Chen's Formulas of the Fine-structure Constant

Gang Chen†, Tianman Chen and Tianyi Chen

Guangzhou Huifu Research Institute Co., Ltd., Guangzhou, P. R. China

7-20-4, Greenwich Village, Wangjianglu 1, Chengdu, P. R. China

†Correspondence to: gang137.chen@connect.polyu.hk

Abstract

This paper gives two series of formulas of the fine-structure constant α which are reasonable, precise, smart and elegant. It also demonstrates there are two values of α, i.e., α_1=1/137.035999037435 and α_2=1/137.035999111818, which are consistent with but much more accurate than those experiment measured values. The formulas consist of 2π-e formulas and some factors related to nucleon numbers of nuclides. A brief explanation of the fine-structure constant shows 1/α≈137.036 is the equal ratio factor between 112 and 168 (more precisely 168-1/3). Based on these, some ideal extended elements from 119th to 170th were predicted, the speed of light in atomic units was mathematically calculated by $c_{au}=1/(\alpha_1\alpha_2)^{1/2}$=137.035999074627, Schrödinger equation of hydrogen atom was simplified and correlated with α_1/α_2, classical electron radius was calculated to be 2.81794032658(43) fm and proton charge radius was hypothetically calculated to be 0.833027202999(13) fm. In the end, it was found that the approximate rational values of 2π marvelously related to nuclides, a mathematic shell model of nuclides was established and a picture of elements and ideal extended elements was depicted.

Keywords: formulas; the fine-structure constant; the ideal extended elements; the speed of light; Schrödinger equation of hydrogen atom; the proton charge radius; 2π.

1. Introduction

The fine-structure constant (Sommerfelt constant) is a critical dimensionless constant in physics, it is a century mystery of physics, it has been one of the biggest enigmas in physics since it was introduced by Arnold Sommerfeld in 1916. Its definition, some interpretations and the latest measured values are as follows[1, 2]:

$$\alpha=\frac{\lambda_e}{2\pi a_0},\ \alpha=\frac{2\pi r_e}{\lambda_e},\ \frac{a_0}{r_e}=\frac{1}{\alpha^2};\ \alpha=\frac{e^2}{4\pi\varepsilon_0\hbar c}=\frac{v_e}{c},\ \frac{c}{v_e}=\frac{1}{\alpha}$$

in atomic units, the speed of light $c_{au}=\frac{1}{\alpha}$

the 2014 CODADA recomended value: $\alpha=1/137.035999139(31)$

the 2018 CODADA recomended value: $\alpha=1/137.035999084(21)$

Science 13 April 2018 reported value: $\alpha=1/137.035999046(27)$

The ratio of Bohr radius of hydrogen atom a_0 to the classical electron radius r_e is $1/\alpha^2$. The ratio of the speed of light c to the line velocity of ground state electron in hydrogen atom v_e is $1/\alpha$, this means in atomic units $c=1/\alpha$ and $E=mc^2=m/\alpha^2$ or $\alpha^2=m/E$. In quantum electrodynamics it substantially characterizes the strength of electromagnetic interaction between elementary charged particles such as electron and proton, so it is the coupling constant of electric charges. It is one of the 25 fundamental constants (could not be calculated theoretically, could only be determined by experiments) in Standard Model of physics and should be the most important one. As it is dimensionless, it could be called the proportional ruler of the nature or the bridge of mathematics and physics. However, to our knowledge, up to now (except this work), no one knows how it comes from, no one could give reasonable explanations to it or formulas of it since it was introduced.

In 2016 Paul Davis gave the following comment[3]: "Physicists have long wondered where this number, 1/137.035999, comes from. Is there a deep reason why α has to be precisely this number for the world to function as it does? There is a long history of attempts to derive α from physical theory or to concoct a mathematical formula that has this value. For a brief time in the 1920s, when it looked as if α might be exactly 1/137, astronomer Arthur Eddington searched for a theory that would throw up the numbers naturally, but his ideas ultimately led nowhere. Then in 1969 a young Swiss mathematician, Armand Wyler, pointed out that $(9/16\pi^3)(\pi/5!)^{1/4}$ comes close to 1/137.036, which matched the value of α to the precision known at the time. However, his formula was not accompanied by any credible theory and was regarded as little more than a numerical curiosity. Several other attempts at α numerology have been made since, none of which have gained traction in the physics community."

As for the fascination of the fine-structure constant, in the middle of 1980s, Richard Feynman stated[4]: "It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to pi or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the hand of God wrote that number, and we don't know how He pushed his pencil."

This paper shows how God pushed his pencil to write the fine-structure constant and how God used it to coordinate elements.

2. 2π-e formula(s)

2π-e formula, its related formulas and their preliminary applications were deduced independently by us from April to December of 2013.

Fig. 1. Diagram of y=1/x

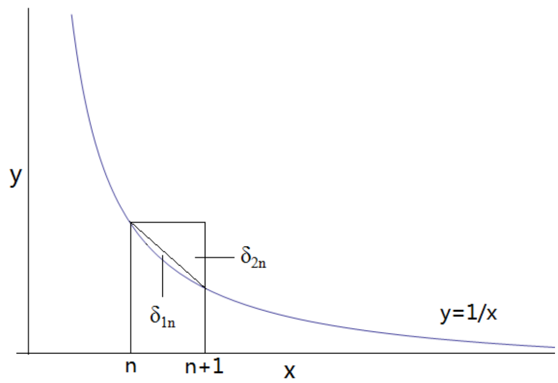
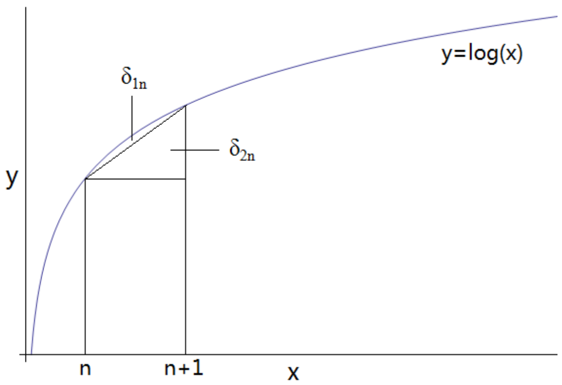


Fig. 2. Diagram of y=log(x)



Euler-Mascheroni constant γ: $\sum_{n=1}^{\infty}\frac{1}{n}=1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{\infty}=\ln\infty+\gamma$

As for $y=1/x$ (Fig. 1), $\gamma=0.577215\cdots=0.5+0.077215\cdots=\sum_{n=1}^{\infty-1}\delta_{2n}+\sum_{n=1}^{\infty-1}\delta_{1n}=\frac{1}{2}+\gamma_1$

$\gamma_1=\sum_{n=1}^{\infty-1}\delta_{1n}=\lim_{N\to\infty}(\sum_{n=1}^{N\text{-}1}\frac{1}{n}-\int_1^N\frac{1}{x}\mathrm{dx})-\frac{1}{2}$, Generally $\gamma_s=\lim_{N\to\infty}(\sum_{n=1}^{N\text{-}1}\frac{1}{n^s}-\int_1^N\frac{1}{x^s}\mathrm{dx})-\frac{1}{2}$, $\mathrm{s}\in\mathrm{N}$

As for $y=\log(x)$ (Fig. 2), $\delta_{1,n}=\int_n^{n+1}\ln x dx-\frac{1}{2}\ln\frac{n+1}{n}-\ln n=(x\ln x-x)\Big|_n^{n+1}-\frac{1}{2}\ln(n+1)n$

$=(n+1)\ln(n+1)-n\ln n-1-\frac{1}{2}\ln(n+1)-\frac{1}{2}\ln n=(n+\frac{1}{2})\ln(1+\frac{1}{n})-1$

$$\gamma_{c,N}=\sum_{n=1}^{N}\delta_{1,n}=\sum_{n=1}^{N}[(n+\frac{1}{2})\ln(1+\frac{1}{n})-1]=\sum_{n=1}^{N}\ln\frac{(1+\frac{1}{n})^{(n+\frac{1}{2})}}{e}=\ln\prod_{n=1}^{N}\frac{(1+\frac{1}{n})^{(n+\frac{1}{2})}}{e}$$

$$\gamma_c=\gamma_{c,\infty}=\sum_{n=1}^{\infty}\delta_{1,n}=\lim_{N\to\infty}(\int_1^{N+1}\log(\mathrm{x})\,\mathrm{dx}-\sum_{n=1}^{N}\log(n)-\frac{\log(\mathrm{N+1})}{2})$$

$$=\sum_{n=1}^{\infty}[(n+\frac{1}{2})\ln(1+\frac{1}{n})-1]=\sum_{n=1}^{\infty}\ln\frac{(1+\frac{1}{n})^{(n+\frac{1}{2})}}{e}=\ln\prod_{n=1}^{\infty}\frac{(1+\frac{1}{n})^{(n+\frac{1}{2})}}{e}$$

$$\ln N!=\sum_{n=1}^{N}\ln n=\int_1^{N+1}\ln x dx-\sum_{n=1}^{N}\delta_{1,n}-\sum_{n=1}^{N}\delta_{2,n}=(x\ln x-x)\Big|_1^{N+1}-\ln e^{\gamma_{c,N}}-\sum_{n=1}^{N}\frac{\ln(n+1)-\ln n}{2}$$

$$=(N+1)\ln\frac{(N+1)}{e}+\ln\frac{e}{e^{\gamma_{c,N}}}-\frac{1}{2}\ln(N+1)=\ln[\frac{e^{1-\gamma_{c,N}}}{\sqrt{N+1}}(\frac{N+1}{e})^{(N+1)}]$$

$N!=\frac{e^{1-\gamma_{c,N}}}{\sqrt{N+1}}(\frac{N+1}{e})^{(N+1)}$, compared to Stirling formula : $N!\sim\sqrt{2\pi N}(\frac{N}{e})^N$

$(N+1)!=(N+1)N!\sim\sqrt{2\pi(N+1)}(\frac{N+1}{e})^{N+1}$, $N!\sim\frac{\sqrt{2\pi}}{\sqrt{N+1}}(\frac{N+1}{e})^{N+1}$

Compared to previous formula, gives $\sqrt{2\pi}\sim e^{1-\gamma_{c,N}}$ or $2\pi=(\frac{e}{e^{\gamma_c}})^2$

$2\pi-e$ formula(s): $2\pi=(\frac{e}{e^{\gamma_c}})^2=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots$, $(2\pi)_k=(\frac{e}{e^{\gamma_{c/k}}})^2=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}$

$\gamma_c=0.0810614668$, $e^{\gamma_c}=1.0844375$

2π-e formula is an expanding form of Stirling formula. To our knowledge, it was first deduced by us. If it was new, it could be named Chen's 2π-e formula.

3. Some Formulas Related to 2π-e Formula

The following formulas which correlate each other and has similar form could be called Chen's natural group formulas, and the form is called natural group.

$$1 = 4\gamma_c + \frac{4\gamma_1}{1(1+1)} + \frac{4\gamma_2}{2(2+1)} + \frac{4\gamma_3}{3(3+1)} + \cdots$$

$$= |B|\frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{|B_{2n}|(\pi/2)^{2n}}{(2n)!} = \sum_{n=1}^{\infty}\frac{|B_{2n}|\pi^{2n}}{(2n)!} = -|B|\frac{3\pi}{2} + \sum_{n=1}^{\infty}\frac{|B_{2n}|(3\pi/2)^{2n}}{(2n)!}$$

$$N \sim -\frac{3}{2}|B| + \sum_{n=1}^{\mathrm{N}}\frac{|B_{2n}|(2\pi)^{2n}}{2(2n)!}$$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$$

$$2\pi = (\frac{e}{e^{\gamma_c}})^2 = e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots$$

B, B_{2n}: the Bernoulli numbers such as $-\frac{1}{2},\ -\frac{1}{6},\ -\frac{1}{30},\ \frac{1}{42},\ -\frac{1}{30},\ \cdots$

$$\gamma_c = \lim_{N\to\infty}(\int_1^{N+1}\log(\mathrm{x})\,\mathrm{dx} - \sum_{n=1}^{N}\log(n) - \frac{\log(\mathrm{N}+1)}{2}) = 0.0810614668$$

$$\gamma_s = \lim_{N\to\infty}(\sum_{n=1}^{N-1}\frac{1}{n^s} - \int_1^N\frac{1}{x^s}\mathrm{dx}) - \frac{1}{2},\quad \mathrm{s}\in\mathrm{N}$$

$$\gamma_1 = 0.077215,\ \gamma_2 = 0.144934,\ \gamma_4 = 0.24899,\ \gamma_8 = 0.36122,\ \gamma_{16} = 0.433349,\ \cdots,\ \gamma_\infty = 0.5$$

$\gamma_c\ \gamma_1\ \gamma_2\ \gamma_3\cdots$ are called Chen's natural group constants (analogue to Bernoulli numbers).

