From neutron and quark stars to black holes

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(Dated: March 18, 2020)

Abstract

New physics and models for the most compact astronomical objects - neutron / quark stars and black holes are proposed. Under the new supersymmetric mirror models, neutron stars at least heavy ones could be born from hot deconfined quark matter in the core with a mass limit less than $2.5M_{\odot}$. Even heavier cores will inevitably collapse into black holes as quark matter with more deconfined quark flavors becomes ever softer during the staged restoration of flavor symmetry. With new understanding of gravity as mean field theories emergent from the underlying quantum theories for providing the smooth background spacetime geometry for quantum particles, the black hole interior can be described well as a perfect fluid of free massless Majorana fermions and gauge bosons under the new genuine 2-d model. In particular, the conformal invariance on a 2-d torus for the black hole gives rise to desired consistent results for the interior microphysics and structures including its temperature, density, and entropy. Conjectures for further studies of the black hole and the early universe are also discussed in the new framework.

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I. INTRODUCTION

Supernovae are believed to be the stellar explosions that leave behind the densest compact objects - neutron stars (NS) and possibly black holes (BH). Neutron stars with masses up to around $2M_\odot$ have been observed [1]. Neutron star merger events via gravitational wave detection have set an NS mass limit of about $2.2M_\odot$ [2]. A new mirror matter model for star evolution also requires the same NS mass limit [3].

The neutron degeneracy pressure alone may not be enough to support a neutron star beyond $0.7M_\odot$ [4, 5]. With more realistic equation of state for hadronic matter including likely quark matter, a cold neutron star could meet the observed mass limit [6, 7]. Quark matter, in particular, strange quark matter could be more stable than the most tightly bound iron and nickel nuclei, i.e., with energy per baryon $\lesssim$ 930 MeV [8–10]. Calculations have shown that a star made of quark matter has a mass limit of about 2-3 $M_\odot$ before it turns into a black hole [11]. It is possible that the observed neutron stars may be composed of sophisticated components including neutron matter in the outer layer and quark matter in the core (see Refs. [1, 6] for recent reviews). Even a new type of stellar explosion called Quark-Nova has been proposed for describing the explosive transition of a neutron star to a quark star [12].

The idea of no light escaping from a sufficiently massive astronomical object like a black hole was first proposed more than two hundred years ago by Michell and Laplace [13, 14]. Shortly after Einstein developed his theory of general relativity (GR), Schwarzschild obtained the first modern black hole solution of Einstein's equations in vacuum [15]. It describes a static spherically symmetric black hole with the Schwarzschild metric as follows,

$$ds^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2(\theta) d\varphi^2 \tag{1}$$

where M is the BH mass and we adopt natural units of $c = \hbar = k_B = 1$ but keep the gravitational constant G explicitly. The apparent singularity at the event horizon $r = 2GM$ does not seem to be physical and can be removed in suitable coordinates. But the singularity at $r = 0$ is genuine indicating the breakdown of GR under such extreme conditions. It may hint that GR under 4-d spacetime does not describe the BH interior within its event horizon.

In 4-d spacetime, Einstein's equations can be written as,

$$\mathcal{R}_{\mu\nu} - \frac{1}{2}\mathcal{R} g_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{2}$$

where Λ is the cosmological constant related to vacuum energy and the energy-momentum tensor $T_{\mu\nu}$ accounts for the other contents of radiation and matter. Its contraction for the Ricci scalar $\mathcal{R}$ can be obtained in 4-d spacetime as,

$$\mathcal{R} + 4\Lambda = -8\pi G T^{\mu}_{\mu} \tag{3}$$

where T^{μ}_{μ} is the trace of $T_{\mu\nu}$.

In 2-d spacetime, the curvature tensor $\mathcal{R}_{\mu\nu}$ has only one independent component and therefore its geometry structure is determined solely by the Ricci scalar $\mathcal{R}$. From Eq. (3), we can then infer a possible field equation of gravity in 2-d spacetime to be,

$$\mathcal{R} + 2\Lambda = -8\pi G T^{\mu}_{\mu}. \tag{4}$$

Such an equation can also be derived under the variational principle of an action [16, 17]. Black holes and their properties in 2-d spacetime have also been studied by solving the 2-d vacuum field equation [18].

