A New Solution of the Friedman Equations.

John Hunter

ABSTRACT

A new cosmological model is suggested. It is a modified Einstein – De Sitter model with a different redshift scale-factor relation. There is discussion of why observational evidence often supports the possibly faulty Concordance Cosmology. In the new model there is no coincidence problem as the cosmological constant is zero and there is a solution to the Hubble tension.

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INTRODUCTION

Since 1915 there has been apparently increasing evidence for a universe with dark matter, dark energy and a period of inflation, the LCDM model. The evidence has been so strong and from so many different types of investigation that the majority of cosmologists have understandably become supporters of the so called 'Concordance Cosmology'.

There are alternatives however, one of which is the subject of this paper. The layout is as follows.

In section 1 there is a brief review of some of the evidence in favour of the Concordance Cosmology (LCDM), some problems are also discussed. In section 2 there is a new cosmological model which has a new redshift scale-factor relation. The main features of the new cosmology are described. By mirroring the layout of the section 1 subsections, it is shown how the observed matter density and apparent cosmological constant might have been incorrectly inferred in LCDM. Section 3 describes a way in which the tension between Hubble constant measurements can be resolved. Philosophical matters and detailed calculations are in the appendices.

Section 1. Concordance Cosmology

Since the development of General Relativity by Einstein in 1915 cosmologists have developed cosmological models compatible with it.

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In 1922 Friedman published his equations [Friedman, 1999]

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \quad (1)$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3} \quad (2)$$

the symbols have their usual meanings and the Hubble parameter is

$$H(z) = \left(\frac{\dot{a}}{a}\right) \quad (3)$$

where z is the redshift of light from distant objects. The matter density is

$$\Omega_m = \frac{\rho}{\rho_{crit}} \quad (4)$$

the critical density is

$$\rho_{crit} = \frac{3H(z)^2}{8\pi G} \quad (5)$$

Concordance cosmology has a solution with a matter density Ω_m a dark energy density parameter $\Omega_\Lambda \approx 1 - \Omega_m$ and four other variable parameters, it has been shown to give a good match to data.

The universe is thought to originate in the Big Bang. A period of inflation followed. The universe then expanded with quantities, including the matter density, being related to the scale factor a.

The redshift is related to the scale factor as

$$a = \frac{1}{1+z} \quad (6)$$

In LCDM the Hubble parameter is

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}$$

H_0 is the local Hubble constant. (7)

The co-moving distance is obtained from

$$D_M = \int_t^0 \frac{c}{a(t)} dt \quad (8)$$

$$\frac{da}{dt} = \frac{da}{dz} \times \frac{dz}{dt} = -\frac{1}{(1+z)^2} \times \frac{dz}{dt} \quad (9)$$

$$H(z) = \frac{\dot{a}}{a} = -\frac{1}{1+z} \times \frac{dz}{dt} \quad (10)$$

So (8) can also be written as

$$D_M = \int_0^z \frac{c}{H(z)} dz \quad (11)$$

Luminosity distance D_L and angular diameter distance D_A are from

$$D_L = (1+z)D_M \quad (12)$$

and

$$D_A = \frac{D_M}{1+z} \quad (13)$$

Alternatives to some of these equations are in section 2.

Below is a list of factors, in approximately historical order, that have led to the development and acceptance of the LCDM model, there is a review in Peebles [Peebles, 2002].

1.1 Redshift

The redshift of light from distant objects seems to indicate an expansion and support for a Big Bang type of model - although not all scientists believed that it was evidence of an expansion e.g. Zwicky [Zwicky,1929] who favoured a 'tired light' model.

1.2 Abundancies of the elements

Big Bang nuclear synthesis (BBN) successfully predicts the abundancies of the elements, adding further support to a Big Bang type of model. Typical values for the baryon density $\Omega_b h^2$ from BBN are 0.02166 [Cooke, 2018] where $H_0 = 100h$ kms^{-1}Mpc^{-1}.

1.3 Cosmic Microwave Background Radiation

The Cosmic Microwave Background Radiation (CMB) was discovered by Penzias and Wilson in 1965 [Durrer, 2015], a 2.73K Planck black body spectrum. It was immediately interpreted as the 'afterglow' from the Big Bang. Alpher and Herman [Alpher, 1967] had predicted a 5K background from Big Bang theory in 1948.

Remaining alternatives such as the Steady State model of Bondi, Gold and Hoyle [Bondi, 1948] and tired light models went out of fashion – although Hoyle, Burbidge and Narlikar [Hoyle, 1993] later published a "Quasi Steady State Cosmology".

1.4 Flatness and horizon problems

The Big Bang model still faced difficulties. The flatness problem is that the universe appears to be near critical density. The horizon problem is that the universe is homogeneous, even on scales so large that that the separate regions should not have had time to communicate with each other.

Inflation was developed in the 1970s and 80s and added to the Big Bang model [Guth, 1980], to help solve these problems.

