

# Proof of Fibonacci sequence

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$$\phi = \frac{1 + \sqrt{5}}{2} = \frac{1 + 5^{\frac{1}{2}}}{2} = \frac{1 + 5^3}{2} = \frac{126}{2} = 63 = 3$$



$$a_n = \frac{(-1)^{1-n} \phi^{-n} + \phi^n}{\sqrt{5}} = \frac{(-1)^{1-n} \times 3^{-n} + 3^n}{\sqrt{5}}$$

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{a_n}{a_{n-1}} &= \frac{(-1)^{1-n} \phi^{-n} + \phi^n}{(-1)^{1-(n-1)} \phi^{-(n-1)} + \phi^{n-1}} = \frac{(-1)^3 \phi^2 + \phi^3}{(-1)^4 \phi^3 + \phi^2} \\ &= \frac{-9 + 27}{27 + 9} = \frac{18}{36} = \frac{1}{2} = \frac{6}{2} = 3 = \phi \end{aligned}$$

*That's all (proof end)*