In the following, we will apply recently developed supersymmetric mirror models (SMM) and new understanding of gravity to study such extreme celestial objects, in particular, black holes under a new framework.

II. SUPERSYMMETRIC MIRROR MODELS

The existence of a mirror sector of the Universe has been conjectured since Lee and Yang published their Nobel Prize-winning work on parity violation [19]. It is conceivable that there exist two sectors of particles sharing the same gravity but governed by two separate gauge groups under 4-d spacetime. Some early works on mirror matter theory had discussed interesting perspectives mainly in cosmology [20–22]. Later attempts to introduce feeble interactions between the two sectors might be too conservative following conventional practices [23–25]. Most recent works [3, 26–32] by taking the essence of mirror matter theory with new understanding of supersymmetry (SUSY) may indeed lead us to new physics beyond the Standard Model we have all been looking for.

Under the new framework, quarks of six flavors could undergo phase transitions via staged quark condensation [28, 30] at temperatures between 10^2 MeV and 10^2 GeV, i.e., between energy scales of QCD and electroweak phase transitions. The spontaneous symmetry breaking process results in an N=4 pseudo-SUSY model (SMM^{4b}) with a gauge SUSY multiplet

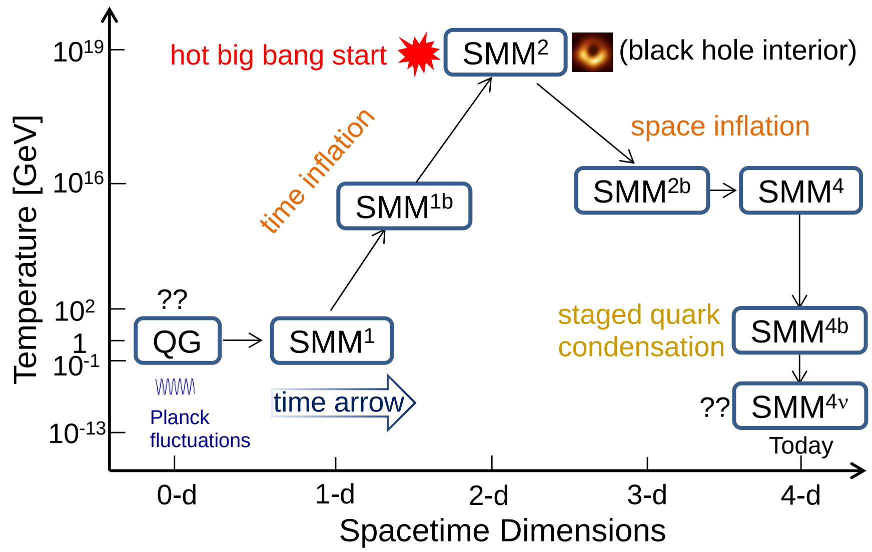

FIG. 1. The schematic diagram (not to scale) is shown for the dynamic hierarchy of quantum gravity (QG) and supersymmetric mirror models (SMM) at various phases of the Universe and spacetime. The superscript number in model name SMM denotes the number of spacetime dimensions while superscript 'b' indicates that the model is for the corresponding spontaneous symmetry breaking process. SMM^{4b} (or a slight variant $\mathrm{SMM}^{4\nu}$ due to possible neutrino condensation) is the model that governs the current Universe. See Refs. [31, 32] for more details.

and three chiral multiplets of chiral neutrinos and six Higgs scalars in each sector [31, 32]. The schematic diagram of these supersymmetric mirror models is presented in Fig.1.