The following are some other formulas related to 2π-e Formula.

$$\sqrt{2\pi} = e^{1-\gamma_c},\quad e = \sqrt{2\pi}e^{\gamma_c} = \sqrt{2\pi}(1 + \sum_{n=1}^{\infty}\frac{{\gamma_c}^n}{n!})$$

$$\gamma_c = \sum_{n=1}^{\infty}[(n+\frac{1}{2})\ln(1+\frac{1}{n}) - 1] = \sum_{n=1}^{\infty}\frac{(2^{2n}-1)|B_{2n}|\pi^{2n} - 2(2n)!}{2(2n+1)!} = \frac{1}{4} - \sum_{s=1}^{\infty}\frac{\gamma_s}{s(s+1)}$$

$$\gamma_g = \sum_{n=1}^{\infty}(n+\frac{1}{2})\ln(1+\frac{1}{n}) - \int_1^{\infty}(x+\frac{1}{2})\ln(1+\frac{1}{x})dx$$

$$\gamma_{cg} = \frac{1}{2}\lim_{N\to\infty}[\sum_{n=1}^{N}\frac{(2^{2n}-1)|B_{2n}|\pi^{2n}}{(2n+1)!} - \ln N]$$

$$\frac{\pi}{2} = (\frac{e}{e^{\gamma_g}})^2,\ e = \sqrt{\frac{\pi}{2}}e^{\gamma_g} = \sqrt{\frac{\pi}{2}}(1 + \sum_{n=1}^{\infty}\frac{{\gamma_g}^n}{n!});\quad \frac{\pi}{2} = (\frac{e^{\gamma/2}}{e^{\gamma_{cg}}})^2,\ \gamma = \ln\frac{\pi}{2} + 2\gamma_{cg}$$

$$\gamma_c = \gamma_g - \ln 2 = 1 - \frac{\gamma}{2} + \gamma_{cg} - \ln 2,\ \gamma_{cg} = \frac{1}{2} + \sum_{s=2}^{\infty}\frac{\gamma_s}{s(s+1)} - \ln 2$$

$$\gamma_c = 0.0810614668,\quad \gamma_g = 0.7742086474,\quad \gamma_{cg} = 0.0628164798$$

$$\frac{\pi}{2} = \sum_{n=1}^{\infty}\frac{|B_{2n}|\pi^{2n}}{2n(2n)!};\quad \sum_{n=1}^{\infty}[\zeta(2n) - 1] = \frac{3}{4},\quad \zeta(2n) = \sum_{k=1}^{\infty}\frac{1}{k^{2n}}$$

$$\sum_{n=1}^{\infty}\frac{1}{n} = \sum_{n=1}^{\infty/2}\frac{|B_{2n}|(2\pi)^{2n}}{2n(2n)!} = \sum_{n=1}^{\infty}\frac{|B_{2n}|(2n2^{2n}+1)\pi^{2n}}{2n(2n+1)!}$$

4. Some Applications of 2π-e Formula and its Related Formulas

(1). 2π-e formula is basically an algebraic expanding of Stirling formula, but it is more meaningful, it exhibits the relationship between 2π and e. In 2π-e formula, γ_c is a real constant with geometric definition like Euler-Mascheroni constant γ. With 2π-e formula and its related formulas, 2π can be calculated from e and vice versa. So it is the real 2π-e relationship formula.

$$2\pi = (\frac{e}{e^{\gamma_c}})^2 = e^2\prod_{n=1}^{\infty}\frac{e^2}{(1+\frac{1}{n})^{2n+1}} = e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots$$

$$e = \sqrt{2\pi}e^{\gamma_c} = \sqrt{2\pi}(1+\sum_{n=1}^{\infty}\frac{\gamma_c^{\ n}}{n!}),\quad \gamma_c = \sum_{n=1}^{\infty}\frac{(2^{2n}-1)|B_{2n}|\pi^{2n}-2(2n)!}{2(2n+1)!}$$

(2). 2π-e formula demonstrates 2π is a natural constant rather than π. π/2 is somewhat fundamental but not as complete as 2π. π is neither fundamental nor complete. In 2001 mathematician Bob Palais said "π is wrong"[5]. 2π-e formula and the Taylor expansion of e have similar form (natural group form), this should give a conclusive proof that 2π is a real natural constant and π is not.

$$2\pi = (\frac{e}{e^{\gamma_c}})^2 = e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots \Rightarrow 2\pi \text{ or } \sqrt{2\pi} \text{ is a natural constant}$$

$$\frac{\pi}{2} = (\frac{e}{e^{\gamma_g}})^2 = (\frac{e^{\gamma/2}}{e^{\gamma_{cg}}})^2 \Rightarrow \frac{\pi}{2} \text{ or } \sqrt{\frac{\pi}{2}} \text{ is almost a natural constant}$$

$$\pi = (\frac{e}{e^{\gamma_c}\sqrt{2}})^2 = (\frac{e\sqrt{2}}{e^{\gamma_g}})^2 \Rightarrow \pi \text{ or } \sqrt{\pi} \text{ is not a natural constant}$$

Table 1 lists some points of view of Piist who support π is a natural constant, Tauist who support 2π is a natural constant and this work which supports the later.

Table 1. Comparison of points of view of Piist, Tauist and this work

	Piist	Tauist	This work
Circumference of a circle	πd	2πR	2πR
Area of a circle	πR^2		(1/2)(2πR)R
Volume of sphere	(4/3) πR^3	(2/3)(2π)R^3	$(2\pi R^2/3)2R$
Volume of n-dimension sphere	$\frac{\pi^{n/2}}{\Gamma(n/2+1)}R^n$	$\frac{(2\pi)^{n/2}}{2^{n/2}\Gamma(n/2+1)}R^n$	$\frac{2\pi R^2}{n}V_{n-2}$
Euler's identity	$e^{i\pi}+1=0$	$e^{2\pi i}=1$	$e^{2\pi i}=1$
Gauss integral	$\int_{-\infty}^{+\infty}e^{-x^2}dx=\sqrt{\pi}$		$\int_{-\infty}^{+\infty}e^{-x^2}dx=\frac{e}{e^{\gamma_c}}\frac{1}{\sqrt{2}}$

(3). As 2π is a square number, the frequent appearing of its square root in some important equations such as Gaussian distribution (normal distribution) and Maxwell –Boltzmann distribution becomes resonable and understandable. And the distributions can be transformed as follows.

$$\text{Standard Normal Distribution: } f(x,0,1)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}=e^{-\frac{x^2+2(1-\gamma_c)}{2}}$$

$$\text{Maxwell – Boltzmann Distribution: } f(v)=\frac{2}{\sqrt{2\pi}}v^2(\frac{m}{kT})^{\frac{3}{2}}e^{-\frac{mv^2}{2kT}}=2(\frac{m}{kT})^{\frac{3}{2}}v^2e^{-(\frac{m}{kT}\frac{v^2}{2}+1-\gamma_c)}$$

(4). Euler's identity (Euler's equation) $e^{i\pi}+1=0$ is called God formula and the most beautiful formula in mathematics. However, as 2π is the real natural constant and π is not, $e^{2\pi i}=1$ should be more beautiful.

(5). $\gamma=\ln(2\pi)+\gamma_{cg}$ may help to prove γ is an irrational number or even a transcendental number.

(6). The natural group formulas help us to establish "Chen's Periodic Table of Elements and Natural Group Theory"[6] (2014-2017).

(7). The mathematic expression of chirality is ±2π. This concept is helpful for us to establish "Chirality and Poetry Model of Atomic Nuclei"[7] (2017/12-2018/3).

(8). Based on the above theories, Chen's theory of the fine-structure constant was deduced (2018/4-6)[8] and has been revised, modified and improved (2018/7-2020/1).

5. Original Inspiration for Formulas of the Fine-structure Constant

1. According to $\alpha=\frac{e^2}{4\pi\varepsilon_0\hbar c}=\frac{\lambda_e}{2\pi a_0}=\frac{2\pi r_e}{\lambda_e}\approx\frac{1}{137.036}$, the formulas of α should relate to 2π.

2. $\frac{137.036}{2\pi}=\frac{137.036}{6.28318}=21.81$, $137.036=21.81\times 2\pi$

3. According to 2π-e formula: $2\pi=(\frac{e}{e^{\gamma_c}})^2=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\frac{e^2}{(\frac{4}{3})^7}\cdots$

2π is a square number, suppose $21.81=x^2$, $x=4.670\approx 14/3$

so: $\frac{1}{\alpha}\approx(\frac{14}{3})^2 2\pi$ or $\alpha\approx(\frac{3}{14})^2\frac{1}{2\pi}$ (Discover: about 2 am on 2018/4/12)

4. Apply with 2π-e formula (in the afternoon of 2018/4/12, a meeting in the morning)

$$\alpha=(\frac{3}{14})^2\frac{1}{(2\pi)_{112}}=(\frac{3}{14})^2\frac{1}{e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{113}{112})^{225}}}=137.035781520\text{, closest to the real value.}$$

As 112 is one of the most important stable numbers and the 112th element $^{285}_{112}Cn^*_{173}$ is the natural end of elements according to our Chen's Chirality and Poetry Model of Atomic Nuclei[6].

So: *Eureka*! Subsequently transformed to: $\alpha=\frac{6^2}{7(2\pi)_{112}}\frac{1}{112}=137.035781520$,

Finally modified to: $\alpha=\frac{6^2}{7(2\pi)_{112}}\frac{1}{112+\frac{1}{75^2}}=137.035999037435$

6. Logical Deduction of Chen's Formulas of the Fine-structure Constant

Physicist Richard Feynman noticed a hydrogen-like atom with Z protons and only one electron, according to Bohr model, the line velocity of the nth rank electron $v_{e/Z/n}$ satisfies:

$$\frac{v_{e/z/n}}{c}=\frac{Ze^2}{n4\pi\varepsilon_0\hbar c}=\frac{Z}{n}\alpha,\ \text{as } v_{e/z/n}\le c,\ \ \alpha=\frac{v_{e/z/n}}{c}\frac{n}{Z}\approx\frac{1}{Z_{\max-\text{ideal}}}=\frac{1}{Fy}=\frac{1}{137}$$

The 137th hydrogen-like element Fy (Feynmanium) is an ideal (imaginative) element,

in reality, the above formula should be modified to: $\alpha=f(\mathrm{Z}_{real})\dfrac{1}{Z_{\max-\text{real}}}$

According to Chen's Chirality and Peotry Model of Atomic Nuclei[6],

$Z_{\max-\text{real}}=112=2\cdot 56$, so $\alpha=f(\mathrm{Z}_{real})\dfrac{1}{Z_{\max-\text{real}}}=f(\mathrm{Z}_{real})\dfrac{1}{112}$

Compared to $\alpha=\dfrac{\lambda_e}{2\pi a_0}$, the formula should have a 2π factor:

$$\alpha=f(\mathrm{Z}_{real})\frac{1}{Z_{\max-\text{real}}}=\frac{n}{\mathrm{m}(2\pi)}\frac{1}{Z_{\max-\text{real}}}=\frac{6^2}{7\cdot(2\pi)}\frac{1}{112}=1/136.8$$

Apply with 2π-e formula: $2\pi=e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\dfrac{e^2}{(\frac{4}{3})^7}\cdots$

the formula is transformed to:

$$\alpha=\frac{n}{\mathrm{m}(2\pi)_k}\frac{1}{Z_{\max-\text{real}}}=\frac{6^2}{7\cdot(2\pi)_{112}}\frac{1}{112}=\frac{6^2}{7\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{113}{112})^{225}}}\frac{1}{112}=1/137.035782$$

Above deduction on 2018/4/12, only $(2\pi)_{112}$ gives the closest value to α,
this coincidence of one part per infinity proves the formula itself is correct.

Added an calibration factor (δ=1/75^2) on 2018/4/20, the accurate formula is:

$$\alpha_1=\frac{\lambda_e}{2\pi a_0}=\frac{6^2}{7\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{113}{112})^{225}}}\frac{1}{112+\dfrac{1}{75^2}}=1/137.035999037435$$

Discover: 2018/4/12; Revise: 2018/4/20 (add 1/75^2 factor)

By the same procedure but compared to $\alpha=\dfrac{2\pi r_e}{\lambda_e}$, the other formula is:

$$\alpha_2=\frac{2\pi r_e}{\lambda_e}=\frac{\mathrm{m}(2\pi)_k}{n}\frac{1}{Z_{\max-\text{real}}}=\frac{13\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{279}{278})^{557}}}{10^2}\frac{1}{112-\dfrac{1}{3\cdot 29\cdot 64}}=1/137.035999111818$$

Discover: 2018/4/24; Revise: 2018/9/18-20 ($280\to 278$, $-\dfrac{1}{39^2}+\dfrac{1}{780^2}\to-\dfrac{1}{3\cdot 29\cdot 64}$)

Another amazing coincidence is 6^2 and 10^2 are square numbers in accordance with $2\pi=(\dfrac{e}{e^{\gamma_c}})^2$

This also demonstrates that α has two values with two kinds of formulas.

As $f(\mathrm{Z}_{real})=\dfrac{n}{\mathrm{m}(2\pi)_k}$ or $f(\mathrm{Z}_{real})=\dfrac{\mathrm{m}(2\pi)_k}{n}$, m n k δ should relate to nucleon numbers of nuclides.

7. The Two Most Important Formulas

The above two formulas for α_1 and α_2 were our first gained formulas and are the most important formulas among their serial formulas which will be given followed in this paper. Calculation to give the values of α_1 and α_2 is shown in **Fig. 3** and **Table 2**.

