The influence of the spacetime compression on the gravitational constant

Zhi Cheng

Abstract: If spacetime are considered as an elastic medium, this elastic medium will be able to be compressed. The analysis in this paper shows that if spacetime is compressed, it will cause a change in the gravitational constant. From the theory of general relativity, it is mass that can compress this elastic medium. In the solar system, 99% of the mass is concentrated in the sun. However, about one thousandth of the mass of the sun is concentrated in Jupiter. The position on the same side of the Sun and Jupiter, that is, near the center of mass of the two bodies of the Sun and Jupiter, is the position where the mass of the solar system is most concentrated in spacetime. Therefore, this part of spacetime will be compressed the most. If a planet like the earth moves to this position, it means that the gravitational constant measured by the experimental device on the earth's surface will change. The estimates in this article show that the gravitational constant of the earth moving between Jupiter and the sun is greater than that of the earth on the other side of the position of the sun and Jupiter. The specific estimated increased value is about 77ppm of the gravitational constant, which is higher than the uncertainty of many current measurements. This paper analyzes the HUST-09 results with the results of theoretical estimates, and believes that this method can explain the difference between the two experimental results.

Keywords: gravitational constant; spacetime compression; planetary orbit

1 Theoretical model and estimation

Due to the great success of Newton's law and the geometrical processing of gravity by general relativity, people think that the gravitational constant should be as precise as other physical constant. However, the actual experimental measurement results are just the opposite. The current experimental measurement values of various gravitational constants have the lowest accuracy among all other physical constants. The measurement error is more than several orders of magnitude of some other physical constants [1]. This prompts us to think about why this problem is caused. According to the general theory of relativity, this article believes that spacetime is an elastic substance, so it can be bent and compressed. What can bend and compress spacetime is mass. Therefore, the existence of mass will squeeze the surrounding spacetime, which directly leads to the change of the measurement scale of spacetime. Taking into account the principle of conservation of energy and the constant speed of light, this change in the measurement scale of spacetime will directly lead to a change in the gravitational constant.

Let us now consider the relationship between the degree of compression of spacetime caused by the change in position between the sun and Jupiter. Figure 1 shows the position between the sun and Jupiter.

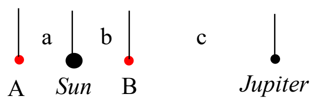

Figure 1. The position of Sun and Jupiter

In Figure 1, the small circle *Jupiter* represents planet Jupiter, and the large circle *Sun* represents the star sun. A and B respectively mark two positions on the earth. It can be seen from the figure that position B is located near the center of mass of the sun and Jupiter. Since the gravitational forces of Jupiter and the sun can cancel each other, the curvature of this spacetime is not as good as that of point A. However, due to the squeezing of the masses of Jupiter and the Sun, the spatial density of point B is greater than that of point A.

The larger the spatial density of point B, it means that the spatial measurement scale of this location is shorter. Although the spacetime of point A is more severely curved, the spatial density is not as large as that of point B. Therefore, the space measurement scale at point A is longer. Consider here that the distance between A and B is large enough from the sun, so that the time is basically flat. Believe that we only need to consider the density of space, and ignore the impact of time changes.

Then we consider the Schwarzschild radius corresponding to a mass. Consider that in Figure 1, points A and B are two different positions of the earth's orbit. The mass of the earth is m_e.

In this way, we can express the Schwarzschild radius of the earth as

$$r_{earth} = \frac{2GM}{c^2}$$

We can notice that in the flat spacetime observation system far away from the position of the sun and Jupiter, the observed Schwarzschild radius of the earth should not change, otherwise the solution of the Schwarzschild radius cannot be derived from the Einstein field equation. The problem now is that if you use the position of the earth at point B as the frame of reference, you will actually find that the Schwarzschild radius of the earth becomes longer. This is because the spatial scale used to measure the Earth's Schwarzschild radius has shrunk. Due to the conservation of energy, the mass of the earth cannot be changed regardless of whether it is at position A or at position B in Figure 1. According to the requirements of relativity, the speed of light is a constant. Therefore, an increase in Schwarzschild's radius means an increase in the gravitational constant G.

This explains why the gravitational constant measured at position B is slightly larger than the gravitational constant measured at position A.

Let's make a simple estimate below.

Let the distance between point A and Sun be a; the distance between Sun and B as b; and the distance between B and Jupiter as c.