At temperatures in between 10^2 and 10^{16} GeV, the exact N=1 gauge SUSY is restored for SMM^4 still in 4-d spacetime with gauge symmetry of $U_f(6) \times SU_c(3) \times SU_w(2) \times U_Y(1)$ for the ordinary sector and $U_f(6)' \times SU_c(3)' \times SU_w(2)' \times U_Y(1)'$ for the mirror sector [31, 32].

At temperatures above 10^{16} GeV, spacetime itself may undergo dimensional phase transition, i.e., going from 4-d to 2-d. The underlying model (SMM^2) becomes a simple N=1 SUSY model in 2-d spacetime with the Lagrangian,

$$\mathcal{L}_{\mathrm{SMM}^2} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} (\lambda_L^\dagger \bar{\sigma}^\mu \partial_\mu \lambda_L + \lambda_R^\dagger \sigma^\mu \partial_\mu \lambda_R) \tag{5}$$

where the $U(1)$ gauge tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the Majorana fermion λ has to be neutral and does not couple to the gauge field A_μ. Both λ and A_μ are massless and have two components or degrees of freedom. They form the simplest $N = 1$ abelian gauge SUSY multiplet $(1, 1/2)$ with the on-shell Lagrangian of Eq. (5).

The phase transition between 2-d and 4-d spacetime is driven by the Majorana condensates or space inflaton scalars ϕ and ϕ' under the N=1 pseudo-SUSY model SMM^{2b} [31]. The phase transition between 1-d and 2-d spacetime or the time inflation process is induced by the "timeron" scalar φ under the following Lagrangian (SMM^{1b}),

$$\mathcal{L}_{\text{SMM}^{1b}} = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m_\varphi^2\varphi^2 - \frac{1}{8}\varphi^4. \tag{6}$$

We will apply these models for understanding the most compact stars including black holes in the following discussions.

III. QUARK STARS

At hot neutron star births in core collapse supernovae or gravitational mergers, temperatures can go as high as a few tens of MeV or even $\sim 10^2$ MeV in current models [6]. In the new theory proposed for evolution of massive stars [3], fusion and neutron enrichment reactions due to $n - n'$ oscillations can make the core much hotter and evenly mixed with both ordinary and mirror particles. In addition, the temperature could be even higher if quark matter, in particular, strange quark matter is more stable in the core than nuclei [9].

Therefore, a hot massive core could reach high enough temperatures to restore the chiral $SU(2)$ or even $SU(3)$ symmetry for up, down, and strange quarks. As studied in staged quark condensation of mirror matter theory [28, 30], such phase transition temperatures could be around 150 MeV where u-, d-, and even s-quarks become deconfined and may form the so-called quark-gluon plasma. For a uniform hot quark-gluon plasma at temperature T, its energy density can be written as,

$$\rho = \frac{\pi^2}{30} g T^4 \tag{7}$$

where $g = n_b + 7/8 n_f$ is the effective number of relativistic degrees of freedom for fermions (e.g., quarks) (n_f) and bosons (e.g., gluons) (n_b).

Now we can find out the upper mass limit for such a uniform hot quark matter core using the mirror matter model. For static spherically symmetric solutions to Einstein's equations without singularities, there exists an upper limit on the mass-to-radius ratio of a star for any equation of state [33],

$$\frac{GM}{R} < \frac{4}{9}. \tag{8}$$

For a uniform core with $\rho = 3M/(4\pi R^3)$, one can easily derive an upper mass limit for hot quark matter stars from Eqs. (7-8),

$$M < \frac{4}{9\pi}\sqrt{\frac{10}{\pi g G^3}}\frac{1}{T^2} \simeq \frac{0.41 M_\odot}{\sqrt{g} T_{\rm GeV}^2} \tag{9}$$

where $M_\odot$ is the solar mass and $T_{\rm GeV}$ is the temperature in GeV. If only chiral $SU(2)$ is restored for u- and d- quarks, we obtain $n_b = 16$ and $n_f = 12$ for each sector and hence $g = 53$ for both ordinary and mirror sectors. Assuming that $T = 0.15$ GeV similar to the QCD phase transition temperature, the mass limit becomes $M < 2.5 M_\odot$.

