I think that it is possible the quantization of the gravity using the quantum field theory (I use the Lewis Ryder book) and the classical theory of fields (I use the Landau-Lifsits book).

It is evident, from the Lifsits book, that the Euler-Lagrange equation for the gravitational field is the Einstein field equation obtained from the Lagrangian of the gravitational field (I write the vacuum solution):

$$\begin{array}{l} G = g^{ik}\left(\Gamma^m_{il}\Gamma^l_{km} - \Gamma^l_{ik}\Gamma^m_{lm}\right) \\ \mathcal{L} = -\frac{\sqrt{-g}}{16\pi k} G \end{array}$$

the generalized variables, used in the Euler-Lagrange equation are:

$$\mathcal{L}\left(g^{ik}\ ,\ \frac{\partial g^{lm}}{\partial x^r}\right)$$

as is evident from the variation of the action:

$$\delta S_g = -\frac{c^3}{16\pi k}\int\left\{\frac{\partial(G\sqrt{-g})}{\partial g^{ik}} - \frac{\partial}{\partial x^l}\frac{\partial(G\sqrt{-g})}{\partial\frac{\partial g^{ik}}{\partial x^l}}\right\}\delta g^{ik} d\Omega$$

so that it is simple to obtain the Hamiltonian[1] from this action if the Christoffel symbols are expressed in the variables subject to variation:

$$\Gamma^i_{kl} = -\frac{1}{2}\left(g_{pk}\frac{\partial g^{ip}}{\partial x^l} + g_{lp}\frac{\partial g^{ip}}{\partial x^k} - g^{ip}g_{kw}g_{lz}\frac{\partial g^{wz}}{\partial x^p}\right)$$

then, the Hamiltonian is

$$\mathcal{H} = \frac{\partial g^{ij}}{\partial x^0}\frac{\partial\mathcal{L}}{\frac{\partial g^{ij}}{\partial x^0}} - \mathcal{L}$$

the Lagrangian is a quadratic function in the derivative of the metric tensor, so that the Hamiltonian is just a quadratic function of the metric tensor; there are no linear terms of the time derivative of the metric tensor:

$$\mathcal{H} = \frac{1}{2}\frac{\partial\mathcal{L}}{\partial\frac{\partial g^{ik}}{\partial x^0}\partial\frac{\partial g^{ik}}{\partial x^0}}(\partial_0 g^{ik})^2 - \frac{1}{2}\frac{\partial\mathcal{L}}{\partial\frac{\partial g^{ij}}{\partial x^\mu}\partial\frac{\partial g^{lm}}{\partial x^\nu}}\partial_\mu g^{ik}\partial_\nu g^{lm}$$

where $\mu \in 1, 2, 3$ and so for each greek letter.

It is evident that the classical quantization is possible, if it is possible to write the time derivative of the metric tensor like a function of the generalized moment (I don't use the Heisenberg picture):

$$\begin{array}{l} \hat{H} = \int\hat{\mathcal{H}}\ d^3\mathbf{x} \\ i\hbar\frac{\partial}{\partial t}\Psi\left(g^{ij}, \partial_\mu g^{kl}\right) = \hat{H}\Psi\left(g^{ij}, \partial_\mu g^{kl}\right) \\ \hat{\pi}_{rs}\Psi(g^{ij}, \partial_\mu g^{kl}) = -i\hbar\frac{\partial}{\partial\partial_0 g^{rs}}\Psi(g^{ij}, \partial_\mu g^{kl}) \end{array}$$

