Time Dependency of Lockdown Measure for Controlling Coronavirus Outbreaks

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Abstract

Based on Real data, we study how long it takes for Coronavirus lockdown measures from the Government to became effective.

Key Words: biology, virology.

1 Introduction

One of the most simple model in Virology is the SIR (Susceptible, Infected, Removed) Model developed by Kermack and McKendrick in 1927, mostly used for didactics but still very effective. A key parameter of the Model is R_0, the basic reproduction number which represent the average number of people an infected transmits the virus to. This Parameter takes in account the the Social separating measure take by the government to block the infection. However, the model is not designed to take into account a variable R_0. In this paper we try to somehow introduce this variable in the model.

2 The SIR Model

The SIR model studies the evolution of the number of people $S(t)$, $I(t)$ and $R(t)$ which are respectively the Susceptible (the ones that can be can be infected), the Infected and the Removed (the ones that have been immunised by the virus or vaccine).

Of course in stationary hypothesis, which is when the period of the outbreak is small enough to consider the total population N constant, we have $S+I+R=N$.

The SIR model is described by the following equations:

$$\left\{ \begin{array}{lcl} x' & = & -\beta xy \\ y' & = & \beta xy - \gamma y \end{array} \right. \quad where \quad \begin{array}{l} x(t) = \frac{S(t)}{N} \\ y(t) = \frac{I(t)}{N} \end{array} \tag{1}$$

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where $R_0 = \frac{\beta}{\gamma}$ and $\gamma = \frac{1}{T_i}$ with T_i is the effective period in which an infected person passes the virus to R_0 other people (in average). For the coronavirus, first estimations of the parameters from various organizations around the world are T_i between 5 and 7 days and R_0 between 2 and 3 people.

With these two parameter the model is fully defined and R_0 is basically responsible for the pick and the total infected at the end of the outbreak while T_i is basically responsible for the duration of it.

If the initial conditions are $x(0) = x_0$ and $y(0) \approx 0$, which is the case for coronavirus, then it is possible to expand x and y to the first order in the equations which became linear in y. By using $x = x_0$ in we get:

$$y' = (\beta x_0 - \gamma)y \tag{2}$$

which has the following solution:

$$y = y_0 e^{rt}; \;\; with \;\; r = \beta x_0 - \gamma = (\frac{\beta x_0}{\gamma} - 1)\gamma = (R_0 x_0 - 1)\gamma \tag{3}$$

We get easily:

$$R_0 = \frac{1}{x_0}\left[1 + rT_i\right] \tag{4}$$

If R_0 is constant, also r does not change and the initial part of the solution $y(t)$ is a constant slope line in a graph where the y axis is in a logarithmic scale. Assuming to have the value for T_i, R_0 can be estimated using the slope of the linear regression of the above curve.

Finally, assuming that T_i does not depend from social distancing measures in place from the government and R_0 changes because of them, we may think to evaluate R_0 with the following function D_0:

$$D_0 = \frac{1}{x_0}\left[1 + \frac{d}{dt} ln[y(t)] T_i\right] \tag{5}$$

Where $y(i)$ are real measured data and for the first outbreak we may choose safely $x_0 = 1$. However, there are some problem with D_0, and this will be discussed in the following paragraph.

3 R_0 Varying with Time

We turn now our attention to a real case which is the outbreak of CoVd-19 of the beginning of 2020. When social separating measure are taken by the government, R_0 changes as a step functionary, from a day to he following, between a value R_i, previous measures, to a value R_f, post measures. However, analysis of real data from real cases show that the estimation Do of the 5 goes down slowly. Fig. 1 shows Do evaluated on real data from the Chinese outbreak (source [2]).

From the picture $D0$ looks having an exponential decreasing trend. We make the hypothesis that D_0 is like a the output of a first order dynamic systems with transfer function:

$$Do(s) = \frac{1}{1 + s\tau} R_0(s) \tag{6}$$

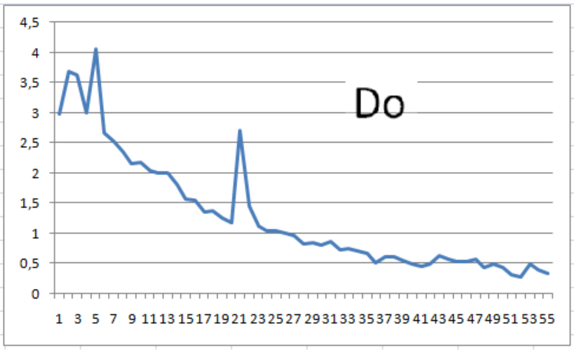

Figure 1: *Do* in China estimated on real data

and responding to the step function $Ro = R_i + (R_f - R_i)u(t)$ where $u(t)$ is the Heaviside step function.

We propose to modify the model, in order to take into account the delay of the system to respond to a change in Ro, as shown in Fig. 2.