If chiral $SU(3)$ for u-, d-, and s-quarks is restored at the same temperature, we have $n_f = 18$ for one sector and $g = 63.5$ for both resulting in $M < 2.3 M_\odot$. In addition, the topological B-violating "quarkiton" process [28] that converts between strange quarks and anti-strange quarks may be significant enough to open up more degrees of freedom. In the extreme scenario, the plasma may include all three types of quarks, e- and μ-leptons and neutrinos, and their anti-particles. Such an extreme case will lead to $g = 116$ and $M < 1.7 M_\odot$. Realistically, the mass limit of a hot neutron/quark star should be somewhere in between these cases and therefore compatible with the observed limit of $\sim 2.2 M_\odot$. Once the newly formed hot compact star cools down, the equation of state for the cold neutron star can be stiff enough to keep it stable [6, 7]. Therefore, a neutron star especially a massive one may be born first as a hot quark star and then cool down into neutron / hadronic matter later.

Note that Eq. (9) indicates that more massive cores (e.g., $> 2.5 M_\odot$) can not be sustained by quark matter as more deconfined quarks (larger g) and higher temperature T will result in an even lower mass limit. As a matter of fact, once the mass is over the limit, more degrees of freedom are set free and the ever softer equation of state will trigger a runaway collapse. This will quickly heat up the core under symmetric mirror models (SMM^{4b} and SMM^4) in 4-d spacetime to a temperature of $\sim 10^{16}$ GeV. At this point, spacetime phase transition occurs going from 4-d to 2-d with much reduced degrees of freedom [31] resulting in a 2-d black hole. In other words, the interior of a black hole is 2-d in nature and should be studied under SMM^2 and a suitable 2-d gravity theory.

IV. EMERGENT GRAVITY IN DIFFERENT SPACETIME DIMENSIONS

A naive inclusion of gravity in quantum field theory could be $\mathcal{S} = \int d^4x\sqrt{-g}(-\frac{1}{16\pi G}\mathcal{R} + \mathcal{L}_m + \mathcal{L}_\phi)$ in 4-d spacetime, where $\mathcal{L}_m$ of the kinetic and mass terms of all fermion and boson fields contributes to the energy-momentum tensor and $\mathcal{L}_\phi$ of the quartic terms of scalar fields determines the cosmological constant or vacuum energy.

For the case of a single scalar field ϕ, it gives a non-vanishing cosmological constant as $\Lambda = \pi G\langle\phi\rangle^4$. However, for more complicated cases like the supersymmetric mirror models of SMM^{2b} and SMM^{4b} [31], we have to distinguish the three different vacuum configurations for ordinary and mirror gauge interactions and gravity, respectively. In particular, the vacuum energy for gravity should be determined by the coherent sum of all scalar fields (even from different gauge sectors) since they share the same gravity / spacetime. As such, the gravitational vacuum energy density is $\rho_\Lambda = \frac{1}{8}\langle\sum_j \phi_j\rangle^4$, which can explain the observed dark energy density of $(10^{-3}\,\mathrm{eV})^4$ under our new model [30].

An immediate idea following this line of thought is that gravity may be emergent from the mean field effects of quantum fluctuations of spacetime after its inflation. General relativity can then be treated as a mean field theory obtained from the mean field action,

$$\mathcal{S}_{\mathrm{grav}} = \int d^4x\sqrt{-g}(-\frac{1}{16\pi G}\mathcal{R} + \langle\mathcal{L}_m\rangle + \langle\mathcal{L}_\phi\rangle) \tag{10}$$

where $\langle\mathcal{L}_\phi\rangle$ requires the coherent sum of all scalar fields for the proper cosmological constant. Under this scenario, gravity just defines the spacetime metric $g^{\mu\nu}$ as the smooth mean geometrical background for quantum particles.