Starting from the second derivative of the Lagrangian it is possible to write the Hamiltonian, and the moments of the Lagrangian:

$$\begin{array}{l} \frac{\partial^2\mathcal{L}}{\partial\frac{\partial g^{ab}}{\partial x^r}\partial\frac{\partial g^{cd}}{\partial x^s}} = g^{ik}\left(\frac{\partial\Gamma^m_{il}}{\partial\frac{\partial g^{ab}}{\partial x^r}}\frac{\partial\Gamma^l_{km}}{\partial\frac{\partial g^{cd}}{\partial x^s}} + \frac{\partial\Gamma^m_{il}}{\partial\frac{\partial g^{cd}}{\partial x^s}}\frac{\partial\Gamma^l_{km}}{\partial\frac{\partial g^{ab}}{\partial x^r}} - \frac{\partial\Gamma^l_{ik}}{\partial\frac{\partial g^{ab}}{\partial x^r}}\frac{\partial\Gamma^m_{lm}}{\partial\frac{\partial g^{cd}}{\partial x^s}} - \frac{\partial\Gamma^l_{ik}}{\partial\frac{\partial g^{cd}}{\partial x^s}}\frac{\partial\Gamma^m_{lm}}{\partial\frac{\partial g^{ab}}{\partial x^r}}\right) = \\ = \delta^s_a\delta^r_c g_{bd} + \delta^s_a\delta^r_d g_{bc} + \delta^s_b\delta^r_c g_{ad} + \delta^s_b\delta^r_d g_{ac} - g_{ac}g_{bd}g^{rs} - g_{ad}g_{bc}g^{rs} - \delta^s_a\delta^r_b g_{cd} - \delta^r_a\delta^s_b g_{cd} - \delta^s_c\delta^r_d g_{ab} - \delta^r_c\delta^s_d g_{ab} + 2g_{ab}g_{cd}g^{rs} \end{array}$$

---

[1] $-\frac{\sqrt{-g}}{16\pi k}$ is a scale factor that is a constant in the calculations which will only be considered, in the Hamiltonian, in the end

then:

$$\frac{\partial^2\mathcal{L}}{\partial\frac{\partial g^{ab}}{\partial x^0}\partial\frac{\partial g^{cd}}{\partial x^s}} = \delta^s_a\delta^0_c g_{bd} + \delta^s_a\delta^0_d g_{bc} + \delta^s_b\delta^0_c g_{ad} + \delta^s_b\delta^0_d g_{ac} - g_{ac}g_{bd}g^{0s} - g_{ad}g_{bc}g^{0s} - \delta^s_a\delta^0_b g_{cd} - \delta^0_a\delta^s_b g_{cd} - \delta^s_c\delta^0_d g_{ab} - \delta^0_c\delta^s_d g_{ab} + 2g_{ab}g_{cd}g^{0s}$$
$$\frac{\partial\mathcal{L}}{\partial\frac{\partial g^{ab}}{\partial x^0}\partial\frac{\partial g^{cd}}{\partial x^0}} = -2\delta^0_a\delta^0_b g_{cd} + \delta^0_a\delta^0_c g_{bd} + \delta^0_a\delta^0_d g_{bc} + \delta^0_b\delta^0_c g_{ad} + \delta^0_b\delta^0_d g_{ac} - 2\delta^0_c\delta^0_d g_{ab} + 2g^{00}g_{ab}g_{cd} - g^{00}g_{ac}g_{bd} - g^{00}g_{ad}g_{bc}$$
$$\frac{\partial\mathcal{L}}{\partial\frac{\partial g^{ab}}{\partial x^0}\partial\frac{\partial g^{ab}}{\partial x^0}} = -2\delta^0_a\delta^0_b g_{ab} + \delta^0_a\delta^0_a g_{bb} + \delta^0_b\delta^0_b g_{aa} + g^{00}g_{ab}g_{ab} - g^{00}g_{aa}g_{bb}$$
$$\frac{\partial\mathcal{L}}{\partial\frac{\partial g^{aa}}{\partial x^0}\partial\frac{\partial g^{aa}}{\partial x^0}} = 0$$
$$\pi_{ab} = \frac{\partial\mathcal{L}}{\partial\frac{\partial g^{ab}}{\partial x^0}\partial\frac{\partial g^{cd}}{\partial x^0}}\frac{\partial g^{cd}}{\partial x^0} + \frac{\partial\mathcal{L}}{\partial\frac{\partial g^{ab}}{\partial x^0}\partial\frac{\partial g^{cd}}{\partial x^\mu}}\frac{\partial g^{cd}}{\partial x^\mu}$$
$$\boxed{\pi_{ab} = \frac{\partial\mathcal{L}}{\partial\frac{\partial g^{ab}}{\partial x^0}\partial\frac{\partial g^{cd}}{\partial x^l}}\partial_l g^{cd} = -\delta^0_a\delta^0_b g_{cd}\partial_0 g^{cd} + \delta^0_a g_{bd}\partial_0 g^{d0} + \delta^0_b g_{ad}\partial_0 g^{d0} - \delta^0_c g_{ab}\partial_0 g^{c0} + g^{00}g_{ab}g_{cd}\partial_0 g^{cd} - g^{00}g_{ac}g_{bd}\partial_0 g^{cd} + F_{ab}}$$

