂ Mg ₁₃	^{28,29,30} ₁₄ Si _{14,15,16}	³¹ ₁₅ P ₁₆
³² ₁₆ S ₁₆	⁴¹ ₁₉ K ₂₂	^{46,47,48,49,50} ₂₂ Ti _{24,25,26,27,28}	⁵⁵ ₂₅ Mn ₃₀	⁶⁶ ₃₀ Zn ₃₆	⁷³ ₃₂ Ge ₄₁	⁹³ ₄₁ Nb ₅₂	^{106,108} ₄₆ Pd _{60,62}		
^{107,109} ₄₇ Ag _{60,62}	^{106,108,112,113,114,116} ₄₈ Cd _{58,60,64,65,66,68}	^{113,115} ₄₉ In _{64,66}	¹¹⁶ ₅₀ Sn ₆₆	¹⁶³ ₆₆ Dy ₉₇	¹⁷³ ₇₀ Yb ₁₀₃				
¹⁷⁹ ₇₂ Hf ₁₀₇	¹⁸⁰ ₇₃ Ta ₁₀₇	²⁶² ₁₀₃ Lr ₁₅₉ *	²⁷⁰ ₁₀₇ Bh ₁₆₃ *	²⁸⁵ ₁₁₂ Cn ₁₇₃ *	²⁸⁶ ₁₁₃ Nh ₁₇₃ ^{ie}	²⁸⁸ ₁₁₅ Nh ₁₇₃ ^{ie}	^{434,435} ₁₇₃ Ch _{261,262} ^{ie}		

11. The Meanings of the Numerical Values of the Fine-structure Constant

$$\alpha_1 = \frac{1}{137.035999037435} = \frac{1}{137 + \frac{1}{27} - \frac{1}{9 \cdot 107} + \frac{1}{4 \cdot 79 \cdot (2 \cdot 23 \cdot 163 + 1)}}$$



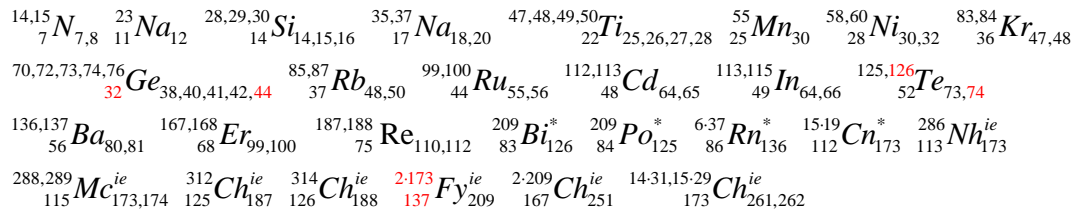
$$\alpha_2 = \frac{1}{137.035999111818} = \frac{1}{137 + \frac{1}{27} - \frac{1}{9 \cdot 107} + \frac{1}{5 \cdot 17 \cdot 137 \cdot 173}}$$



12. Construct Formulas of the Fine-structure Constant with 137 instead of 112 or 173

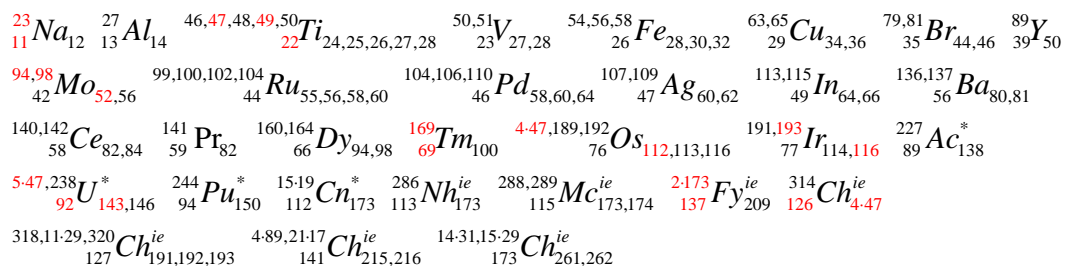
$$\alpha_{1-7(137)} = \frac{4 \cdot 11}{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot 126 - 1}{2 \cdot 125}\right)^{3 \cdot 167}}} \frac{1}{137 + \frac{1}{37 \cdot 173 - \frac{5}{32}}}$$

$$= 1/137.035999037435$$



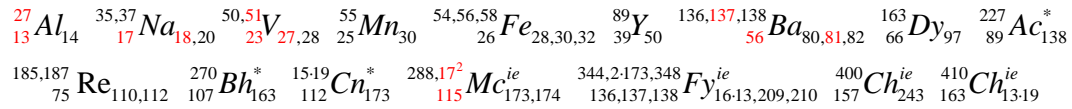
$$\alpha_{1-39(137)} = \frac{5 \cdot 7^2}{39 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{4 \cdot 3 \cdot 13^2 - 1}{2 \cdot (4 \cdot 11 \cdot 23 + 1)}\right)^{3 \cdot 7 \cdot 193}}} \frac{1}{137 + \frac{1}{13 \cdot 29 \cdot (4 \cdot 3 \cdot 5 \cdot 47 - 1)}}$$

$$= 1/137.035999037435$$



$$\alpha_{1-46(137)} = \frac{17^2}{2 \cdot 23 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{3 \cdot 13 \cdot 25}{2 \cdot (2 \cdot 3^5 + 1)}\right)^{1949}}} \frac{1}{137 + \frac{1}{2 \cdot 5 \cdot 13 \cdot (4 \cdot 5 \cdot 163 - 1) - \frac{7}{8}}}$$

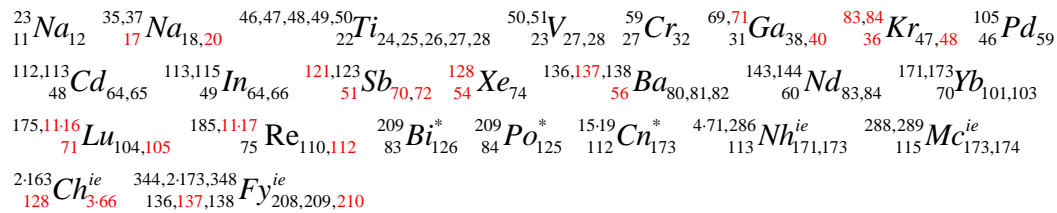
$$= 1/137.035999037435$$



$$\alpha_{1-71(137)}$$

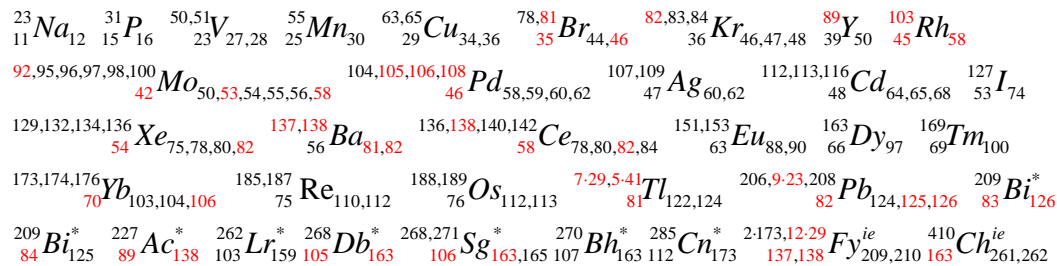
$$= \frac{2 \cdot (2 \cdot 112 - 1)}{71 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{6 \cdot (6 \cdot 11 \cdot 17 + 1) - 1}{16 \cdot (4 \cdot 3 \cdot 5 \cdot 7 + 1)}\right)^{27 \cdot (6 \cdot 83 + 1)}}} \frac{1}{137 + \frac{1}{256 \cdot 11 \cdot (8 \cdot 27 \cdot 7 - 1)}}$$

$$= 1/137.035999037435$$



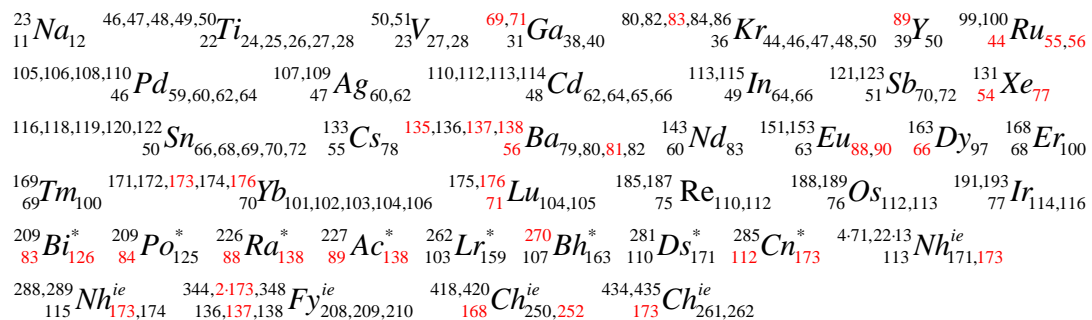
$$\alpha_{1-103(137)} = \frac{8 \cdot 81 - 1}{103 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot 29 \cdot (6 \cdot 7 \cdot 23 + 1)}{15 \cdot (6 \cdot 7 \cdot 89 + 1)}\right)^{23 \cdot (4 \cdot 23 \cdot 53 + 1)}}} \frac{1}{137 + \frac{2 \cdot 41 \cdot 163 + 1}{125 \cdot 10^{11}}}$$

$$= 1/137.035999037435$$



$$\alpha_{1-110(137)} = \frac{4 \cdot 173 - 1}{110 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot (168 \cdot 11 - 1)}{3 \cdot (112 \cdot 11 - 1)}\right)^{83 \cdot 89}}} \frac{1}{137 + \frac{1}{3 \cdot 23 \cdot 71 \cdot (270 - 1)}}$$