Naturally, gravity emerges during the phase transition of 1-d time to 2-d spacetime. Under SMM^{1b} as in Eq. (6), the "timeron" mass can be related to the gravity curvature scalar of the inflating 2-d spacetime as follows,

$$\mathcal{R}_t = -m_\varphi^2 = -\frac{1}{16\pi G} \tag{11}$$

where the curvature scalar $\mathcal{R}_t$ or the gravitational constant G is a running constant during the phase transition going from an initial value of $1/\sqrt{G_0} \sim 1$ GeV to its current value of $1/\sqrt{G} \sim 10^{19}$ GeV. Afterwards, $\mathcal{R}_t$ becomes a fixed constant of $-1/(16\pi G)$.

Consequently, the gravitational action for Einstein's general relativity in 4-d spacetime can be rewritten as,

$$\mathcal{S}_{\mathrm{grav}}^{4d} = \int d^4x\sqrt{-g}(\mathcal{R}_t(\mathcal{R} + 2\Lambda) + \langle\mathcal{L}_m\rangle). \tag{12}$$

Similarly, the gravity theory for 2-d spacetime can be inferred as,

$$\mathcal{S}_{\text{grav}}^{2d} = \int d^2x f(\phi, \phi')\sqrt{-g}(\mathcal{R}_t(\mathcal{R} + 2\Lambda) + \langle \mathcal{L}_m \rangle). \tag{13}$$

where $f(\phi, \phi')$ is a function of the two inflaton scalar fields after integrating out the two unextended space dimensions (which are inflated in the early dynamic universe). The variational principle of the action with respect to the metric $g_{\mu\nu}$ yields no constraint on the Ricci curvature $\mathcal{R}$ but does require that the trace of the energy-momentum tensor $T^\mu_\mu = 0$. On the contrary, varying field ϕ or ϕ' gives the same equation on the Ricci scalar $\mathcal{R}$ as Eq. (4) [16]. Therefore, the eventual gravitational equation for 2-d spacetime is,

$$\mathcal{R} + 2\Lambda = -8\pi G T^\mu_\mu = 0. \tag{14}$$

In 2-d quantum field theory of SMM^2 as in Eq. (5), massless Majorana fermions and $U(1)$ gauge bosons make a perfect fluid resulting in the vanishing trace of $T_{\mu\nu}$ and therefore obey the 2-d gravity / spacetime model of Eq. (14). Next we will see how the interior of a black hole is described in these 2-d models.

V. GENUINE 2-D BLACK HOLES

For a static spherically symmetric black hole, the Schwarzschild metric of Eq. 1) provides the solution of its exterior ($r > R = 2GM$) 4-d spacetime. But the BH interior has to be studied with the genuine 2-d models of Eq. (5) for quantum SMM^2 and Eq. (14) for gravity. Such a new framework, as shown below, presents the BH interior as a perfect fluid of free massless Majorana fermions and gauge bosons in 2-d conformal field theory (CFT) on a torus [34].

SMM^2 of Eq. (5) and the gravity model of Eq. (14) together describe a conformally invariant 2-d quantum and spacetime theory. Its effective number of relativistic degrees of freedom $g = 3$ is identically the central charge c of the Virasoro symmetry algebra in 2-d CFT. The corresponding Weyl anomaly in 2-d CFT [35] can produce a non-vanishing trace of $2\rho_\Lambda = -c/(24\pi G)\mathcal{R}$ due to nontrivial metric, where ρ_Λ can be interpreted as vacuum energy density and related to the cosmological constant via $\Lambda = 8\pi G\rho_\Lambda$ in comparison with Eq. (14). The integration of this trace anomaly, introduced dynamically by the inflaton scalars ϕ and ϕ' [31], is related to a topological invariant - the Euler characteristic [36], which may be important for studying nontrivial spacetime topology during the space inflation process.

However, the static BH interior should be Ricci-flat, i.e, $\mathcal{R} = 0$ and can then be studied in 2-d CFT on a torus. The Euler characteristic of a torus vanishes and makes it consistent that the black hole interior should be static and free of the trace anomaly or $\Lambda = 0$. In this case, the general solution of the 2-d metric can be written as,

$$ds^2 = e^{2\omega(x)}(dt^2 - dx^2) \tag{15}$$

where $\omega(x)$ defines the conformal transformations of the metric.