Fig. 3. Calculation diagram of α_1 and α_2 (2018/4-6)

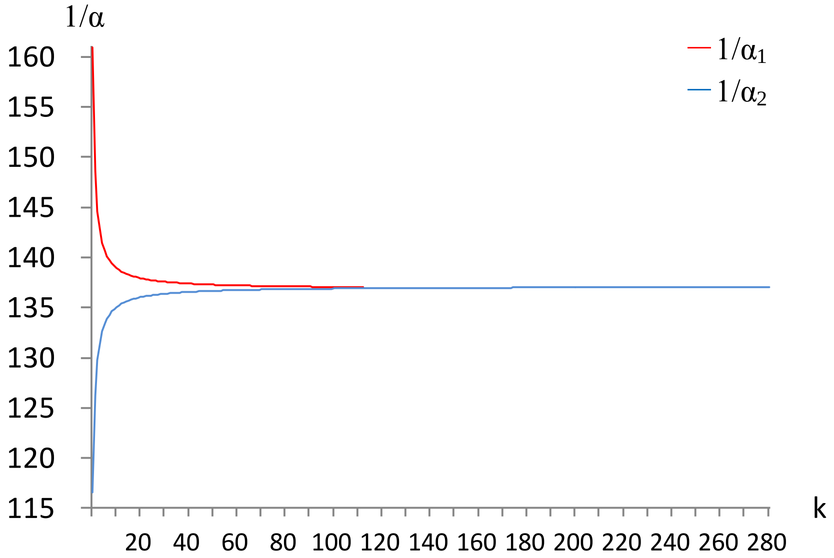


Table 2. Calculation of α_1 and α_2 (2018/4-6)

k	$(2\pi)_k$	$1/\alpha_1$	k	$(2\pi)_k$	$1/\alpha_2$
	7.389056099	160.917477134		7.389056099	116.596364743
1	6.824768754	148.628533230	1	6.824768754	126.236816375
2	6.640803185	144.622165589	2	6.640803185	129.733867427
3	6.549956514	142.643723845	3	6.549956514	131.533251879
4	6.49586908	141.465817857	4	6.49586908	132.628454999
5	6.46000004	140.684668634	5	6.46000004	133.364872233
6	6.434476503	140.128821836	6	6.434476503	133.893888578
7	6.415388754	139.713132398	7	6.415388754	134.292263980
8	6.400576029	139.390543654	8	6.400576029	134.603053878
9	6.388747203	139.132937708	9	6.388747203	134.852272701
10	6.379083388	138.922480953	10	6.379083388	135.056563407
14	6.353377324	138.362659116	14	6.353377324	135.603008624
28	6.319398093	137.622665802	28	6.319398093	136.332142298
56	6.301583891	137.234711452	56	6.301583891	136.717545138
110	6.29262658	137.039640822	112	6.292459356	136.915795771
111	6.292542221	137.037803660	224	6.28784124	137.016353814
112	**6.292459356**	**137.035999037435**	276	6.286966940	137.035408057
113	6.292377945	137.034226098	277	6.286953333	137.035704647
114	6.292297952	137.032484014	**278**	**6.286939823**	**137.035999111818**
			279	6.286926410	137.036291474
			280	6.286913093	137.036581756

In these two formulas (deduced from the modification of Z_{max}), there are some factors which are essentially related to nucleon numbers of some nuclides especially some important stable numbers (stipulated by Chen's Chirality and Poetry Model of Atomic Nuclei[7]) such as 28, 42, 56, 83, 84, 112, 126, 166, 167, 168 *et al.* And these numbers correlate with each others. This kind of relationship is shown in the follows.

A brief illustration of the relationships between the fine-structure constant and nulides:

${}^{136,137,138}_{56}\mathrm{Ba}_{80,81,82}$ ${}^{166,167,168}_{68}Er_{98,99,100}$ ${}^{169}_{69}Tm_{100}$ ${}^{209}_{83}Po^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$

Above nuclides indicate that 136 – 138, which can be called the fine-structure constant numbers, definitely relate to 112 and 166 – 168 (double of 56 and 83 – 84, the most stable numbers in nuclides).

$$\alpha_1=\frac{6^2}{7\cdot(2\pi)_{112}}\frac{1}{112+\frac{1}{75^2}}=\frac{6^2}{7\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{113}{112})^{225}}}\frac{1}{112+\frac{1}{75^2}}=1/137.035999037435$$

Relations to nuclides $(7\cdot(2\pi)_{112}\approx 44;\ {}^{nucleon}_{proton}X_{nutron})$: ${}^{7}_{3}Li_4$ ${}^{9}_{4}Be_5$ ${}^{11}_{5}B_6$ ${}^{12}_{6}C_6$ ${}^{14}_{7}N_7$ ${}^{19}_{9}F_{10}$ ${}^{24}_{12}Mg_{12}$ ${}^{28}_{14}Si_{14}$ ${}^{52}_{24}Cr_{28}$ ${}^{55}_{25}Mg_{30}$ ${}^{64}_{28}Ni_{36}$ ${}^{75}_{33}As_{42}$ ${}^{84}_{36}Kr_{48}$ ${}^{96,98}_{42}Mo_{54,56}$ ${}^{100}_{44}Ru_{56}$ ${}^{88}_{38}Sr_{50}$ ${}^{112}_{48}Cd_{64}$ ${}^{122}_{50}Sn_{72}$ ${}^{125}_{52}Te_{73}$ ${}^{129}_{54}Xe_{75}$ ${}^{136,137,138}_{56}\mathrm{Ba}_{80,81,82}$ ${}^{144}_{60}Nd_{84}$ ${}^{150}_{62}Sm_{88}$ ${}^{180}_{72}Hf_{108}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$

$$\alpha_2=\frac{13\cdot(2\pi)_{278}}{10^2}\frac{1}{112-\frac{1}{3\cdot 29\cdot 64}}=\frac{13\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{9\cdot 31}{278})^{557}}}{10^2}\frac{1}{112-\frac{1}{3\cdot 29\cdot 64}}=1/137.035999111818$$

Relations to nuclides $(13\cdot(2\pi)_{278}\approx 82;\ {}^{nucleon}_{proton}\mathrm{X}_{nutron})$: ${}^{20}_{10}Ne_{10}$ ${}^{27}_{13}\mathrm{Al}_{14}$ ${}^{29}_{14}\mathrm{Si}_{15}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57,58}_{26}\mathrm{Fe}_{28,30,31,32}$ ${}^{58}_{28}Ni_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{93}_{41}Nb_{52}$ ${}^{100}_{44}Ru_{56}$ ${}^{112,116}_{48}Cd_{64,68}$ ${}^{116}_{50}Sn_{66}$ ${}^{128}_{52}Te_{76}$ ${}^{136,137,38}_{56}\mathrm{Ba}_{80,81,82}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$ ${}^{149}_{62}Sm_{87}$ ${}^{156,157,160}_{64}Gd_{92,93,96}$ ${}^{166-168}_{68}Er_{98-100}$ ${}^{169}_{69}Tm_{100}$ ${}^{192}_{76}Os_{116}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{237}_{93}Np^{*}_{144}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{278+7}_{112}Cn^{*}_{166+7}$

The value of the front part of each above formula is almost equal to $1/(3/2)^{1/2}$ (because 112 is the element natural proton end and 168 is the element natural neutron end as shown in ${}_{112}\mathrm{Cn}_{168+5}$), so the formulas can be transformed to the follows.

$$\alpha_1=\alpha_{1-(3/2)}=\frac{1}{(\frac{3}{2}-\frac{1}{3\cdot 112+1}+\frac{1}{2^2\cdot 3\cdot 5^3\cdot 13\cdot 23-\frac{30}{64}})^{\frac{1}{2}}}\frac{1}{112+\frac{1}{75^2}}=1/137.035999037435$$

2019 / 4 / 25　　Relations to nuclides：

${}^{27}_{13}Al_{14}$ ${}^{27}_{15}P_{16}$ ${}^{40-48}_{20}Ca_{20-28}$ ${}^{51}_{23}V_{28}$ ${}^{55}_{25}Mn_{30}$ ${}^{56}_{26}Fe_{30}$ ${}^{64}_{30}Zn_{34}$ ${}^{72,74,76}_{32}Ge_{40,42,44}$ ${}^{89}_{39}Y_{50}$ ${}^{100}_{44}Ru_{56}$ ${}^{106,110}_{46}Pd_{60,64}$ ${}^{112}_{48}Cd_{64}$ ${}^{118,119}_{50}Sn_{68,69}$ ${}^{124-130}_{52}Te_{72-78}$ ${}^{155-160}_{64}Gd_{91-96}$ ${}^{169}_{69}Tm_{100}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{207}_{82}Pb_{125}$ ${}^{209}_{84}Po^{*}_{125}$

$$\alpha_2=\alpha_{2-(3/2)}=\frac{1}{(\frac{3}{2}-\frac{1}{3\cdot 112+1}+\frac{1}{2\cdot 7\cdot 11\cdot 19\cdot 29+\frac{36}{75^2}})^{\frac{1}{2}}}\frac{1}{112-\frac{1}{3\cdot 29\cdot 64}}=1/137.035999111818$$

2019 / 4 / 25　　Relations to nuclides:

${}^{28}_{14}Si_{14}$ ${}^{50}_{22}Ti_{28}$ ${}^{58}_{28}Ni_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{88}_{38}Ni_{50}$ ${}^{100,102}_{44}Ru_{56,58}$ ${}^{112,116}_{48}Cd_{64,68}$ ${}^{116,118,119}_{50}Sn_{66,68,69}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$ ${}^{180}_{72}Hf_{108}$ ${}^{187}_{75}\mathrm{Re}_{112}$ ${}^{188,192}_{76}Os_{112,116}$ ${}^{193}_{77}\mathrm{Ir}_{116}$ ${}^{194}_{78}\mathrm{Pt}_{116}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{223}_{87}Fr^{*}_{136}$

8. The Integrated Fine-structure Constant

Multiplication of α_1 and α_2 should almost divide out the 2π factors and give 3/2 and 112×112 factors, this means $\alpha_1\alpha_2$ is almost equal to 112×168, so we define $\alpha_c=(\alpha_1\alpha_2)^{1/2}$ as the integrated fine-structure constant or Chen's fine-structure constant.

$$\frac{1}{\alpha_c{}^2}=\frac{1}{\alpha_1\alpha_2}=\frac{2\pi a_0}{\lambda_e}\frac{\lambda_e}{2\pi r_e}=\frac{a_0}{r_e}=(\frac{c}{v_e})^2$$

$$=112\times(168-\frac{1}{3}+\frac{1}{12\cdot 47}-\frac{1}{6\cdot 29\cdot 53\cdot 59-79/47})\quad 2018/6/8\text{-}9,\ 9/18\text{-}19,\ 2019/4/19$$

$$=136(138+\frac{1}{2}-\frac{1}{10\cdot 29}+\frac{1}{12\cdot 53\cdot(6\cdot 53-1)-27/47})\quad 2019/4/17\text{-}19$$

$$=137(137+\frac{1}{13}-\frac{1}{7\cdot 29}+\frac{1}{32\cdot 33\cdot 89+16/49})\quad 2019/4/17\text{-}19$$

$$=112\cdot 167.668437878408=18778.865042381$$

$${}^{27}_{13}\mathrm{Al}_{14}\ {}^{29}_{14}Si_{15}\ {}^{47,49}_{22}Ti_{25,27}\ {}^{53}_{24}\mathrm{Cr}_{29}\ {}^{54,56,58}_{26}\mathrm{Fe}_{28,30,32}\ {}^{59}_{27}\mathrm{Co}_{32}\ {}^{58,60,61}_{28}\mathrm{Ni}_{30,32,33}\ {}^{63,65}_{29}\mathrm{Cu}_{34,36}\ {}^{79}_{35}\mathrm{Br}_{44}\ {}^{87}_{38}\mathrm{Sr}_{49}$$

$${}^{100,102}_{44}\mathrm{Ru}_{56,58}\ {}^{112}_{48}Cd_{64}\ {}^{113}_{49}In_{64}\ {}^{135-138}_{56}\mathrm{Ba}_{79-82}\ {}^{136,138}_{58}Ce_{78,80}\ {}^{3\cdot 47}_{59}\mathrm{Pr}_{82}\ {}^{158,160}_{64}\mathrm{Gd}_{94,96}\ {}^{159}_{65}Tb_{94}\ {}^{166,168}_{68}Er_{98,100}$$

$${}^{174}_{70}Yb_{104}\ {}^{188}_{76}Os_{112}\ {}^{197}_{79}Au_{118}\ {}^{203}_{81}\mathrm{Tl}_{122}\ {}^{223}_{87}Fr^{*}_{136}\ {}^{226}_{88}Ra^{*}_{138}\ {}^{227}_{89}Ar^{*}_{138}\ {}^{262}_{103}Lr^{*}_{159}\ {}^{285}_{112}Cn^{*}_{173}\ {}^{293}_{116}Lv^{ie}_{177}$$

$$\alpha_c{}^2=\alpha_1\alpha_2=[\frac{6^2}{7\cdot(2\pi)_{112}}\frac{1}{112+\frac{1}{75^2}}][\frac{13\cdot(2\pi)_{278}}{10^2}\frac{1}{112-\frac{1}{3\cdot 29\cdot 64}}]$$

$$=\frac{13\cdot 3^2}{7\cdot 5^2}\frac{e^2}{(\frac{2\cdot 3\cdot 19}{113})^{227}}\frac{e^2}{(\frac{115}{114})^{229}}\cdots\frac{e^2}{(\frac{9\cdot 31}{2\cdot 139})^{557}}\frac{1}{112^2-\frac{1}{30^2\cdot 5}+\frac{1}{60^2\cdot 15}-\frac{1}{120^2\cdot 15\cdot 29}}$$

$$=1/18778.865042381\quad 2019/12/14$$

$${}^{27}_{13}Al_{14}\ {}^{31}_{15}P_{16}\ {}^{39}_{19}K_{20}\ {}^{55}_{25}Mn_{30}\ {}^{54,56,57,58}_{26}Fe_{28,30,31,32}\ {}^{63,65}_{29}\mathrm{Cu}_{34,36}\ {}^{69,71}_{31}Ga_{38,40}\ {}^{79,81}_{35}Br_{44,46}\ {}^{87}_{38}Sm_{49}$$

