Planck Length and Speed of Gravity (Light) from Z, without Prior Knowledge of G, $\hbar$, or c

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Abstract

For more than hundred years, it has been assumed that one needs to know the Newton gravitational constant G, the Planck constant $\hbar$, and the speed of light c to find the Planck length. Here we demonstrate that the Planck length can be found without any knowledge of G, $\hbar$, or c, simply by observing the change in the frequency of a laser beam in a gravity field at two altitudes. When this is done, we also show that the speed of light (gravity) easily can be extracted from any observable gravity phenomena. Further, we show that all observable gravity phenomena can be predicted using just these two constants, in addition to one variable that is dependent on the size of the gravity mass and the distance from the center of the gravity object. This lies in contrast to the standard theory, which holds that we need the three constants Max Planck suggested were the important universal constants, namely G, $\hbar$, and c; in that formulation, we also need a variable for the mass size, and the radius. Based on our new findings, we get both a reduction in the number of constants required and a simplification of understanding gravity that is directly linked to the Planck scale.

We discuss how this has a number of important implications that could even constitute a breakthrough in unifying quantum mechanics with gravity. Our analysis strongly indicates that standard physics uses two different mass definitions without being actively aware of it. The standard kg mass is used in all non-gravitational physics. Apparently, we are using the same mass in gravity, but we claim that the more complete mass is hidden in the multiplication of G and M. Based on this view, we will see that only two universal constants are needed, namely c and l_p, to do all gravity predictions compared to the G, h, and c in the standard view of physics. In order to unify gravity with quantum mechanics, we need to use this "embedded" mass definition from gravity, which also impacts the rest of physics.

Since 1922, a series of physicists have thought that the Planck length would play a major role in making progress in the understanding of gravity, particularly in the hope of unifying quantum mechanics with gravity. Although there have been a series of attempts to incorporate the Planck length in quantum gravity, little theoretical progress has been accomplished. However, with this recent discovery, we have reasons to think that a piece of the puzzle has emerged. We will continue our analysis and welcome other researchers to scrutinize our findings over time before drawing final conclusions.

Key Words: Planck length, Planck units, gravity, quantum gravity.

1 Introduction

The idea that fundamental units exist goes back at least to Stoney [1], who suggested in 1883 that there existed some fundamental natural units, which he derived from elementary charge, together with the speed of light and the Newton gravitational constant. These are known today as the Stoney units. It is fair to say that the Stoney units were overtaken by the units that Max Planck [2, 3] introduced in 1899 and 1906. Max Planck assumed there were three important universal constants, namely the Newton gravitational constant G, the Planck constant $\hbar$, and the speed of light c. Based on dimensional analysis, Planck calculated what he thought were a fundamental length $l_p = \sqrt{\frac{G\hbar}{c^3}}$, time $t_p = \sqrt{\frac{G\hbar}{c^5}}$, mass $m_p = \sqrt{\frac{\hbar c}{G}}$, and energy $E_p = \sqrt{\frac{\hbar c^5}{G}}$ (or Planck temperature $E_p k_B$, where k_B is the Boltzmann constant). Standard physics, therefore, assumes that one must know the three universal constants G, $\hbar$, and c to find or more precisely calculate the Planck units. We will strongly challenge that view here. We [4] have recently shown that the Planck length can be found without any knowledge of G and h, and we go one step forward and show that the Planck length can be found without any prior knowledge of G, h, and c. We will show that this leads to an ability to extract the speed of light (gravity?) from any gravitational phenomena without any prior knowledge of G, h, and c. From the speed of gravity c_{gw} and the Planck length alone, we can again predict all observable gravity phenomena. This gives new insight about the interpretation of the Planck length and the Planck scale in relation to gravity. We will claim that detecting any gravity phenomena is detecting the Planck scale; this also contrasts with standard theory.

It is also useful to have a short historical background on the history of the Planck lengths relation to gravity before we show how we can find it without G, $\hbar$, and c. In 1916, Einstein suggested that to move forward in the understanding of gravity, one had to develop a quantum gravity theory, but he did not link it to the Planck length at that time. In 1922, Eddington[1] [5] was likely the first to claim that the Planck length had to play an important role in advancing our understanding of quantum gravity. Actually, the claims of Planck and Eddington that the Planck length (and other Planck units) could play an important role in gaining deeper insights in physics were ridiculed by other famous physicists at that time, Bridgman [6] is one example, but he was not alone in his skepticism. For a long time, there was little interest in the Planck units in relation to gravity. In fact, several decades after Einstein pointed out the necessity of a quantum-gravitational theory in 1916, there was not much interest in exploring quantum gravity either, although one of the reasons for its neglect was because there were so many other interesting areas for research at that time, see [7] for more about this period.

The consensus among physicists working on quantum gravity theory today is that there must be a connection to the Planck length. There is also a generally accepted notion that the Planck length represents the shortest possible length, see [8, 9], for example. In quantum gravity theory, one predicts that there are Lorentz symmetry violations at the Planck scale. One of the challenges is that the Planck energy is very high, far above what can be achieved in the Large Hadron Collider. However, several physicists have suggested that one could look for vague effects from the Planck scale that hypothetically also spillover at lower energies, but despite extensive search for evidence of this occurring, there have been no findings so far [10]. In spite of the view that the Planck scale has not been detected and may not be in the foreseeable future, we are still not even close to be able to build accelerators that give energy scales close to the Planck scale. Some could claim the situation is similar to the ether theory of the past century; if the ether cannot be detected, why not simply abandon the ether (as Einstein did in 1905). However, our findings in this paper strongly support that the Planck length, and therefore the Planck scale are the very essence of gravity, together with the speed of gravity that we can extract from gravity observations with no prior knowledge of h, G, or c.

2 Mass and the Compton Wavelength

One important element for our work will be to understand the Compton wavelength in relation to all masses, not only the electron. In 1923, Compton [11] introduced the Compton wave for a rest-mass; it is given by the following formula

$$\lambda = \frac{h}{mc} \tag{1}$$

where h is the Planck constant, and c is the speed of light in vacuum; this is a well-tested formula in relation to electrons. However, we will claim the formula is valid for any mass, and will discuss that in greater detail here. Solving Formula 1 with respect to the mass, we get

$$m = \frac{h}{\lambda}\frac{1}{c} = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{2}$$

That is, any mass in terms of kg can be described by this formula if we know its Compton wavelength, the Planck constant, and the speed of light. It is worth paying attention to the fact that only the Compton wavelength will be different for different masses. When we are only interested in the mass in terms of kg, we are not concerned about properties such as charge, but only about a way to describe the mass size. Some will likely protest here and claim that only elementary particles can have a Compton wavelength and that the Compton formula therefore cannot hold for any mass. We agree that likely only elementary particles such as electrons have a physical Compton wavelength, but larger masses consist of many elementary particles, so there must be a way to add the Compton waves from elementary particles in a composite mass such that it is consistent with other parts of physics. The Compton wave from elementary particles making up a larger mass can be added the following way

$$\lambda = \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \cdots + \frac{1}{\lambda_N}} \tag{3}$$

[1] Who was the first to confirm the GR prediction of gravitational light bending experimentally.

