Excircles of a triangle with two parallel sides

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Abstract. For a triangle ABC, the radius of the excircle touching CA from the side opposite to B equals 0 if BC and CA are parallel.

Keywords. excircle, triangle with parallel sides, division by zero.

Let us consider the excircle of a triangle ABC touching CA from the side opposite to B (see Figure 1). We show that the radius of this circle equal 0 if BC and CA are parallel.

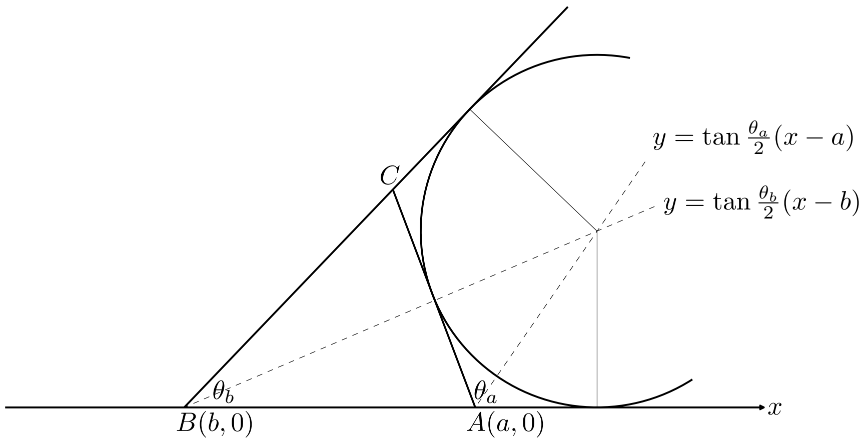

Figure 1.

We use a rectangular coordinate system such that A and B have coordinates $(a, 0)$ and $(b, 0)$, respectively, where we assume that the point C lies on the region $y > 0$. Let θ_a (resp. θ_b) be the angles between $\overrightarrow{BA}$ and $\overrightarrow{AC}$ (resp. $\overrightarrow{BC}$). Then the center of the excircle coincides with the point of intersection of the two lines expressed by $y = \tan\frac{\theta_a}{2}(x-a)$ and $y = \tan\frac{\theta_b}{2}(x-b)$. The coordinates of the point are give by:

$$\left(\frac{a\tan\frac{\theta_a}{2} - b\tan\frac{\theta_b}{2}}{\tan\frac{\theta_a}{2} - \tan\frac{\theta_b}{2}}, (a-b)\frac{\sin\frac{\theta_a}{2}\sin\frac{\theta_b}{2}}{\sin\frac{\theta_a-\theta_b}{2}}\right),$$

where notice that the y-coordinate gives the radius of the excircle. Now we fix the points A and B and consider the case $\theta_a = \theta_b$ (see Figure 2). Then by the definition of division by zero, $a/0 = 0$ for any number a [1], we see that the center of the excircle has coordinates $(0, 0)$. Therefore the exradius equals 0.

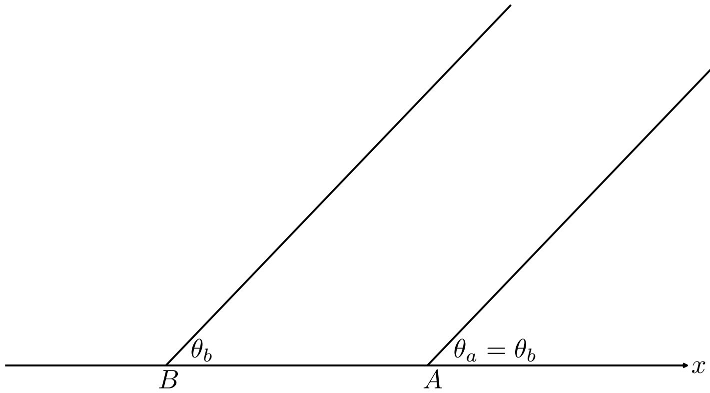

Figure 2.

Remark. The essential part of this note is the fact that the point of intersection of two parallel lines coincides with the origin. The fact is alrady pointed out in [2].

References

[1] M. Kuroda, H. Michiwaki, S. Saitoh, M. Yamane, New meanings of the division by zero and interpretations on $100/0 = 0$ and on $0/0 = 0$, Int. J. Appl. Math., **27**(2) (2014) 191–198.

[2] S. Saitoh, Division by zero calculus (draft).