$${}^{89}_{39}Y_{50}\ {}^{93}_{41}Nb_{52}\ {}^{112-120-124}_{50}Sn_{62-70-74}\ {}^{135-138}_{56}Ba_{79-82}\ {}^{139}_{57}La_{82}\ {}^{136,138}_{58}Ce_{78,80}\ {}^{144,145}_{60}Nd_{83,84}\ {}^{157}_{64}Gd_{93}$$

$${}^{200}_{80}Hg_{120}\ {}^{209}_{83}Bi^{*}_{126}\ {}^{223}_{87}Fr^{*}_{136}\ {}^{237}_{93}Np^{*}_{144}\ {}^{278+7}_{112}Cn^{*}_{166+7}\ {}^{284}_{113}Nh^{ie}_{171=9\cdot 19}$$

$$\alpha_c{}^2=\alpha_1\alpha_2=\frac{1}{(\frac{3}{2}-\frac{1}{3\cdot 112+1}+\frac{1}{7\cdot 19\cdot 29\cdot 37-\frac{25}{44}})}\frac{1}{112+\frac{1}{75^2}}\frac{1}{112-\frac{1}{3\cdot 29\cdot 64}}$$

$$=1/18778.865042381\quad 2019/12/14$$

$${}^{39}_{19}K_{20}\ {}^{47,50}_{22}Ti_{25,28}\ {}^{55}_{25}Mn_{30}\ {}^{63,65}_{29}\mathrm{Cu}_{34,36}\ {}^{85,87}_{37}Rb_{48,50}\ {}^{87,88}_{38}Sr_{49,50}\ {}^{99,100,102,104}_{44}Ru_{55,56,58,60}\ {}^{112}_{48}Cd_{64}$$

$${}^{112,114,115,116,120,124}_{50}Sn_{62,64,65,66,70,74}\ {}^{5\cdot 37,11\cdot 17}_{75}\mathrm{Re}_{110,112}\ {}^{223}_{87}Fr^{*}_{136}\ {}^{226}_{88}Ra^{*}_{138}$$

9. A Brief Explanation of the Fine-structure Constant

According to Chen's Chirality and Poetry Model of Atomic Nuclei[7], the ratio of neutron number N to proton number Z in nuclides increases from 1/1 to 3/2 (eventually slightly above 3/2) along with the increasing of atomic number, for example, from ${}_{14}\mathrm{Si}_{14}$, ${}_{26}\mathrm{Fe}_{30}$, ${}_{29}\mathrm{Cu}_{34,36}$, ${}_{56}\mathrm{Ba}_{82}$, ${}_{84}\mathrm{Po}_{125}$ to ${}_{112}\mathrm{Cn}^{*}_{168+5}$. In this process, $(3/2)^{1/2}$ will act as a transition foothold. As for nuclide ${}_{112}\mathrm{Cn}_{168+5}$ with Z=112, N=168+5 and 168/112=3/2, 137 is just right their $(3/2)^{1/2}$ times intermediate stage. This should be why 137 exists and what's the real meaning of 137.

$$\frac{112}{1/\alpha_1} \approx \frac{1/\alpha_2}{168-1/3} \quad \text{or} \quad \frac{112}{137.036} \approx \frac{137.036}{168-1/3}$$

$$137.036^2 \approx 112 \cdot (168-1/3)$$

$$112 \cdot (\frac{3}{2} - \frac{1}{336+1})^{\frac{1}{2}} \approx 137.036, \; 137.036 \cdot (\frac{3}{2} - \frac{1}{336+1})^{\frac{1}{2}} \approx 168-1/3$$

$$\frac{112}{1/\alpha_1} \approx \frac{1/\alpha_2}{168} \quad \text{or} \quad \frac{112}{137} \approx \frac{137}{168}$$

$$137^2 \approx 112 \cdot 168, \; 112(\frac{3}{2})^{\frac{1}{2}} \approx 137, \; 137(\frac{3}{2})^{\frac{1}{2}} \approx 168$$

The relationships between the fine-structure constant and elements are mainly reflected by correlation of nucleon numbers of 56, 68-69, 82, 82-84, 112, 136-138, and 166-168 which are derived from the three key numbers 112, 137 and 168, and by correlation of nucleon numbers of the other factors in Chen's formulas of the fine-structure constant. The former type is illustrated as follows.

$^{56}_{26}Fe_{30}$ $^{63,65}_{29}Cu_{34,36}$ $^{68}_{30}Zn_{38}$ $^{69}_{31}Ga_{38}$ $^{75}_{33}As_{42}$ $^{82,83,84}_{36}Kr_{46,47,48}$ $^{98}_{42}Mo_{56}$ $^{110-112-114,116}_{48}Gd_{62-66,68}$ $^{118,119}_{50}Sn_{68,69}$ $^{125,126}_{52}Te_{73,74}$

$^{136,137,138}_{56}Ba_{80,81,82}$ $^{142,143,144}_{60}Nd_{82,83,84}$ $^{148-150,152,154}_{62}Sm_{66-68,90,92}$ $^{151,153}_{63}Eu_{88,90}$ $^{166,167,168}_{68}Er_{98,99,100}$ $^{169}_{69}Tm_{100}$ $^{185,187}_{75}Re_{110,112}$

$^{186-188-190}_{76}Os_{110-112-114}$ $^{208}_{82}Pb_{126}$ $^{209}_{83}Bi^*_{126}$ $^{209}_{84}Po^*_{125}$ $^{210}_{85}At^*_{125}$ $^{222}_{86}Rn^*_{136}$ $^{223}_{87}Fr^*_{136}$ $^{226}_{88}Ra^*_{138}$ $^{227}_{89}Ac^*_{138}$ $^{232}_{90}Th^*_{142}$ $^{231}_{91}Pa^*_{140}$ $^{238}_{92}U^*_{146}$ $^{237}_{93}Np^*_{144}$

$^{244}_{94}Pu^*_{150}$ $^{243}_{95}Am^*_{148}$ $^{247}_{96}Cm^*_{151}$ $^{251}_{98}Cf^*_{153}$ $^{252}_{99}Es^*_{153}$ $^{257}_{100}Fm^*_{157}$ $^{258}_{101}Md^*_{157}$ $^{259}_{102}No^*_{157}$ $^{268}_{105}Db^*_{163}$ $^{269}_{106}Sg^*_{163}$ $^{270}_{107}Bh^*_{163}$ $^{277}_{108}Hs^*_{169}$ $^{285}_{112}Cn^*_{173}$ $^{294}_{118}Og^*_{176}$

$^{312,314}_{125,126}Ch^{ie}_{187,188}$ $^{344-348}_{136-138}Fy^{ie}_{208-210}$ $^{366-376}_{144-149}Ch^{ie}_{222-227}$ $^{384,386,392,394}_{153,154,155,156}Ch^{ie}_{231,232,237,238}$ $^{400,402}_{157,158}Ch^{ie}_{243,244}$ $^{410}_{163}Ch^{ie}_{247}$ $^{416-420}_{166,168}Ch^{ie}_{250/252}$ $^{426}_{169}Ch^{ie}_{257}$

Several clusters of ideal extended elements (*ie*) Fy and Ch are hence predicted.

10. Comparison to Experiment Determined Values

The above two calculated values of the fine-structure constant, i.e., α_1=1/137.035999037435 and α_2=1/137.035999111818 are consistent with those experiment measured values[2], but much more accurate with several more digits.

The above theoretical analysis and formulas also demonstrate there are two different values of the fine-structure constant, i.e. α_1 and α_2. Accordingly, we have found that up to now the experiment determinations of α have almost proved this because the α ranges measured by two different but accurate methods couldn't overlap each other [2]. It seems that the time comes to a critical point to prove there are two values of the fine-structure constant theoretically and experimentally.

11. Theoretical Calculation of the Speed of Light

In atomic units, the line velocity of the ground state electron in hydrogen atom can be assigned as the natural unit of speed ($v_{e/au}$=1), then the speed of light becomes the reciprocal of the fine-structure constant, i.e., $c_{au}=1/\alpha$=137.035999. However, we have demonstrated that there are two values of α, but the speed of light shouldn't have two values, so by referring to Maxwell's formula of calculating the speed of electromagnetic wave or light, it should be reasonable to suppose the speed of light to be the integrated fine-structure constant, i.e., $c_{au}=1/\alpha_c= 1/(\alpha_1\alpha_2)^{1/2}$=137.035999074627. It means we've theoretically/mathematically calculated the speed of light, the formula is intrinsically consistent with Maxwell's formula, and the value is much accurate.

In atomic units ($e=m_e=\hbar=1$ and $\varepsilon_0=\frac{1}{4\pi}$), as $\frac{c}{v_e}=\frac{1}{\alpha}$, $v_{e/au}=\frac{e^2}{4\pi\varepsilon_0 m_e c^2}=1$

As there shouldn't be two c, so it should be: $c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_1\alpha_2}}$ (*au*: atomic units)

Compared to Maxwell Formula $c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}$, $c_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_1\alpha_2}}$ should be reasonable.

$$\mathrm{c}_{au}=\frac{1}{\sqrt{\mu_{0/au}\varepsilon_{0/au}}},\ \mu_{0/au}\varepsilon_{0/au}=\alpha_1\alpha_2,\ \mu_{0/au}=4\pi\alpha_1\alpha_2 \qquad (2019/11/30)$$

So the theroretical formula of the speed of light in atomic units is as follows:

$$\mathrm{c}_{au}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_1\alpha_2}}=\frac{1}{\sqrt{\left(\frac{6^2}{7\cdot(2\pi)_{112}}\frac{1}{112+\frac{1}{75^2}}\right)\left(\frac{13\cdot(2\pi)_{278}}{10^2}\frac{1}{112-\frac{1}{3\cdot29\cdot64}}\right)}}$$

$$=\sqrt{\frac{7\cdot5^2}{13\cdot3^2}\frac{(2\pi)_{112}}{(2\pi)_{278}}\left(112^2-\frac{1}{30^2\cdot5}+\frac{1}{60^2\cdot15}-\frac{1}{120^2\cdot15\cdot29}\right)}$$

$$=\sqrt{\frac{\frac{5\cdot17}{11}-\frac{10}{11\cdot23}}{\frac{36}{7}-\frac{1}{7\cdot19}}\frac{(2\pi)_{12389}}{(2\pi)_{28186}}\left(112^2-\frac{2\cdot7^2\cdot43\cdot67+\frac{5}{7}}{5^2\times10^{10}}\right)}$$

$$=\sqrt{\left(\frac{2^3\cdot25\cdot17}{3^2\cdot11\cdot23}+\frac{2\cdot25\cdot17}{3^2\cdot11\cdot23\cdot97}\right)\frac{(2\pi)_{34450}}{(2\pi)_{28186}}\left(112^2-\frac{2^5\cdot3\cdot7\cdot13}{10^{10}}\right)}$$

$$=\sqrt{\left(\frac{3}{2}-\frac{1}{3\cdot112+1}+\frac{1}{7\cdot19\cdot29\cdot37-\frac{25}{44}}\right)\left(112^2-\frac{1}{30^2\cdot5}+\frac{1}{60^2\cdot15}-\frac{1}{120^2\cdot15\cdot29}\right)}$$

$$=\sqrt{\frac{3}{2}}\left(112-\frac{1}{3^2}+\frac{1}{12^2\cdot13-\frac{30\cdot19}{100}-\frac{1}{125\cdot100}}\right)=\sqrt{\frac{3}{2}-\frac{1}{3\cdot112+1}+\frac{1}{14\cdot53\cdot193-\frac{33}{2\cdot47}}}\times112$$

$$=\sqrt{112\times\left(168-\frac{1}{3}+\frac{1}{12\cdot47}-\frac{1}{6\cdot29\cdot53\cdot59-79/47}\right)}$$

$$=\sqrt{137.035999037435\times137.035999111818}=137.035999074627$$

Note: $112/278\approx27/67$, $12389/28186\approx11/25$, $34450/28186\approx11/9\approx66/29$

Discover: 2019/12/16; Revise and Supplement: 2020/1/5–8

12. The Special 29 and 75 Factors

In the above formulas some factors especially 29 and 75 appear several times. This feature should be analyzed and explained. Accompanying N/Z ratio from 1/1 to slightly above 3/2 along with the increasing of atomic number, $_{29}Cu_{34,36}$ is the critical point of N/Z ratio approaching $(3/2)^{1/2}$ and $_{75}Re_{110,112}$ is the critical point of N/Z ratio approaching 3/2 (**Table 3**, **Fig. 4** and **Fig. 5**), so 29 and 75 are important factors and hence frequently appear in the formulas.