For many this may be unfamiliar territory, but it leads to the same standard aggregation rule of smaller masses into a larger mass, $m = m_1 + m_2 + m_3$. In other words, we have

$$
\begin{aligned}
m &= m_1 + m_2 + m_3 && (4)\\
\frac{\hbar}{\lambda}\frac{1}{c} &= \frac{\hbar}{\bar{\lambda}_1}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_1}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_1}\frac{1}{c} \\
\frac{\hbar}{\frac{1}{\frac{1}{\lambda_1}+\frac{1}{\lambda_2}+\frac{1}{\lambda_3}}}\frac{1}{c} &= \frac{\hbar}{\bar{\lambda}_1}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_1}\frac{1}{c} + \frac{\hbar}{\bar{\lambda}_1}\frac{1}{c} && (5)
\end{aligned}
$$

This mean that the Compton wave from a large mass simply represents the aggregate of the Compton waves from the masses making up the larger mass, and Formulas 1 and 2 are therefore valid for all masses. We are fully aware that any mass larger than the Planck mass then will have a Compton wavelength shorter than the Planck length. We will claim that no Compton wavelength that is shorter than the Planck length can exist, but there is no conflict here, as the Compton wavelengths of larger objects are compositions of many Compton wavelengths that follows the addition rule of formula 3. That is, elementary particles still cannot have a Compton wavelength shorter than the Planck length, so this means masses larger than a Planck mass must consist of more than one elementary particle. That a mass is smaller than a Planck mass naturally does not mean it only consists of one elementary particle, as the proton, for example, is known to consist of several elementary particles, so our theory is fully consistent with standard theory in this respect.

We can even find the Compton wave from a larger mass without knowing the Planck constant. In order to do this, we will start by finding the Compton wavelength of an electron. This we can do from Compton scattering, where one shoot photons at an electron and measures the wavelength of the electron before and after; based on this, we can find the Compton wavelength of the electron

$$
\begin{aligned}
\lambda_1 - \lambda_2 &= \frac{h}{mc}(1-\cos\theta) \\
\lambda_1 - \lambda_2 &= \frac{h}{\frac{h}{\lambda_e}\frac{1}{c}c}(1-\cos\theta) \\
\lambda_1 - \lambda_2 &= \lambda_e(1-\cos\theta) \\
\lambda_e &= \frac{\lambda_1 - \lambda_2}{1-\cos\theta} && (6)
\end{aligned}
$$

That is, we need only knowledge of the photon's wavelength before and after the electron hit to find the electron's Compton wavelength. We do not need to know the Planck constant, or the electron mass to find the electron's Compton wavelength. Next we can find the Compton wavelength of a proton without knowing its mass or knowing the Planck constant by using a cyclotron. The angular cyclotron velocity is given by

$$\omega = \frac{v}{r} = \frac{qB}{m} \tag{7}$$

Since electrons and protons have the same charge, the cyclotron ratio is equal to their mass ratio, we have

$$\frac{\omega_P}{\omega_e} = \frac{\frac{qB}{m_P}}{\frac{qB}{m_e}} = \frac{m_e}{m_P} = \frac{\lambda_P}{\lambda_e} \tag{8}$$

If we know the cyclotron frequency of the electron and the proton in addition to the Compton wavelength of the electron, then iwe know Compton wavelength of the proton. The following papers [12, 13] used cyclotron resonance experiments to find the proton to electron mass ratio, $\frac{\omega_P}{\omega_e} = \frac{\lambda_e}{\lambda_P} = \frac{m_P}{m_e}$; And they found it is about 1836.15247.

What is important to understand at this stage is that we can find the Compton wavelengths of electrons and protons without any knowledge of the Planck constant. This also means that we can find the Compton wavelength of any larger mass, as it must consist of many protons. So, if we can count the number of protons in a mass, then we know its Compton wavelength without having to know its kg mass or the Planck constant. Most atoms also consist of neutrons, but they have almost the same mass as a proton, so we can treat them as protons for simplicity. The Compton wavelength of the large mass will be an aggregated Compton wavelength given by formula 3. The important takeaway is that we can find the Compton wavelength of any mass without any knowledge of the Planck constant. This will play a key role when we want to find the Planck length from red-shift.

To find the Compton wavelength of the Earth, we could first find the Compton wavelength of an electron by Compton scattering. Next we could find the Compton wavelength of a proton by a cyclotron. We could then count the number of protons in the Earth. Theoretically, this is "easily" possible, but in practice it is impossible due to the enormous resources it would require. Luckily, the Compton wavelength is also proportional to gravitational acceleration. In a Cavendish [14] apparatus, we can find the gravitational acceleration between the small spheres and the larger spheres through the following formula (see Appendix A for full derivation)

$$g = \frac{L4\pi^2\theta}{T^2} \tag{9}$$

where L is the distance between the small balls, θ is the measured angle, and T is the measured oscillation period. Even a small Cavendish apparatus can do this accurately today. Unlike when, for example, we are using a Cavendish apparatus to find G, we need no knowledge of the Planck constant here; why this is the case will become clear later on. Picture 1 shows a modern Cavendish apparatus, where old invented mechanics are combined with fine electronics to measure the angle and oscillation period very accurately; these are then feed directly into a computer with a USB cable. The g we extract here is the gravitational acceleration from the large lead ball for the distance R to the center of the small lead ball. Later in this article it will be shown that we are not even indirectly dependent on G here. Further, the ratio of the reduced Compton wavelengths of different objects are proportional to their gravitational acceleration fields in the following way

$$\frac{g_1 R_1^2}{g_2 R^2} = \frac{\frac{GM_1}{R_1^2}R_1^2}{\frac{GM_2}{R_2^2}R_2^2} = \frac{M_1}{M_2} = \frac{\frac{\hbar}{\bar{\lambda}_1}\frac{1}{c}}{\frac{\hbar}{\bar{\lambda}_2}\frac{1}{c}} = \frac{\bar{\lambda}_2}{\bar{\lambda}_1} \tag{10}$$