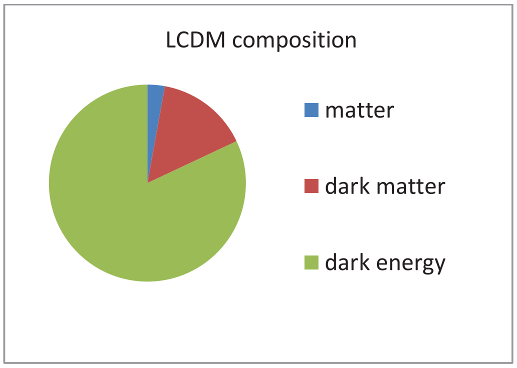
1.5 Matter density

Evidence for a low matter density came from the motion of stars around galaxies and from clusters of galaxies [Ferramacho, 2006], [Allen, 2007]. It seems that as well as visible matter there must be dark matter. Even including dark matter, the total matter density seemed to be much less than one, a typical value for Ω_m from galaxy clusters being 0.2-0.3. So the next piece of evidence for LCDM was readily received.

1.6 Supernovae and the cosmological constant.

The Luminosity distances of supernovae seem to show an accelerating expansion [Riess, 1998] and [Perlmutter, 1998]. LCDM could accommodate this data with a cosmological constant. The cosmological constant was also necessary to account for the low matter density and have a flat universe where the density components add up to one. The pie chart in Figure 1 shows the composition of the universe for the LCDM model.

Figure 1 Composition of the universe, LCDM



1.7 Anisotropies in the CMBR

Measurements of the size of the acoustic peaks of the power spectrum of the anisotropies in the Cosmic Microwave Background Radiation have enabled parameters in the LCDM model to be measured, WMAP [Hinshaw, 2012] finding Ω_m= 0.281, Ω_Λ= 0.719, H_0 = 69.7kms^{-1}Mpc^{-1} and PLANCK [Aghanim, 2018] Ω_m= 0.315, Ω_Λ= 0.68, $H_0 = 67.4$ kms^{-1}Mpc^{-1}

1.8 Baryon Acoustic Oscillations

A relatively new method, BAO observations involve measurements of the sound horizon, which are thought to provide a 'standard ruler' [Aubourg, 2014], for a review see Bassett [Bassett, 2009]. BAO on the whole, support LCDM. This method is often combined with CMB and supernovae data. Using an 'inverse distance ladder' approach, the results give a low Hubble constant typically of 67.3 kms^{-1}Mpc^{-1}

1.9 Hubble parameter data

By measuring the differential ageing of galaxies at different redshifts [Jimenez, 2001], the Hubble parameter can be found from equation (10)

$$H(z) = \frac{\dot{a}}{a} = -\frac{1}{1+z} \times \frac{dz}{dt}$$

The results are consistent with LCDM but also compatible with $H(z) = (1+z)H_0$. Data from different groups correlated with CMB and BAO data [Farooq, 2016] show a transition from deceleration to acceleration at about z = 0.78

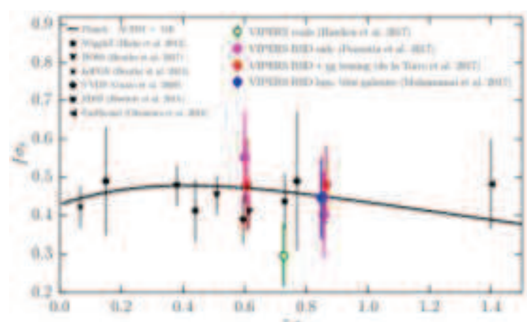
1.10 Measurements of the Hubble constant

Recent local methods of determining the Hubble constant give 74.03 kms^{-1}Mpc^{-1} [Riess, 2019], in tension, 4.4σ, with values from CMB and BAO. Local measurements do not appear to agree with the value for H_0 deduced by LCDM. This is the so called 'Hubble tension'.

1.11 Large Scale Structure

Measurements of the growth of large scale structure measure the quantity $f\sigma_8(z)$ where $f(z)$ is the growth factor and σ_8 is the amplitude of the matter fluctuations on a scale of 8 Mpc.

Figure 2 Growth parameter against redshift



The graph of Figure 2 [Guzzo, 2018] shows the LCDM match to the growth data. The new model (section 2.11) gives a good match with a constant $f\sigma_8(z)$ of about 0.46.

The above is quite a long list! The fact that LCDM successfully accounts for nearly all these observations is an astounding success. There is also a degree of consistency, with apparent evidence for dark energy and dark matter coming from different types of investigation. The fact that scientists have designed and carried out such experiments, with such precision, is a credit to the world's physicists, astronomers, cosmologists and engineers.

We have now entered the era of precision cosmology and we are in a position, with further upcoming experiments, to subject LCDM to even more detailed scrutiny.

However there are some problems with LCDM. No matter how successfully it matches the observations, it cannot be denied that, philosophically speaking, its foundations are not so sound. It relies on three phenomena which have no theoretical understanding and have not been subject to local tests - dark energy, dark matter and inflation.

Whilst observationally, on the whole, LCDM has been successful, there is the Hubble tension that has recently risen to over 4 standard deviations. There is also the Lyman Alpha Forest anomaly, LCDM not giving a good match to BAO data at z=2.34. The beginning of the universe is also a problem - there would have been a state of infinite density and pressure. More matter than antimatter would have to be created.