Table 3. N/Z ratios of the Elements (2019/4/23)

	Z	N	N/Z		Z	N	N/Z		Z	N	N/Z		Z	N	N/Z
H	1	0	0	Ga	31	38.80	1.25	Pm	61	84	1.38	Pa*	91	140	1.54
He	2	2.00	1.00	Ge	32	40.71	1.27	Sm	62	88.45	1.43	U*	92	146	1.59
Li	3	3.92	1.31	As	33	42	1.27	Eu	63	89.04	1.41	Np*	93	144	1.55
Be	4	5	1.25	Se	34	45.05	1.33	Gd	64	93.33	1.46	Pu*	94	150	1.60
B	5	5.80	1.16	Br	35	44.98	1.29	Tb	65	94	1.45	Am*	95	148	1.56
C	**6**	**6.01**	**1.00**	Kr	36	47.89	1.33	Dy	66	96.57	1.46	Cm*	96	151	1.57
N	7	7.00	1.00	Rb	37	48.56	1.31	Ho	67	98	1.46	Bk*	97	150	1.55
O	8	8.00	1.00	Sr	38	49.71	1.31	Er	68	99.33	1.46	Cf*	98	153	1.56
F	9	10	1.11	Y	39	50	1.28	Tm	69	100	1.45	Es*	99	153	1.55
Ne	10	10.19	1.02	Zr	40	51.32	1.28	Yb	70	103.11	1.47	Fm*	100	157	1.57
Na	11	12	1.09	Nb	41	52	1.27	Lu	71	104.03	1.47	Md*	101	157	1.55
Mg	12	12.32	1.03	Mo	42	54.04	1.29	Hf	72	106.54	1.48	No*	102	157	1.54
Al	13	14	1.08	Td	43	55	1.28	Ta	73	108	1.48	Lr*	103	159	1.54
Si	14	14.11	1.01	Ru	44	57.16	1.30	W	74	109.89	1.49	Rf*	104	161	1.55
P	15	16	1.07	Rh	45	58	1.29	**Re**	**75**	**111.25**	**1.48**	Db*	105	163	1.55
S	16	16.09	1.01	Pd	46	60.51	1.32	Os	76	114.27	1.50	Sg*	106	165	1.56
Cl	17	18.48	1.09	Ag	47	60.96	1.30	Ir	77	115.25	1.50	Bh*	107	163	1.52
Ar	18	21.99	1.22	Cd	48	64.52	1.34	Pt	78	117.12	1.50	Hs*	108	169	1.56
K	19	20.13	1.06	In	49	65.91	1.35	Au	79	118	1.49	Mt*	109	167	1.53
Ca	20	20.12	1.01	Sn	50	68.81	1.38	Hg	80	120.62	1.51	Ds*	110	171	1.55
Sc	21	24	1.14	Sb	51	70.86	1.39	Tl	81	123.41	1.52	Rg*	111	169	1.52
Ti	22	25.92	1.18	Te	52	75.70	1.46	Pb	82	125.24	1.53	**Cn***	**112**	**173**	**1.54**
V	23	28	1.22	I	53	74	1.40	Bi*	83	126	1.52	Nh*	113	171	1.51
Cr	24	28.06	1.17	Xe	54	77.39	1.43	Po*	84	125	1.49	Fl*	114	175	1.54
Mn	25	30	1.20	Cs	55	78	1.42	At*	85	125	1.47	Mc*	115	173	1.50
Fe	26	29.91	1.15	Ba	56	81.42	1.45	Rn*	86	136	1.58	Lv*	116	177	1.53
Co	27	32	1.19	La	57	82	1.44	Fr*	87	136	1.56	Ts*	117	177	1.51
Ni	28	30.76	1.10	Ce	58	82.21	1.42	Ra*	88	138	1.57	Og*	118	176	1.49
Cu	**29**	**34.62**	**1.19**	Pr	59	82	1.39	Ac*	89	138	1.55				
Zn	30	35.45	1.18	Nd	60	84.41	1.41	Th*	90	142	1.58				

Z: atomic number, N: average neutron number or neutron number of the most stable isotope.

1. N/Z from 1/1 ($_{6}$C) to slightly above 3/2 (such as $_{112}$Cn which is the natural end of elements demonstrated by Chen's Chirality and Poetry Model of Atomic Nuclei[7]).
2. For $_{29}$Cu, N/Z ratio 1.19 is near to $(3/2)^{1/2}$=1.22，slightly less is because of stability effect.
3. For $_{75}$Re, N/Z ratio 1.48 is near to 3/2=1.50，slightly less is because of stability effect.
4. From $_{6}$C to $_{112}$Cn, the middle of N/Z 1.5 range is at (76.5-5)/(112-5)=0.668≈2/3 position.

Fig. 4 and **Fig. 5** shows that stability effect of nucleon number 64 makes the neutron numbers of $_{29}$Cu's isotopes are relatively less (34 and 36) than normal so that its N/Z ratio is a little less than $(3/2)^{1/2}$ which is otherwise it should be. Also the stability effect of nucleon numbers 110 and 112 make the neutron numbers of $_{75}$Re's nuclides are relatively less (110 and 112) than normal so that its N/Z ratio is a little less than 3/2 which otherwise it should be.

Fig. 4. Complete Graph of N/Z Ratios of Elements (2019/4/23-24)

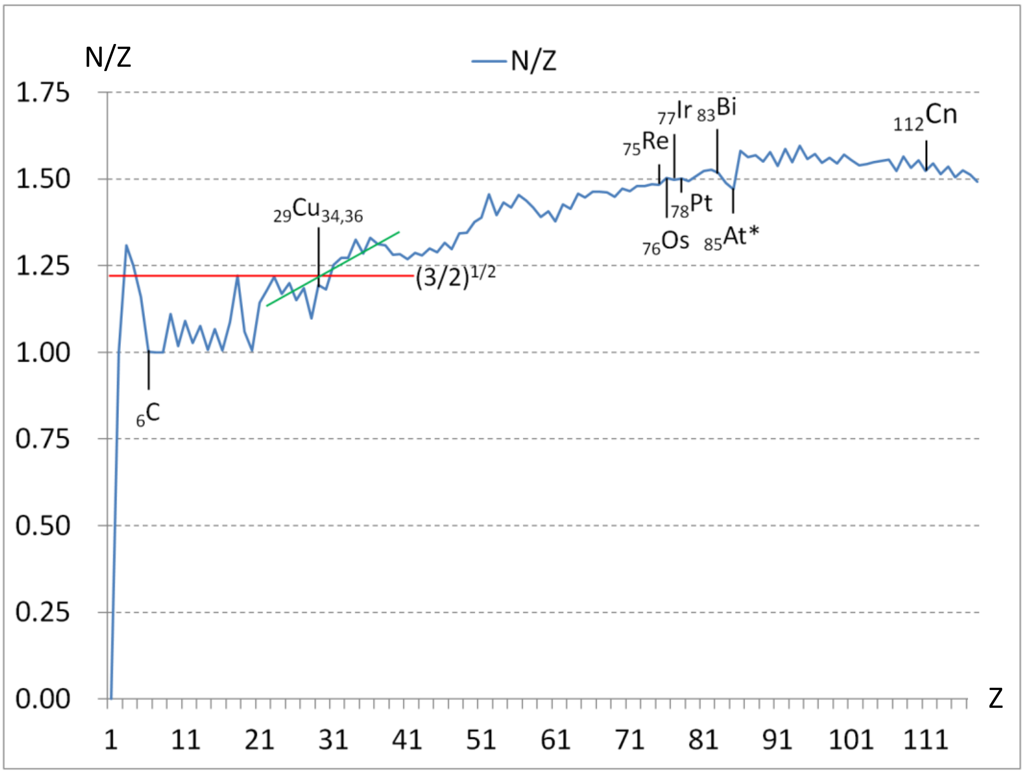
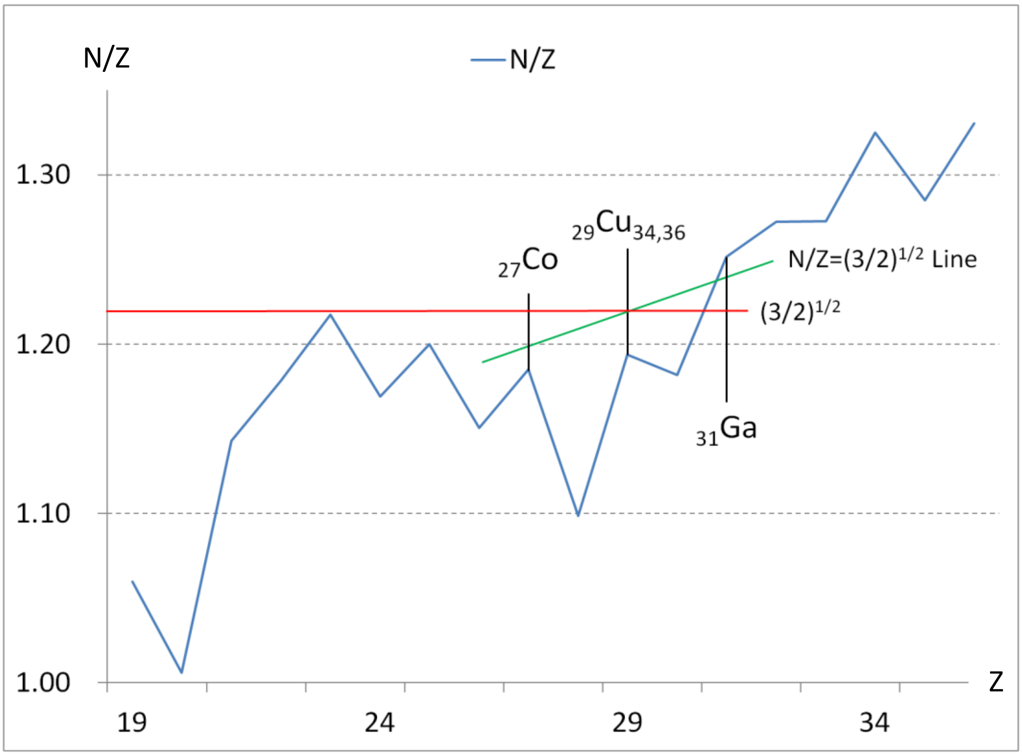


Fig. 5. Partially Amplified Graph of N/Z Ratios of Elements (2019/4/24)

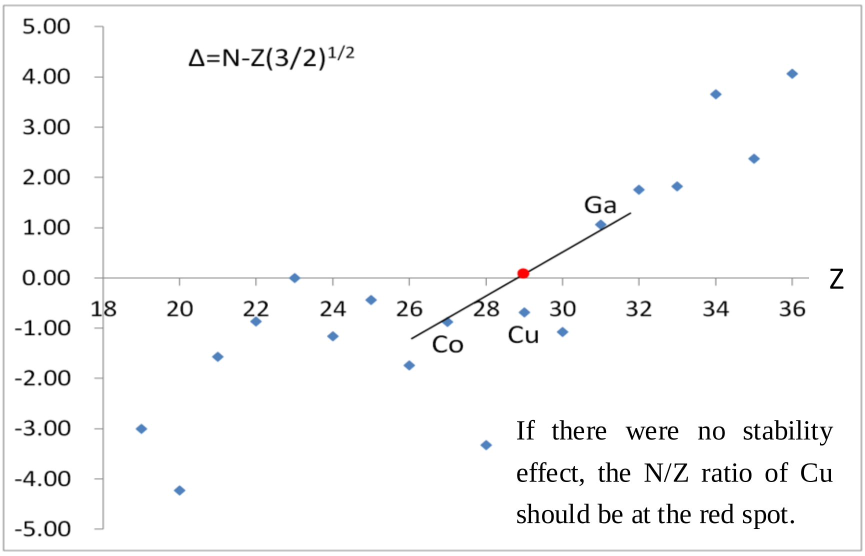


The general trend of N/Z ratio of elements is from 1/1 ($_{6}C_{6}$) to slightly above 3/2 ($_{112}Cn_{173}$) definitely. However, the increasing process is not smooth, the N/Z ratio rising fluctuates consecutively. According to Chen's Chirality and Poetry Model of Atomic Nuclei[7], there are some stable numbers (magic numbers) which can bring about this kind of fluctuation (**Table 4** and **Fig. 6**).

Table 4. Effect of Stable Numbers on N/Z ratio's fluctuation (2019/4/22)

Element	Z	N(Average)	$Z(3/2)^{1/2}$	$N-Z(3/2)^{1/2}$	Stable Number
K	19	20.13	23.27	-3.17	20
Ca	20	20.12	24.49	-4.41	20+20
Sc	21	24	25.72	-1.74	
Ti	22	25.92	26.94	-1.07	22+26=48
V	23	28.00	28.17	-0.23	28
Cr	24	28.06	29.39	-1.39	28
Mn	25	30	30.62	-0.68	
Fe	26	29.91	31.84	-1.99	26+30=56
Co	27	32.00	33.07	-1.14	
Ni	28	30.76	34.29	-3.60	28+30=58、28+32=60
Cu	**29**	**34.62**	**35.52**	**-0.97**	**64**
Zn	30	35.45	36.74	-1.36	30+34=64、30+36=66
Ga	31	38.80	37.97	0.75	
Ge	32	40.71	39.19	1.44	32+40=72
As	33	42.00	40.42	1.50	
Se	34	45.05	41.64	3.30	34+46=80
Br	35	44.98	42.87	2.03	
Kr	36	47.89	44.09	3.71	36+48=84

Fig. 6. Effect of Stable Numbers on N/Z ratio's fluctuation (2019/4/22-23)



13. α_1/α_2 in Schrödinger Equation of Hydrogen Atom

Stationary Schrodinger Equation $-\frac{\hbar^2}{2m}\nabla^2\psi+U\psi=E\psi$, applied to hydron atom:

$\nabla^2\psi+\frac{2m_e}{\hbar^2}(E+\frac{e^2}{4\pi\varepsilon_0 r})\psi=0,\ E=-\frac{m_e e^4}{2n^2(4\pi\varepsilon_0)^2\hbar^2}$, do substitution and simlification:

$\frac{2m_e}{\hbar^2}(\frac{m_e e^4}{2n^2(4\pi\varepsilon_0)^2\hbar^2}-\frac{e^2}{4\pi\varepsilon_0 r})\psi=\nabla^2\psi,\quad [\frac{1}{n^2}(\frac{m_e e^2}{4\pi\varepsilon_0\hbar^2})^2-\frac{2}{r}\frac{m_e e^2}{4\pi\varepsilon_0\hbar^2}]\psi=\nabla^2\psi,$

$[\frac{1}{n^2}(\frac{e^2}{4\pi\varepsilon_0\hbar c}\frac{m_e c}{\hbar})^2-\frac{2}{r}\frac{e^2}{4\pi\varepsilon_0\hbar c}\frac{m_e c}{\hbar}]\psi=\nabla^2\psi,$

As $\sqrt{\alpha_1\alpha_2}=\frac{v_e}{c}=\frac{e^2}{4\pi\varepsilon_0\hbar c}$, $\lambda_e=\frac{h}{m_e c}$ and $\alpha_1=\frac{\lambda_e}{2\pi a_0}$:

$[\frac{1}{n^2}(\sqrt{\alpha_1\alpha_2}\frac{2\pi}{\lambda_e})^2-\frac{2}{r}\sqrt{\alpha_1\alpha_2}\frac{2\pi}{\lambda_e}]\psi=\nabla^2\psi,$

$[\frac{1}{n^2(\lambda_e/2\pi/\sqrt{\alpha_1\alpha_2})^2}-\frac{2}{(\lambda_e/2\pi/\sqrt{\alpha_1\alpha_2})r}]\psi=\nabla^2\psi,$

$[\frac{1}{n^2 a_0{}^2(\alpha_1/\alpha_2)}-\frac{2}{a_0 r\sqrt{\alpha_1/\alpha_2}}]\psi=\nabla^2\psi$

As $\alpha_1/\alpha_2\approx1$, simplyfied to: $[\frac{1}{n^2 a_0{}^2}-\frac{2}{a_0 r}]\psi=\nabla^2\psi$ (factor 2 seems not beautiful)

In atomic units (au: $e=m_e=\hbar=1$ and $\varepsilon_0=\frac{1}{4\pi}$),

$a_{0/au}=\frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}=1,\ v_{e/au}=\frac{e^2}{4\pi\varepsilon_0\hbar}=1,\ c_{au}=\frac{v_{e/au}}{\alpha_c}=\frac{1}{\alpha_c}=\frac{1}{\sqrt{\alpha_1\alpha_2}}$

$[\frac{1}{n^2(\alpha_1/\alpha_2)}-\frac{2}{r_{au}\sqrt{\alpha_1/\alpha_2}}]\psi=\nabla_{au}{}^2\psi$, or $(\frac{c_{au}{}^2}{\alpha_1{}^2 n^2}-\frac{2c_{au}}{\alpha_1 r_{au}})\psi=\nabla_{au}{}^2\psi$

the above equation could be called Schrodinger-Chen eqution of hydarogen atom, the later form of the equation shows factor 2 is still reasonable and beautiful.

As $\alpha_1/\alpha_2\approx1$, simplyfied to: $[\frac{1}{n^2}-\frac{2}{r_{au}}]\psi=\nabla_{au}{}^2\psi$

Discover: 2018/4-6; Revise: 2019/12/13 (add au form)

$\alpha_1/\alpha_2=\frac{137.035999111818}{137.035999037435}=1.0000000005428=1+\frac{23\cdot59}{25\cdot10^{11}}=(1+\frac{23\cdot59}{50\cdot10^{11}})^2$

$\sqrt{\alpha_1/\alpha_2}=1+\frac{23\cdot59}{50\cdot10^{11}}=1.000000002714$

Relations to nuclides: $^{23}_{11}Na_{12}$ $^{50,51}_{23}V_{27,28}$ $^{55}_{25}Mn_{30}$ $^{99,100}_{44}\mathrm{Ru}_{55,56}$ $^{105}_{46}Pd_{59}$ $^{137}_{56}Ba_{81}$ $^{118+1}_{50}Sn_{69}$ $^{141}_{59}\mathrm{Pr}_{82}$ $^{169}_{69}Tm_{100}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{169}_{88}Ra^{*}_{137}$

2019/8/28–29

Solution of Schrödinger equation of hydrogen atom gives some quantum numbers such as n, l and m_l which determine the electron shell structure and the chemical properties of atoms. That means Schrödinger equation of hydrogen atom is the base of chemical periodicity of elements. On the other hand, from above analysis, we have already demonstrated the formulas of the fine-structure constant α are derived from Chen's Chirality and Poetry Model of Atomic Nuclei[7] and hence mainly connected to the stability of atomic nuclei. So, a question is whether and how α is connected to Schrödinger Equation of hydrogen atom. This question should reveal the connection of the theory of electron shell of atoms and the theory of nuclei of elements. The above deduction provides the answer. The fine-structure constant α relates to Schrödinger Equation of hydrogen atom in α_1/α_2 way which is subtle and negligible but could show the equation is really reasonable and beautiful.

14. The Two Kinds of General Formulas of the Fine-structure Constant

Based on the above two formulas of α_1 and α_2, it should be reasonable to assume there are two kinds of serial formulas of α_1 and α_2 which are listed in follows. Among these formulas, the above two first discovered formulas are the most fundamental and important. Some formulas both with a big m and an extra large k should be more important referring to the trend of the approximate values of α.

Approximate formulas:

$$\alpha_{1-m'} = \frac{n}{m\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}}\frac{1}{112} \approx 1/137.036$$

$$\alpha_{2-m'} = \frac{m\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}}{n}\frac{1}{112} \approx 1/137.036$$

Accurate Formulas:

$$\alpha_{1-m} = \frac{n}{m\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}}\frac{1}{112+\delta_1}$$

$$= 1/137.035999037435$$

$$\alpha_{2-m} = \frac{m\cdot e^2 \frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}}{n}\frac{1}{112-\delta_2}$$

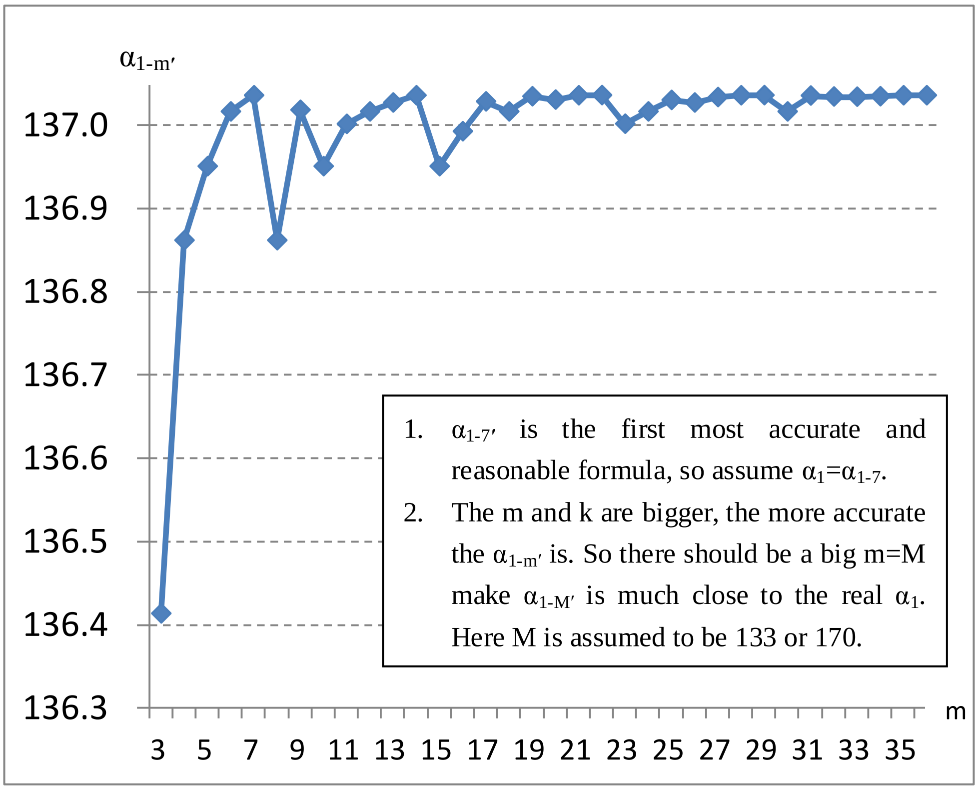
$$= 1/137.035999111818$$

Discover: 2019 / 6 / 27; Revise: 2019/7 / 2-3

Table 5. Parameters and Results of Approximate Formulas of α_1 (2019/7/2)

m	n	k	$\alpha_{1\text{-}m'}$	m	n	k	$\alpha_{1\text{-}m'}$
1	6	1	122.265854937	24	124	27	137.016359405
2	11	2	135.230901223	**25**	129	34	137.030171763
3	**16**	4	136.413250690	26	134	46	137.027100696
4	21	7	136.861626741	27	139	66	137.033636049
5	26	13	136.950569252	28	144	112	137.035781520
6	31	27	137.016359405	29	149	**321**	137.035917078
7	**36**	**112**	**137.035781520**	30	155	27	137.016359405
8	42	7	136.861626741	31	**160**	32	137.035453560
9	47	9	137.018237882	**32**	165	40	137.034309209
10	52	13	136.950569252	33	170	52	137.034083409
11	57	18	137.001388822	34	**175**	72	137.034617877
12	62	27	137.016359405	35	180	112	137.035781520
13	67	46	137.027100696	**36**	185	**236**	137.035810961
14	72	112	137.035781520	43	221	**200**	137.035845637
15	78	13	136.950569252	50	257	**181**	137.035307038
16	83	16	136.992590996	59	303	**2645**	137.035986189
17	**88**	20	137.028423583	**81**	**416**	**1605**	137.035992406
18	93	27	137.016359405	**96**	493	**5806**	137.035998789
19	**98**	37	137.034579883	103	**529**	**1310**	137.035994308
20	103	58	137.030572071	**133**	**683**	**12389**	**137.035999034**
21	108	112	137.035781520	140	719	**1923**	137.035994882
22	**113**	**782**	**137.035967638**	155	**796**	**3988**	137.035997989
23	119	22	137.001596764	**170**	**873**	**34450**	**137.035999031**

Fig. 7. Results of Approximate Formulas of α_1 (2019/7/2)



$$\alpha_{1-1}=\frac{6}{1\cdot e^{2}(\frac{2}{1})^{2}}\frac{1}{112+\frac{17}{2}-\frac{1}{40}+\frac{1}{6\cdot 23\cdot 25-\frac{36}{55}}}=1/137.035999037434$$

${}^{12}_{6}C_{6}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$

${}^{40,42,43,44,46,48}_{20}Ca_{20,22,23,24,26,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{74,78,80,82}_{34}Se_{40,44,46,48}$ ${}^{80,82,84,86}_{36}Kr_{44,46,48,50}$

${}^{90,91,96}_{40}Zr_{50,51,56}$ ${}^{102,106}_{46}Pd_{56,60}$ ${}^{116,118,119,120,122}_{50}Sn_{66,68,69,70,72}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{288=8\cdot 36}_{115=5\cdot 23}Mc^{ie}_{173}$

$$\alpha_{1-2}=\frac{11}{2\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}}\frac{1}{112+\frac{3}{2}-\frac{1}{200}+\frac{1}{5\cdot(3\cdot 42+1)\cdot(6\cdot 37-1)+\frac{2}{7}}}=1/137.035999037435$$

${}^{10,11}_{5}B_{5,6}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{79}_{35}Br_{44}$ ${}^{87}_{37}Rb_{50}$ ${}^{92,97,98}_{42}Mo_{50,55,56}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{90}_{40}Zr_{50}$ ${}^{200}_{80}Hg_{120}$ ${}^{330}_{130}Ch^{ie}_{200}$

$$\alpha_{1-3}=\frac{4^{2}}{3\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\frac{e^{2}}{(\frac{4}{3})^{7}}\frac{e^{2}}{(\frac{5}{4})^{9}}}\frac{1}{112+1-\frac{1}{2}+\frac{1}{88}-\frac{1}{13\cdot(2\cdot 9\cdot 5\cdot 13+1)-\frac{2}{73}}}=1/137.035999037435$$

${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{40}_{18}Ar_{22}$ ${}^{45}_{21}Sc_{24}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$

${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{73}_{32}Ge_{41}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{112}_{48}Cd_{64}$ ${}^{125}_{52}Te_{73}$ ${}^{151,153}_{63}Eu_{88,90}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{226}_{88}Ra^{*}_{138}$

$$\alpha_{1-4}=\frac{21}{2^{2}\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{8}{7})^{15}}}\frac{1}{112+\frac{1}{7}-\frac{1}{8\cdot 19\cdot 41-\frac{75}{98}}}=1/137.035999037435$$

${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{75}_{33}As_{42}$ ${}^{98}_{42}Mo_{56}$ ${}^{129}_{54}Ke_{75}$ ${}^{208}_{82}Pb_{126}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$

${}^{326,326/328=8\cdot 41}_{128,129}Ch^{ie}_{198,197/199}$ ${}^{378=18\cdot 21,380,382}_{150,151,152=8\cdot 19}Ch^{ie}_{228=12\cdot 19,229,230}$

$$\alpha_{1-5}=\frac{26}{5\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{14}{13})^{27}}}\frac{1}{112+\frac{1}{14}-\frac{1}{9\cdot 71}+\frac{1}{67\cdot(75\cdot 100-1)+\frac{1}{10}}}$$

$$=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{100}_{44}Ru_{56}$ ${}^{117}_{50}Sn_{67}$ ${}^{126,128}_{52}Te_{74,76}$ ${}^{165}_{67}Ho_{98}$ ${}^{168}_{68}\mathrm{Re}_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{185,186}_{75}\mathrm{Re}_{110,112}$