This means that to find the Compton wavelength of the Earth, we "only" need to count the number of protons in the large lead ball, as we then will have the Compton wavelength of the large lead ball by aggregating the Compton wavelengths of the protons, that we again obtained from the cyclotron, measuring the Compton wave of the electron first. Now all we require for obtaining the Compton wavelength of the Earth is to measure the gravitational acceleration field of the Earth. Naturally, it is a formidable task to count the number of atoms, even in a half kg size lead ball or silicon crystal ball, but this is basically what one has worked on recently in silicon crystal spheres, to get a more precise definition of the kilogram, and we understand this method is linked to counting atoms in a very uniform medium[2] [15, 16]. Even though it is challenging, it is not impossible. It is easy to think, albeit mistakenly, that we need G here, as we normally have $g = \frac{GM}{r^2}$, but that is to predict g when one knows the mass in kg as well as G. Here we simply measure g in a Cavendish apparatus, as well as from an object we drop on the surface on the Earth: none of these methods require knowledge of G or M, and we will explain exactly why later in this paper.

Normally, it is the de Broglie [17, 18] wave that has been linked to matter in theoretical work, and less so the Compton wavelength. The de Broglie wave is $\lambda_b = \frac{h}{mv\gamma}$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$. It is not mathematically defined for a rest-mass particle $v = 0$, as this would involve dividing by zero. However, one could try to argue that due to Heisenberg's [19, 20] uncertainty principle, a mass can never stand still; as we let v converge to 0 the de Broglie wave converges to infinity. Several physicists have interpreted this as meaning that the de Broglie wavelength is infinite when $v = 0$ (or v converge to zero), see, for example, [21]. To have a particle that has a matter wave that spreads out infinitely sounds very strange. The Compton wavelength, on the other hand, is always well-defined for rest-mass particles, and also for any mass that has length at what we can call subatomic length scales, while the de Broglie wavelength easily has macroscopic and even close to infinite lengths at low velocities. Such waves have never been observed, but are predicted from the de Broglie matter wave formulation. Interestingly, the de Broglie wave can interpreted as simply a mathematical derivative of the Compton wave, where we always have $\lambda_b = \lambda\frac{c}{v}$. We will claim there is no need for two types of matter waves, and that the Compton wave is the more important of the two, since it always is well-defined; this is discussed in greater detail by Haug [22]. What is important here is that we can find the Compton wave from masses without knowing the Planck constant, as this is a key to not being dependent on the Planck constant to later find the Planck length. One does not need to agree on our interpretation of the de Broglie wave, but it is evident that one can express all rest-masses in kg from the Compton wave as $m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$, if one does not accept this, we would be interested in seeing a counter-example to disprove our claims.

[2] We are not experts on silicon spheres and whether or not one is able to count the atoms within them without relying on the Planck constant and the speed of light, but we mention this to illustrate the idea that counting atoms in macroscopic objects is indeed something that several areas of physics are involved with currently.

3 Measuring the Planck Length without Relying on Knowledge of Any Constant

The standard gravitational red-shift is given by

$$Z = \frac{\sqrt{1-\frac{r_s}{R_L}}}{\sqrt{1-\frac{r_s}{R_h}}} - 1 = \frac{\sqrt{1-\frac{2g_L}{R_L c^2}}}{\sqrt{1-\frac{2g_h}{R_h c^2}}} - 1 \tag{11}$$

where r_s is the Schwarzschild radius and Z is the observed gravitational red-shift from a laser beam sent from radius R_h to radius R_L. That is, R_L and R_h are the distance from the center of the Earth from two altitudes where the measurements take place (the sender frequency and receiver frequency), where $R_L < R_h$. Further g_h and g_L are the gravitational acceleration at these altitudes. Both the radiuses and the gravitational acceleration fields we can measure with no knowledge of G, h, and c. Gravitational red-shift in a weak gravitational field (the Earth) was confirmed by 1959 by the well-known Pound-Rebka [23] experiment, and have been repeated. In a weak gravitational field where $\frac{r_s}{R_h} << 1$, we can approximate this very well by using the first term of a Taylor series expansion, this gives

$$Z \approx \frac{1-\frac{1}{2}\frac{g_L}{R_L c^2}}{1-\frac{1}{2}\frac{g_h}{R_h c^2}} - 1 \tag{12}$$

Solved with respect to c gives (see Appendix B for a detailed derivation)

$$c \approx \sqrt{\frac{g_L R_L + g_L R_L Z - g_h R_h}{Z}} \tag{13}$$

Or the exact solution that also holds for a strong gravitational field is

$$c = \frac{\sqrt{g_L R_L Z^2 + 2g_L R_L Z + g_L R_L - 2g_h R_h}}{\sqrt{Z^2+2Z}} \tag{14}$$

Further, from Newton theory we have that the orbital velocity is given by

$$v_o = \sqrt{\frac{\mu}{r}} \tag{15}$$

where $\mu = GM$ is the gravitational parameter. If we solve the Planck length formula,$l_p = \sqrt{\frac{G\hbar}{c^3}}$ with respect to G we get $G = \frac{l_p^2 c^3}{\hbar}$. The mass in kg we have shown can be described as $m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$. This means we can rewrite the gravitational parameter as

$$\mu = GM = \frac{l_p^2 c^3}{\hbar} \times \frac{\hbar}{\bar{\lambda}}\frac{1}{c} = c^2 \frac{l_p^2}{\bar{\lambda}} \tag{16}$$

Inputing this in the orbital velocity formula, we get

$$v_o = \sqrt{\frac{c^2 l_p^2}{\bar{\lambda} R}} = c l_p \sqrt{\frac{1}{\bar{\lambda} R}} \tag{17}$$

solved with respect to l_p gives

$$l_p = \frac{v_o}{c}\sqrt{\bar{\lambda} R} \tag{18}$$

Next we replace the speed of light with $c = \frac{\sqrt{g_L R_L Z^2 + 2g_L R_L Z + g_L R_L - 2g_h R_h}}{\sqrt{Z^2+2Z}}$

$$
\begin{aligned}
l_p &= \frac{v_o}{\frac{\sqrt{g_L R_L Z^2+2g_L R_L Z+g_L R_L-2g_h R_h}}{\sqrt{Z^2+2Z}}}\sqrt{R_L\bar{\lambda}_E} \\
l_p &= \frac{\sqrt{g_L R_L}\sqrt{Z^2+2Z}}{\sqrt{g_L R_L Z^2+2g_L R_L Z+g_L R_L-2g_h R_h}}\sqrt{R_L\bar{\lambda}_E} \\
l_p &= \frac{\sqrt{R_L\bar{\lambda}_E}\sqrt{Z^2+2Z}}{\sqrt{Z^2+2Z+1-2\frac{g_h}{g_L}\frac{R_h}{R_L}}} \\
l_p &= \frac{\sqrt{R_L\bar{\lambda}_E(Z^2+2Z)}}{\sqrt{Z^2+2Z+1-2\frac{R_L}{R_h}}}
\end{aligned} \tag{19}
$$