$$= 1/137.035999037435$$



$$\alpha_{2-7(137)} = \frac{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{6 \cdot 199}{8 \cdot 149 + 1}\right)^{7 \cdot 11 \cdot 31}}}{4 \cdot 11} \frac{1}{137 - \frac{1}{17 \cdot (4 \cdot 5 \cdot 7 \cdot 31 + 1) - \frac{13}{3 \cdot 19}}}$$

$$= \frac{7 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{6 \cdot 199}{8 \cdot 149 + 1}\right)^{7 \cdot 11 \cdot 31}}}{4 \cdot 11} \frac{1}{137 - \frac{1}{17 \cdot (2 \cdot 13 \cdot 167 - 1) - \frac{13}{3 \cdot 19}}}$$

$$= 1/137.035999111818$$

¹⁹ F ₁₀	²³ Na ₁₁	²⁷ Al ₁₃	^{28,29,30} Si _{14,15,16}	^{35,37} Cl _{17,18,20}	^{39,40,41} K _{19,20,21,22}	⁴⁸ Ti ₂₂	^{54,56,57,58} Fe _{26,28,30,31,32}
^{58,60,61,62,64} Ni _{28,30,32,33,34,36}	^{69,71} Ga _{31,38,40}	^{76,77,78,80} Se _{34,42,43,44,46}	^{79,81} Br _{35,44,46}	^{83,84} Kr _{36,47,48}	⁸⁹ Y ₃₉		
^{99,100,101,102,104} Ru _{44,55,56,57,58,60}	^{110,112,113,114,116} Cd _{48,62,64,65,66,68}	^{113,115} In _{49,64,66}	^{121,123} Sb _{51,70,72}				
^{122,124,128,130} Te _{52,70,72,76,78}	^{130,131,132} Xe _{54,76,77,78}	^{136,137} Ba _{56,80,81}	¹⁴⁹ Sm _{62,87}	^{167,168} Er _{68,99,100}			
^{171,172,173,174,176} Yb _{70,101,102,103,104,106}	^{185,187} Re _{75,110,112}	^{186,187,188,189} Os _{76,110,111,112,113,114}	^{191,193} Ir _{77,114,116}				
^{192,195,196} Pt _{78,114,117,118}	^{199,204} Hg _{80,7,17,124}	²⁰⁹ Bi _{83,126}	²⁰⁹ Bi _{84,125}	²²⁷ Ac _{89,138}	¹⁵⁻¹⁹ Cn _{112,173}	^{284,22-13} Nh _{113,9-19,173}	
³¹⁰ Ch _{124,6-31}	^{326,328} Ch _{129,197,199}	^{344,2-173} Fy _{136,137,208,209}	³⁷⁶ Ch _{149,227}	²²⁻¹⁹ Ch _{167,251}	^{14-31,435} Ch _{173,9-19,262}		

13. The Meanings of the Numerical Value of the Speed of Light in Atomic Unites

$$c_{au} = 137.035999074627 = 137 + \frac{1}{27} - \frac{1}{9 \cdot 107} + \frac{1}{8 \cdot 137 \cdot (4 \cdot 7 \cdot 71 - 1)}$$

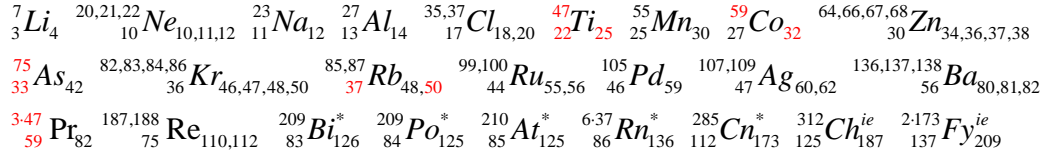
$$c_{au} = 137.035999074627 = 137 + \frac{1}{27} - \frac{1}{9 \cdot 107} + \frac{1}{3 \cdot 17 \cdot (4 \cdot 7 \cdot 25 \cdot 61 + 1)}$$

⁷ Li ₃	⁹ Be ₄	^{10,11} B _{5,6}	^{12,13} C _{6,7}	^{14,15} N _{7,8}	^{16,17,18} O _{8,9,10}	¹⁹ F ₉	^{20,21,22} Ne _{10,11,12}	²⁷ Al ₁₃	^{28,29,30} Si _{14,15,16}
³¹ P ₁₅	^{32,33,34,36} S _{16,17,18,20}	^{35,37} Cl _{17,18,20}	^{39,40,41} K _{19,20,21,22}	⁴⁵ Sc ₂₁	^{47,49} Ti _{22,25,27}	^{50,51} V _{23,27,28}	⁵⁵ Mn ₂₅	⁵⁹ Co ₂₇	³²
^{58,60,61,62,64} Ni _{28,30,32,33,34,36}	^{63,65} Cu _{29,34,36}	^{64,66,68} Zn _{30,34,36,38}	^{69,71} Ga _{31,38,40}	^{70,72,74} Ge _{32,38,40,42}	^{79,81} Br _{35,44,46}				
^{83,84,86} Kr _{36,47,48,50}	^{85,87} Rb _{37,48,50}	⁸⁹ Y ₃₉	^{96,97,98} Mo _{42,54,55,56}	^{97,98,99} Tc _{43,54,55,56}	¹⁰³ Rh _{45,58}	^{105,106,108} Pd _{46,59,60,62}			
^{96,98,99,100,102,104} Ru _{44,52,54,55,56,58,60}	^{107,109} Ag _{47,60,62}	^{112,113} Cd _{48,64,65}	^{113,115} In _{49,64,66}	^{118,119,122} Sn _{50,68,69,72}	¹³¹ Xe _{54,77}				
^{121,123} Sb _{51,70,72}	^{5-27,8-17,137,138} Ba _{56,79,80,81,82}	^{140,142} Ce _{58,82,84}	^{145,146,147} Pm _{61,84,85,86}	¹⁵⁷ Gd _{64,93}	^{162,163} Dy _{66,96,97}				
¹⁶⁸ Er _{68,100}	¹⁶⁹ Tm _{69,100}	^{170,171,172,173,174} Yb _{70,100,101,102,103,104}	^{175,176} Lu _{71,104,105}	^{3-59,179,180} Hf _{72,105,107,108}	^{185,187} Re _{75,110,112}				
^{188,189} Os _{76,112,113}	^{203,205} Tl _{81,122,124}	²⁰⁹ Bi _{83,126}	²⁰⁹ Po _{84,125}	²¹⁰ At _{85,125}	²²² Rn _{86,136}	²²³ Fr _{87,136}	²²⁶ Ra _{88,138}	²²⁷ Ac _{89,138}	
²³² Th _{90,142}	²⁴⁷ Bk _{97,150}	²⁵⁷ Fm _{100,157}	²⁵⁸ Md _{101,157}	²⁵⁹ No _{102,157}	²⁶² Lr _{103,159}	²⁶⁸ Db _{105,163}	^{269,271} Sg _{106,163,165}	²⁷⁰ Bh _{107,163}	
¹⁵⁻¹⁹ Cn _{112,173}	^{4-71,286} Nh _{113,171,173}	^{288,289} Mc _{115,173,174}	³⁰⁴ Ch _{122,182}	^{344,2-173,348} Fy _{136,137,138,208,209,210}	^{2-179,360} Ch _{142,216,218}	¹⁴⁻²⁷ Ch _{150,228}			
⁴⁰⁰ Ch _{157,243}	⁴¹⁰ Ch _{163,247}	⁶⁻⁷¹ Ch _{169,257}	^{33-13,430} Ch _{171,6-43,7-37}	^{430,16-27} Ch _{172,6-43,260}	^{14-31,15-29} Ch _{173,9-29,262}				

14. Other Formulas of the Fine-structure Constant with 137

$$\alpha_{1-7(137)\text{-Wallis}} = \frac{11}{7 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{750}{751} \frac{16 \cdot 47}{2 \cdot 3 \cdot 125 + 1}\right)} \frac{1}{137 + \frac{1}{13 \cdot 37 \cdot 59 - \frac{4 \cdot 11}{3 \cdot 25}}}$$

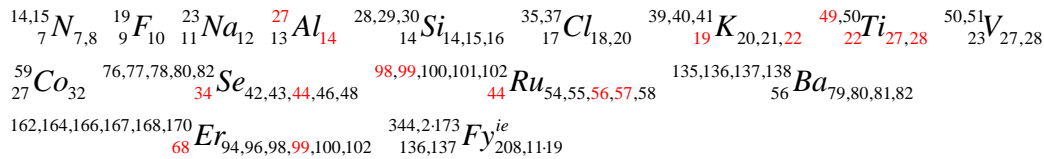
$$= 1/137.035999037435$$



$$\alpha_{1-7(137)\text{-GL}} = \frac{11}{2 \cdot 7 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot (2 \cdot 7 \cdot 17 + 1) + 1}\right)} \frac{1}{4 \cdot 11 \cdot (4 \cdot 7^2 + 1) + \frac{19}{27}}$$

$$= \frac{11}{2 \cdot 7 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot (2 \cdot 7 \cdot 17 + 1) + 1}\right)} \frac{1}{4 \cdot 11 \cdot (2 \cdot 9 \cdot 11 - 1) + \frac{19}{27}}$$