Perhaps just as serious is the coincidence problem. LCDM does not account for why the matter density parameter and the dark energy density parameter should be the same order of magnitude at the present time.

There has also been a kind of inter-dependency of methods, whereby conclusions from one e.g. supernovae data is used to bolster another, e.g. BAO, or BAO data is used to calibrate CMB data.

In the next section a model is suggested which might help with some of the issues above. After introducing the main features of the model, the subsections in section 2 mirror those of section 1, to enable any changes necessary to be discussed. Section 2 illustrates how it's possible for concordance cosmology to be incorrect, yet still appear to match observations.

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Section 2. An alternative Solution

There are other solutions of the Friedman equations. If we are guided by the principle of simplicity we can look for a flat solution, $k = 0$, with a constant expansion parameter H

$$a = e^{-Ht} \tag{14}$$

an expanding solution, with the convention that t is positive into the past.

Equations (1) and (2) reduce to

$$3H^2 = -4\pi G\left(\rho + \frac{3p}{c^2}\right) + \Lambda c^2 \tag{15}$$

$$3H^2 = 8\pi G\rho + \Lambda c^2 \tag{16}$$

with zero cosmological constant, $\Lambda = 0$,

$$\rho = \frac{3H^2}{8\pi G} \tag{17}$$

the universe is at the critical density

$$G = \frac{3H^2}{8\pi\rho} \tag{18}$$

and

$$p = -\rho c^2 \tag{19}$$

With similarities to the Einstein - De Sitter [Einstein 1932] model. Is such a situation possible? It seems as though, that when the universe expands, the matter density should reduce and the above solution should be ruled out. However observations show that (17) is true, at least approximately and at the present time.

This is really the flatness problem as noted by Dicke [Dicke, 1970]. It's similar to the 'coincidence problem', that the matter density has a value ~ 0.5 so that the dark energy density has a similar value, in a flat universe. This important 'coincidence' has not been properly accounted for.

If we look for a way that (17) is always true, we can find the following interpretation.

In LCDM the scale factor determines the distance between galaxies - there is now an important distinction with the new model.

The changing scale factor a now applies not only to space, but to matter too, and all distances. This includes atoms, people, all distances, the sizes of stars, galaxies and the distance between all objects now depend on the scale factor. All physical constants which contain length dimensions are changed by the changing scale factor too.

Every quantity Q with n length dimensions changes according to Qa^n so, for example

Table 1 Changes of physical quantities

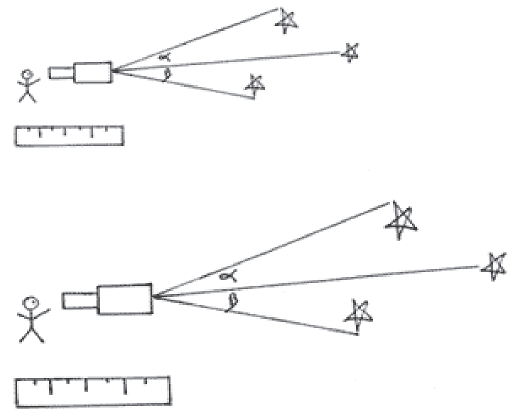
Physical Quantity	Change with time
Planck's constant h	he^{-2Ht}
Masses m	m constant
Fine structure constant α	α constant
Gravitational constant G	Ge^{-3Ht}
Pressure p	pe^{Ht}
Speed of light c	ce^{-Ht}
Density ρ	ρe^{3Ht}

With t being applied in the cosmologists sense, positive into the past. Equations (1), (2) and (17) (and in fact all physics equations true today), remain valid with this system, it's a kind of global conformal transformation, but an ongoing transformation.

This model universe is expanding yet static. Expanding in the sense that there is a continuous expansion (of all length scales) – but static in the sense that it would be impossible for any observer to measure the expansion locally. For them the universe could be regarded as static - for example G appears constant in time if measured locally.

Measurements at a distance would also yield a null result. If we tried to measure any change in the fine structure constant α [Murphy, 2016], for distant stars, since Plancks constant h, the relative permittivity of free space ε_0 and the speed of light c all change, the exponential factors cancel and α is left unchanged.

Figure 3 Cartoon to show the expanding universe



Attempts to measure the changes of other physical quantities at a distance would result either in an unchanged time measurement, (as it has no length dimension) an unchanged angle (also no length dimension) or a ratio of lengths – also unchanged.

The universe would always appear to be at critical density from (17), if the correct value of the expansion constant H is used (see section 2.1).

In appendix A it's suggested that gravity is *caused* by the expansion, with G having the value to conserve energy as the expansion occurs – hence equation (17) . It's also suggested that the strength of gravity reduces when the mass to radius ratio of a region of matter approaches c^2/G.

The pressure term arises as there is a balance between gravity and the explosions that occur for any region of matter which becomes so dense that the mass to radius ratio approaches c^2/G, such as the centres of galaxies.

So although static on the largest scales, there is a great deal of motion on a smaller scale. Peculiar velocities occur, also the interaction and flowing of matter. The Big Bang (or Bangs) is included in this model and occur when large quantities of matter collapse under gravity and then 'bounce'. So although apparently static in terms of scale factor, the universe is in a dynamic equilibrium.