That is, the Planck length can be found from simply measuring the frequency of a laser beam at two altitudes, as this gives Z, and also we need to know the reduced Compton wavelength of the Earth $\bar{\lambda}_E$, that both can be found without any knowledge of G or the Planck constant or c. We are, from a single beam of light in a gravity field, finding both the speed of light and the Planck length, a rather remarkable feat. We will soon see that these are the only two constants required to predict any observable gravity phenomena.

Since we can approximate the speed of light from $c \approx \sqrt{\frac{g_L R_L+g_L R_L Z-g_h R_h}{Z}}$ as shown above; we can also make a more compact approximation formula for the Planck length that still should be very accurate in a weak gravity field, such as on the surface on the Earth. Then we get

$$
\begin{aligned}
l_p &= \frac{R_L}{c}\sqrt{g_L\bar{\lambda}} \\
l_p &\approx \frac{R_L}{\sqrt{\frac{g_L R_L+g_L R_L Z-g_h R_h}{Z}}}\sqrt{g_L\bar{\lambda}_E} \\
l_p &\approx \frac{\sqrt{R_L\bar{\lambda}_E Z}}{\sqrt{1+Z-\frac{R_L}{R_h}}}
\end{aligned} \tag{20}
$$

Since we normally have $Z << \frac{R_L}{R_h}$, this can be simplified further to $l_p \approx \frac{\sqrt{R_L\bar{\lambda}_E Z}}{\sqrt{1-\frac{R_L}{R_h}}}$. This means we can measure the Planck length by observing the change in wavelength in a gravitational field and the Compton wave of the gravitational object. This means we can measure the Planck length without any knowledge of G, $\hbar$, or c. This is in strong contrast to the assumptions in standard physics, where it is assumed we only can derive the Planck units using dimensional analysis from G, c, and $\hbar$.

Table 1 shows a series of ways in which we can find the Planck length without any prior knowledge of G, $\hbar$, and in some cases also without knowledge of c. Basically from any observable gravity phenomena, we can extract the Planck length with no knowledge of G and $\hbar$ if we know the speed of light (gravity?).

4 Are We Still Not Indirectly Dependent on G?

In several of the formulas in Table 1, we see that the Planck length is dependent on the gravitational acceleration g. And the gravitational acceleration formula is known as $g = \frac{GM}{R^2}$, so does this not mean we simply are dependent on G indirectly? First of all, we clearly do not need to know G to measure g. And as we will soon see, we also do not even need to know G to predict g.

We have that the Planck length is given by $l_p = \sqrt{\frac{G\hbar}{c^3}}$, as first described by Max Planck. There is nothing wrong mathematically with solving this formula with respect to G; this gives $G = \frac{l_p^2 c^3}{\hbar}$. Haug [24] has suggested that the Newton gravity constant is a universal composite constant of exactly this form. However, if one needs to know G first to find the Planck length, this would just lead to a circular unsolvable problem, or more precisely then it would be very clear that the Planck length is a function of G, and not the other way around. Nevertheless, we have demonstrated that we can find the Planck length from red-shift with no prior knowledge off G, $\hbar$, and c.

If we look at the kg mass as $m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$ (see section 2) then to know G and m, we need to know the following constants, G, $\hbar$, and c. And to know GM, we still need to know G, $\hbar$, and c, in other words, the three constants

Planck Length	Prediction	Easily applicable in practice?
From redshift	$l_p = \frac{\sqrt{R_L \bar{\lambda}_E (Z^2+2Z)}}{\sqrt{2+Z^2+2Z-\frac{R_L}{R_h}}}$	No need G, c or $\hbar$
From redshift Weak field approximation	$l_p \approx \frac{\sqrt{R_L \bar{\lambda}_E Z}}{\sqrt{1+Z-\frac{R_L}{R_h}}} \approx \frac{\sqrt{R_L \bar{\lambda}_E Z}}{\sqrt{1-\frac{R_L}{R_h}}}$	No need G, c, or $\hbar$
From time dilation	$l_p = \frac{\sqrt{R_L \lambda_E (T_h^2-T_L^2)}}{\sqrt{2T_h^2-2T_L^2\frac{R_L}{R_h}}}$	No need G, c or $\hbar$
From time dilation	$l_p \approx \frac{\sqrt{R_L \lambda_E (T_h-T_L)}}{\sqrt{T_h-T_L\frac{R_L}{R_h}}}$	No need G, c or $\hbar$
From gravitational light bending	$l_p = \frac{\sqrt{\delta \bar{\lambda} R}}{2}$	No need G, c or $\hbar$
From gravitational acceleration field	$l_p = \frac{R}{c}\sqrt{g\bar{\lambda}}$	No need G or $\hbar$
From orbital velocity	$l_p = \frac{\sqrt{R\bar{\lambda}}}{c} v_o$	No need G or $\hbar$
From escape velocity	$l_p = \frac{\sqrt{R\bar{\lambda}}}{c}\frac{v_e}{2}$	No need for G or $\hbar$
From From grandfather pendulum clock	$l_p = \frac{Rf}{c}\sqrt{2\pi L\lambda}$	No need for G or $\hbar$
From Newton force spring	$l_p = \frac{R}{c}\sqrt{\frac{kx\lambda}{2\pi m}} = \frac{R}{c}\sqrt{\frac{g\lambda}{2\pi}}$	No need G or $\hbar$
From two colliding elastic balls "Newton cradle"	$l_p \approx R\frac{v_{out}}{c}\sqrt{\frac{\bar{\lambda}}{2H}}$	No need G or $\hbar$ H is here hight of ball drop

Table 1: Ways to measure (extract) the Planck length from gravitational observations. We see that we never need to know G or $\hbar$ to find the Planck length. In the Newton cradle, v_{out} is the velocity of the second ball after taking a hit by a ball dropped from height h.