Suppose the mass of the earth is m_e. For the earth at point B, the sum of the gravitational potential energy of the sun and Jupiter is:

$$V_B = -Gm_e\left(\frac{M}{b} + \frac{m}{c}\right)$$

In the same way, for the earth at point A, the sum of the gravitational potential energy of the sun and Jupiter is

$$V_A = -Gm_e\left[\frac{M}{a} + \frac{m}{a+b+c}\right]$$

Here, **we assume that the magnitude of the spatial measurement scale change is proportional to the gravitational potential energy**, then

$$\frac{r_{earth}}{r'_{earth}} = \frac{V_A}{V_B} = \frac{\frac{M}{a} + \frac{m}{a+b+c}}{\frac{M}{b} + \frac{m}{c}} = \frac{G}{G'}$$

For convenience, suppose the earth moves around the sun in a circular orbit, then $a=b$

such

$$G' = \frac{1 + \frac{m}{M}\frac{a}{c}}{1 + \frac{m}{M}\frac{a}{2a+c}}G$$

Then substitute various parameters into the above formula, where

$$M = 2 \times 10^{30} kg$$

$$m = 2 \times 10^{27} kg$$

$$a = b = 1.5 \times 10^{11} m$$

$$a + c = 7.8 \times 10^{11} m$$

It can be calculated

$$G' = \frac{1 + \frac{1}{1000}\frac{1.5}{6.3}}{1 + \frac{1}{1000}\frac{1.5}{9.3}}G = 1.000077G = (1 + 7.7 \times 10^{-5})G$$

It can be seen that the biggest system uncertainty of the gravitational constant measured on the earth due to the different orbital position of Jupiter is about 77ppm. This uncertainty range is larger than the current most accurate measurement accuracy of the gravitational constant [2]. Therefore, the variation range of the gravitational constant can be roughly determined as

$$(6.674184 \pm 0.00051) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

2 Analysis of actual gravitational constant measurement data

In 2007, Jupiter's closest approach to Earth was about June 5th, and in 2008 it was July 9th. Therefore, if the gravitational constant is measured around these few days, the gravitational constant measured by the same experimental device should be able to reach the maximum value. In another period of time, such as the experiment of measuring the gravitational constant in December 2007, the value of the gravitational constant measured during this period of time should theoretically be the smallest.

I noticed that in the 2010 paper [3] of Luo's group, two experiments to measure the gravitational constant have detailed time records. The first experiment was from March 21, 2007 to May 20, 2007, and from April 19, 2008 to May 10, 2008. It can be seen that this period is relatively close to the position where the Earth and Jupiter are in the same side of the Sun. The earth is in the position of point A in Figure 2.

The second experiment was from August 25, 2008 to September 28, 2008, and from October 8, 2008 to November 16, 2008. The earth is in the position of point B in Figure 2.

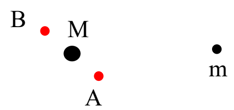

Figure 2. A: Experiment 1; B: Experiment 2

Since A is closer to Jupiter, theoretically the result of the first experiment measurement will be larger than the result of the second experiment.

The actual situation is that the gravitational constant value measured by Luo's group in the first experiment is:

$$(6.67352 \pm 0.00019) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

The gravitational constant measured in the second experiment is:

$$(6.67346 \pm 0.00021) \times 10^{-11} \mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

It can be seen that the gravitational constant measured in the first experiment is slightly larger than the value in the second experiment. Considering that the points A and B in Fig. 2 deviate from the minimum and maximum values of the model in this paper, the calculated system uncertainty will be smaller. Therefore, if the experiment can take the systematic uncertainty proposed in this article into account, it can be found that the result is still within the experimental error range.

3 Discussion

The measurement of the gravitational constant is very difficult. On the one hand, it is very difficult to improve the accuracy of experimental measurements. On the other hand, judging from the results currently available, the gap between the results of measurements at different times and locations is also very large. This also shows that the factors affecting the gravitational constant may far exceed the various factors we already know.

When a planet is in motion, due to the squeezing effect of mass on spacetime, the spacetime density around it also changes accordingly. One result of the change in spacetime density is the change of gravitational constant.

This article further infers that, for the earth, Jupiter's motion will cause changes in the spatial density around the earth's orbit. Therefore, the difference of Jupiter's position directly leads to different gravitational constants measured on the earth's surface.

At present, judging from the very limited results of the papers with specific experimental dates, there are no results that contradict the theoretical speculations of this article. One of the most typical examples is the results measured by Luo's group at two different times from 2007 to 2008. The theory of this article can explain the difference between the two experimental results to a certain extent.

References

[1] Rothleitner, C. & Schlamminger, S. (2017). Measurements of the Newtonian constant of gravitation, G. Rev. Sci. Instrum. 88, 111101.
[2] Luo, J. et al. (2018). Measurements of the gravitational constant using two independent methods. Nature, 560(7720), 582-588.
[3] Luo, J. et al. (2010). New determination of the gravitational constant g with time-of-swing method. Physical Review D.