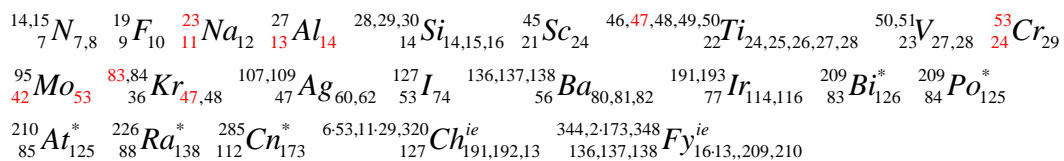
$$= 1/137.035999037435$$



$$\alpha_{1-7(137)\text{-NC}} = \frac{4 \cdot 11}{7 \cdot (2\pi)_{\text{NC-5}}} \frac{1}{137 + \frac{1}{23} - \frac{1}{2 \cdot (4 \cdot 3 \cdot 5 \cdot 7 + 1)} + \frac{1}{2 \cdot 3 \cdot 7 \cdot (2 \cdot 5 \cdot 53 \cdot 83 + 1)}}$$

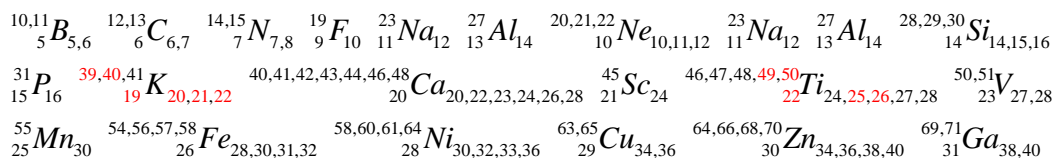
$$= \frac{4 \cdot 11}{7 \cdot (2\pi)_{\text{NC-5}}} \frac{1}{137 + \frac{1}{23} - \frac{1}{2 \cdot (4 \cdot 3 \cdot 5 \cdot 7 + 1)} + \frac{1}{2 \cdot 3 \cdot 7 \cdot (8 \cdot 9 \cdot 13 \cdot 47 + 1)}}$$

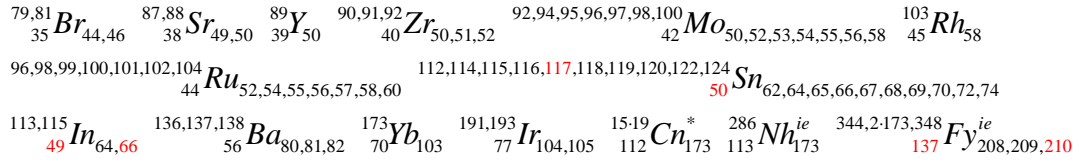
$$= 1/137.035999037435$$



$$\alpha_{1-39(137)\text{-Wallis}} = \frac{5 \cdot 7^2}{4 \cdot 39 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{6080}{6081} \frac{2 \cdot (2 \cdot 9 \cdot 13^2 - 1)}{2 \cdot 32 \cdot 5 \cdot 19 + 1}\right)} \frac{1}{2 \cdot 3 \cdot 5^2 \cdot 7 \cdot 11 - \frac{1}{10}}$$

$$= 1/137.035999037435$$

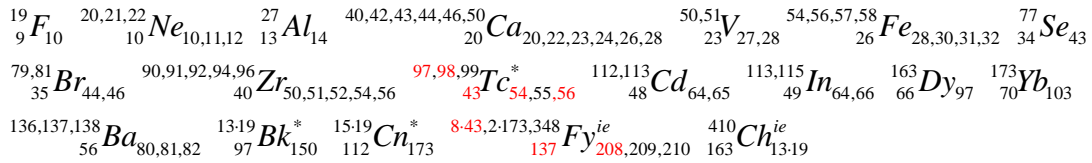




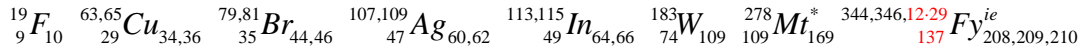
$$\alpha_{1-39(137)-GL} = \frac{5 \cdot 7^2}{8 \cdot 39 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 9 \cdot 5 \cdot 43 + 1}\right)} \frac{1}{137 + \frac{1}{8 \cdot 5 \cdot 7 \cdot (2 \cdot 5 \cdot 23 \cdot 27 - 1)}}$$

$$= \frac{5 \cdot 7^2}{8 \cdot 39 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 9 \cdot 5 \cdot 43 + 1}\right)} \frac{1}{137 + \frac{1}{8 \cdot 5 \cdot 7 \cdot (64 \cdot 97 + 1)}}$$

$$= 1/137.035999037435$$



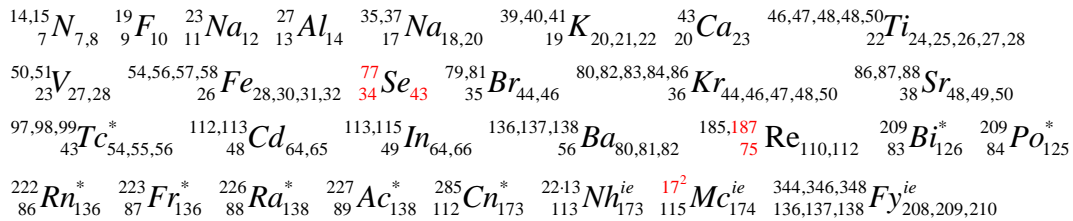
$$\alpha_{1-39(137)-NC} = \frac{5 \cdot 7^2}{39 \cdot (2\pi)_9} \frac{1}{137 + \frac{109 \cdot (12 \cdot 29 + 1)}{4 \cdot 10^{11}}} = 1/137.035999037435$$



$$\alpha_{1-46(137)-Wallis}$$

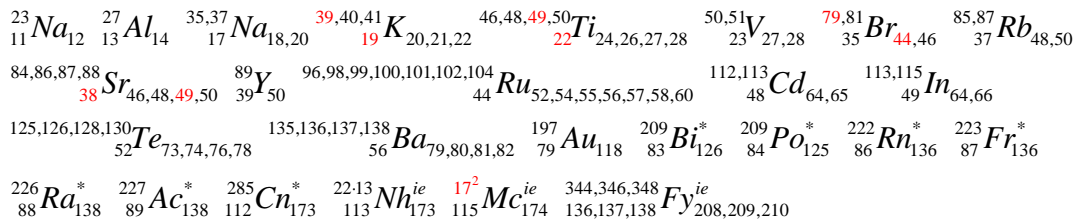
$$= \frac{17^2}{8 \cdot 23 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{2924}{9 \cdot 25 \cdot 13} \frac{2 \cdot 7 \cdot 11 \cdot 19}{2 \cdot 2 \cdot 17 \cdot 43 + 1}\right)} \frac{1}{137 + \frac{1}{4 \cdot 3 \cdot 11 \cdot 17 \cdot 43 - \frac{13}{17}}}$$

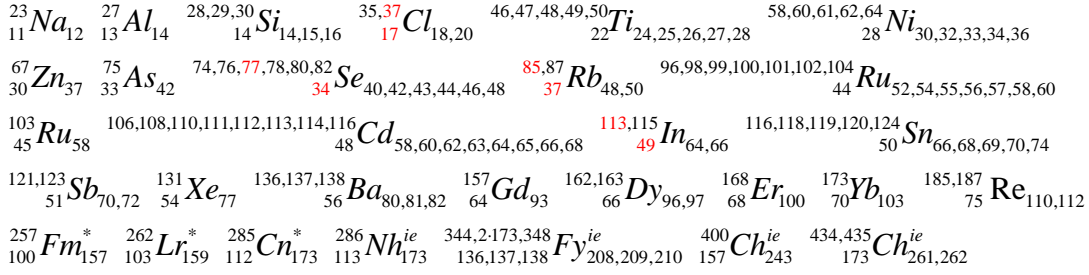
$$= 1/137.035999037435$$



$$\alpha_{1-46(137)-GL} = \frac{17^2}{16 \cdot 23 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 49 \cdot 19 + 1}\right)} \frac{1}{137 + \frac{1}{16 \cdot 3 \cdot 17 \cdot 79 - \frac{3 \cdot 13}{4 \cdot 11}}}$$

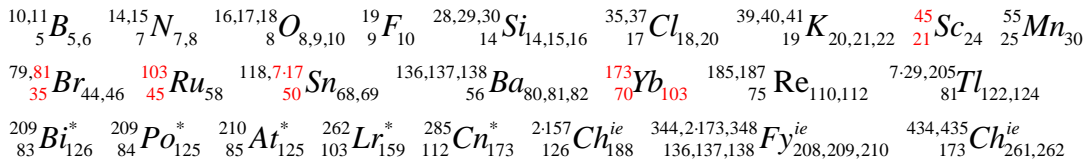
$$= 1/137.035999037435$$



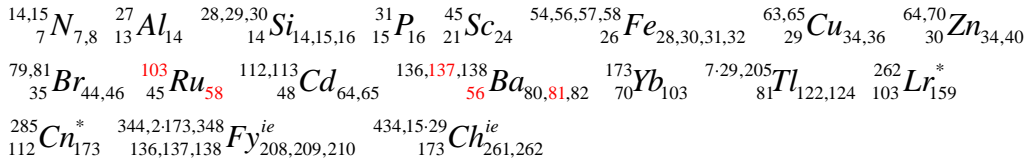


$$\alpha_{1-103(137)-GL} = \frac{8 \cdot 81 - 1}{8 \cdot 103 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot (2 \cdot 9 \cdot 25 \cdot 7 \cdot 17 + 1)}\right)} \frac{1}{137 + \frac{19 \cdot 173}{5 \cdot 10^{11}}}$$