${}^{257}_{100}Fm^{*}_{157}$ ${}^{284=4\cdot 71}_{113}Nh^{ie}_{171=9\cdot 19}$ ${}^{340=20\cdot 17}_{134=2\cdot 67,135=5\cdot 27}Ch^{ie}_{206,205=5\cdot 41}$ ${}^{426=6\cdot 71}_{169}Ch^{ie}_{257}$

$$\alpha_{1-6}=\frac{31}{6\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{28}{27})^{55}}}\frac{1}{112+\frac{1}{2\cdot 31}-\frac{1}{3\cdot 11\cdot 13\cdot 31-\frac{43}{4\cdot 27}}}=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{75}_{33}As_{42}$ ${}^{89}_{39}Y_{50}$ ${}^{97,98,99}_{43}Tc^{*}_{54,55,56}$ ${}^{108}_{46}Pd_{62}$ ${}^{148,150}_{62}Sm_{86,88}$ ${}^{222}_{86}Rn^{*}_{136}$

${}^{384=3\cdot 128,386}_{153=9\cdot 17,154=14\cdot 11}Ch^{ie}_{7\cdot 33,8\cdot 29}$ ${}^{392,394}_{155=5\cdot 31,156=12\cdot 13}Ch^{ie}_{237,238}$

$$\alpha_{1}=\alpha_{1-7}=\frac{6^{2}}{7\cdot e^{2}\frac{e^{2}}{(\frac{2}{1})^{3}}\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{113}{112})^{225}}}\frac{1}{112+\frac{1}{75^{2}}}=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{55}_{25}Mn_{30}$ ${}^{56}_{26}Fe_{30}$ ${}^{75}_{33}As_{42}$ ${}^{84}_{36}Kr_{48}$ ${}^{96,98}_{42}Mo_{54,56}$ ${}^{100}_{44}Ru_{56}$ ${}^{103}_{45}Rh_{58}$ ${}^{125}_{52}Te_{73}$ ${}^{129}_{54}Xe_{75}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$

${}^{209}_{84}Po^{*}_{125}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{171=9\cdot 19}$ ${}^{312,314}_{125,126}Ch^{ie}_{187,188}$ ${}^{366,372}_{144,147}Ch^{ie}_{222,225}$

$$\alpha_{1-9} = \frac{47}{3^2 \cdot e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{10}{9})^{19}}} \frac{1}{112 + \frac{1}{4 \cdot 17} - \frac{1}{2(8 \cdot 9 \cdot 37 - 1) + \frac{3 \cdot 17}{157}}} = 1/137.035999037436$$

${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{106}_{51}Sb_{72}$ ${}^{152,168}_{68}Er_{94,100}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{258=6\cdot 43}_{101}Md^{*}_{157}$ ${}^{259=7\cdot 37}_{102=6\cdot 17}Md^{*}_{157}$
${}^{312,2\cdot 157}_{125,9\cdot 14}Ch^{ie}_{11\cdot 17,4\cdot 47}$ ${}^{400}_{157}Ch^{ie}_{3\cdot 81}$

$$\alpha_{1-11} = \frac{3 \cdot 19}{11 \cdot e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{19}{18})^{37}}} \frac{1}{112 + \frac{1}{35} - \frac{1}{88 \cdot 41 - \frac{5 \cdot 53}{22 \cdot 13}}} = 1/137.035999037435$$

${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{41}_{19}Ar_{22}$ ${}^{48}_{22}Ti_{26}$ ${}^{56,57}_{26}Fe_{30,31}$ ${}^{79}_{35}Br_{44}$ ${}^{93}_{41}Nb_{52}$ ${}^{99,100,101}_{44}Ru_{55,56,57}$ ${}^{127}_{53}I_{74}$ ${}^{139}_{57}La_{82}$ ${}^{180,186}_{74}W_{106,112}$
${}^{235}_{92}U^{*}_{143}$ ${}^{289}_{114}Fl^{ie}_{175}$

$$\alpha_{1-13} = \frac{67}{13 \cdot e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{47}{2 \cdot 23})^{3 \cdot 31}}} \frac{1}{112 + \frac{1}{137} - \frac{1}{6(2 \cdot 27 \cdot 59 + 1) + \frac{9}{50}}} = 1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{59}_{27}Co_{32}$ ${}^{89}_{39}Y_{50}$ ${}^{94}_{40}Zr_{54}$ ${}^{93}_{41}Nb_{52}$ ${}^{94,96}_{42}Mo_{52,54}$ ${}^{105}_{46}Pd_{59}$ ${}^{107,109}_{47}Ag_{60,62}$
${}^{9\cdot 13,118,119}_{50}Sn_{67,68,69}$ ${}^{130-132}_{54}Xe_{76-78}$ ${}^{137}_{56}Ba_{81}$ ${}^{3\cdot 47}_{59}\mathrm{Pr}_{82}$ ${}^{157}_{64}Gd_{93}$ ${}^{165}_{67}Ho_{98}$ ${}^{196}_{78}Pt_{118}$ ${}^{198,200,201}_{80}Hg_{118,120,121}$
${}^{209}_{83}Bi^{*}_{126}$ ${}^{294}_{117=9\cdot 13}Ts^{ie}_{177=3\cdot 59}$ ${}^{330,4\cdot 83}_{130,131}Ch^{ie}_{200,201}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-16} = \frac{83}{4^2 \cdot e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{17}{16})^{33}}} \frac{1}{112 + \frac{1}{28} - \frac{1}{6 \cdot (18 \cdot 41 + 1) + \frac{173}{2 \cdot (2 \cdot 75 - 1)}}} = 1/137.035999037435$$

${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{60,61,62,64}_{28}Ni_{32,33,34,36}$ ${}^{73}_{32}Ge_{41}$ ${}^{75}_{33}As_{42}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{129}_{54}Xe_{75}$ ${}^{138}_{56}Ba_{82}$ ${}^{173}_{70}Yb_{103}$ ${}^{11\cdot 17}_{75}Re_{112}$
${}^{208}_{82}Pt_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Bi^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$ ${}^{288}_{115}Mc^{ie}_{173}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{418}_{166,168}Ch^{ie}_{252,250}$

$$\alpha_{1-17} = \frac{2^2 \cdot 22}{17 \cdot e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{21}{20})^{41}}} \frac{1}{112 + \frac{1}{137} - \frac{1}{2 \cdot 19 \cdot 23 \cdot 59 - \frac{30}{100}}} = 1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{38,40}_{18}Ar_{20,22}$ ${}^{39,41}_{19}K_{20,22}$ ${}^{40,42,43,54}_{20}Ca_{20,22,23,34}$ ${}^{46,50}_{22}Ti_{24,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{64,68}_{30}Zn_{34,38}$
${}^{78,80,82}_{34}Se_{44,46,48}$ ${}^{80,82,84}_{36}Kr_{44,46,48}$ ${}^{84,88}_{38}Sr_{46,50}$ ${}^{100}_{44}Ru_{56}$ ${}^{105,106,108}_{46}Pd_{59,60,64}$ ${}^{118}_{50}Sn_{68}$ ${}^{137}_{56}Ba_{81}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{142,150}_{60}Nd_{82,88}$
${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{179,180}_{72}Hf_{107,108}$ ${}^{184,188}_{76}Hf_{108,112}$ ${}^{208}_{82}Pt_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{238}_{92}U^{*}_{146}$ ${}^{14\cdot 21}_{118}Og^{ie}_{16\cdot 11}$
${}^{340=20\cdot 17}_{134,135=5\cdot 27}Ch^{ie}_{206,205=5\cdot 41}$ ${}^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$\alpha_{1-19} = \frac{2 \cdot 7^2}{19 \cdot e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{38}{37})^{75}}} \frac{1}{112 + \frac{1}{2 \cdot (8 \cdot 54 - 1) + \frac{54}{19^2}}} = 1/137.035999037440$$

${}^{49}_{22}Ti_{27}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{87}_{38}Sr_{49}$ ${}^{96,98}_{42}Mo_{54,56}$ ${}^{128,129,130}_{54}Xe_{74,75,76}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{378=18\cdot 21,380,382}_{150,151,152=8\cdot 19}Ch^{ie}_{228=12\cdot 19,229,230}$

$$\alpha_{1-20} = \frac{103}{2^2 \cdot 5 \cdot e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{59}{2 \cdot 29})^{9 \cdot 13}}} \frac{1}{112 + \frac{1}{32 \cdot 45 \cdot 79 + \frac{22}{3 \cdot 17}}} = 1/137.035999037435$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38,40}_{18}Ar_{18,20,22}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{70,72,76}_{32}Ge_{38,40,44}$ ${}^{89}_{39}Y_{50}$ ${}^{103}_{45}Rh_{58}$ ${}^{135,136}_{56}Ba_{79,80}$ ${}^{136,138}_{58}Ce_{78,80}$ ${}^{197}_{79}Au_{118}$ ${}^{173}_{70}Yb_{103}$
${}^{14\cdot 21}_{118}Og^{ie}_{16\cdot 11}$ ${}^{384=3\cdot 128,386}_{153=9\cdot 17,154=14\cdot 11}Ch^{ie}_{21\cdot 11,8\cdot 29}$ ${}^{394,394}_{155,156=12\cdot 13}Ch^{ie}_{3\cdot 79,14\cdot 17}$ ${}^{402}_{158}Ch^{ie}_{244}$

$$\alpha_{1-22}=\frac{113}{22\cdot e^{2}\,\frac{e^{2}}{(\frac{2}{1})^{3}}\,\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{27\cdot 29}{2\cdot 17\cdot 23})^{5\cdot(2\cdot 157-1)}}}\,\frac{1}{112+\frac{1}{2\cdot[2\cdot 3\cdot 17\cdot(10\cdot 19+1)+1]+\frac{29}{49}}}$$

$$=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{39}_{19}K_{20}$ ${}^{49,50}_{22}Ti_{27,28}$ ${}^{50,51}_{23}Cd_{27,28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{77,78,80}_{34}\mathrm{Se}_{43,44,46}$ ${}^{98-102}_{44}Ru_{54-58}$ ${}^{104,106}_{46}Pd_{58,60}$ ${}^{113,115}_{49}In_{64,66}$

${}^{185,11\cdot 17}_{75}\mathrm{Re}_{110,112}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{258=6\cdot 43}_{101}Md^{*}_{157}$ ${}^{259=7\cdot 37}_{102=6\cdot 17}No^{*}_{157}$ ${}^{284}_{113}Nh^{ie}_{171=9\cdot 19}$ ${}^{22\cdot 17}_{148}Ch^{ie}_{226}$?

${}^{384=3\cdot 128,386}_{153=9\cdot 17,154=14\cdot 11}Ch^{ie}_{21\cdot 11,8\cdot 29}$ ${}^{400}_{157}Ch^{ie}_{243=9\cdot 27}$

$$\alpha_{1-23}=\frac{7\cdot 17}{23\cdot e^{2}\,\frac{e^{2}}{(\frac{2}{1})^{3}}\,\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{23}{22})^{45}}}\,\frac{1}{112+\frac{1}{35}-\frac{1}{43\cdot 52-\frac{2\cdot 29}{16\cdot 17-1}}}=1/137.035999037435$$

${}^{32,33,34}_{16}S_{16,17,18}$ ${}^{35}_{17}Cl_{18}$ ${}^{51}_{23}Cd_{28}$ ${}^{56}_{26}Fe_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{238=14\cdot 17}_{92=4\cdot 23}U^{*}_{146}$ ${}^{288}_{115}Mc^{ie}_{173}$

${}^{298,300}_{119,120}Ch^{ie}_{179,180=4\cdot 45}$ ${}^{384=3\cdot 128,386}_{153=9\cdot 17,154=14\cdot 11}Ch^{ie}_{21\cdot 11,8\cdot 29}$

$$\alpha_{1-25}=\frac{3\cdot 43}{5^{2}\cdot e^{2}\,\frac{e^{2}}{(\frac{2}{1})^{3}}\,\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{35}{34})^{3\cdot 23}}}\,\frac{1}{112+\frac{1}{11\cdot 19}-\frac{1}{13^{2}(16\cdot 17-1)+\frac{11}{25}}}=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{25}_{12}Mg_{13}$ ${}^{27}_{13}Al_{14}$ ${}^{34}_{16}S_{18}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,41}_{19}K_{20,22}$ ${}^{47,49}_{22}Ti_{25,26}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{78,79,81}_{34}Se_{43,44,46}$

${}^{79,81}_{35}Br_{44.46}$ ${}^{98,99}_{43}Tc^{*}_{55,56}$ ${}^{100}_{44}Ru_{56}$ ${}^{169}_{69}Tm_{100}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{24\cdot 13,314}_{125,126}Ch^{ie}_{11\cdot 17,188}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{418}_{168}Ch^{ie}_{250}$

$$\alpha_{1-27}=\frac{139}{27\cdot e^{2}\,\frac{e^{2}}{(\frac{2}{1})^{3}}\,\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{67}{66})^{7\cdot 19}}}\,\frac{1}{112+\frac{1}{11\cdot 47+\frac{18}{23}+\frac{1}{6\cdot 23\cdot 137}}}=1/137.035999037435$$

${}^{19}_{9}F_{10}$ ${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{40}_{18}Ar_{22}$ ${}^{41}_{19}K_{22}$ ${}^{47,49,50}_{22}Ti_{25,27,28}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{131,132,134}_{54}Xe_{77,78,80}$ ${}^{133}_{55}Cs_{78}$ ${}^{137}_{56}Ba_{81}$