The following subsections have the same structure as the subsections of section 1.

2.1 Redshift

The Big Bang (or Bangs) is thought to have occurred long ago and motions of the galaxies through space can now be ignored. The continuing expansion of all length scales causes a redshift as follows.

If the energy of a photon emitted (subscript 1) from a distant star towards an observer is conserved.

$$h_0 f_0 = h_1 f_1 \qquad (20)$$

Since Planck's constant was lower in the past, there is a redshift of received light according to

$$f_0 = f_1 \frac{h_1}{h_0} \qquad (21)$$

$$\lambda_0 = \lambda_1 e^{2Ht} \qquad (22)$$

This leads to a new redshift – scale factor relation. The redshift of received light is given by

$$z = \frac{\lambda_1 e^{2Ht} - \lambda_1}{\lambda_1} \qquad (23)$$

$$1 + z = e^{2Ht} = \frac{1}{a^2} \qquad (24)$$

so

$$a = \frac{1}{\sqrt{1+z}} \qquad (25)$$

An object, a distance d away, would have an apparent velocity v, depending on the redshift.

$$\frac{v}{c} = z = e^{2Hd/c} - 1 \approx \frac{2Hd}{c} \qquad (26)$$

$$v = 2Hd \qquad (27)$$

comparing with Hubble's law, the expansion constant is half of the Hubble constant approximately 37.5 $\mathrm{kms^{-1}Mpc^{-1}}$

$$H = \frac{H_0}{2} \qquad (28)$$

Bassett [Bassett, 2013] mentions models where there is a 'redshift remapping' and discusses possible observational constraints on models that violate distance duality. Distance duality is preserved in this model (section 2.6). Wojtak [Wojtak, 2016] demonstrate that redshift remappings are degenerate with dark energy and they mention that a metric based interpretation of redshift guarantees agreement with the observed black body spectrum of the CMB.

The alternative equations (25) and (28) lead to changes in other formulae commonly used in cosmology and as Basset cautions, this can lead to changes in data points. Although similar in some ways to the Big Bang model with an expanding universe, the model has different formulae for luminosity and angular diameter distance. These are derived in section 2.6

2.2 Abundancies of the elements

The Big Bang(s) occurred in this model so the success of the predictions of the proportions of the elements is still retained.

Measurements of deuterium abundance from quasars and Big Bang nuclear synthesis (BBN) gave the baryon density $\Omega_b h^2 = 0.02166$ (section 1.2). With the new value for h of about 0.375, half of the traditional value, the new value for Ω_b is four times larger, about 15 - 16% of the universe.

2.3 Cosmic Microwave Background Radiation

This still occurs in a similar way to the Big Bang model, due to the explosions that occur when collapsing matter reaches a high enough mass to radius ratio, c^2/G.

2.4 Flatness and horizon problems

The flatness problem does not occur in this model. The universe is naturally always at critical density, equations (17) and (18)

$$\rho = \frac{3H^2}{8\pi G}$$

and

$$G = \frac{3H^2}{8\pi \rho}$$

The interpretation of this, discussed more in Appendix A, is that gravity is caused by the expansion. It has the strength necessary to conserve energy as the universe expands.

The matter density is always 1.0 although could be measured to be 0.25 (depending on the method used) as the H_0 would be used in the denominator of (4) instead of the real expansion constant H.

The horizon problem is a problem for the Big Bang model, with a definite start for time. In the new model there is no beginning of time, and no particular single Big Bang event, although, over the last few billion years, there would have been many enormous explosive events, and perhaps one larger than the others.

2.5 Matter density

The low matter density (0.2-0.3) inferred from the X-ray gas mass fraction, was mentioned in section 1.5. The matter density is from

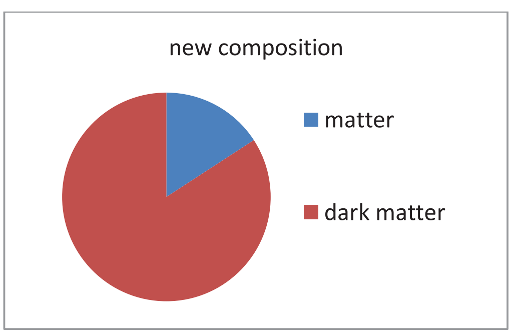
$$\Omega_m = \frac{\Omega_b}{f_{gas}(1+0.19h^{0.5})} \quad (29)$$

[Allen 2002] The denominator contains h and f_{gas}. f_{gas} is determined by measuring the mass of gas in a cluster from X-ray luminosity and the total mass of the cluster. The situation is complicated as both d_A and ρ_{crit} are used in the estimate of f_{gas}.

The main change needed, however, would be to the numerator. Ω_b is calculated from an $\Omega_b h^2$ value of 0.0205 (in 2001), obtained from the Deuterium to Hydrogen ratio in quasars. Hence Ω_b should be four times larger, this increases Ω_M by a similar factor to approximately 1.0.

The same factor four change applies to matter density from the CMB as discussed in section 2.7 Some dark matter still seems necessary. Figure 4 shows the composition of the universe in the new model.