the F_{ab} terms are:

$$F_{ab} = \frac{\partial^2\mathcal{L}}{\partial\frac{\partial g^{ab}}{\partial x^0}\partial\frac{\partial g^{cd}}{\partial x^\mu}}\partial_\mu g^{cd}$$
$$\boxed{F_{ab} = (\delta^\mu_a\delta^0_c g_{bd} + \delta^\mu_a\delta^0_d g_{bc} + \delta^\mu_b\delta^0_c g_{ad} + \delta^\mu_b\delta^0_d g_{ac} - g_{ac}g_{bd}g^{0\mu} - g_{ad}g_{bc}g^{0\mu} - \delta^\mu_a\delta^0_b g_{cd} - \delta^0_a\delta^\mu_b g_{cd} - \delta^\mu_c\delta^0_d g_{ab} - \delta^0_c\delta^\mu_d g_{ab} + 2g_{ab}g_{cd}g^{\mu 0})\partial_\mu g^{cd}}$$

it is important to obtain a minimum number of terms in the momentum, so that it is more simple to obtain the time derivative of the metric tensor like a function of the generalized moments:

$$\partial_0 g^{ab} = P^{ab} + Q^{ab.cd}\pi_{cd}$$

The Hamiltonian is:

$$\mathcal{H} = \boxed{g^{00}(g_{ab}g_{ab} - g_{aa}g_{bb})\frac{\partial g^{ab}}{\partial x^0}\frac{\partial g^{ab}}{\partial x^0}} - \tfrac{1}{2}(+\delta^\beta_a\delta^\alpha_c g_{bd} + \delta^\beta_a\delta^\alpha_d g_{bc} + \delta^\beta_b\delta^\alpha_c g_{ad} + \delta^\beta_b\delta^\alpha_d g_{ac} - g_{ac}g_{bd}g^{\alpha\beta} - g_{ad}g_{bc}g^{\alpha\beta} - \delta^\beta_a\delta^\alpha_b g_{cd} - \delta^\alpha_a\delta^\beta_b g_{cd}+$$
$$-\delta^\beta_c\delta^\alpha_d g_{ab} - \delta^\alpha_c\delta^\beta_d g_{ab} + 2g_{ab}g_{cd}g^{\alpha\beta})\tfrac{\partial g^{ab}}{\partial x^\alpha}\tfrac{\partial g^{cd}}{\partial x^\beta} = g^{00}(g_{ab}g_{ab} - g_{aa}g_{bb})\partial_0 g^{ab}\partial_0 g^{ab} - 2g_{cd}\partial_\beta g^{\alpha d}\partial_\alpha g^{\beta c} + 2g_{cd}\partial_\beta g^{cd}\partial_\alpha g^{\alpha\beta}+$$
$$+g_{ac}g_{bd}g^{\alpha\beta}\partial_\alpha g^{ab}\partial_\beta g^{cd} - g_{ab}g_{cd}g^{\alpha\beta}\partial_\alpha g^{ab}\partial_\beta g^{cd}$$

it is possible to write the time derivative of the metric tensor using a reduced number of terms:

$$\frac{\partial^2\mathcal{L}}{\partial\frac{\partial g^{ab}}{\partial x^0}\partial\frac{\partial g^{cd}}{\partial x^0}}\frac{\partial g^{cd}}{\partial x^0} = \Theta_{ab} = -F_{ab} + \pi_{ab} = -\delta^0_a\delta^0_b g_{cd}\partial_0 g^{cd} + \delta^0_a g_{bc}\partial_0 g^{c0} + \delta^0_b g_{ac}\partial_0 g^{c0} - g_{ab}\partial_0 g^{00} + g^{00}g_{ab}g_{cd}\partial_0 g^{cd} - g^{00}g_{ac}g_{bd}\partial_0 g^{cd}$$
$$\frac{\Theta^a_a}{2} = -\partial_0 g^{00} + g^{00}g_{cd}\partial_0 g^{cd} = (g^{00}g_{00} - 1)\partial_0 g^{00} + 2g^{00}g_{\mu 0}\partial_0 g^{\mu 0} + g^{00}g_{\mu\nu}\partial_0 g^{\mu\nu}$$
$$\boxed{\Lambda_{ab} = \Theta_{ab} - \frac{g_{ab}\Theta^m_m}{2} = \delta^0_a g_{bc}\partial_0 g^{c0} + \delta^0_b g_{ac}\partial_0 g^{c0} - \delta^0_a\delta^0_b g_{cd}\partial_0 g^{cd} - g^{00}g_{ac}g_{bd}\partial_0 g^{cd}}$$
$$\boxed{\Lambda_{00} = 2g_{c0}\partial_0 g^{c0} - (g_{cd} + g^{00}g_{c0}g_{d0})\partial_0 g^{cd} = g_{00}(1 - g^{00}g_{00})\partial_0 g^{00} + g^{00}g_{00}g_{\alpha 0}\partial_0 g^{\alpha 0} - (g_{\alpha\beta} + g^{00}g_{\alpha 0}g_{\beta 0})\partial_0 g^{\alpha\beta}}$$
$$\Lambda_{\mu\nu} = -g^{00}g_{\mu c}g_{\nu d}\partial_0 g^{cd} = g^{00}g_{\mu c}g^{cd}\partial_0 g_{\nu d} = g^{00}\partial_0 g_{\nu\mu}$$
$$\frac{\Lambda_{\mu\nu}}{g^{00}} = \partial_0 g_{\mu\nu}$$
$$g^{\alpha\nu}g^{\beta\mu}\partial_0 g_{\mu\nu} = \frac{\Lambda^{\alpha\beta}}{g^{00}}$$
$$\boxed{\partial_0 g^{\alpha\beta} = -\frac{\Lambda^{\alpha\beta}}{g^{00}} = -g^{\alpha\mu}g^{\beta\nu}\frac{\Lambda_{\mu\nu}}{g^{00}}}$$

if a new terms Υ_0^a is introduced then the calculations, to evaluate the remaining components of the metric tensor, are simpler:

$$\begin{array}{l}
\Lambda_{a0} = (\delta_a^0 g_{c0} + g_{ac})\partial_0 g^{c0} - (\delta_a^0 g_{cd} + g^{00} g_{ac} g_{d0})\partial_0 g^{cd} = (\delta_a^0 g_{00} + g_{a0})\partial_0 g^{00} + (\delta_a^0 g_{\alpha 0} + g_{a\alpha})\partial_0 g^{\alpha 0} - (\delta_a^0 g_{00} + g^{00} g_{a0} g_{00})\partial_0 g^{00} + \\
-(\delta_a^0 g_{\alpha 0} + g^{00} g_{a\alpha} g_{00})\partial_0 g^{\alpha 0} - (\delta_a^0 g_{\alpha 0} + g^{00} g_{a0} g_{\alpha 0})\partial_0 g^{\alpha 0} - (\delta_a^0 g_{\alpha\beta} + g^{00} g_{a\alpha} g_{\beta 0})\partial_0 g^{\alpha\beta} = g_{a0}(1 - g^{00} g_{00})\partial_0 g^{00} + \\
+(g_{a\alpha} - \delta_a^0 g_{\alpha 0} - g^{00} g_{a\alpha} g_{00} - g^{00} g_{a0} g_{\alpha 0})\partial_0 g^{\alpha 0} - (\delta_a^0 g_{\alpha\beta} + g^{00} g_{a\alpha} g_{\beta 0})\partial_0 g^{\alpha\beta} \\
\Upsilon_{a0} = \Lambda_{a0} + (\delta_a^0 g_{\alpha\beta} + g^{00} g_{a\alpha} g_{\beta 0})\partial_0 g^{\alpha\beta} = g_{a0}(1 - g^{00} g_{00})\partial_0 g^{00} + \left(g_{a\alpha} - \delta_a^0 g_{\alpha 0} - g^{00} g_{00} g_{a\alpha} - g^{00} g_{a0} g_{\alpha 0}\right)\partial_0 g^{\alpha 0} \\
\boxed{\Upsilon_{\kappa 0} = \Lambda_{\kappa 0} - g_{\kappa\alpha} g_{\beta 0} g^{\alpha\mu} g^{\beta\nu} \Lambda_{\mu\nu} = g_{\kappa 0}(1 - g^{00} g_{00})\partial_0 g^{00} + [g_{\kappa\alpha}(1 - g^{00} g_{00}) - g^{00} g_{\kappa 0} g_{\alpha 0}]\partial_0 g^{\alpha 0}}
\end{array}$$

$$\boxed{\Upsilon_{00} = \Lambda_{00} - \left(\frac{g_{\alpha\beta}}{g^{00}} + g_{\alpha 0} g_{\beta 0}\right) g^{\alpha\mu} g^{\beta\nu} \Lambda_{\mu\nu} = g_{00}(1 - g^{00} g_{00})\partial_0 g^{00} - 2 g^{00} g_{00} g_{\alpha 0}\partial_0 g^{\alpha 0}}$$

then it is possible to obtain the $\partial_0 g^{a0}$ as a $\Upsilon(\Lambda(\Theta(\pi^{lm})))$ function:

$$\begin{pmatrix} \Upsilon_{10} - \frac{g_{10}}{g_{00}}\Upsilon_{00} \\ \Upsilon_{20} - \frac{g_{20}}{g_{00}}\Upsilon_{00} \\ \Upsilon_{30} - \frac{g_{30}}{g_{00}}\Upsilon_{00} \end{pmatrix} = \begin{pmatrix} g_{11}(1 - g^{00} g_{00}) + g^{00} g_{10} g_{10} & g_{12}(1 - g^{00} g_{00}) + g^{00} g_{10} g_{20} & g_{13}(1 - g^{00} g_{00}) + g^{00} g_{10} g_{30} \\ g_{21}(1 - g^{00} g_{00}) + g^{00} g_{20} g_{10} & g_{22}(1 - g^{00} g_{00}) + g^{00} g_{20} g_{20} & g_{23}(1 - g^{00} g_{00}) + g^{00} g_{20} g_{30} \\ g_{31}(1 - g^{00} g_{00}) + g^{00} g_{30} g_{10} & g_{32}(1 - g^{00} g_{00}) + g^{00} g_{30} g_{20} & g_{33}(1 - g^{00} g_{00}) + g^{00} g_{30} g_{30} \end{pmatrix} \begin{pmatrix} \partial_0 g^{10} \\ \partial_0 g^{20} \\ \partial_0 g^{30} \end{pmatrix}$$

the determinant of the linear system is in general not null:

$$\begin{vmatrix} g_{11}(1 - g^{00} g_{00}) + g^{00} g_{10} g_{10} & g_{12}(1 - g^{00} g_{00}) + g^{00} g_{10} g_{20} & g_{13}(1 - g^{00} g_{00}) + g^{00} g_{10} g_{30} \\ g_{21}(1 - g^{00} g_{00}) + g^{00} g_{20} g_{10} & g_{22}(1 - g^{00} g_{00}) + g^{00} g_{20} g_{20} & g_{23}(1 - g^{00} g_{00}) + g^{00} g_{20} g_{30} \\ g_{31}(1 - g^{00} g_{00}) + g^{00} g_{30} g_{10} & g_{32}(1 - g^{00} g_{00}) + g^{00} g_{30} g_{20} & g_{33}(1 - g^{00} g_{00}) + g^{00} g_{30} g_{30} \end{vmatrix} = (1 - g^{00} g_{00})^2(\gamma - g^{00} g) \geq 0$$

where $g = |g_{ab}|$ and $\gamma = |g_{\alpha\beta}|$, then it is possible to obtain the transformation between time derivatives of the metric tensor and generalized moments $\partial_0 g^{ab} = P^{ab} + Q^{ab.cd}\pi_{cd}$.

$$\begin{array}{l}
\partial_0 g^{\kappa 0} = \frac{\sum_{\alpha_1\alpha_2\alpha_3\beta_1\beta_2} \epsilon_{\kappa\beta_1\beta_2}\epsilon_{\alpha_1\alpha_2\alpha_3}\left[g_{\alpha_1\beta_1}(1-g^{00}g_{00})+g^{00}g_{\alpha_1 0}g_{\beta_1 0}\right]\left[g_{\alpha_2\beta_2}(1-g^{00}g_{00})+g^{00}g_{\alpha_2 0}g_{\beta_2 0}\right]\left[\Upsilon_{\alpha_3 0}-\frac{g_{\alpha_3 0}}{g_{00}}\Upsilon_{00}\right]}{2(1-g^{00}g_{00})^2(\gamma-g^{00}g)} = \\
= \frac{\sum_{\alpha_1\alpha_2\alpha_3\beta_1\beta_2} \epsilon_{\kappa\beta_1\beta_2}\epsilon_{\alpha_1\alpha_2\alpha_3} g_{\alpha_2\beta_2}\left[g_{\alpha_1\beta_1}(1-g^{00}g_{00})+2g^{00}g_{\alpha_1 0}g_{\beta_1 0}\right]\left[\Upsilon_{\alpha_3 0}-\frac{g_{\alpha_3 0}}{g_{00}}\Upsilon_{00}\right]}{2(1-g^{00}g_{00})(\gamma-g^{00}g)}
\end{array}$$

It is simple to obtain the Hamiltonian like a π_{ab} function:

$$\begin{array}{l}
\frac{16\pi k}{c^3\sqrt{-g}}\mathcal{H} = g^{00}(g_{aa}g_{bb} - g_{ab}g_{ab})\partial_0 g^{\alpha\beta}\partial_0 g^{\alpha\beta} + 2g_{cd}\partial_\beta g^{\alpha d}\partial_\alpha g^{\beta c} - 2g_{cd}\partial_\beta g^{cd}\partial_\alpha g^{\alpha\beta} - g_{ac}g_{bd}g^{\alpha\beta}\partial_\alpha g^{ab}\partial_\beta g^{cd} + g_{ab}g_{cd}g^{\alpha\beta}\partial_\alpha g^{ab}\partial_\beta g^{cd} = \\
= \boxed{\begin{array}{l} g^{00}(g_{aa}g_{bb} - g_{ab}g_{ab})\left(P^{ab} + Q^{ab.cd}\pi_{cd}\right)^2 + \\ +2g_{cd}\partial_\beta g^{\alpha d}\partial_\alpha g^{\beta c} - 2g_{cd}\partial_\beta g^{cd}\partial_\alpha g^{\alpha\beta} - g_{ac}g_{bd}g^{\alpha\beta}\partial_\alpha g^{ab}\partial_\beta g^{cd} + g_{ab}g_{cd}g^{\alpha\beta}\partial_\alpha g^{ab}\partial_\beta g^{cd} \end{array}}
\end{array}$$