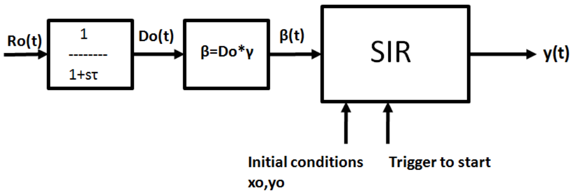

Figure 2: Simulation with variable Ro

4 Evaluation of the Time Constant

A first approximation of the value of thr time constant τ can be done directly from the pot of Do. For example, from real data of the Chinese outbreak in Wuhan of Jan. 2020 (source [2]), after the Lockdown Do has gone from a value of $D_i = 2.53$ to a value of about $R_f = 0.5$. Since the time constant (assuming a first order system) is the time required for Do to decrease by 63.2% of the interval $(R_i - R_f)$, the time constant can be evaluates as the number of days required to Do to get to a value of about 1.26.

From the data and based on the above consideration we get a time constant of exactly:

$$\tau = 14\, days \tag{7}$$

5 Evaluation of Ro in the Early Days

We want to evaluate Ro from the early days after a social separating measure has been applied to see if it has been effective. Give the above assumption of a first order system behaviour, the value of $Do(t)$ is function of the three parameter, R_i, R_f and τ as follows:

$$f_{Do}(t, R_i, R_f, \tau) = R_i + (R_f - R_i)\left(1 - e^{-\frac{t}{\tau}}\right) \tag{8}$$

Given the real data $Do(t)$, the above three parameters can be evaluated mimimising the functional:

$$J = \int |Do(t) - f_{Do}(t, R_i, R_f, \tau)|\, dt \tag{9}$$

where the above integral is rather a discrete sum since data are known with a time sampling of a day.

6 Characterization of Outbreaks

We have evaluated the above three parameters from real data of the Chinese outbreak in Wuhan (source [2]) and for the Italian outbreak (source [1]) of the beginning of 2020 and we have found as follows:

Chinese outbreak:

$$\begin{cases} R_i &= 2.467 \ [people] \\ R_f &= 0.252 \ [people] \\ \tau &= 16.229 \ [days] \end{cases} \tag{10}$$

The following figures shows the comparison between real data and relevant characterizing curve:

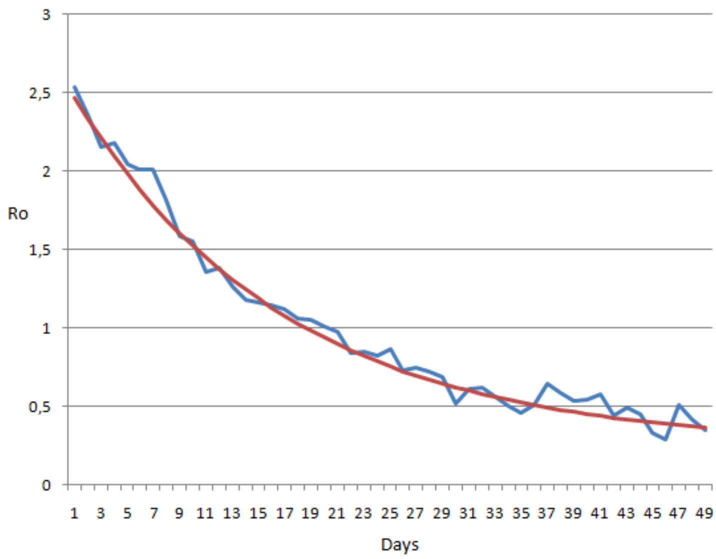

Figure 3: Chinese Coronavirus Outbreak

Italian outbreak:

$$\begin{cases} R_i &= 2.331 \; [people] \\ R_f &= 0.802 \; [poeple] \\ \tau &= 18.123 \; [days] \end{cases} \tag{11}$$

The following figures shows the comparison between real data and relevant characterizing curve:

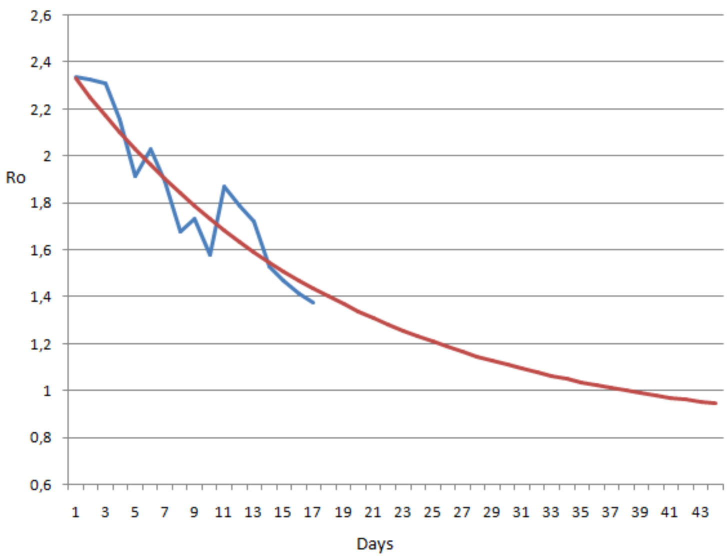

Figure 4: Italian Coronavirus Outbreak

In the above figure, real data (blue line) are reported till the data of writing of this paper (i.e. 25/03/20). The red line shows the theoretical evolution of the outbreak if the trend of the data continues to follow the same pattern. It has to be noted that the real data are affected by a lot of noise which has a major effect of the final parameters.

References

[1] Italian outbreak *www.ilsole24ore.com.*

[2] Chinese outbreak data *www.worldometers.info.*