Max Planck assumed were the most important constants. However, if we are aware that the gravity constant is a composite constant, then to know G and M, we still need to know three fundamental constants: l_p, $\hbar$, and c, so one could suspect we only have replaced one constant with another one, simply by some change in how to express units. But that is not the case, which is clear when we now look at GM. Since $G = \frac{l_p^2 c^3}{\hbar}$ and the mass both contain the Planck constant, one in the denominator and the other in the numerator, these will always cancel when we multiply G with M. That is, to know $GM = \frac{l_p^2 c^3}{\hbar} \times \frac{\hbar}{\bar{\lambda}}\frac{1}{c} = c^2 \frac{l_p^2}{\bar{\lambda}}$, we only need to know two constants, namely c and l_p, rather than three constants, but only if we understand G is a composite constant and in addition, we are able to measure l_p without first knowing G, as we have demonstrated. Based on this view, we will claim that G likely contains the Planck constant itself in order to get rid of the Planck constant in the mass, something we will get back to shortly.

Some will likely react here, as Newton was not aware of the Planck constant, the speed of light, or the Planck length in his time. First of all, Newton never invented such a constant; his gravity force formula, that he only stated in words in the Principia, was actually: $F = \frac{Mm}{R^2}$, and not the well-known "Newton" formula: $F = G\frac{Mm}{R^2}$. Further, the gravitational constant was not introduced by Cavendish in 1798, even though a Cavendish apparatus can be used to find it when we use the kg mass definition.

The gravitational constant was actually first introduced in a footnote in a paper published in 1873 by Cornu and Baille [25], not so long after the kg had become the new mass/weight definition. One can speculate on why it was first introduced in a footnote. We actually suspect that the authors must have been aware that introducing a gravitational constant would lead to a formula different from what Newton had stated originally, and that it was therefore "safer" to put it in a footnote. Who would have wanted to be the first to indicate the Newtonian formula was incomplete, or lacked a constant? The gravitational constant first had the notation f, and was only called G in 1894 by Boys [26]. Planck [27] used the notation f for the gravity constant as late as in 1928, but the G notation became the "standard" from the 1930s onward. What notation was used for the gravity constant is naturally not of any real importance here, we just mention it to show the history of how we got to the formula that is known as the Newton gravity formula today. What is important is that one needed a gravity constant with the dimensions of G to calibrate the formula when using the kg mass definition. When using the kg definition of mass, one first needs to calibrate the formula to one observation, for example in a Cavendish apparatus to find G. One can then use the formula to derive and predict other gravity phenomena. The gravitational constant has also been carried over to the general relativity theory of Einstein [28].

Newton was actually able to predict the relative mass size in a series of planets without any gravitational constant, see [29], for example. We will show that the gravitational constant contains missing information that is needed to calibrate the Newton formula when one uses a kg mass definition. That is, the gravitational constant is not unique, and perhaps not even that fundamental, as it is linked to the kg mass definition. That G came before the Planck length and the Planck constant, and that the Planck length can be calculated from G (and two other constants) does not mean it is more fundamental than the Planck length. There is nothing wrong with claiming G is a universal composite constant and, as we have seen, the Planck constant embedded in G is needed to get the Planck constant out of the mass, and the Planck length into the mass. But can this actually be correct? In the Newton formula, we have GMm and not just GM, and even when assuming the gravitational constant is a composite constant, the Planck constant does not cancel out for $GMm = \frac{l_p^2 c^3}{\hbar} \times \frac{\hbar}{\lambda_1}\frac{1}{c} \times \frac{\hbar}{\lambda_1}\frac{1}{c} = \hbar c \frac{l_p}{\lambda_1}\frac{l_p}{\lambda_2}$. However, we will claim GMm is never used in predicting any observable gravity phenomena. The small mass in GMm always cancels out when one derives observable gravity phenomena from the Newton formula; the Newton formula only calculates the gravitational effects from one mass and it is a one body problem formula. In a two-body formula, the gravity parameter is changed from $\mu = GM$ to $\mu = G(M_1 + GM_2) = GM_1 + GM_2$. So, when we are working with gravitational effects from two masses, we still have GM and not GMm. The second mass in the Newton formula is only used for derivations where it actually always cancels out before we get a formula that can predict any observable gravity phenomena, as is shown in Table 2.

Important insight:		
Gravitational constant	$G = \frac{l_p^2 c^3}{\hbar}$	Need $\hbar$, l_p and c
Mass (kg definition)	$M = \frac{\hbar}{\lambda}\frac{1}{c}$	Need $\hbar$ and c
Gravitational constant times mass	$GM = c^2 \frac{l_p^2}{\lambda}$	Only need l_p and c
Non "observable" predictions: (contains GMm)		
Gravity force	$F = G\frac{Mm}{R^2} = \frac{\hbar c}{R^2}\frac{l_p}{\lambda_M}\frac{l_p}{\lambda_m}$	(needs $\hbar$, c and l_p or G)
Observable predictions: (contains only GM)		
Frequency Newton spring	$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi R}\sqrt{\frac{GM}{x}} = \frac{c}{2\pi R}\sqrt{\frac{l_p^2}{x\lambda}}$	Only need l_p and c
Periodicity Pendulum (clock)	$T = 2\pi\sqrt{\frac{L}{g}} = T = 2\pi R\sqrt{\frac{L}{GM}} = T = \frac{2\pi R}{c}\sqrt{\frac{L\lambda}{l_p^2}}$	Only need l_p and c
Gravity acceleration	$g = \frac{GM}{R^2} = \frac{c^2}{R^2}\frac{l_p^2}{\lambda}$	Only need l_p and c
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}} = c\sqrt{\frac{l_p^2}{R\lambda}}$	Only need l_p and c
Time dilation	$T_R = T_f\sqrt{1 - \sqrt{\frac{2GM}{R}}^2/c^2} = T_f\sqrt{1 - \frac{2l_p^2}{R\lambda}}$	Only need l_p
Gravitational red-shift	$z = \frac{\sqrt{1-\frac{2GM}{R_1c^2}}}{\sqrt{1-\frac{2GM}{R_2c^2}}} - 1 = \frac{\sqrt{1-\frac{2l_p^2}{R_1\lambda}}}{\sqrt{1-\frac{2l_p^2}{R_2\lambda}}} - 1$	Only need l_p
Gravitational red-shift	$z_\infty(r) \approx \frac{GM}{c^2R} = \frac{l_p^2}{R\lambda}$	Only need l_p
Gravitational deflection	$\delta = \frac{4GM}{c^2R} = \frac{4}{R}\frac{l_p^2}{\lambda}$	Only need l_p
Advance of perihelion	$\frac{6\pi GM}{a(1-e^2)c^2} = \frac{6\pi}{a(1-e^2)}\frac{l_p^2}{\lambda}$	Only need l_p and c
Velocity ball Newton cradle	$v_{out} = \sqrt{2gH} = \frac{cl_p}{R}\sqrt{2H/\lambda}$	Only need l_p and c
Indirectly/"hypothetical" observable predictions: (contains only GM)		
Escape velocity	$v_e = \sqrt{\frac{2GM}{R}} = c\sqrt{2\frac{l_p^2}{R\lambda}}$	Only need l_p and c
Schwarzschild radius	$r_s = \frac{2GM}{c^2} = 2\frac{l_p^2}{\lambda}$	Only need l_p and c