$$= 1/137.035999037435$$



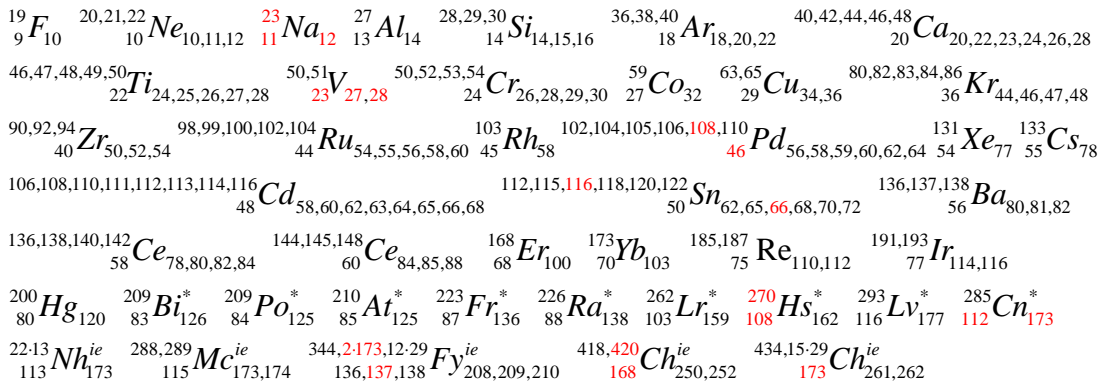
$$\alpha_{1-103(137)-NC} = \frac{8 \cdot 81 - 1}{103 \cdot (2\pi)_{NC-29}} \frac{1}{137 + \frac{1}{32 \cdot 3 \cdot 7 \cdot 13 \cdot 29 + \frac{2}{15}}} = 1/137.035999037435$$



$\alpha_{1-110(137)-Wallis}$

$$= \frac{4 \cdot 173 - 1}{4 \cdot 110 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{11080}{11081} \frac{2 \cdot 3 \cdot (168 \cdot 11 - 1)}{2 \cdot 8 \cdot 5 \cdot (12 \cdot 23 + 1)}\right)} \frac{1}{137 + \frac{1}{4 \cdot 27 \cdot 29 \cdot (420 + 1)}}$$

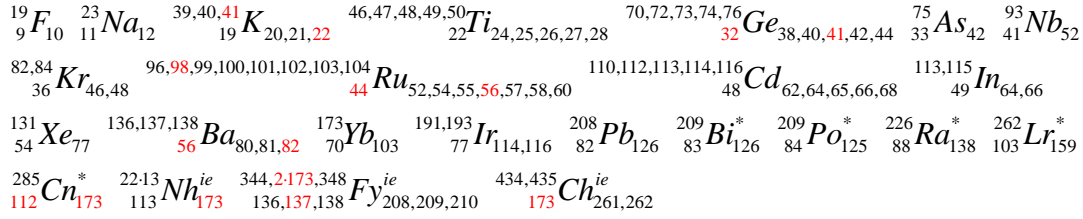
$$= 1/137.035999037435$$



$\alpha_{1-110(137)-GL}$

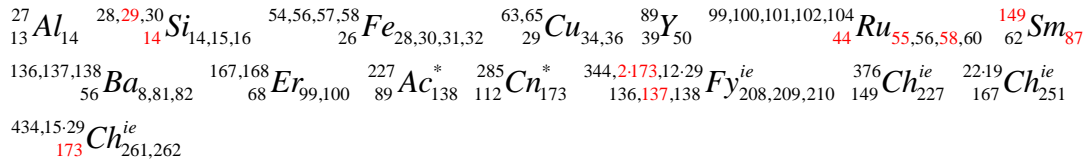
$$= \frac{4 \cdot 173 - 1}{8 \cdot 110 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot (8 \cdot 9 \cdot 49 - 1)}\right)} \frac{1}{137 + \frac{1}{3 \cdot 7 \cdot 11 \cdot (8 \cdot 3 \cdot 5 \cdot 41 - 1)}}$$

$$= 1/137.035999037435$$



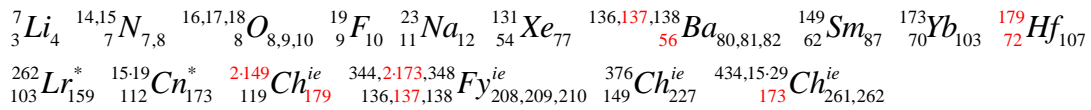
$$\alpha_{1-110(137)-NC} = \frac{4 \cdot 173 - 1}{110 \cdot (2\pi)_{NC-13}} \frac{1}{137 + \frac{1}{3 \cdot 149} - \frac{1}{29 \cdot (2 \cdot 3 \cdot 13 \cdot 137 + 1) + \frac{4}{7}}}$$

$$= \frac{4 \cdot 173 - 1}{110 \cdot (2\pi)_{NC-13}} \frac{1}{137 + \frac{1}{3 \cdot 149} - \frac{1}{29 \cdot (64 \cdot 167 - 1) + \frac{4}{7}}} = 1/137.035999037435$$



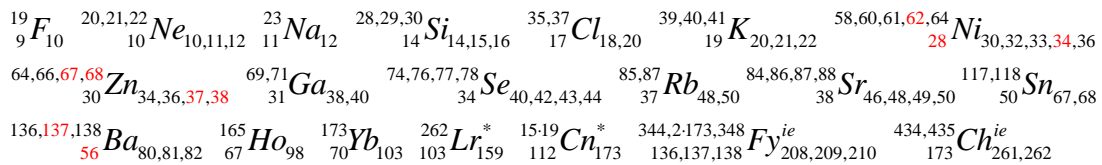
$$\alpha_{2-7(137)-wallis} = \frac{7 \cdot (2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{3580}{3579} \frac{4 \cdot 5 \cdot 179}{2 \cdot (4 \cdot 3 \cdot 149 + 1) + 1})}{11} \frac{1}{137 - \frac{1}{32 \cdot 9 \cdot 7 \cdot 173 + 1}}$$

$$= 1/137.035999111818$$

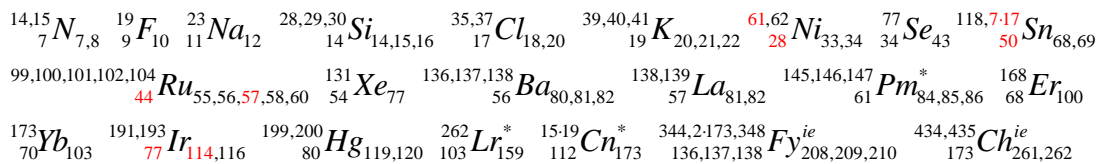


$$\alpha_{2-7(137)-GL} = \frac{2 \cdot 7 \cdot (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 2 \cdot 17 \cdot 67 + 1})}{11} \frac{1}{137 - \frac{1}{4 \cdot 7 \cdot 19 \cdot (2 \cdot 9 \cdot 31 - 1) - \frac{10}{37}}}$$

$$= 1/137.035999111818$$



$$\alpha_{2-7(137)-NC} = \frac{7 \cdot (2\pi)_{NC-9}}{4 \cdot 11} \frac{1}{137 - \frac{1}{7 \cdot 17} + \frac{1}{3 \cdot 19 \cdot (8 \cdot 61 - 1) - \frac{61}{100}}} = 1/137.035999111818$$



15. Formulas of the Fine-structure Constant with 83

As the 83th element $_{83}\text{Bi}$ should be the end of stable elements and the start of radioactive elements (except $_{43}\text{Tc}$ and $_{61}\text{Pm}$) in the periodic table of elements, some formulas of the fine-structure constant could be constructed with the factor 83 instead of 112, 173 and 137 as follows.

$$\alpha_{1-31(83)} = \frac{2 \cdot 59}{31 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{9 \cdot 83}{2 \cdot (2 \cdot 11 \cdot 17 - 1)}\right)^{1493}}} \frac{1}{83 + \frac{1}{3 \cdot 29 \cdot (6 \cdot 83 - 1) - \frac{19}{2 \cdot 37}}}$$

$$= \frac{2 \cdot 59}{31 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{9 \cdot 83}{2 \cdot (12 \cdot 31 + 1)}\right)^{1493}}} \frac{1}{83 + \frac{1}{3 \cdot 29 \cdot (16 \cdot 31 + 1) - \frac{19}{2 \cdot 37}}}$$

$$= 1/137.035999037435$$

$_{9}^{19}\text{F}$ $_{11}^{23}\text{Na}$ $_{12}^{31}\text{P}$ $_{15}^{35,37}\text{Cl}$ $_{17}^{18,20}$ $_{18}^{36,38,40}\text{Ar}$ $_{18,20,22}^{39,40,41}\text{K}$ $_{19}^{46,47,48,49,50}\text{Ti}$ $_{20,21,22}^{59}\text{Co}$ $_{22}^{54,56,57,58}\text{Fe}$ $_{26}^{28,30,31,32}\text{Co}$ $_{29}^{63,65}\text{Zn}$ $_{30}^{64,66,67,68}\text{Ga}$ $_{31}^{69,71}\text{Se}$ $_{34}^{74,76,77,78,80,82}\text{Kr}$ $_{36}^{82,83,84}\text{Rb}$ $_{37}^{85,87}\text{Sr}$ $_{48,50}^{84,86,87,88}\text{Ru}$ $_{44}^{99,100,102}\text{Pd}$ $_{46}^{105,107}\text{Ag}$ $_{47}^{107,109}\text{Cd}$ $_{48}^{62,63,64,65,66,68}\text{Sn}$ $_{50}^{2 \cdot 59}\text{Te}$ $_{52}^{126}\text{Ba}$ $_{56}^{136,137,138}\text{Ce}$ $_{58}^{140,142}\text{Pr}$ $_{59}^{3 \cdot 47}\text{Nd}$ $_{60}^{143}\text{Sm}$ $_{62}^{149}\text{Er}$ $_{68}^{166,167,168,170}\text{Yb}$ $_{70}^{173}\text{W}$ $_{74}^{184,6 \cdot 31}\text{Re}$ $_{75}^{185,11 \cdot 17}\text{Bi}$ $_{77}^{209}\text{Po}^*$ $_{83}^{285}\text{Cn}^*$ $_{112}^{286}\text{Nh}^{ie}$ $_{113}^{294}\text{Og}^{ie}$ $_{116}^{344,2 \cdot 173,12 \cdot 29}\text{Fy}^{ie}$