We will present the views of the BH interior by both a distant exterior observer and an interior one. The two reference frames are connected by the global scale transformation under 2-d CFT. There are two quantities that the two observers see the same: proper energy density ρ and total entropy $S_{\rm BH}$.

The picture seen by the exterior observer is fairly simple for a Schwarzschild black hole with mass M and radius $R = 2GM$. The BH interior can be regarded as a 2-d torus with two circumferences of $2R$ (space) and $2\pi R$ (time), which can also be viewed externally as a dual of the event horizon surface, notably with the same area $A = 4\pi R^2$. Its conformally invariant 2-d energy density is,

$$\rho = \frac{M}{2R} = \frac{1}{4G} \tag{16}$$

which is the same as the CFT Casimir energy density $\rho_c = E_c/(2R) = c/(12G)$ with no negative sign under the condition of $c = g = 3$. This 2-d structure with a constant energy density provides a natural explanation of the linear relation between mass and radius of the black hole.

The entropy of an $n+1$ dimensional CFT on $R \times S^n$ can be given by the Cardy-Verlinde formula [37, 38],

$$S = \frac{2\pi R}{n}\sqrt{E_c(2E - E_c)} \tag{17}$$

where $E_c = 2Rc/(12G)$ is the Casimir energy in 2-d CFT (i.e., $n = 1$) and the total energy $E = 2R\rho$ is exactly the same as E_c if the central charge $c = g = 3$. Under the BH conditions of $n = 1$ and $c = g = 3$, Eq. (17) becomes the well-known Bekenstein-Hawking entropy [39, 40],

$$S_{\rm BH} = \frac{A}{4G}. \tag{18}$$

Based on Maldacena's AdS/CFT correspondence [41] and Witten's arguments [42], Verlinde proposed a more general entropy formula for the same CFT on $R \times S^n$ as [38],

$$S = \frac{cA}{12nG} \tag{19}$$

which again gives the same entropy as Eq. (18) for $n = 1$ and $c = g = 3$.

The BH entropy can also have an equivalent thermodynamic explanation. Namely, the exterior observer sees no free energy from the black hole, i.e., its partition function $Z = 1$. As such, the entropy is solely determined by its total energy / mass M and temperature $1/\beta$ where $\beta = 2\pi R$ is the Euclidean time period or the time circumference of the torus. Then, the same BH entropy can be easily calculated from

$$S_{\rm BH} = \ln(Z) + \beta M = \beta M = \frac{A}{4G} \tag{20}$$

where, notably, the interior BH temperature of $1/\beta$ viewed by the exterior observer is twice as high as the surface temperature that is responsible for Hawking radiation [40, 43].

Before we present the interior view, it is worth stressing that in this work the BH interior resides in genuine two-dimensional spacetime unlike other toy 2-d model studies. The BH interior is composed of a perfect fluid of free Majorana fermions and gauge bosons governed by SMM^2 of Eq. (5). As viewed by the interior observer, its energy and entropy densities in 2-d spacetime after momentum space integration can be written as,

$$\rho = \frac{\pi}{6} g T_{\rm in}^2 = \frac{\pi}{2} T_{\rm in}^2 \tag{21}$$

$$s = \frac{\rho}{T_{\rm in}} = \frac{\pi}{2} T_{\rm in} \tag{22}$$

where $g = n_b + 1/2 n_f = 3$ is the effective number of relativistic degrees of freedom for Majorana fermions ($n_f = 2$) and gauge bosons ($n_b = 2$) and $T_{\rm in}$ is the interior temperature. Such an expression of 2-d entropy density in Eq. (22) has also been obtained in general 2-d conformal field theory in the high temperature limit [37, 44].