Figure 4 Composition of the universe, new model



2.6 Supernovae and the cosmological constant

A new formula for luminosity distance is derived.

The co-moving distance is still (8)

$$D_M = \int_t^0 \frac{c}{a(t)} dt$$

Using (25)

$$a = \frac{1}{\sqrt{1+Z}}$$

$$\frac{da}{dt} = \frac{da}{dz} \times \frac{dz}{dt} = -\frac{1}{2(1+z)^{3/2}} \times \frac{dz}{dt} \quad (30)$$

$$H(z) = H = \frac{\dot{a}}{a} = \frac{-1}{2(1+z)} \times \frac{dz}{dt} \quad (31)$$

$$dt = \frac{-1}{2H(1+z)} dz \quad (32)$$

$$D_M = \int_0^z \frac{c}{2H\sqrt{1+z}} dz \quad (33)$$

(33) is different to equation (11), so

$$D_M = \frac{2c}{H_0}\left(\sqrt{1+z} - 1\right) \quad (34)$$

Flux depends on the Luminosity distance as

$$F = \frac{L}{4\pi D_L^2} \quad (35)$$

There are two factors diminishing the flux arriving, which both depend on $(1+z)$, (not $\sqrt{1+z}$), the energy of the photon and the increased time of arrival due to the redshift.

Luminosity distance is D_M multiplied by the factor $(1+z)$.

Angular diameter distance is from D_M divided by $\sqrt{1+z}$. This would be true for a cosmology with redshift scale-factor relation (25), with the objects retaining their usual sizes. As the object being viewed was smaller (in the past) relative to the observer, the angular diameter distance is reduced by another factor $\sqrt{1+z}$

$$D_A = \frac{2c}{H_0(1+z))}(\sqrt{1+z} - 1) \quad (36)$$

$$D_L = \frac{2c}{H_0}(1+z)(\sqrt{1+z} - 1) \quad (37)$$

Distance duality still applies.

Plotting the distance moduli for supernovae binned data [Betoule, 2014] gives Figure 5. It shows the new model, top curve. LCDM with a matter density of 0.3 is the middle curve, and matter density of 1.0 is the bottom curve.

Figure 5 New model, LCDM Ω_m = 0.3 and 1.0

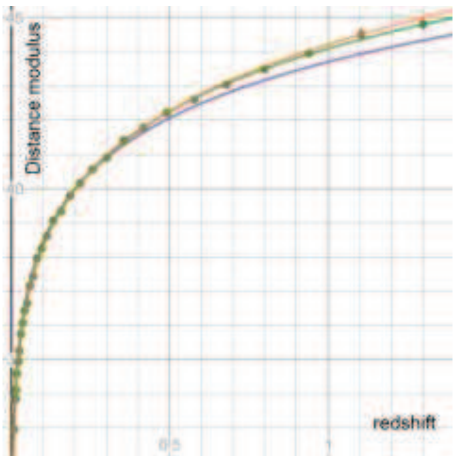
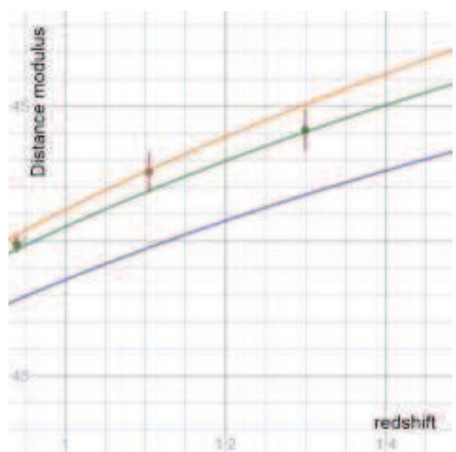


Figure 6 An enlargement of Figure 5



LCDM has two variable parameters, Ω_m and H_0. The new model has only one, H_0.

By looking at the binomial expansion of LCDM and the new model, we would expect LCDM to give a best fit Ω_m of slightly under 1/3.

For the new model

$$D_M = \frac{c}{2H}\left(z - \frac{z^2}{4} + \cdots\right) \qquad (38)$$

For LCDM

$$D_M = \frac{c}{H_0}\left(z - \frac{3mz^2}{4} + \cdots\right) \qquad (39)$$

Details are in Appendix B. By comparing (38) and (39) there is a match if $3m = 1$, where m is short for Ω_m. Most of the data points are at low values of z, so if LCDM is not correct but is varying the matter density to match data from the new model, we would expect it to predict $\Omega_m = 1/3$.

The matter density inferred from supernovae [Abbot, 2018] is 0.331.

2.7 Anisotropies in the CMBR

As mentioned in section 1.7, WMAP9 [Hinshaw, 2012] finds Ω_m= 0.2815 from $\Omega_m h^2 = 0.1368 \pm 0.005$ and a h value of 0.697 ± 0.02. If h is halved, Ω_m becomes four times as large, giving an Ω_m value of 1.126 ± 0.10.

There is a similar situation for PLANCK data.

