Bekenstein-Hawking black hole entropy, Hawking temperature, and the Unruh effect: insight from the laws of thermodynamics – a synopsis

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Abstract: The laws of thermodynamics play a central role in scientific inquiry, guiding physics as to the validity of hypothesized claims. It is for this reason that quantities of thermodynamic relevance must retain their character wherever they appear. Temperature, for example, must always be intensive, a requirement set by the 0^{th} law. Otherwise, the very definition of temperature is compromised. Similarly, entropy must remain extensive, in order to conform to the 2^{nd} law. These rules must be observed whenever a system is large enough to be characterized by macroscopic quantities, such as volume or area. This explains why ensembles comprised of just a few atoms cannot be considered thermodynamic systems. In this regard, black holes are hypothesized to be large systems, characterized by the Schwarzschild radius (r_s=$2GM/c^2$) and its associated 'horizon' area ($A = 4\pi r_s^2$), where G, M, and c represent the universal constant of gravitation, the mass of the black hole, and the speed of light in vacuum, respectively. It can be readily demonstrated that Bekenstein-Hawking black hole entropy is non-extensive, while the Hawking and the Unruh temperatures are non-intensive. As a result, the associated equations violate the laws of thermodynamics and can hold no place in the physical sciences.

I. INTRODUCTION

Thermodynamic equations must be dimensionally balanced and, furthermore, they must also be thermodynamically balanced. Thus, if the thermodynamic character of one side of such an equation is intensive or extensive, then the other side must also be intensive or extensive, respectively. Any formulation involving thermodynamic coordinates that violates this balance is inadmissible.

It is readily apparent that the equations which describe black hole thermodynamics violate the rules that entropy must be expressed as an extensive property and temperature as an intensive property. Black hole entropy is not extensive. Black hole and Unruh temperatures are not intensive. As a result, they are completely detached from any link to thermodynamics.

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II. BEKENSTEIN-HAWKING BLACK HOLE ENTROPY

The Bekenstein-Hawking black hole entropy equation is

$$S = \frac{\pi c^3 k_B}{2hG} A, \tag{1}$$

where S is entropy, c is the speed of light in vacuum, k_B is Boltzmann's constant, h is Planck's constant, G is the universal constant of gravitation, and A is the area of the event horizon. In the case of an uncharged, non-rotating black hole, $A = 4\pi r_s^2$, where $r_s = 2GM/c^2$ (so-called 'Schwarzschild radius). Equation (1) then becomes

$$S = \frac{8\pi^2 k_B G}{hc} M^2. \tag{2}$$

Although mass M is always extensive, M^2 is not extensive. Since entropy S is extensive, Eqs.(1) and (2) violate the laws of thermodynamics and are, therefore, inadmissible.

III. BLACK HOLE TEMPERATURE AND HAWKING RADIATION

For an uncharged non-rotating black hole, Hawking radiation is said to correspond to a blackbody spectrum at a temperature T_H given by

$$T_H = \frac{\hbar c^3}{8\pi k_B GM}, \tag{3}$$

where $\hbar$ is the reduced Planck's constant, c is the speed of light in vacuum, G is the universal constant of gravitation, k_B is Boltzmann's constant, and M is the mass of the black hole. Equation (3) is etched into the gravestone of Stephen Hawking in Westminster Abbey. However, temperature is an intensive property so it cannot be made to depend on the mass of a system, an extensive property, without an associated extensive property, like volume, which in combination with M leads to an intensive property. The left side of Eq.(3) is intensive but the right side is not, because it varies as $1/M$. The equation for Hawking temperature violates the laws of thermodynamics and is, therefore, invalid. Furthermore, the production of a blackbody spectrum depends absolutely on the presence of a physical vibrational lattice, as is well-known throughout metrology. The idea that such a blackbody spectrum can be generated from thermal equilibrium considerations alone is false. As a result, black holes cannot be reconciled with the known laws of thermodynamics and Hawking radiation does not exist.