Table 2: The table shows that any gravity observations we can observe contain GM and not GMm; GM contains and needs less information than is required to find G and M. The L in the line for pendulum clock is the length of the pendulum, and H is the height of the ball drop in a two ball Newton cradle.

5 We can also extract the speed of light (gravity?) from any gravity phenomena without knowledge of G and h

In this section, we will show that we can extract the speed of light from any observed gravity phenomena without prior knowledge of the speed of light and also no knowledge of G or $\hbar$. This is demonstrated in several cases simply by using Newton gravitational mechanics. Table 3 shows a series of ways to extract the speed of light from gravity observations. From some observations, we need no knowledge of physical constants at all. In other observations, we need to know the Planck length, but never do we need knowledge of G or $\hbar$. We can extract both the speed of light and the Planck length from gravity observations with no knowledge of G, c, or h.

From observation:	**Solved with respect to** c	**Comment constants needed etc.**
Gravitational light bending	$c=\sqrt{\frac{4gR}{\delta}}$	No constant needed
Red-shift	$c=\frac{\sqrt{Z^2+2Z+2-2\frac{R_L}{R_h}}}{\sqrt{\frac{Z^2}{R_L g_L}+\frac{2Z}{R_L g_L}}}$	No constant needed
Red-shift (weak field)	$c\approx\frac{\sqrt{1+Z-\frac{R_L}{R_h}}}{\sqrt{\frac{Z}{R_L g_L}}}\approx\frac{\sqrt{1-\frac{R_L}{R_h}}}{\sqrt{\frac{Z}{R_L g_L}}}$	No constant needed
Grandfather pendulum clock	$c=\frac{Rf}{l_p}\sqrt{2\pi L\lambda}$.	Need l_p
	where: f : pendulum frequency, L length of pendulum.	
Gravitational acceleration	$c=\sqrt{\frac{g\bar{\lambda}R^2}{lp^2}}$	Need l_p
Orbital velocity	$c=v_o\sqrt{\frac{\bar{\lambda}R}{l_p^2}}$	Need l_p
Two colliding balls (Newton 'cradle')	$c\approx\frac{R}{l_p}v_{out}\sqrt{\frac{\bar{\lambda}}{2H}}$ h : hight of ball drop.	Need l_p
"Non"-observables		
Escape velocity	$c=v_e\sqrt{\frac{\bar{\lambda}R}{2l_p^2}}$	Need l_p

Table 3: Ways to measure (extract) the speed of light/gravity from gravitational observations. As we can see, we need no knowledge of G or $\hbar$ to find the speed of light, In a series of measurements, we do need to know the Planck length, but the Planck length can, as we have shown, be extracted from red-shift with no knowledge of G, $\hbar$, or c.

That we can extract the speed of light from any gravity phenomena without prior knowledge of G, h, or c indicates that this could be described as the speed of gravity c_{gw}, perhaps. It is also still a mystery in standard theory why the speed of gravity is assumed to be the same of light. In 1890, Lévy [30] suggested that the speed of gravity likely was equal to the speed of light. In 1904, Poincaré [31] argued that the speed of gravity could not be larger than the speed of light (in a vacuum). In Einstein's general relativity, it is assumed that the speed of gravity is equal to the speed of light, see [32, 33], for example. However, there are still several gravity theories that try to argue that the speed of gravity is faster than the speed of light, for example, [34, 35]. More research on the speed of gravity is therefore of great importance to clarify this, and to develop a better understanding of gravity.

It has been assumed that one only can measure the speed of gravity by detecting the hypothetical gravitons, although they have not been detected yet, or by measuring the speed of gravitational waves. LIGO announced that it has detected gravitational waves, but there is still uncertainty concerning the velocity of them; based on interpretation of LIGO measurements, there is a very wide confidence interval. For example, Cornish, Blas, and Nardini [36] assert, based on LIGO data, that the speed of gravitational waves with 90% certainty must be in the interval $0.55c < c_{gw} < 1.42c$. Even if this strongly points in the direction that the speed of gravity is in the ball-park of the speed of light, this is still an incredibly wide confidence interval. Doing standard gravity measurements, as described in this section, will give us a much narrower confidence interval for the speed of gravity, if the speed we obtain here is directly linked to the speed of gravity. Why should all observable gravity phenomena only need the Planck length, or the Planck length and the speed of light, in addition to a variable that is dependent on mass size and the distance from the object if the speed of light was not directly linked to gravity, see Table 2. And how can we extract the speed of light from any gravity phenomena with no prior knowledge of G, h, or c? Some may argue that our derivations are trivial and obvious, or that they give no new insight into gravity theory. However, to our knowledge, no one has shown that the speed of light can be extracted from any gravity phenomena without relying on knowledge of G, h, and c. Naturally, if one has G, one can extract the speed of light from G, if one know the Planck length and the speed of light. But we are extracting the speed of light from gravity observations with no knowledge of any of these constants, so this is significant different, even if trivial mathematically.

One can naturally discuss if our formulas are extracting the speed of gravity or just the speed of light, and we encourage further discussion. This is an extremely important question to settle. If we can fully confirm that the speed of gravity is indirectly or directly linked to the speed of light, then we can exclude a series of alternative gravitational theories claiming that the speed of gravity is faster than c. In addition, if we are able to measure the speed of gravity accurately, then this can likely help us understand gravity even better, particularly around whether or not there is a link between gravity and electromagnetism. In our view, to detect the speed of light or the speed of gravity may actually be the same thing. The speed of light is embedded, not only in electromagnetism through the speed of electromagnetic "waves", but it is also embedded internally in any mass, because how else could we describe the size of any kg mass with the formula $m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$, if it not the case that the speed of light plays an important role in mass?