So we can see why LCDM often gives values for the matter density varying between 0.25 and 0.333. Data from supernovae tend to give matter density values towards 0.333, values from $\Omega_m h^2$ give approximately 0.25

Often studies combine data from different methods and decide on a value between 0.25 and 0.333. In a flat universe the dark energy density parameter Ω_Λ is deduced to be between 0.667 and 0.75, but is really 0.

2.8 Baryon Acoustic Oscillations

Does the new model lead to any changes for BAO?

The BAO observations measure the standard ruler both parallel and perpendicular to the line of sight. Parallel measurements constrain $c/H(z)$ and perpendicular measurements constrain d_A

In section 2.6 eqn (11) was changed to (33)

$$D_M = \int_0^z \frac{c}{2H\sqrt{1+z}} dz$$

This might affect the interpretation of the parallel measurements. The parallel component has been identified as a probable source of the Hubble tension [Wu, 2020]. This tension is the subject of section 3.

Reports [Alam, 2016] on the values of d_M from BAO data are in Table 2, with the predictions from the new model using (34)

$$D_M = \frac{2c}{H_0}(\sqrt{1+z}-1)$$

With $H_0 = 70$ kms^{-1}Mpc^{-1}

Table 2 BAO measurements

Redshift	Average D_M measurement	Prediction of the new model
0.38	1509 ± 24	1497
0.51	1974 ± 28	1961
0.61	2298 ± 36	2304

2.9 Hubble parameter data

From the reworking of equations in section 2.6 equation (10) became (31)

$$2H(z) = \frac{\dot{a}}{a} = = -\frac{1}{(1+z)} \times \frac{dz}{dt}$$

i.e. the formulae for finding the Hubble parameter and H_0 are still valid, but the method measures twice the real expansion parameter, $2H$.

There is a problem however. Time dilation has not been taken into account when applying the method. The measured dt would be lower than the real dt by a factor $(1+z)$. For example if it appears that 0.1Gyr has passed in the ageing of galaxies then really 0.2Gyr might have passed and the real dt should be bigger. This reduces $H(z)$ by a factor $(1+z)$, so instead of measurements showing an apparent $H(z) = H_0(1+z)$ relation, the data is compatible with a constant $H(z)$.

Table 3 Hubble parameter data

z	$H(z)$	$H(z)/1+z$
0.199	75.0	62.6
0.24	79.69	64.3
0.35	82.1	60.8
0.57	92.4	57.8
0.6	87.9	54.9
0.73	92.3	53.3
2.34	222	66.5

Table 3 shows data from different groups, compiled by Meng [Meng, 2015]. Some of the data points (with lowest standard deviation compared to redshift) are chosen to illustrate the approximate $H(z) = (1+z)H_0$ relation, the errors in the middle column are up to ± 7 kms^{-1}Mpc^{-1}

The method uses spectroscopic dating - often a spectral feature D(4000) [Moresco, 2020] to determine ages. However galaxies and stars will evolve slower with time dilation and all features depending on their evolution will be affected, similarly with any age differences. So the last column is showing us the real $H(z)$, or $2H$.

2.10 Measurements of the Hubble constant

There are some changes to the methods used to find the value of H_0. They are discussed in the next section.

2.11 Large Scale Structure

Figure 2 of section 1.11 shows an apparently constant value for $f\sigma_8(z)$ with little or no variation with redshift. In the new model, $f(\Omega_m)$ is predicted to be constant at $-1+\sqrt{5/2} \approx 0.5811$ (Appendix D) σ_8 is about 0.8 so $f\sigma_8(z)$ is constant ≈ 0.46 a good match to the data of Figure 2.

The data for an apparently low matter density universe comes from different sources, but since they usually depend on values of $\Omega_b h^2$ or $\Omega_m h^2$, these matter densities could be four times as large, if the expansion parameter is halved. Supernovae data can be matched by the alternative model with constant H and fewer variable parameters. These factors indicate that it might be possible to have a solution to the Friedman equations with a cosmological constant of zero.

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Section 3. The Hubble Tension

Over recent years there has been tension in the values of the Hubble constant measured locally or via an inverse distance ladder approach. For a review [Riess, 2020]. The tension has now reached 4.4σ with local measurements being higher, around 74 kms^{-1}Mpc^{-1}, whereas the inverse distance ladder method, with BAO, being lower around 67.3 kms^{-1}Mpc^{-1}

The value for H_0 is derived in three different ways, from CMB data, BAO and the local distance method. Some changes are needed but compatibility is found at 75-76 kms^{-1}Mpc^{-1}.

3.1 The CMB data of PLANCK and WMAP

From Planck data [Aghanim, 2018], $\Omega_m h^2$ = 0.1430 if we assume that $\Omega_m = 1$ then $h = 0.37815$ and $H_0 = 75.63$ kms^{-1}Mpc^{-1}

From WMAP [Hinshaw, 2012], the values are $\Omega_m h^2 = 0.1367$, if $\Omega_m = 1$ then $h = 0.3697$ and $H_0 = 73.95$ kms^{-1}Mpc^{-1}

3.2 BAO measurements

BAO measurements parallel to the line of sight constrain $H(z)$. Due to the changes made in section 2.6, equation (11) became equation (33) and we can determine the real expansion parameter by putting

$$\frac{c}{2H_{true}\sqrt{1+z}} = \frac{c}{H_{LCDM}(z)} \qquad (40)$$

This is because the LCDM modelling for $H(z)$ would have had to include the $\sqrt{1+z}$ factor, but the true value wouldn't.