时空压缩对引力常数的影响

Zhi Cheng

摘要：如果考虑时空是一种弹性介质，则这种弹性介质将能够被压缩。本文的分析表明，如果时空被压缩，则将引起引力常数的变化。从广义相对论的理论来看，能够压缩这种弹性介质的是质量。在太阳系中，99%的质量集中在太阳。但是还有大约占太阳质量千分之一的质量集中在木星。在太阳和木星同一侧的位置，即太阳和木星二体的质心位置附近，是太阳系时空中质量最集中的位置。因此这部分的时空将被压缩的最严重。如果像地球这样的行星运行到这个位置，则意味着在地球表面实验装置所测量出来的引力常数将发生变化。本文的估算表明地球运行到木星和太阳之间的引力常数要大于地球位于太阳和木星位置的另一侧。具体估算数值大约为 77ppm，要高于目前很多测量值的不确定性。本文用理论估算的结果对 HUST-09 的结果进行分析，认为这种方法可以解释两次实验结果的差异问题。

关键词：引力常数；时空压缩；行星轨道

1 理论模型和估算

由于牛顿定律的巨大成功以及广义相对论对引力进行几何化处理，使得人们认为引力常数应该像物理常数那样获得一个非常精确的数值。然而实际的实验测量结果却正好相反，目前各种引力常数的实验测量数值是所有其他物理常数中精度最低的。测量误差是其他物理常数的十几个数量级[1]。这促使我们思考为什么会造成这样的问题。本文根据广义相对论，认为时空是一种弹性物质，因此可以被弯曲和压缩。能够弯曲和压缩时空的是质量。因此质量的存在将挤压周围的时空，这直接导致了时空测量尺度的变化。考虑到能量守恒以及光速不变的原理，这种时空测量尺度的变化将直接导致引力常数的改变。

现在我们来考虑太阳和木星之间的位置变化所引起的时空被压缩的程度之间关系。图 1 显示了太阳和木星之间的位置。

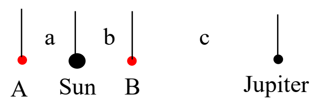

Figure 1. The position of Sun and Jupiter

图 1 中小圆圈 Jupiter 表示木星，大圆圈 Sun 表示太阳。A 和 B 分别标记了地球的两个位置。从图中可以看出 B 位置位于太阳和木星的质心附近，由于木星和太阳的引力可以相互抵消，因此在这一块时空的弯曲程度反而不如 A 点。但是由于木星和太阳质量的挤压，导致 B 点的空间密度要大于 A 点的时空密度。

B 点空间密度越大，意味着这一位置的空间度量尺度要短一些。而 A 点时空虽然弯曲的比较严重，但是空间密度反而不如 B 点大，因此在 A 点位置空间的度量尺度要长一些。这里考虑 A 和 B 两点离太阳的距离足够大，这样时间基本上还是平坦的。相信这样我们只需要考虑空间的密度，而忽略时间变化的影响。

然后我们再考虑对应一个质量的史瓦西半径。考虑到图 1 中，A 和 B 两点分别是地球运行的两个不同的位置。地球的质量是 m_e.

这样我们可以将地球的史瓦西半径表示出来：

$$r_{earth} = \frac{2GM}{c^2}$$

我们可以注意到，在远离太阳和木星位置的平坦时空观察系，观察到的地球的史瓦西半径不应该出现变化，否则就无法由爱因斯坦场方程导出史瓦西半径的解。现在的问题就是如果按照地球位于 B 点的位置作为参照系来进行测量，实际上会发现地球的史瓦西半径变长了。这是因为用来度量地球史瓦西半径的空间尺度缩小了。由于能量守恒，不管是在图 1 中的 A 位置还是在 B 位置，地球的质量都不可以改变。而根据相对论要求，光速是一个常数。因此史瓦西半径的增大意味着引力常数 G 的增大。

这就解释了为什么在 B 位置测量出来的引力常数要略大于在 A 位置测量出来的引力常数。

下面我们来做一个简单的估算。

先设 A 点到 Sun 之间的距离为 a; Sun 到 B 之间的距离为 b; B 到 Jupiter 之间的距离为 c.