Using the two views of the energy density in Eqs. (16,21), we can obtain the temperature of the BH interior,

$$T_{\rm in} = \frac{1}{\sqrt{2\pi G}} \simeq \frac{0.4}{\sqrt{G}} \tag{23}$$

which is constant and just below the Planck energy of $1/\sqrt{G} \sim 10^{19}$ GeV but well above the space inflation energy scale of 10^{16} GeV. This means that the interior temperature does meet the criteria for SMM^2 in 2-d spacetime and this BH model is self-consistent. The constant energy density and temperature also indicate that no further collapse is possible and there is no upper limit for the BH mass under this model.

Now we need to find the interior size of the black hole in order to calculate the total entropy from the entropy density of Eq. (22). From the view of the interior observer, the torus is conformally dilated as follows,

$$2R \to L_s \equiv e^{\omega} 2R, \quad 2\pi R \to L_t \equiv e^{-\omega} 2\pi R \tag{24}$$

where L_s and L_t are the different space and time scales (or sizes of the torus) for the interior observer under the global dilation or scale transformation. From $L_t = 1/T_{\text{in}}$, we can obtain the dilation factor as,

$$e^{\omega} = \sqrt{\frac{2\pi}{G}} R \simeq 4.58 \times 10^{38} \frac{M}{M_{\odot}}. \tag{25}$$

It is now easy to calculate the entropy from the point of view of the interior observer,

$$S_{\text{BH}} = e^{\omega} 2Rs = \frac{A}{4G} \tag{26}$$

which is the same Bekenstein–Hawking entropy. To demonstrate the enormous BH size seen by the interior observer, we can calculate it for one solar mass to be $e^{\omega} 2R \sim 10^{42}$ m in contrast to the size of the observable universe (10^{27} m).

VI. OUTLOOK AND FURTHER DISCUSSIONS

Under this work, the thermodynamic properties of a black hole originate from the quantum nature in 2-d spacetime. More studies could be stimulated for further understanding of the black hole and the universe. For example, Eq. (19) shows a very intriguing feature: in order to conserve the entropy, the central charge $c = g$ has to increase linearly with the space dimension number n. This indicates that three generations of quantum particles in 4-d spacetime may be tied to the three space dimensions they reside in. To apply it to the dynamic evolution of the universe [31, 32], we need to consider that global and gauge symmetries can reduce the effective volume or surface area. Therefore, n should be replaced with the dimension $n(T)$ of unbroken gauge groups and the factor $m(T) = 2$ when the mirror symmetry is unbroken for the entropy of the universe,

$$S = \frac{g(T)}{n(T)m(T)} \frac{A}{12G} \sim \frac{A}{4G} \tag{27}$$

where the effective number of relativistic degrees of freedom $g(T)$ is slightly above 3 in the current universe and $n(T) \sim 1$ for the only unbroken and unconfined $U_{\rm EM}(1)$ interaction ($SU(2)$ is broken and $SU(3)$ is confined). In the early universe at $T \gtrsim 10^2$ GeV, the observed gauge symmetry of $U_f(6) \times SU_c(3) \times SU_w(2) \times U_Y(1)$ [30] leads to $n(T) = 48$ and $g(T) = 144$. The eventual constant factor in Eq. (27) or conservation of entropy is tied to supersymmetry that matches the degrees of freedom between fermions and bosons.

Another example is on the apparent singularity at the event horizon $r = 2GM$ that is most likely a topological singularity describing a transition from the exterior 4-d spacetime to the interior 2-d spacetime. Techniques especially topological ones developed in studies of CFT, string theory, loop quantum gravity, and other quantum gravity approaches could be applied here and possibly elsewhere such as the space inflation process of the early universe - phase transition from 2-d to 4-d spacetime.

ACKNOWLEDGMENTS

This work is supported in part by the National Science Foundation under grant No. PHY-1713857, OISE-1927130 (IReNA), and the Joint Institute for Nuclear Astrophysics (JINA-CEE, www.jinaweb.org), NSF-PFC under grant No. PHY-1430152. Funding from the faculty research support program at the University of Notre Dame is also acknowledged.

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