Also, one can ask why we can extract the speed of light (gravity?) from Newtonian gravity phenomena (and not only GR phenomena), such as gravitational acceleration only by knowing one other constant[3], which is the Planck length. As we have demonstrated, we can extract the Planck length from gravity phenomena without knowledge of any other constant. Gravity acceleration came long before Einstein's general relativity theory; it is a formula we get from Newton gravitational theory. Newton claimed that gravity was instantaneous, so most physicists today claim that, embedded in Newton gravitational theory is the assumption of infinite gravity velocity, or that there may be no assumptions about the velocity of gravity at all. We will challenge this view. We have shown the speed of light is embedded in both the kg mass and in the gravitational constant. When we multiply G with M, which is needed in all gravity phenomena, the hidden and embedded speed of light do not cancel out like the Planck constant, $GM = \frac{l_p^2 c^3}{\hbar} \times \frac{\hbar}{\bar{\lambda}}\frac{1}{c} = c^2\frac{l_p^2}{\bar{\lambda}}$. We will claim the speed of light is hidden and embedded even in the Newton gravitational theory. We naturally do not in any way mean to suggest that Newton knew about this; after all, he did not even invent or use the gravitational constant, and the speed of light was not known at that time. Yet, in order to calibrate Newtons gravitational theory to gravity phenomena, one need to capture the essence of gravity; if not, the model would likely fail to predict anything. And the essence of gravity seems to be linked to the Planck length and the speed of light (gravity). It is impossible to predict observable gravity phenomena without them. But this does not mean we need to know it to perform gravity predictions; we can do so without knowing it by hiding them in constants that we calibrate to a gravity phenomenon and then use such constants to predict other gravity phenomena. The fact that Newton said he thought gravity was instant, does not automatically mean this is what is embedded in his theory.

We either have to embed the Planck length and the speed of light (gravity) in some other constant, such as G, even without knowing we are doing so, since G is not calculated, but calibrated, or we have to work with the Planck length and the speed of gravity more directly. The speed of light and the Planck length can also be embedded directly in an alternative mass definition that we will look at soon. When we understand what the essence of gravity is, then we can work with l_p and c directly. We can use the formulas on the right in Table 2 to predict gravitational phenomenon; we get a constant reduction, and a simpler theory that can predict the same as the standard theory.

6 Is Todays Mass Definition Incomplete?

Only the speed of light and the Planck length are needed to predict gravity phenomena, together with variables such as the radius from the center of the gravity object, and the Compton wavelength of the mass. All masses are affected by and causes gravity. We cannot describe the standard kg mass using fundamental constants and quantum variables without the Planck constant; this is supported by the fact that the kg recently has been redefined in terms of the Planck constant using the Watt Balance, see [37–39]. So, if the kg and the Planck constant are part of the very essence of mass, and at the same time we do not need the Planck constant to predict any observable gravity phenomena, but actually indirectly get it out by multiplying G with M, then the Planck constant is possibly not exactly what we think it is.

Any rest-mass in terms of kg is fully defined by $m = \frac{\hbar}{\bar{\lambda}}\frac{1}{c}$. That is, the standard mass definition only contains one of the two constants needed to describe and predict gravity phenomena, namely c. It also contains the Planck constant $\hbar$ that not is needed, and lacks the Planck length. However, when we multiply the gravitational constant by the standard mass, we get $Gm = \frac{l_p^2 c^3}{\hbar} \times \frac{\hbar}{\bar{\lambda}}\frac{1}{c} = c^3\frac{l_p}{c}\frac{l_p}{\bar{\lambda}}$. This mass, like the standard mass, contains the Compton wave, but instead of the Planck constant, it has the Planck length. We will claim this is a mass multiplied by the speed of light cubed, where c^3 actually can be seen as a gravitational constant. The gravitational constant is need to turn the output from the mass divided by radius squared to acceleration, as GM/R^2 is the gravitational acceleration. Haug has recently suggested that all masses should be redefined as

$$\bar{m} = \frac{G}{c^3}m = \frac{l_p}{c}\frac{l_p}{\bar{\lambda}} \tag{21}$$

That is, we have gotten rid of the Planck constant and inserted the Planck length into the mass. The mass is now a time dimension. Haug has called this mass definition collision-time. It is how long a mass is in a collision state per Planck second. Haug [22] has suggested that this mass definition implicitly exists in both Newton (and likely also

[3]And naturally a variable dependent on the mass size, and the distance from the center of the gravity object R

Einstein) gravity, and must also be incorporated in other parts of physics to unify gravity with quantum mechanics. The standard energy and mass definition is quantized through $\hbar$ times a frequency, but the standard definition does not take into account the spatial dimension of the ultimate building blocks of energy and matter, which is linked to the Planck length. This is not important for electromagnetism, except at the Planck scale, but it is the very essence of gravity. It is also not difficult to understand that it is basically impossible to get theories using different mass definitions to fit together in one theory. Naturally, standard physics appears to use the same mass definition everywhere, but we encourage the physics community to look at this with new eyes, namely that Gm can be seen as a way to convert a incomplete mass definition to a mass that also incorporates the spatial dimensions of the ultimate building blocks of all matter, which we claim is linked to the Planck length. In fact, standard two-body gravitational theory gives an indication of this, here the gravitational parameter is $\mu = G(m_1 + m_2) = Gm_1 + Gm_2$; in other words, all masses from which we calculate gravitational effects must be multiplied by G, which corresponds to looking at the mass as $\bar{m} = \frac{l_p}{c}\frac{l_p}{\lambda}$ multiplied with c^3 and divided by r^2 to get gravitational acceleration. When switching to this mass definition, we can write the Newton gravity formula as

$$F = c^3 \frac{\bar{M}\bar{m}}{R^2} \tag{22}$$

Even if the gravitational force now gives different dimensional input than the modern version of the Newton formula, it will still lead to exactly the same predictable output as standard Newton theory. From the quantum perspective, this only contains two constants, namely c and l_p, compared to the modern Newton formula that embedded (and hidden) contains three constants, namely G, $\hbar$, and c, or if we have a deeper understanding, l_p, $\hbar$, and c.