设地球质量为 m_e, 对于地球位于 B 点，太阳和木星引力势能总和为：

$$V_B = -Gm_e\left(\frac{M}{b} + \frac{m}{c}\right)$$

同样方法，对于地球位于 A 点，太阳和木星引力势能总和为：

$$V_A = -Gm_e\left[\frac{M}{a} + \frac{m}{a+b+c}\right]$$

假设空间度量尺度变化幅度跟引力势能成正比，则

$$\frac{r_{earth}}{r'_{earth}} = \frac{V_A}{V_B} = \frac{\frac{M}{a} + \frac{m}{a+b+c}}{\frac{M}{b} + \frac{m}{c}} = \frac{G}{G'}$$

为了方便，假设地球绕太阳运动是一个圆形轨道，则 $a=b$

这样：

$$G' = \frac{1+\frac{m}{M}\frac{a}{c}}{1+\frac{m}{M}\frac{a}{2a+c}}G$$

然后将各种参数代入上述公式，其中

$$M = 2\times 10^{30}kg$$

$$m = 2\times 10^{27}kg$$

$$a = b = 1.5\times 10^{11}m$$

$$a + c = 7.8\times 10^{11}m$$

可以计算出：

$$G' = \frac{1+\frac{1}{1000}\frac{1.5}{6.3}}{1+\frac{1}{1000}\frac{1.5}{9.38}}G = 1.000077G = (1+7.7\times 10^{-5})G$$

可以看出因为木星轨道位置的不同导致地球上测量到的引力常数所产生的相对误差大约为77ppm. 这个误差范围比目前最精确的引力常数测量精度[2]要大。因此引力常数的变化范围大致可确定为：

$$(6.674184 \pm 0.00051)\times 10^{-11}\mathrm{m}^3/\mathrm{kg}\cdot \mathrm{s}^2$$

2 对实际的引力常数测量数据的分析

在 2007 年木星与地球最接近的时间大约是 6 月 5 日，2008 年则是 7 月 9 日。因此如果引力常数的测量是在这几天附近，则同一种实验装置测量出来的引力常数值应该能够达到最大值。而在另一段时间，比如在 2007 年 12 月进行引力常数测量的实验，则这段时间测量出来的引力常数值理论上应该是最小的。

对于 HUST-09 的结果，我注意到罗俊小组在 2010 年的论文[3]中两次测量引力常数的实验有详细的时间记录。其中第一次实验是在 2007 年 3 月 21 日到 2007 年 5 月 20 日，2008 年 4 月 19 日到 2008 年 5 月 10 日。可以看出，这段时间比较接近地球和木星在同一侧轨道的情况。如图 2 中的 A 点位置。

第二次实验是在 2008 年 8 月 25 日到 2008 年 9 月 28 日，2008 年 10 月 8 日到 2008 年 11 月 16 日。如图 2 中的 B 点位置。

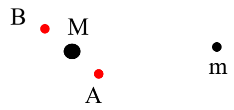

Figure 2. A: Experiment 1; B: Experiment 2

由于 A 更接近木星，因此理论上第一次实验测量的结果会比第二次实验的结果要大一些。

实际情况是，第一次实验罗俊小组测量的引力常数数值为：

$$(6.67352 \pm 0.00019) \times 10^{-11}\mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

第二次实验测得的引力常数为：

$$(6.67346 \pm 0.00021) \times 10^{-11}\mathrm{m}^3/\mathrm{kg} \cdot \mathrm{s}^2$$

可以看出第一次实验测得的引力常数要略大于第二次的数值。再考虑图 2 中的 A 点和 B 点偏离本文模型的最小值和最大值，所计算出来系统误差会小一些。因此如果该实验能够将本文提出的这个系统误差考虑进去，可以发现结果仍在实验误差范围之内。

3 讨论

引力常数的测量是非常困难的。一方面是要提高实验测量的精度非常困难，另一方面，从目前已有的结果来看，不同时间不同地点测量的结果差距也非常大。这也说明，影响引力常数的因素可能远超我们现在已经知道的各种因素。

而行星在运动的时候，由于质量对时空的挤压效应，导致其周围的时空密度也随之而变化，时空密度的改变导致的一个结果就是引力常数的变化。

本文进一步推断，认为对于地球来说，木星的运动将引起地球轨道周围的空间密度的变化。因此木星位置的不同，直接导致了地球表面所测量的引力常数会有所不同。

目前来看，从非常有限的公布具体实验日期的论文结果来看，并没有跟本文的理论推测相矛盾的结果。其中最典型的一个例子就是罗俊小组在 2007 至 2008 年两次不同时间测量出来的结果。本文的理论可以在一定程度上解释这两次实验结果的差异。

参考文献

[1] Rothleitner, C. & Schlamminger, S. (2017). Measurements of the Newtonian constant of

gravitation, G. Rev. Sci. Instrum. 88, 111101.
[2] Luo, J. et al. (2018). Measurements of the gravitational constant using two independent methods. Nature, 560(7720), 582-588.
[3] Luo, J. et al. (2010). New determination of the gravitational constant g with time-of-swing method. Physical Review D.