The data point with lowest standard deviation [Aubourg 2015] is at $z = 0.57$, the modelling would need to pass near that point, so

$$2H_{true}\sqrt{1.57} = H_{LCDM}(0.57) \qquad (41)$$

from (7) with $\Omega_m = 0.321$, $\Omega_k = 0$

$$H_{LCDM}(0.57) = 67.3\sqrt{0.321(1.57)^3 + 1 - 0.321}$$

This in an approximation and depends on the parameters, the above are from Planck data [Aghanim, 2018], but there is some choice here.

BAO is not used alone to find H_0 the method is often used with supernovae or CMB data, but typically it's found that (41) makes $2H_{true}$ about 9 to 11% larger than previous BAO results, about 74 ± 1.2 kms^{-1}Mpc^{-1}

3.3 Direct local distance method

Riess [Riess, 2019] recently measured the value of H_0 directly and found 74.03 kms^{-1}Mpc^{-1}

Details of the calculations used are in [Riess, 2016]. The derived value for H_0 is proportional to

$$X = 1 + \frac{1}{2}(1-q_0)z - \frac{1}{6}[1 - q_0 - 3{q_0}^2 + j_0]z^2$$

Equation (42), see also equations (4) and (5) of [Riess, 2016], q_0 is the deceleration parameter

$$q_0 = -\frac{a\ddot{a}}{\dot{a}^2} \qquad (43)$$

A value of q_0 of - 0.55 is used to determine the local value of H_0 and a jerk parameter $j_0 = 1$

In the new model from (14)

$$a = e^{-Ht}$$

so $q_0 = -1$ and the jerk parameter is still $j_0 = 1$

For the new model X simplifies to

$$X_{new} = 1 + z \qquad (44)$$

For the local method it simplifies to

$$X_{local} = 1 + 0.775z - 0.27375z^2 \qquad (45)$$

X is then determined from 600 supernovae with redshifts between 0.023 and 0.15 (Figure 8 of [Riess, 2016], page 47). The average redshift is approximately $z = 0.06$ and from (44) and (45) X_{new}/X_{local} at $z = 0.06$ is 1.01385

So the new estimate of H_0 is $74.03 \times 1.01385 =$ 75.06 kms^{-1}Mpc^{-1}

3.4 A combined estimate for the Hubble constant

The amended data from the methods above and lensing data from H0LiCOW [Wong 2019] are summarised in Table 4.

Table 4 Values of the Hubble constant

Method	The Hubble Constant ($kms^{-1}Mpc^{-1}$)
CMB (WMAP)	73.95 ± 1.4
CMB (Planck)	75.63 ± 0.29
BAO	74.0 ± 2.0
Local	75.06 ± 1.4
Lensing	73.3 ± 1.8

The errors are subjective, depending on the complexity of the method. The errors on the CMB measurements have been lowered as now the only uncertainty is from the measurement of $\Omega_m h^2$, for Planck it's as low as 0.0011. For BAO the process seems to involve many uncertainties. The local method and lensing errors are unchanged.

It's clear that the amended local method and the amended CMB method, especially Planck, seem most accurate and agreement can be found at a value for H_0 of 75-76 $kms^{-1}Mpc^{-1}$.

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Appendix A. The strength of gravity

The type of expansion proposed in this model ensures conservation of energy as the universe expands - (without any slowing of the expansion due to gravity).

Imagine a mass m, it's rest energy varies during the expansion (in the absence of gravity) as

$$mc^2 e^{2Ht} \quad \text{(A1)}$$

Energy would not be conserved.

With gravity included, the total energy of the mass varies as

$$\left(mc^2 - \frac{GMm}{R}\right) e^{2Ht} \quad \text{(A2)}$$

where the second term represents contributions from the rest of the universe of mass M and radius R. Small numerical constants are omitted for simplicity. Energy can be conserved if the quantity in the bracket of (A2) $= 0$.

$$G = \frac{Rc^2}{M} \quad \text{(A3)}$$

Formula (18), from the Friedman equations can be regarded as a necessary condition.

$$G = \frac{3H^2}{8\pi\rho}$$

It is to conserve energy as the universe expands. Gravity and the value of G is *caused* by the expansion, but does not change the rate of expansion, which is constant.

For a large stationary mass

$$\left(mc^2 - \frac{GMm}{R} - \frac{Gm^2}{r}\right) e^{2Ht} \quad \text{(A4)}$$

And using (A3)

$$G_{reduced} = \frac{c^2}{\frac{M}{R} + \frac{m}{r}} = \frac{G}{1 + \frac{Gm}{rc^2}} \quad \text{(A5)}$$

a reduction in the strength of gravity for regions of matter where m/r approaches c^2/G. These arguments give an indication that a future gravitational theory, based on General Relativity, should include the feature that the strength of gravity reduces for dense regions of matter.

