One-page refutation of abc-conjecture

Dmitri Martila

*Tartu University, 4 Tähe Street, 51010 Tartu, Estonia**

(Dated: July 15, 2020)

Abstract

Relying on the validity of Dr. Dahmen's peer-reviewed result, I am refuting the abc-conjecture even without explicit mentioning prime numbers.

*Electronic address: eestidima@gmail.com

I. AND BECAUSE MISTAKES AND FAKES SHALL ABOUND, THE WAY OF TRUTH WILL BE EVIL SPOKEN OF

This section can be removed from the paper on request of the referee. It is not meant as a proposal to modify the peer-review process, but as an argument for the referee to use goodwill.

The goal “to find mistakes” could be a bad attitude. The final goal should be to enjoy reading the publication. If flaws are seen, they must be reported. However, this report should be given without any laughs and sadistic enjoyment. Instead, the flaws should be reported with some sadness.

The psychologists have conducted a social experiment: they told the probants that the man on the photo is a serial killer. The probants testified that he is looking like one. The next day they told another group of probants that the man on the same photo is an American national hero; these probants have confirmed his heroic look.

In conclusion, having the “mistakes desire” as your default position while reading the manuscript of an unknown author increases the chances for the paper to be unjustly rejected. The scientific skepticism should be the readiness to deal with mistakes, but not the expectation – by desire – to find them.

Why do I ask as an author for detailed reports from the referee system? The referee must convince me that I have done mistakes. Otherwise, I would not accept them. Yes, it seems like living in an “utopian” perfect world. But I cannot repent a hypothetical mistake. I can only repent if the mistake is demonstrated to me and I am convinced that it is not the usual fake-news, trolling or bullying. This research principle is my personal “guiding star” during my quest for the objective truth. As an example, the absolute majority of scientists have accepted the proof for Goldbach’s weak conjecture, but not all of the scientists have accepted it yet, mainly because it is not published in a journal. [5] Therefore, one needs to have personal convictions and opinions to move forward. [6]

To navigate in Science, you need to have a personal point of view and convictions you should not rush to abandon. Otherwise, you will soon be disoriented. Only then you will realize the objective truth. That is the subjective search for the objective truth because you are choosing what is right and what is not.

II. INTRODUCTION

The abc conjecture (also known as the Oesterlé-Masser conjecture) is a conjecture in number theory, first proposed by Joseph Oesterlé (1988) and David Masser (1985). Many famous conjectures and theorems in number theory would follow immediately from the abc conjecture or its versions. Dr. Goldfeld described the abc conjecture as “the most important unsolved problem in Diophantine analysis” [2]. Various attempts to prove the abc conjecture have been made, but none are currently accepted by the mainstream mathematical community and as of 2020, the conjecture is still largely regarded as unproven [3].

The abc-conjecture says the following. For every positive real number ϵ, there exist only finitely many triples (a, b, c) of coprime positive integers, with $a + b = c$, such that $c > (\mathrm{rad}(a\,b\,c))^{1+\epsilon}$.

III. REFUTATION

In the literature, the triples (a, b, c) which satisfy inequality

$$c > \mathrm{rad}(abc) \tag{1}$$

are called the abc-hits [1]. The number of abc-hits for all $c \leq X$ is denoted as $V(X)$.

In the present paper, let’s call the triples (a, b, c) satisfying the inequality

$$c > (\mathrm{rad}(abc))^{1+\epsilon} \tag{2}$$

the martila-hits. The number of martila-hits for all $c \leq X$ is denoted as $n(X)$. The abc-conjecture is true if $n(X) < \infty$ even in the limit $X \to \infty$.

In Ref. [1] the number $N(X)$ counts the number of abc-hits. Hereby, the condition for martila-hits is imposed. In other words, $N(X)$ is the number of triples satisfying the system of three inequalities (1), (2) and $c \leq X$.

Thus, the number $N(X)$ counts the number of martila-hits instead. Indeed, if condition (2) for martila-hits is satisfied, the condition (1) for abc-hits is satisfied as well [7] because $\mathrm{rad}(abc) < (\mathrm{rad}(abc))^{1+\epsilon}$ for any $\epsilon > 0$. Therefore, one has

$$n(X) < V(X)\,. \tag{3}$$

All this means that for $N(X)$ in Ref. [1] one has

$$N(X) \equiv n(X)\,. \tag{4}$$

However, from the abstract of Ref. [1] one obtains in the limit $X \to \infty$

$$N(X) = \infty\,, \quad 0 \le \epsilon < 1/2\,. \tag{5}$$

Thus, the abc-conjecture is false.