$$\alpha_{1-31(83)\text{-Wallis}} = \frac{59}{2 \cdot 31 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{2238}{2239} \frac{4 \cdot 5 \cdot 112}{2 \cdot 3 \cdot (2 \cdot 11 \cdot 17 - 1) + 1}\right)} \frac{1}{83 + \frac{1}{2 \cdot 83 \cdot (3 \cdot 128 - 1) - \frac{5}{11}}}$$

$$= 1/137.035999037435$$

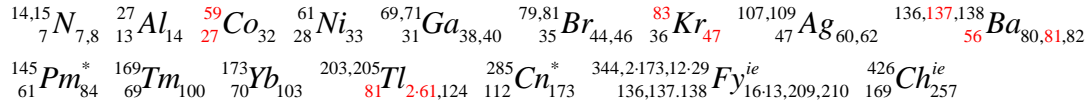
$_{5}^{10,11}\text{B}$ $_{5,6}^{20,21,22}\text{Ne}$ $_{10,11,12}^{23}\text{Na}$ $_{11}^{112}\text{Cd}$ $_{48}^{185,11 \cdot 17}\text{Re}$ $_{75}^{285}\text{Cn}^*$ $_{112}^{344,2 \cdot 173,12 \cdot 29}\text{Fy}^{ie}$

$$\alpha_{1-31(83)\text{-GL}} = \frac{59}{4 \cdot 31 \cdot \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2 \cdot 16 \cdot 89 + 1}\right)} \frac{1}{83 + \frac{1}{13 \cdot 17 \cdot (14 \cdot 73 - 1) - \frac{13}{4 \cdot 7}}}$$

$$= 1/137.035999037435$$

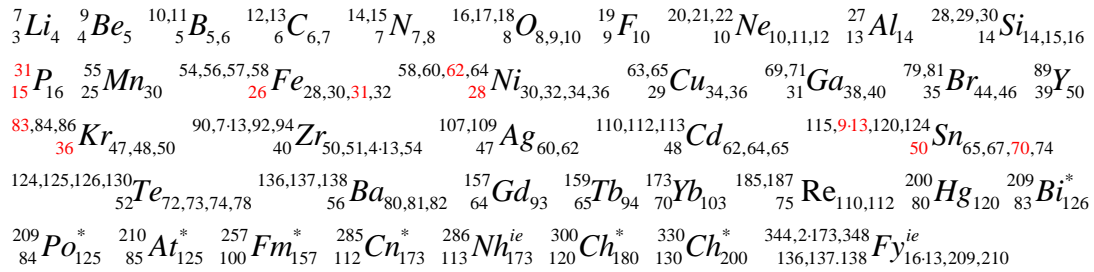
$_{13}^{27}\text{Al}$ $_{14}^{28,29,30}\text{Si}$ $_{14,15,16}^{54,56,57,58}\text{Fe}$ $_{26}^{59}\text{Co}$ $_{27}^{58,60,62,64}\text{Ni}$ $_{28}^{73}\text{Ge}$ $_{30,32,34,36}^{89}\text{Y}$ $_{32}^{125}\text{Te}$ $_{32}^{136,137,138}\text{Ba}$ $_{56}^{151,153}\text{Eu}$ $_{63}^{227}\text{Ac}^*$ $_{89}^{285}\text{Cn}^*$ $_{112}^{24 \cdot 13}\text{Ch}^{ie}$ $_{117}^{344,2 \cdot 173,12 \cdot 29}\text{Fy}^{ie}$

$$\alpha_{1-31(83)-NC} = \frac{2 \cdot 59}{31 \cdot (2\pi)_{NC-7}} \frac{1}{83 + \frac{1}{13^2} - \frac{1}{2 \cdot (4 \cdot 81 \cdot 47 - 1) - \frac{47}{61}}} = 1/137.035999037435$$



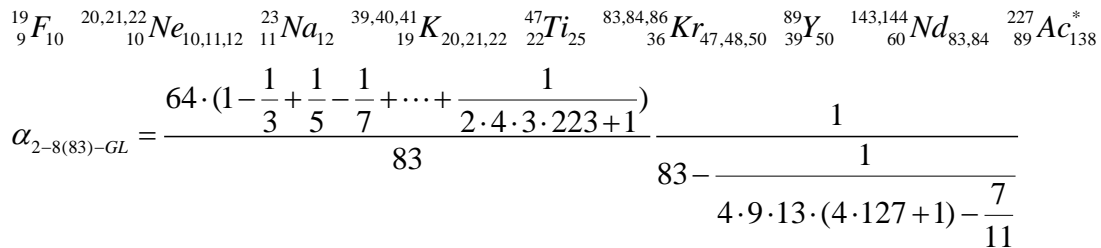
$$\alpha_{2-8(83)} = \frac{8 \cdot e^2 \frac{e^2}{\left(\frac{2}{1}\right)^3} \frac{e^2}{\left(\frac{3}{2}\right)^5} \frac{e^2}{\left(\frac{4}{3}\right)^7} \dots \frac{e^2}{\left(\frac{2 \cdot (4 \cdot 25 \cdot 7 + 1)}{3 \cdot (4 \cdot 9 \cdot 13 - 1)}\right)^{2803}}}{83} \frac{1}{83 - \frac{1}{2 \cdot 31 \cdot (2 \cdot 3 \cdot 5 \cdot 7 \cdot 13 + 1) - \frac{1}{10}}}$$

$$= 1/137.035999111818$$

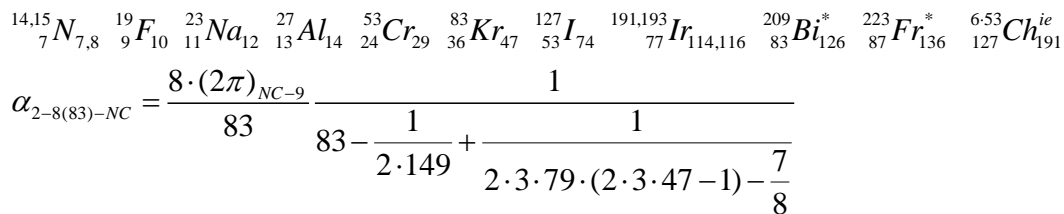


$$\alpha_{2-8(83)-Wallis} = \frac{32 \cdot \left(2 \frac{2}{3} \frac{4}{3} \frac{4}{5} \frac{6}{5} \frac{6}{7} \frac{8}{7} \dots \frac{4202}{4203} \frac{4 \cdot (2 \cdot 3 \cdot 25 \cdot 7 + 1)}{2 \cdot 11 \cdot (2 \cdot 5 \cdot 19 + 1)}\right)}{83} \frac{1}{83 - \frac{1}{4 \cdot 7 \cdot 89 \cdot (4 \cdot 83 - 1)}}$$

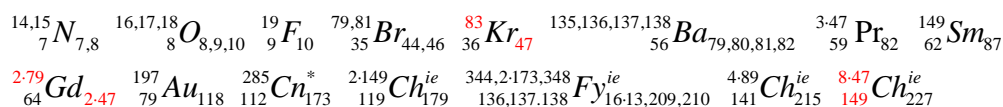
$$= 1/137.035999111818$$



$$= 1/137.035999111818$$



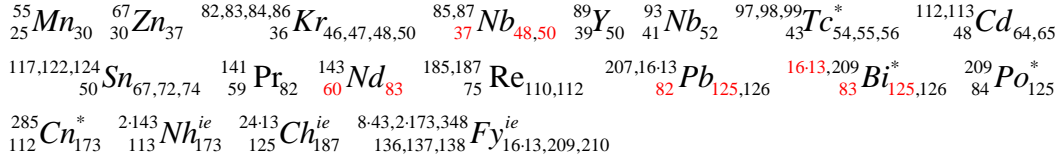
$$= 1/137.035999111818$$



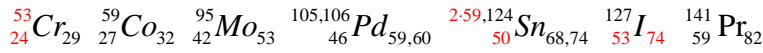
16. Formulas of the Fine-structure Constant with 83²

$$\alpha_{1-(50)/83^2} = \frac{50 + \frac{1}{3} - \frac{1}{16} + \frac{1}{43 \cdot 37} - \frac{4 \cdot 3 \cdot 5 \cdot 41 - 1}{125 \cdot 10^{10}}}{83^2} = 1/137.035999037435$$

Note: $\frac{1}{3} - \frac{1}{16} = \frac{13}{16 \cdot 3}$

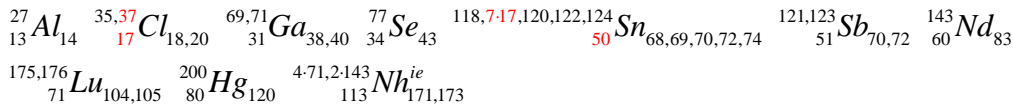


$$\alpha_{1-(50)/83^2} = \frac{50 + \frac{1}{3} - \frac{1}{16} + \frac{1}{43 \cdot 37 + \frac{59 \cdot (4 \cdot 53 - 1)}{25 \cdot 10^5}}}{83^2} = 1/137.035999037435$$