In the case of the Kerr-Newman black hole (i.e. a charged and rotating black hole), the Hawking temperature is given by

$$T_H = \frac{\hbar c\sqrt{\frac{G^2M^2}{c^4}-\frac{J^2}{M^2c^2}-\frac{Gq^2}{4\pi\varepsilon_o c^4}}}{4\pi k_B\left[\frac{GM}{c^2}\left(\frac{GM}{c^2}+\sqrt{\frac{G^2M^2}{c^4}-\frac{J^2}{M^2c^2}-\frac{Gq^2}{4\pi\varepsilon_o c^4}}\right)-\frac{Gq^2}{8\pi\varepsilon_o c^4}\right]}, \tag{4}$$

where J is angular momentum, and q is electric charge. If $q = 0$,

$$T_H = \frac{\hbar c\sqrt{\frac{G^2M^2}{c^4}-\frac{J^2}{M^2c^2}}}{4\pi k_B\left[\frac{GM}{c^2}\left(\frac{GM}{c^2}+\sqrt{\frac{G^2M^2}{c^4}-\frac{J^2}{M^2c^2}}\right)\right]}. \tag{5}$$

If $J = 0$,

$$T_H = \frac{\hbar c\sqrt{\frac{G^2M^2}{c^4}-\frac{Gq^2}{4\pi\varepsilon_o c^4}}}{4\pi k_B\left[\frac{GM}{c^2}\left(\frac{GM}{c^2}+\sqrt{\frac{G^2M^2}{c^4}-\frac{Gq^2}{4\pi\varepsilon_o c^4}}\right)-\frac{Gq^2}{8\pi\varepsilon_o c^4}\right]}. \tag{7}$$

Equations (4), (5), and (6) all render temperature non-intensive, in violation of the laws of thermodynamics. They are all therefore, invalid.

IV. THE UNRUH EFFECT

According to this theoretical effect, "*From the point of view of an accelerating observer or detector, empty space contains a gas of particles at a temperature proportional to the acceleration*".[3] The Unruh temperature is given by,

$$T_{Unruh} = \frac{ha}{4\pi^2 ck_B}, \tag{8}$$

where a is acceleration. Setting

$$a = g = \frac{GM}{r_S^2} = \frac{c^4}{4GM}, \tag{9}$$

and substituting into Eq. (8) gives Hawking's black hole temperature Eq.(3). Setting

$$a = \frac{GM}{r^2\sqrt{1-\frac{2GM}{c^2r}}}, \tag{10}$$

[3] S. A. Fulling, G. E. A. Matsas, *Scholarpedia*, 9(10):31789, (2014), http://www.scholarpedia.org/article/Unruh_effect

then

$$T_{Unruh} = \frac{h}{4\pi^2 ck_B}\frac{GM}{r^2\sqrt{1-2GM/c^2r}}, \tag{11}$$

which diverges as $r \rightarrow r_s = 2GM/c^2$. The Unruh temperature diverges "*in precisely the same way as for an accelerating detector in flat space-time. In both cases the temperature diverges as a/2π*".[4] Equations (8) and (11) render temperature non-intensive, in violation of the laws of thermodynamics. They are therefore invalid.

V. PHYSICS BY HYPOTHESIS

"*Define the generalized entropy,* S', *to be the sum of the ordinary entropy, S, of matter outside a black hole plus the black hole entropy*

$$S' \equiv S + S_{bh}$$

Finally, replace the ordinary second law of thermodynamics by the generalized second law (GSL): The total generalized entropy of the universe never decreases with time

$$\Delta S' \geq 0.\text{''}$$[5]

The generalized entropy is thus defined as additive, in accordance with the laws of thermodynamics, but contains a component, S_{bh}, that is not additive; in other words, the generalized entropy is both extensive and not extensive. The 'generalised entropy,' S', and the Bekenstein-Hawking black hole entropy are contradictory, and false.

V. CONCLUSIONS

Bekenstein-Hawking black hole entropy, black hole temperature, and the Unruh effect, stand in violation of the laws of thermodynamics. They are therefore invalid. Hawking radiation does not exist. Cosmological generalised entropy is a logical contradiction and therefore invalid.

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[4] W. G. Unruh, *Phys. Rev. D*, **14**, 4, p.870, (1976).

[5] R. M. Wald, The Thermodynamics of Black Holes, *Living Rev. in Relativity*, 4, (2006), arXiv:gr-qc/9912119v2 2000