The above formula is for a large stationery mass, and work is still ongoing to decide whether it applies to masses in motion in the same way. It's interesting to wonder however, that, as the earth gets nearer or further away from the sun over a year, whether a cyclic change in G would be measured.

We would expect, from (A5), an annual cyclic variation in G of amplitude $1.69 \times 10^{-10} G$. In [Matsuo, 2013] such a variation of the Earth's gravity field has been noted, (Fig 1a) of Matsuo. It's from Satellite Laser Ranging (SLR) data - and has the same amplitude and period. These variations are being interpreted as being due to mass redistributions of ice and water.

Appendix B. Binomial Expansions

Binomial expansion for the new model, from (34)

$$D_M = \frac{2c}{H_0}(\sqrt{1+z} - 1)$$

omitting the c/H_0 and for small z

$$= 2\left(1 + \frac{1}{2}z - \frac{1}{4}z^2 - 1\right) \quad \text{(B1)}$$

$$= z - \frac{1}{4}z^2 \quad \text{(B2)}$$

For LCDM from (11)

$$D_M = \int_0^z \frac{c}{H(z)} dz$$

$$= \int_0^z \frac{c}{H_0\sqrt{\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}} dz \quad \text{(B3)}$$

$$= \int_0^z (m(1+z)^3 + 1 - m)^{-1/2} dz \quad \text{(B4)}$$

a flat universe approximation, again omitting the c/H_0 and using m for Ω_m gives (B5)

$$= \int_0^z (m(1 + 3z + 3z^2 + \cdots) + 1 - m)^{-1/2} dz$$

$$= \int_0^z (1 + 3mz + 3mz^2)^{-1/2} dz \quad \text{(B6)}$$

$$= \int_0^z \left(1 - \frac{3}{2}mz + \cdots\right) dz \quad \text{(B7)}$$

$$= z - \frac{3m}{4}z^2 \quad \text{(B8)}$$

Comparing (B2) and (B8) there is a match for low z if Ω_m is 1/3. Most of the supernovae are at low z.

Higher values of z have a match, (using equal Hubble's constants) at values under 1/3.

Using graphical software we can see that at z = 0.5 it's $\Omega_m = 0.25$, z = 1.0 it's $\Omega_m = 0.19$ and at z = 1.5 it's $\Omega_m = 0.155$ so we would expect supernovae data to give values for Ω_m of slightly under 1/3.

Appendix C. Time symmetry and antimatter

The universe in the new model is infinitely old. There are explosive events due to collapsing matter reaching the critical mass/radius ratio, as described in Appendix A and section 2. However there is no definite beginning of time. If we were to be able to film such a universe and then run the film backwards, it would look the same - on the largest scales. This type of universe obeys the 'perfect cosmological principle'. The universe is isotropic, homogeneous and will always look the same.

Other areas of physics have laws with time symmetry. If we go forward in time far enough - will the region of the universe near us collapse and then 'bounce' in a type of Big Bang?

In the cosmologically reversed time universe, matter collapses, and bounces back out in a colossal explosion (just as our Big Bang would have originated from collapsing matter). Stars galaxies and planets would form and life would evolve (as the fundamental laws of electromagnetism and atomic physics are time symmetric).

If we do a time reversal

$$t \to -t \quad \text{(C1)}$$

$$H \to -H \quad \text{(C2)}$$

the equations of the new model look the same, e.g. (14)

$$a = e^{-Ht}$$

equation (18) shows that gravity would still be attractive.

$$G = \frac{3H^2}{8\pi\rho}$$

The model is unchanged by a change of time direction.

To conserve the measured CPT symmetry, a time reversed universe would have a charge conjugation and a parity reversal. It would be an antimatter universe.

One of the problems of Big Bang cosmology is that we are left with the question of why there is more matter than antimatter.

In the new cosmology, there is no cosmological distinction between past and future. So there is no distinction between a $+t$ matter universe and a $-t$ antimatter universe. It is valid to claim that we are living in the $-t$ antimatter universe.

So perhaps the fact that we normally only observe matter isn't a mystery after all. Even in a time reversed universe, we would still observe time flowing forward and observe matter to be normal matter (instead of antimatter), there would be a redshift and gravity would still be attractive.

Appendix D. LSS - Growth factor

From perturbation theory, in LCDM the growth factor D is a solution of

$$\ddot{D} + 2H(z)\dot{D} - \tfrac{3}{2}\Omega_m H_0^2(1+z)^3 D = 0 \quad \text{(D1)}$$

In the new model Ω_m remains constant at 1.0

and $H(z) = H$

$$\ddot{D} + 2H\dot{D} - \tfrac{3}{2}H^2 D = 0 \quad \text{(D2)}$$

for a solution of the form $D = e^{kHt}$

$$k^2 + 2k - \tfrac{3}{2} = 0 \quad \text{(D3)}$$

$$k = \sqrt{(5/2)} - 1 \approx 0.5811 \quad \text{(D4)}$$

$$f(\Omega_{m)} = \frac{1}{H}\frac{\dot{D}}{D} = k \quad \text{(D5)}$$

so in the new model $f(z)$ is constant at about 0.5811