$$\frac{16\pi k}{c^3\sqrt{-g}}\hat{\mathcal{H}} = \boxed{\begin{array}{l} g^{00}(g_{aa}g_{bb} - g_{ab}g_{ab})\left(P^{ab}P^{ab} - 2i\hbar\, P^{ab}Q^{ab.cd}\frac{\partial}{\partial g^{cd}} - \hbar^2\, Q^{ab.cd}Q^{ab.ef}\frac{\partial^2}{\partial g^{ab}\partial g^{ef}}\right) + \\ +2g_{cd}\partial_\beta g^{\alpha d}\partial_\alpha g^{\beta c} - 2g_{cd}\partial_\beta g^{cd}\partial_\alpha g^{\alpha\beta} - g_{ac}g_{bd}g^{\alpha\beta}\partial_\alpha g^{ab}\partial_\beta g^{cd} + g_{ab}g_{cd}g^{\alpha\beta}\partial_\alpha g^{ab}\partial_\beta g^{cd} \end{array}}$$

this is the Hamiltonian of the gravitational field, that it is quantizable (even if not uniquely), and it is similar to the Wheeler-De Witt equation.

There is a simple isotropic solution with a constant metric tensor in the space, and with null space derivative of the metric tensor:

$$
\begin{array}{l}
g^{\mu\nu}(x^0) = \left(\begin{array}{cccc} g^{00}(x_0) & 0 & 0 & 0 \\ 0 & g^{11}(x_0) & g^{12}(x_0) & g^{13}(x_0) \\ 0 & g^{21}(x_0) & g^{22}(x_0) & g^{23}(x_0) \\ 0 & g^{31}(x_0) & g^{32}(x_0) & g^{33}(x_0) \end{array} \right) \\
\boxed{g^{ij} = g^{ij(0)} + h^{ij} t} \\
\partial_\mu g^{ik} = 0 \\
F_{ab} = 0 \\
\Theta_{\mu\nu} = \pi_{\mu\nu} = -g_{\mu\nu}\partial_0 g^{00} + g^{00} g_{\mu\nu} g_{cd} \partial_0 g^{cd} - g^{00} g_{\mu c} g_{\nu d} \partial_0 g^{cd} \\
\Lambda_{ab} = \Theta_{ab} - g_{ab}\frac{\Theta^m_m}{2} = \pi_{ab} - g_{ab}\frac{g^{ms}\pi_{ms}}{2} = \pi_{ab} - g_{ab}\frac{g^{00}\pi_{00}}{2} - g_{ab}\frac{g^{\gamma\delta}\pi_{\gamma\delta}}{2} \\
\partial_0 g^{\alpha 0} = 0 \\
\partial_0 g^{\alpha\beta} = -g^{\alpha\mu} g^{\beta\nu}\frac{\Lambda_{\mu\nu}}{g^{00}} = -\frac{g^{\alpha\mu} g^{\beta\nu}}{g^{00}}\pi_{\mu\nu} + g_{\mu\nu}\frac{g^{\alpha\mu} g^{\beta\nu}}{2}\pi_{00} + g_{\mu\nu}\frac{g^{\alpha\mu} g^{\beta\nu} g^{\gamma\delta}}{2g^{00}}\pi_{\gamma\delta}
\end{array}
$$

the relativistic quantum equation is[2]