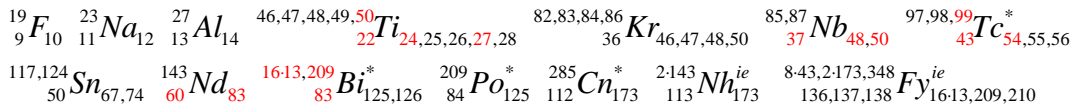


$$\alpha_{1-(50)/83^2} = \frac{50 + \frac{1}{3} - \frac{1}{16} + \frac{1}{43 \cdot 37 + \frac{1}{200 + \frac{2 \cdot 7}{17}}}}{83^2} = \frac{50 + \frac{1}{3} - \frac{1}{16} + \frac{1}{43 \cdot 37 + \frac{17}{2 \cdot 3 \cdot (8 \cdot 71 + 1)}}}{83^2}$$

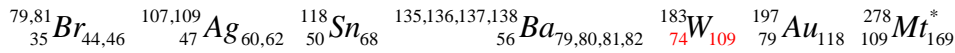
$$= 1/137.035999037435$$



$$\alpha_{2-(50)/83^2} = \frac{50 + \frac{1}{3} - \frac{1}{16} + \frac{1}{43 \cdot 37} - \frac{27 \cdot 11 \cdot (2 \cdot 9 \cdot 11 - 1)}{2 \cdot 10^{12}}}{83^2} = 1/137.035999111818$$

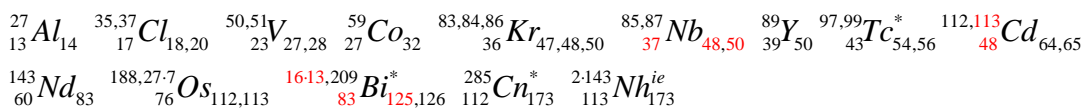


$$\alpha_{2-(50)/83^2} = \frac{50 + \frac{1}{3} - \frac{1}{16} + \frac{1}{43 \cdot 37 + \frac{43 \cdot 79 \cdot 109}{5 \cdot 10^6}}}{83^2} = 1/137.035999111818$$



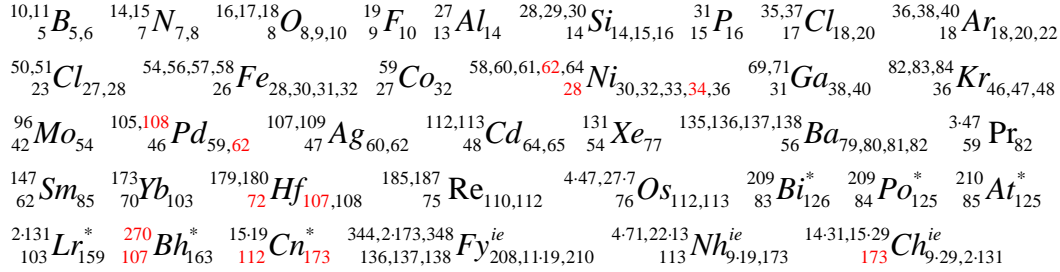
$$\alpha_{2-(50)/83^2} = \frac{50 + \frac{1}{3} - \frac{1}{16} + \frac{1}{43 \cdot 37 + \frac{2}{27 + \frac{7}{1000}}}}{83^2} = \frac{50 + \frac{13}{16 \cdot 3} + \frac{1}{43 \cdot 37 + \frac{16 \cdot 125}{113 \cdot (2 \cdot 7 \cdot 17 + 1)}}}{83^2}$$

$$= 1/137.035999111818$$

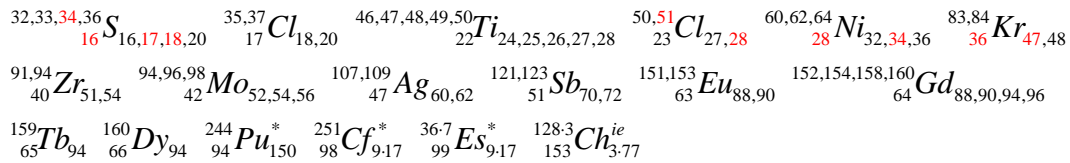


17. Formulas of the Fine-structure Constant with 112×173

$$\alpha_{1-(141)/112 \cdot 173} = \frac{3 \cdot 47 + \frac{1}{2} - \frac{1}{9} + \frac{1}{8 \cdot 27} - \frac{1}{2 \cdot 31 \cdot (2 \cdot 5 \cdot 107 - 1) - \frac{5}{17}}}{112 \cdot 173} = 1/137.035999037435$$



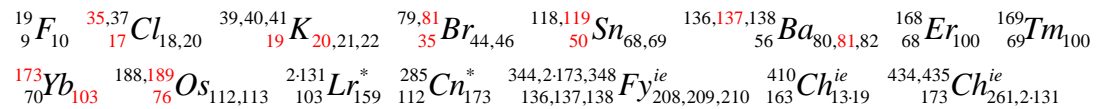
$$\alpha_{2-(141)/112 \cdot 173} = \frac{3 \cdot 47 + \frac{1}{2} - \frac{1}{9} + \frac{1}{8 \cdot 27} - \frac{1}{9 \cdot 17 \cdot (16 \cdot 27 - 1) - \frac{2 \cdot 17}{47}}}{112 \cdot 173} = 1/137.035999111818$$



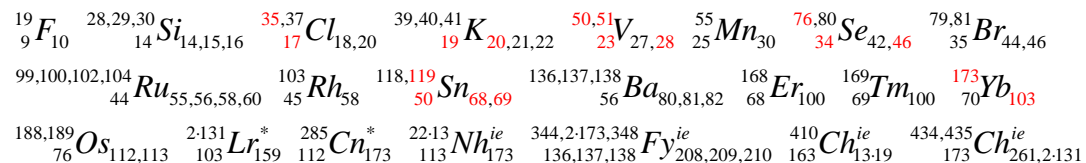
18. Formulas of the Fine-structure Constant with 163×173

$163 \times 173 \approx 167.9^2 \approx 168^2$, so in the world of nuclides, 163 should also be a sub-stable number, or 163 and 173 could be regarded as twin numbers split from 168, and hence some formulas of the fine-structure constant could be constructed with 163×173 as follows.

$$\alpha_{1-(206)/163 \cdot 173} = \frac{2 \cdot 103 + \frac{1}{4} - \frac{1}{5 \cdot 7} + \frac{1}{4 \cdot 25 \cdot 19 + \frac{2 \cdot 7 \cdot 17 - 1}{2 \cdot 81 \cdot 7}}}{163 \cdot 173} = 1/137.035999037435$$

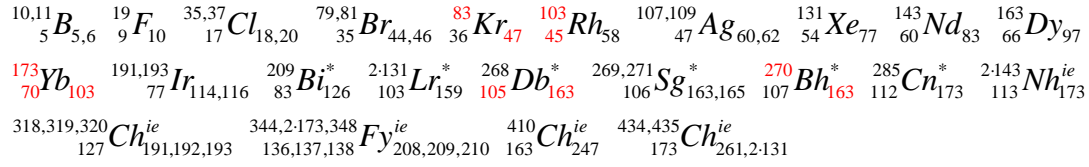


$$\alpha_{2-(206)/163 \cdot 173} = \frac{2 \cdot 103 + \frac{1}{4} - \frac{1}{5 \cdot 7} + \frac{1}{4 \cdot 25 \cdot 19 - \frac{2 \cdot 23}{2 \cdot 7 \cdot 17 - 1}}}{163 \cdot 173} = 1/137.035999111818$$



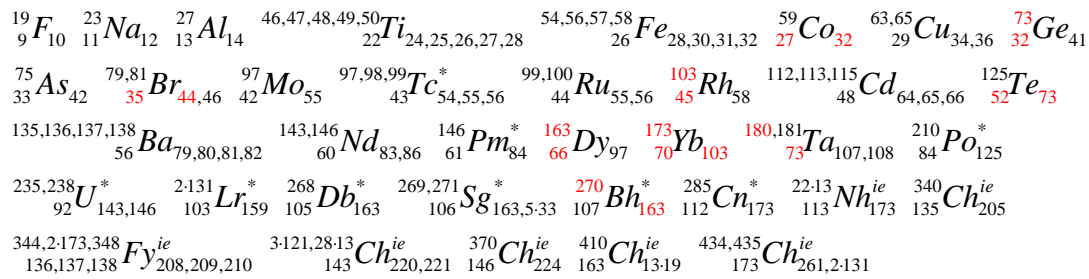
$$\alpha_{1-(206)/163-173} = \frac{2 \cdot 103 + \frac{1}{4} - \frac{1}{5 \cdot 7} + \frac{1}{9 \cdot (2 \cdot 3 \cdot 5 \cdot 7 + 1)} - \frac{1}{4 \cdot 47 \cdot 83 \cdot 191}}{163 \cdot 173}$$

$$= 1/137.035999037435$$



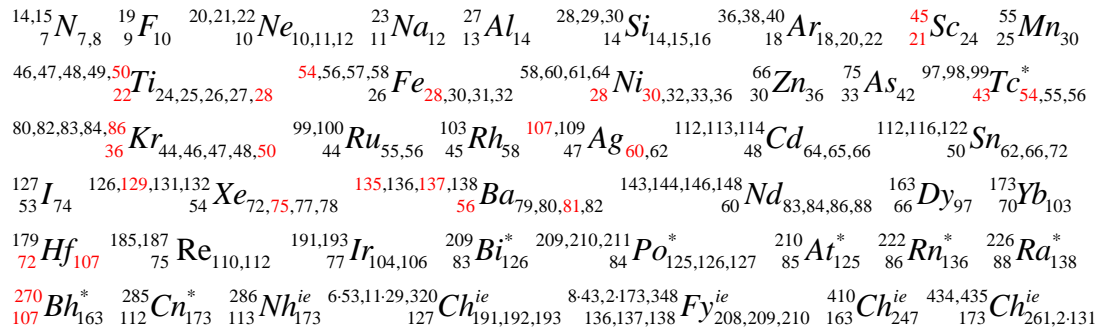
$$\alpha_{2-(206)/163-173} = \frac{2 \cdot 103 + \frac{1}{4} - \frac{1}{5 \cdot 7} + \frac{1}{2 \cdot 13 \cdot 73} - \frac{1}{27 \cdot 5 \cdot 7 \cdot (64 \cdot 3 \cdot 11 - 1)}}{163 \cdot 173}$$