7 There Are Two Important Constants for Mass and Gravity

We will claim there are only two important constants for gravity and the understanding of mass, particularly in relation to gravity. One is the speed of light, and the other is the Planck length. Both can be found from gravity observations only, with no prior knowledge of G, $\hbar$, or c. In a weak field, we have

$$c \approx \frac{\sqrt{1 + Z - \frac{R_L}{R_h}}}{\sqrt{\frac{Z}{R_L g_L}}} \approx \frac{\sqrt{1 - \frac{R_L}{R_h}}}{\sqrt{\frac{Z}{R_L g_L}}}, \qquad l_p \approx \frac{\sqrt{R_L \bar{\lambda}_E Z}}{\sqrt{1 + Z - \frac{R_L}{R_h}}} \approx \frac{\sqrt{R_L \bar{\lambda}_E Z}}{\sqrt{1 - \frac{R_L}{R_h}}}$$

Further, we have

$$c = \frac{R_L\sqrt{\bar{\lambda}_E g_L}}{l_p}, \qquad l_p = \frac{R_L\sqrt{\bar{\lambda}_E g_L}}{c}$$

In other words, there are no need to know G, $\hbar$, or the speed of light c to measure these two variables from gravity phenomena. When we first have extracted the Planck length and the speed of light (gravity) from observable gravity phenomena, we can predict all other gravity phenomena, still with no knowledge of G and $\hbar$. And again we are not simply hiding the G in g, as G is not needed to measure or even predict g.

8 Summary

We have shown that the Planck length and the speed of light (gravity) can be extracted from gravity observations with no prior knowledge of G, $\hbar$ and c. We have further shown that likely all observable gravity phenomena are only dependent on these two constants, plus one variable dependent on the mass size of the gravity object and the distance to the gravity object. In our view, the gravitational constant is a universal composite constant of the form $G = \frac{l_p^2 c^3}{\hbar}$. However, the only reason it contains the Planck constant is to cancel out the Planck constant in the mass, and to get the Planck length into the mass. This is naturally unknown to standard physics, but is, in our view, what is hidden even in Newtonian gravity, as well as in Einstein gravity. The standard mass definition and energy definition are lacking important information. Anything that is observable in gravity can only be predicted by GM, and we claim the multiplication with a gravity constant G is actually used to convert the incomplete mass definition into a mass definition that accounts for gravity $GM = \frac{l_p^2 c^3}{\hbar} \times \frac{\hbar}{\bar{\lambda}}\frac{1}{c} = c^3 \times \frac{l_p}{c}\frac{l_p}{\bar{\lambda}}$. This means GM can be written as $c^3\bar{M}$ instead of GM, where $\bar{M} = \frac{1}{l_p}\frac{l_p}{\bar{\lambda}}$ is our new mass definition. This is more than a cosmetic makeover, as it relies on less physical constants; G and $\hbar$ are basically replaced by the Planck length, and in addition we need c, as also is needed in standard theory. Only when this new mass definition is implemented in non-gravity physics can one unify quantum mechanics with gravity [22], something we have recently claimed to have done. Naturally, this should be studied by a series of researchers before one draws any firm conclusions. Still, we think it is a promising approach that is worth further study by the physics community.

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Appendix A

In 1798, Cavendish [14] was the first to use a so-called Cavendish apparatus to find the density of the Earth. He actually did not use it to find the gravitational constant, as the gravitational constant first was suggested in 1873. Here we will use a Cavendish apparatus to find the gravitational acceleration from the large lead ball acting on the small ball in the apparatus. A Cavendish apparatus consists of two small balls and two larger balls, all made of lead, for example. The torque (moment of force) is given by

$$\kappa\theta \tag{23}$$

where κ is the torsion coefficient of the suspending wire. Further, θ is the deflection angle of the balance. Next we have the following well-known relationship

$$\kappa\theta = LF \tag{24}$$

where L is the length between the two small balls in the apparatus and F can be set equal to the gravitational force given by

$$F = G\frac{Mm}{r^2} \tag{25}$$

This means we must have

$$\kappa\theta = LG\frac{Mm}{r^2} \tag{26}$$

We also have that the natural resonant oscillation period of a torsion balance is given by

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \tag{27}$$

Further, the moment of inertia I of the balance is given by

$$I = m\left(\frac{L}{2}\right)^2 + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2} \tag{28}$$

from this we have

$$T = 2\pi\sqrt{\frac{mL^2}{2\kappa}} \tag{29}$$

next we solve this with respect to κ and get

$$\begin{aligned} \frac{T^2}{2^2\pi^2} &= \frac{mL^2}{2\kappa} \\ \kappa &= \frac{mL^2}{2\frac{T^2}{2^2\pi^2}} \\ \kappa &= \frac{mL^2 2\pi^2}{T^2} \end{aligned} \tag{30}$$

Then in equation 26, we are replacing κ with this expression, and solving with respect to $g = \frac{GM}{r^2}$, this gives

$$\begin{aligned} \frac{mL^2 2\pi^2\theta}{T^2} &= LG\frac{Mm}{r^2} \\ \frac{L^2 2\pi^2\theta}{T^2} &= LG\frac{M}{r^2} \\ \frac{L^2\pi^2\theta}{T^2} &= G\frac{M}{r^2} \\ g = \frac{L^2\pi^2\theta}{T^2} \end{aligned} \tag{31}$$

Appendix B

Here we show in detail how to derive the speed of light from gravitational red-shift. From Einstein's general relativity theory, we have

$$Z = \frac{\sqrt{1 - \frac{2g_L}{R_L c^2}}}{\sqrt{1 - \frac{2g_h}{R_h c^2}}} - 1 \tag{32}$$

when $\frac{2g_L}{R_L c^2} << 1$, as it is in a weak gravitational field, we can approximate $\sqrt{1 - \frac{2g_L}{R_L c^2}}$ with the first term of a Taylor series expansion $\sqrt{1 - \frac{2g_L}{R_L c^2}} \approx= 1 - \frac{g_L}{R_L c^2}$, and this gives the well-known

$$Z \approx \frac{1 - \frac{g_L}{R_L c^2}}{1 - \frac{g_h}{R_h c^2}} - 1 \tag{33}$$

Solved with respect to c, this gives

$$\begin{aligned} Z &\approx \frac{c^2 - g_h R_h}{c^2 - g_L R_L} - 1 \\ Z(c^2 - g_L R_L) &\approx c^2 - g_h r_h - (c^2 - g_L R_L) \\ c &\approx \sqrt{\frac{g_L R_L + g_L R_L Z - g_h R_h}{Z}} \end{aligned} \tag{34}$$