$$
\begin{array}{l}
i\hbar\frac{\partial}{\partial t}\Psi(g^{ij}, \partial_k g^{ij}) = \frac{c^3\sqrt{-g}}{16\pi k} g^{00}(g_{\alpha\alpha} g_{\beta\beta} - g_{\alpha\beta} g_{\alpha\beta}) \left[-\frac{g^{\alpha\mu} g^{\beta\nu}}{g^{00}}\pi_{\mu\nu} + g_{\mu\nu}\frac{g^{\alpha\mu} g^{\beta\nu}}{2}\pi_{00} + g_{\mu\nu}\frac{g^{\alpha\mu} g^{\beta\nu} g^{\gamma\delta}}{2g^{00}}\pi_{\gamma\delta} \right]^2 \Psi(g^{ij}, \partial_k g^{ij}) \\
i\hbar\frac{\partial}{\partial t}\Psi(g^{ij}, \partial_k g^{ij}) = -\frac{\hbar^2 c^3\sqrt{-g}}{16\pi k} g^{00}(g_{\alpha\alpha} g_{\beta\beta} - g_{\alpha\beta} g_{\alpha\beta}) \left[\frac{g^{\alpha\mu} g^{\beta\nu}}{g^{00}}\frac{\partial}{\partial g^{\mu\nu}} - g_{\mu\nu}\frac{g^{\alpha\mu} g^{\beta\nu}}{2}\frac{\partial}{\partial g^{00}} - g_{\mu\nu}\frac{g^{\alpha\mu} g^{\beta\nu} g^{\gamma\delta}}{2g^{00}}\frac{\partial}{\partial g^{\gamma\delta}} \right]^2 \Psi(g^{ij}, \partial_k g^{ij}) \\
\Psi = e^{i(\hbar k_{\alpha\beta} g^{\alpha\beta} + \hbar k_{00} g^{00} - Et)/\hbar} \\
E - \hbar k_{ij} h^{ij} = -\frac{\hbar^2 c^3\sqrt{-g}}{16\pi k} g^{00}(g_{\alpha\alpha} g_{\beta\beta} - g_{\alpha\beta} g_{\alpha\beta}) \left[\frac{g^{\alpha\mu} g^{\beta\nu}}{g^{00}} k_{\mu\nu} - g_{\mu\nu}\frac{g^{\alpha\mu} g^{\beta\nu}}{2} k_{00} - g_{\mu\nu}\frac{g^{\alpha\mu} g^{\beta\nu} g^{\gamma\delta}}{2g^{00}} k_{\gamma\delta} \right]^2
\end{array}
$$

these are the energies of the particle in an isotropic space.

This solution is true only for a metric tensor that change in a constant way in the time; I think that this is a dead end for the theory (it allows to obtain only trivial solutions).

It is only possible, in general, to use the Heisenberg picture for the second quantization[3]:

$$
\begin{array}{l}
\hat{H} = \int \hat{\mathcal{H}}\, d^3\mathbf{x} \\
i\hbar\frac{dg^{ij}}{dt} = \left[g^{ij}, \hat{\mathcal{H}} \right] \\
i\hbar\frac{d\pi_{ij}}{dt} = \left[\pi_{ij}, \hat{\mathcal{H}} \right] \\
\left[\pi_{ij}(\mathbf{x}, t), g^{ij}(\mathbf{x}, t) \right] = -i\hbar\delta^3(\mathbf{x} - \mathbf{x}') \\
g_{ij} = \frac{g}{3!}\sum_{\alpha_1\alpha_2\alpha_3\beta_1\beta_2\beta_3} \epsilon_{i\alpha_1\alpha_2\alpha_3}\epsilon_{j\beta_1\beta_2\beta_3} g^{\alpha_1\beta_1} g^{\alpha_2\beta_2} g^{\alpha_3\beta_3}
\end{array}
$$

and

$$g^{ij}(\mathbf{x}, t) = \int d^3\mathbf{x}\, \mathcal{N}_{\mathbf{k}}\, [a^{ij}(\mathbf{k}) e^{i(\mathbf{k}\cdot\mathbf{x} - \omega_{\mathbf{k}})} + a^{ij\dagger}(\mathbf{k}) e^{-i(\mathbf{k}\cdot\mathbf{x} - \omega_{\mathbf{k}})}]$$

where $a^{ij}(\mathbf{k})$ and $a^{ij\dagger}(\mathbf{k})$ are the destruction and creation operators for the graviton.

[2] for this simple metric tensor

[3] it is written the $g_{ij}(g^{lm})$ so that it is possible to obtain the commutator rules for the covariant tensors with the moments