$$= 1/137.035999111818$$



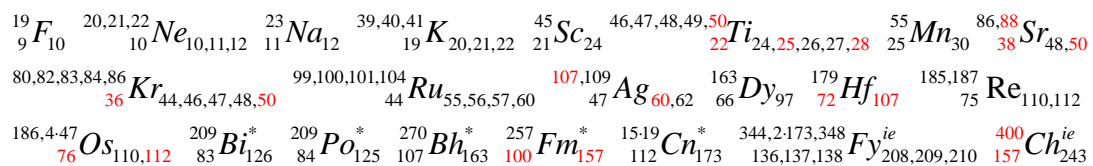
19. Formulas of the Fine-structure Constant with 36/112 and 100 × 112

$$\alpha_{1-36/(44) \cdot 112} = \frac{36}{(44 + \frac{1}{3 \cdot 7} - \frac{1}{4 \cdot 7 \cdot 107} + \frac{1}{2 \cdot 27 \cdot 25 \cdot 43 \cdot 127}) \cdot 112} = 1/137.035999037435$$

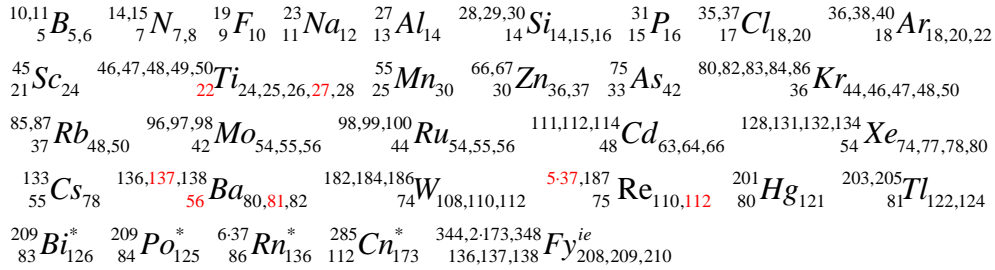


$$\alpha_{2-36/(44) \cdot 112} = \frac{36}{(44 + \frac{1}{3 \cdot 7} - \frac{1}{4 \cdot 7 \cdot 107} + \frac{1}{3 \cdot 7 \cdot 157 \cdot (4 \cdot 25 \cdot 19 + 1)}) \cdot 112}$$

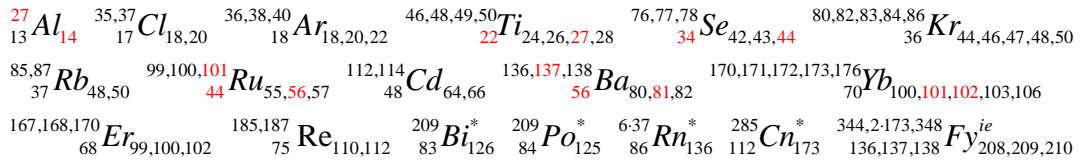
$$= 1/137.035999111818$$



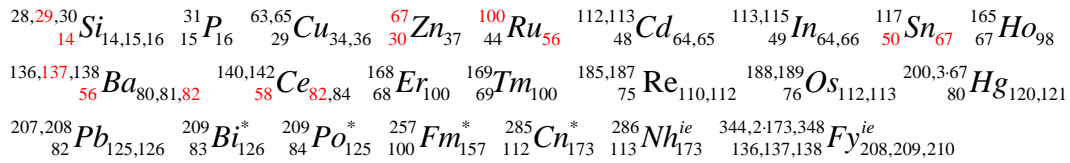
$$\alpha_{1-36/(44)\cdot 112} = \frac{36}{\left(44 + \frac{1}{3 \cdot 7} - \frac{1}{81 \cdot 37 + \frac{11^2}{3 \cdot 5 \cdot 37}}\right) \cdot 112} = 1/137.035999037435$$



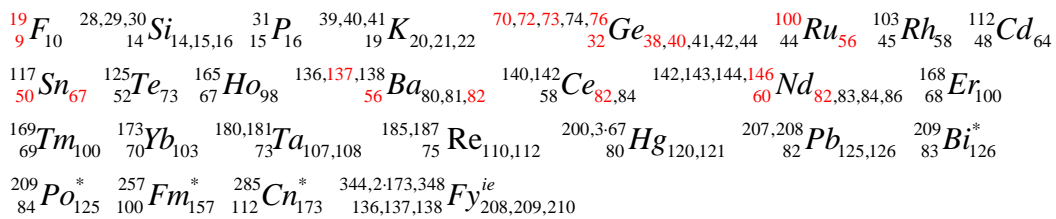
$$\alpha_{2-36/(44)\cdot 112} = \frac{36}{\left(44 + \frac{1}{3 \cdot 7} - \frac{1}{81 \cdot 37 + \frac{2 \cdot 9 \cdot 17}{7 \cdot 101}}\right) \cdot 112} = 1/137.035999111818$$



$$\alpha_{1-(82)/100\cdot 112} = \frac{82 - \frac{1}{3} + \frac{1}{3 \cdot 5} - \frac{1}{5 \cdot 67} + \frac{1}{2 \cdot 7 \cdot 29 \cdot (8 \cdot 5 \cdot 113 - 1)}}{100 \cdot 112} = 1/137.035999037435$$



$$\alpha_{2-(82)/100\cdot 112} = \frac{82 - \frac{1}{3} + \frac{1}{3 \cdot 5} - \frac{1}{5 \cdot 67} + \frac{1}{32 \cdot 9 \cdot 5 \cdot 19 \cdot 73}}{100 \cdot 112} = 1/137.035999111818$$



20. Other Formulas of the Fine-structure Constant with some square numbers

$$\alpha_{1-(29)/36\cdot 112} = \frac{29 + \frac{1}{2} - \frac{1}{12} + \frac{1}{3 \cdot 53} - \frac{1}{3 \cdot 19 \cdot (2 \cdot 17^2 - 1) - \frac{2 \cdot 19}{5 \cdot 37}}}{36 \cdot 112} = 1/137.035999037435$$

$$\alpha_{2-(29)/36\cdot 112} = \frac{29 + \frac{1}{2} - \frac{1}{12} + \frac{1}{3 \cdot 53} - \frac{1}{8 \cdot 7 \cdot (4 \cdot 3 \cdot 49 - 1) - \frac{33}{70}}}{36 \cdot 112} = 1/137.035999111818$$

$$\alpha_{1-81/(66) \cdot 112} = \frac{66 + \frac{1}{4} - \frac{1}{4 \cdot 5} + \frac{1}{2 \cdot 9 \cdot 5 \cdot 7 + 1} - \frac{1}{64 \cdot 13^2 \cdot 41 - \frac{1}{7}}}{81 \cdot 112} = 1/137.035999037435$$

$$\alpha_{2-81/(66) \cdot 112} = \frac{66 + \frac{1}{4} - \frac{1}{4 \cdot 5} + \frac{1}{2 \cdot 9 \cdot 5 \cdot 7 + 1} - \frac{1}{4 \cdot 9 \cdot 125 \cdot 97 + \frac{1}{8}}}{81 \cdot 112} = 1/137.035999111818$$

$$\alpha_{1-23^2/(8 \cdot 81) \cdot 112} = \frac{23^2}{(8 \cdot 81 - 1 + \frac{1}{3} - \frac{1}{12} + \frac{1}{23 \cdot 112 - \frac{5 \cdot 89}{2 \cdot 13 \cdot 23 + 1}}) \cdot 112} = 1/137.035999037435$$

$$\alpha_{2-23^2/(8 \cdot 81) \cdot 112} = \frac{23^2}{(8 \cdot 81 - 1 + \frac{1}{3} - \frac{1}{4 \cdot 3} + \frac{1}{31 \cdot 83 - \frac{3 \cdot 5}{4 \cdot 53}}) \cdot 112} = 1/137.035999111818$$

$$\alpha_{1-(82)/100 \cdot 112} = \frac{82 - \frac{1}{3} + \frac{1}{3 \cdot 5} - \frac{1}{5 \cdot 67} + \frac{1}{2 \cdot 7 \cdot 29 \cdot (8 \cdot 5 \cdot 113 - 1)}}{100 \cdot 112} = 1/137.035999037435$$

$$\alpha_{2-(82)/100 \cdot 112} = \frac{82 - \frac{1}{3} + \frac{1}{3 \cdot 5} - \frac{1}{5 \cdot 67} + \frac{1}{32 \cdot 9 \cdot 5 \cdot 19 \cdot 73}}{100 \cdot 112} = 1/137.035999111818$$

$$\alpha_{1-49/(60) \cdot 112} = \frac{49}{(60 - \frac{1}{3 \cdot 7} + \frac{1}{128 \cdot 9 - \frac{131}{173 + \frac{3 \cdot 19}{100^2}}}) \cdot 112}$$

$$= \frac{49}{(60 - \frac{1}{3 \cdot 7} + \frac{1}{2 \cdot 25 \cdot 23} - \frac{1}{64 \cdot 5 \cdot (256 \cdot 13 + 1)}) \cdot 112} = 1/137.035999037435$$

$$\alpha_{2-49/(60) \cdot 112} = \frac{49}{(60 - \frac{1}{3 \cdot 7} + \frac{1}{2 \cdot 25 \cdot 23} - \frac{1}{16 \cdot 7 \cdot 59 \cdot 167}) \cdot 112} = 1/137.035999111818$$

$$\alpha_{1-(5 \cdot 157)/31^2 \cdot 112} = \frac{5 \cdot 157 + \frac{1}{2} - \frac{1}{2 \cdot 7} + \frac{1}{2 \cdot 5 \cdot 17 \cdot 73 + \frac{50}{69}}}{31^2 \cdot 112} = 1/137.035999037435$$

$$\alpha_{2-(5 \cdot 157)/31^2 \cdot 112} = \frac{5 \cdot 157 + \frac{1}{2} - \frac{1}{2 \cdot 7} + \frac{1}{4 \cdot (16 \cdot 3 \cdot 5 \cdot 13 - 1) + \frac{37}{50}}}{31^2 \cdot 112} = 1/137.035999111818$$

21. Formulas of the Fine-structure Constant in Continued Fraction Form

Some natural constant such as φ , e and 2π can be expressed as continued fraction expansions as follows.