IV. ADDITIONAL EVIDENCES

From Eqs. (3), (4) and (5) taken at $\epsilon \neq 0$ follows that $V(X) = \infty$, $X \to \infty$. The latter effect coincides with Ref. [1], because at $\epsilon = 0$ one has $N(X) \equiv V(X)$.

A. Evidence from Nitaj's paper

Let us define a function $k(\epsilon)$ such as

$$k(\epsilon) = \max_{a,b,c}\left(\frac{c}{(\mathrm{rad}(abc))^{1+\epsilon}}\right), \tag{6}$$

where $\epsilon > 0$, the operator $\max\limits_{a,b,c} f(a,b,c)$ inserts all allowed values of a, b, c into $f(a,b,c)$ and selects the maximum of $f(a,b,c)$, e.g. $\max\limits_{a,b,c} 5 = 5$. Obviously, the abc-conjecture is then equivalent to $k(\epsilon) < \infty$ for any $\epsilon \neq 0$.

Indeed, a possible formulation of abc-conjecture is $c < K(\epsilon)\,(\mathrm{rad}(abc))^{1+\epsilon}$ for all allowed values of a, b, c. In this case one has

$$k(\epsilon) \le K(\epsilon)\,. \tag{7}$$

The $k(\epsilon)$ is the minimum possible $K(\epsilon)$.

Proposition 2.7 proven in Ref. [4] is given in my notation by [8]

$$k(\epsilon) \sim \max_{n}\left(\frac{2^{n\,(1+\epsilon)}}{(c_n)^{\epsilon}}\right), \tag{8}$$

where in Proposition 2.2 (remark 2) c_n was understood to be $c_n \sim 2^{2\,n}$. Therefore, if $0 \le \epsilon < 1/2$, e.g. $\epsilon = 0.2$, one has $k(\epsilon) = \infty$.

Amazingly, this range $0 \le \epsilon < 1/2$ of abc-conjecture failure coincides with the one derived above: Eq.(5).

V. DISCUSSION

A referee might say: "ϵ in Dahmen's expression for $N(X)$ is not the same ϵ in the expression of the martila-hits." If this is the case, by taking the arbitrary free constant ϵ to infinity, we would obtain $N(X) \geq 0$ for any X. In this case, this would provide the entire result of Ref. [1] which turns out to be absolutely trivial. This would be an absurd accusation as Ref. [1] is peer-reviewed. In the case $\epsilon = 1/2$, the result of Ref. [1] would be trivial as well, $N(X) > 0$ for any X. Therefore, the referee is in delusion.

---

[1] Sander R. Dahmen, "Lower bounds for numbers of ABC-hits", Journal Number Theory 128 (6), 1864–1873 (2008). http://www.math.leidenuniv.nl/~desmit/doc/abchits.pdf

[2] D. Goldfeld, "Beyond the last theorem", Math Horizons 4 (September), 26–34 (1996).

[3] D. Castelvecchi, "Mathematical proof that rocked number theory will be published", Nature (3 April 2020).

[4] Abderrahmane Nitaj, "Aspects expérimentaux de la conjecture abc", Séminaire de Théorie des Nombres de Paris (1993–1994), London Math. Soc. Lecture Note Ser. 235, 145–156 (1996), Cambridge Univ. Press. https://doi.org/10.1017/CBO9780511662003.007

[5] Harald A. Helfgott, "The ternary Goldbach conjecture is true", arXiv:1312.7748 [math.NT].

[6] Massimiliano Proietti *et. al.*, Experimental test of local observer-independence, Science Advances 5(9), eaaw9832 (2019), arXiv:1902.05080 [quant-ph]; Ian T. Durham, Observer-independence in the presence of a horizon, arXiv:1902.09028 [quant-ph]

[7] It is always $\mathrm{rad}(abc) \geq 2$, as $abc \geq 2$ for positive integers $a, b \geq 1$, $c = a + b$

[8] If the form of some function $w(n, \epsilon)$ is given, then despite the talk about limit $\epsilon \to 0$, there is information about the $\epsilon \neq 0$ as well.