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}}$$

$$\varphi = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}} = 1.618\dots$$

$$e = 2 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{6 + \dots}}}}}}}} = 2.71828\dots$$

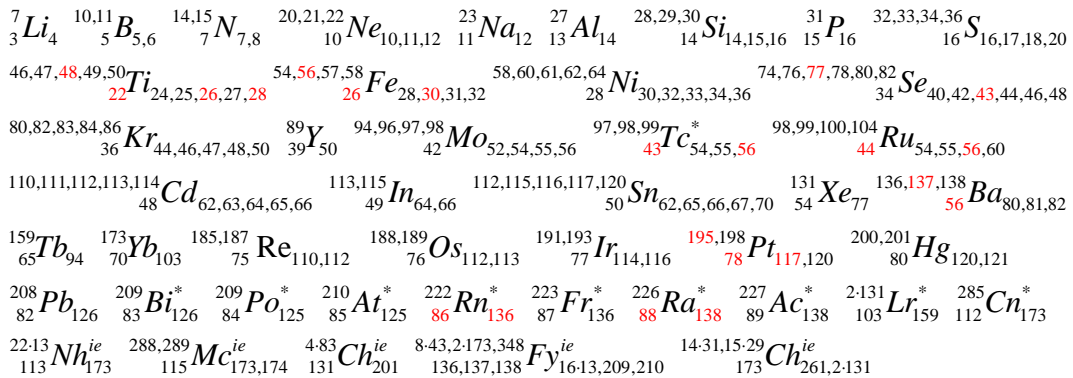
$$2\pi = 6 + \frac{2}{6 + \frac{9}{6 + \frac{25}{6 + \frac{49}{6 + \dots}}}} = 6.283185\dots$$

$$\frac{\pi}{2} = 1 + \frac{1}{1 + \frac{1}{1/2 + \frac{1}{1/3 + \frac{1}{1/4 + \frac{1}{\dots}}}}} = 1.570796\dots$$

As we have demonstrated the formulas of the fine-structure constant α are related to e , 2π and nuclides (φ is related to nuclides too), it should be reasonable to express α in continued fraction form as follows. When these continued fractions are calculated, they miraculously give some specific integers which should correspond to some typical nuclides.

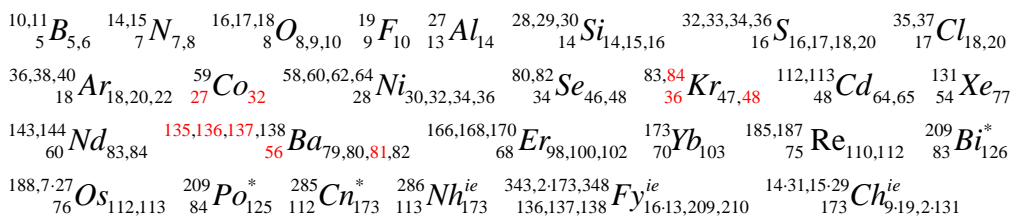
$$\alpha_1 = \frac{1}{137 + \frac{1}{28 - \frac{1}{4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{16 + \frac{1}{15 + \frac{1}{10 + \frac{1}{1 + \frac{1}{2}}}}}}}}}}}} = \frac{1}{137 + \frac{1}{28 - \frac{13 \cdot (112 \cdot 11 - 1)}{3 \cdot 5 \cdot (112 \cdot 43 + 1)}}}$$

$$= 1/137.035999037435$$



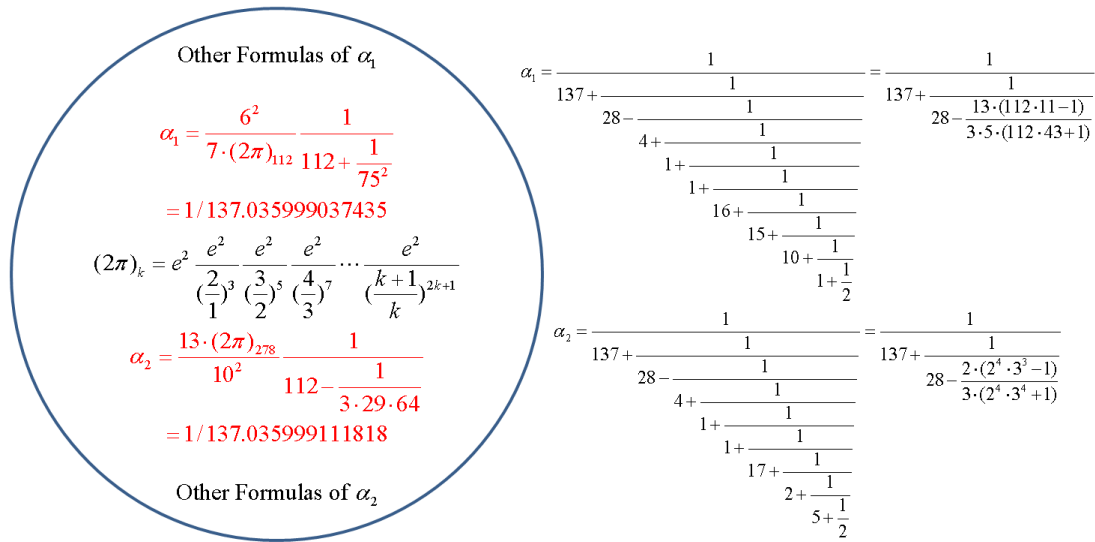
$$\alpha_2 = \frac{1}{137 + \frac{1}{28 - \frac{1}{4 + \frac{1}{1 + \frac{1}{1 + \frac{1}{17 + \frac{1}{2 + \frac{1}{5 + \frac{1}{2}}}}}}}}}} = \frac{1}{137 + \frac{1}{28 - \frac{2 \cdot (2^4 \cdot 3^3 - 1)}{3 \cdot (2^4 \cdot 3^4 + 1)}}}$$

$$= 1/137.035999111818$$



22. Set of Formulas of the Fine-structure Constant

We had found many formulas of the fine-structure constant with hard efforts for more than two years. In the end of this paper, we realized that all of these formulas should constitute a mathematical set in which some formulas exhibit much more importance. The set and the four most important formulas are depicted as follows (**Fig. 2**). The set is designed as a circle, the two originally found formulas should be close to the center of the circle, and the two last found continued fraction formulas should be on the circumference of the circle.



Set of Formulas of the Fine-structure Constant

Gang Chen, Tianman Chen, Tianyi Chen
2018/4/12-2020/8/18

Fig. 2

References

1. G. Chen, T-M. Chen and T-Y. Chen, viXra:2002.0203
(<https://vixra.org/abs/2002.0203>).
2. G. Chen, T-M. Chen and T-Y. Chen, Copyright Registration, Chirality and Poetry Model of Atomic Nuclei, GuoZuoDengZi-2018-L-00421847.

Acknowledgements

Yichang Huifu Silicon Material Co., Ltd., Guangzhou Huifu Research Institute Co., Ltd. and Yichang Huifu Nanometer Material Co., Ltd. have been giving Dr. Gang Chen a part-time employment since Dec. 2018. Thank these companies for their financial support. Specially thank Dr. Yuelin Wang and other colleagues of these companies for their appreciation, support and help.

Appendix I: Research History

1	1-10	2020/7/8-23
2	11-12	2020/7/11-12
3	12-14	2018/8/21-9/6
3-1	13	2018/8/21-25
3-2	13	2018/9/2-6
3-3	14	2018/9/2-6
3-4	14	2018/9/5-6
4	14-15	2020/6/28-30
5	16	2020/6/30-7/1
6	16-17	2020/6/30-7/1,7/5
7	17-18	2020/7/3-5
8	18-19	2020/7/20-21
9	19	2020/7/21-25
10	19-20	2020/7/27-28
11	20	2020/7/28
12	20-22	2020/7/28-30
13	22	2020/8/2
14	23-27	2020/8/4-8
15	28-29	2020/8/11-13
16	30	2020/8/13-14
17	31	2020/8/14
18	31-32	2020/8/15-17
19	32-33	2020/8/15-17
20	33-35	2020/8/15-16
21	35-36	2020/8/17
22	37	2020/8/18
Preparing this paper	1-39	2020/7/8-2020/8/19

Notes: Dates were recorded according to Beijing Time; *ie* means ideal extended elements; *GL* means Gregory-Leibniz formula.