${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{334,2\cdot 168}_{132,133=7\cdot 19}Ch^{ie}_{6\cdot 37,203=7\cdot 29}$ ${}^{340}_{134=2\cdot 67,135=5\cdot 27}Ch^{ie}_{206,205}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{348=12\cdot 29}_{139}Ch^{ie}_{209}$

$$\alpha_{1-29}=\frac{149}{29\cdot e^{2}\,\frac{e^{2}}{(\frac{2}{1})^{3}}\,\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{14\cdot 23}{3\cdot 107})^{643}}}\,\frac{1}{112+\frac{1}{6\cdot 8\cdot(12\cdot 26-1)+\frac{11}{18}}}=1/137.035999037434$$

${}^{12}_{6}C_{6}$ ${}^{23}_{11}Na_{12}$ ${}^{28,29}_{14}Si_{14,15}$ ${}^{40}_{18}Ar_{22}$ ${}^{51}_{23}V_{28}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{100}_{44}Ru_{56}$ ${}^{106,108}_{46}Pd_{60,62}$ ${}^{107}_{47}Ag_{60}$

${}^{112,114}_{48}Cd_{64,66}$ ${}^{149}_{62}Sm_{87}$ ${}^{179}_{72}Hf_{107}$ ${}^{2\cdot 149,300}_{119=7\cdot 17,120}Ch^{ie}_{179,180}$ ${}^{8\cdot 47}_{149}Ch^{ie}_{227}$

$$\alpha_{1-31}=\frac{4^{2}\cdot 10}{31\cdot e^{2}\,\frac{e^{2}}{(\frac{2}{1})^{3}}\,\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{33}{32})^{65}}}\,\frac{1}{112+\frac{1}{12\cdot 11\cdot 17-\frac{4\cdot 49}{5\cdot 41}}}=1/137.035999037434$$

${}^{71}_{31}Ga_{40}$ ${}^{73,74,76}_{32}Ge_{41,42,44}$ ${}^{75}_{33}As_{42}$ ${}^{74,76,77,78,80,82}_{34}Se_{40,42,43,44,46,48}$ ${}^{93}_{41}Nb_{52}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{372=12\cdot 31}_{147=3\cdot 49}Ch^{ie}_{225}$

${}^{8\cdot 49,394}_{155=5\cdot 31,156=12\cdot 13}Ch^{ie}_{237,14\cdot 17}$

$$\alpha_{1-32}=\frac{15\cdot 11}{2\cdot 4^{2}\cdot e^{2}\,\frac{e^{2}}{(\frac{2}{1})^{3}}\,\frac{e^{2}}{(\frac{3}{2})^{5}}\cdots\frac{e^{2}}{(\frac{41}{40})^{81}}}\,\frac{1}{112+\frac{1}{25\cdot 29-\frac{5\cdot 83}{19\cdot 23}}}=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{31}_{15}P_{16}$ ${}^{39}_{19}K_{20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{70,72,73,76}_{32}Ge_{38,40,41,44}$ ${}^{75}_{33}As_{42}$

${}^{7\cdot 19}_{55}Cs_{78}$ ${}^{11\cdot 13}_{60}Nd_{83}$ ${}^{208,11\cdot 19}_{83}Bi^{*}_{125,126}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$ ${}^{15\cdot 19}_{112}Cn^{ie}_{173}$ ${}^{416=32\cdot 13,418=22\cdot 19}_{166}Ch^{ie}_{250,252}$

$$\alpha_{1-33}=\frac{170}{33\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{53}{52})^{105}}}\frac{1}{112+\dfrac{1}{22\cdot 29+\dfrac{4\cdot 73}{5\cdot 83}}}=1/137.035999037436$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{47,48,50}_{22}Ti_{25,26,28}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{75}_{33}As_{42}$ ${}^{77,78}_{34}Se_{43,44}$ ${}^{120,122,125}_{52}Te_{68,70,73}$

${}^{140,142}_{58}Ce_{82.84}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{140,142}_{58}Ce_{82.84}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{384=3\cdot128,386}_{153=9\cdot17,154=14\cdot11}Ch^{ie}_{21\cdot11,8\cdot29}$ ${}^{418}_{166}Ch^{ie}_{252}$

$$\alpha_{1-34}=\frac{7\cdot 5^2}{34\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{73}{72})^{5\cdot 29}}}\frac{1}{112+\dfrac{1}{15\cdot 59+\dfrac{13}{15}+\dfrac{1}{3\cdot(2\cdot 15\cdot 17-1)}}}=1/137.035999037435$$

${}^{35,37}_{17}Cl_{18,20}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{64}_{30}Zn_{34}$ ${}^{78,80}_{34}Se_{44,46}$ ${}^{79,80}_{35}Br_{44,46}$ ${}^{89}_{39}Y_{50}$ ${}^{124,125}_{52}Te_{72,73}$ ${}^{5\cdot29}_{60}Nd_{85}$

$$\alpha_{1-36}=\frac{5\cdot 37}{6^2\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{3\cdot 79}{4\cdot 59})^{11\cdot 43}}}\frac{1}{112+\dfrac{1}{5\cdot(31\cdot 42-1)+\dfrac{3\cdot 31}{14\cdot 13}}}=1/137.035999037436$$

${}^{27}_{13}Al_{14}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{78,80,84,86}_{36}Kr_{42,44,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{402}_{158}Ch^{ie}_{244}$ ${}^{392,394}_{155=5\cdot31,156=12\cdot13}Ch^{ie}_{3\cdot79,14\cdot17}$

$$\alpha_{1-43}=\frac{13\cdot 17}{43\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{3\cdot 67}{200})^{401}}}\frac{1}{112+\dfrac{1}{8\cdot(12\cdot 83+1)+\dfrac{4}{3\cdot 13}}}=1/137.035999037436$$

${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{77}_{34}Se_{43}$ ${}^{83}_{36}Kr_{47}$ ${}^{89}_{39}Y_{50}$ ${}^{97.98}_{43}Tc^{*}_{54,56}$ ${}^{9\cdot13}_{50}Sn_{67}$ ${}^{11\cdot13}_{60}Nd_{83}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$

${}^{200}_{80}Hg_{120}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{330,4\cdot83}_{130,131}Ch^{ie}_{200,201}$ ${}^{340=20\cdot17}_{134=2\cdot67,135}Ch^{ie}_{206,205}$ ${}^{402=6\cdot67}_{158}Ch^{ie}_{244}$ ${}^{418}_{166}Ch^{ie}_{252}$

$$\alpha_{1-50}=\frac{2\cdot 257}{100\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{14\cdot 13}{181})^{3\cdot 11^2}}}\frac{1}{112+\dfrac{1}{29\cdot 61+\dfrac{157}{16\cdot 11}}}=1/137.035999037436$$

${}^{27}_{13}Al_{14}$ ${}^{47,48,50}_{22}Ti_{25,26,28}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{75}_{33}As_{42}$ ${}^{99,100}_{44}Ru_{55,56}$ ${}^{116,120}_{50}Sn_{66,70}$ ${}^{5\cdot29}_{61}\mathrm{Pm}^{*}_{84}$ ${}^{157}_{64}Gd_{93}$ ${}^{226}_{88}Ra^{*}_{138}$

${}^{257}_{100}\mathrm{Fm}^{*}_{157}$ ${}^{258=6\cdot43}_{101}Md^{*}_{157}$ ${}^{259=7\cdot37}_{102=6\cdot17}Md^{*}_{157}$ ${}^{14\cdot21}_{118}Og^{ie}_{176=16\cdot11}$ ${}^{302}_{121}Ch^{ie}_{181}$ ${}^{400,402}_{157,158}Ch^{ie}_{243.244=4\cdot61}$ ${}^{426}_{169}Ch^{ie}_{257}$

$$\alpha_{1-59}=\frac{3\cdot 101}{59\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{2\cdot 27\cdot 49}{5\cdot 23^2})^{11\cdot 13\cdot 37}}}\frac{1}{112+\dfrac{1}{48\cdot 64\cdot 31-\dfrac{17}{81}}}=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{37}_{17}Cl_{20}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{59}_{27}Co_{32}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{105}_{46}Pd_{59}$ ${}^{112}_{48}Cd_{64}$ ${}^{113,115}_{49}In_{64,66}$

${}^{137}_{56}Ba_{81}$ ${}^{205}_{81}Tl_{124}$ ${}^{235=5\cdot47,238=14\cdot17}_{92=4\cdot23}U^{*}_{143=11\cdot13,146}$ ${}^{293}_{116}Lv^{ie}_{177=3\cdot59}$ ${}^{294=6\cdot49}_{117=9\cdot13}Ts^{ie}_{177=3\cdot59}$ ${}^{6\cdot49}_{118=2\cdot59}Og^{ie}_{176=16\cdot11}$

${}^{8\cdot49,394}_{155=5\cdot31,156=12\cdot13}Ch^{ie}_{237,14\cdot17}$

$$\alpha_{1-81}=\frac{4^2\cdot 26}{9^2\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{22\cdot 73}{15\cdot 107})^{13^2\cdot 19}}}\frac{1}{112+\dfrac{1}{2\cdot 81\cdot 17\cdot 67+\dfrac{35}{88}}}=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{35}_{17}Cl_{18}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{59}_{27}Co_{32}$ ${}^{78,80,82}_{34}Se_{44,46,48}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{137}_{56}Ba_{81}$ ${}^{165}_{67}Ho_{98}$ ${}^{180,181}_{73}Ta_{107,108}$

${}^{340}_{134,135}Ch^{ie}_{206,205}$ ${}^{402}_{158}Ch^{ie}_{244}$

$$\alpha_{1-96} = \frac{17\cdot 29}{4^2\cdot 6\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{16\cdot 3\cdot 11^2-1}{27\cdot 5\cdot 43+1})^{79\cdot 147}}}\,\frac{1}{112+\frac{163\cdot(8\cdot 21\cdot 37+1)}{50\cdot 10^{11}}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{45}_{21}Sc_{24}$ ${}^{50,52,53,54}_{24}Cr_{26,28,29,30}$ ${}^{59}_{27}Co_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{75}_{27}As_{42}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{100}_{44}Ru_{56}$ ${}^{112}_{48}Cd_{64}$

${}^{116,118,120,124}_{50}Sn_{66,68,70,74}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{270}_{107}Bh^{*}_{163}$

${}^{326=2\cdot163}_{128,129=3\cdot43}Ch^{ie}_{198,197}$ ${}^{372}_{147}Ch^{ie}_{225}$ ${}^{384=3\cdot128,386}_{153=9\cdot17,154=14\cdot11}Ch^{ie}_{21\cdot11,8\cdot29}$ ${}^{410}_{163}Ch^{ie}_{247}$

$$\alpha_{1-103} = \frac{23^2}{103\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{3\cdot 19\cdot 23}{7\cdot 11\cdot 17+1})^{2621}}}\,\frac{1}{112+\frac{1}{6\cdot(12\cdot(8\cdot(64\cdot 7+1)+1)+1)+\frac{3}{4}}}$$

$$=1/137.035999037435$$

${}^{7}_{3}Li_{4}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{23}_{11}K_{12}$ ${}^{39}_{19}K_{20}$ ${}^{51}_{23}V_{28}$ ${}^{70,72,74}_{32}Ge_{38,40,42}$ ${}^{103}_{45}Rh_{58}$ ${}^{102,104,106,110}_{46}Pd_{56,58,60,64}$ ${}^{112}_{48}Cd_{64}$ ${}^{131}_{54}Xe_{77}$ ${}^{156}_{64}Gd_{92}$ ${}^{173}_{70}Yb_{103}$

$$\alpha_{1-133} = \frac{683}{133\cdot(2\pi)_{12389}}\,\frac{1}{112+\frac{14651}{50\cdot 10^{11}}} = \frac{6^2\cdot 19-1}{7\cdot 19\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{59\cdot 210}{13\cdot(17\cdot 56+1)})^{71\cdot(12\cdot 29+1)}}}\,\frac{1}{112+\frac{7^2\cdot 13\cdot 23}{50\cdot 10^{11}}}$$

$$=1/137.035999037435$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{56}_{26}Fe_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{84}_{36}Kr_{48}$ ${}^{105}_{46}Pd_{59}$ ${}^{5\cdot23}_{49}In_{66}$

${}^{116,118,119,120,122}_{50}Sn_{66,68,69,70,72}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{289=17^2}_{114=6\cdot19}Fl^{ie}_{175}$ ${}^{14\cdot21}_{118}Og^{ie}_{176}$ ${}^{334,2\cdot168}_{132,133=7\cdot19}Ch^{ie}_{202,203=7\cdot29}$

$$\alpha_{1-140} = \frac{6^2\cdot 20-1}{140\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{4\cdot 13\cdot 37}{3\cdot(64\cdot 10+1)})^{3847}}}\,\frac{1}{112+\frac{1}{4\cdot 9\cdot(2\cdot 3\cdot 29\cdot 47+1)+\frac{29}{54}}}$$

$$=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}So_{14,15.16}$ ${}^{58,60,64}_{28}Ni_{30,32,36}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{107,109}_{37}Rb_{48,50}$ ${}^{85,87}_{47}Ag_{60,62}$ ${}^{112}_{48}Cd_{64}$

$$\alpha_{1-155} = \frac{2^2\cdot 199}{5\cdot 31\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{[\frac{19\cdot 210-1}{3^2\cdot(2\cdot 13\cdot 17+1)+1}]^{7977}}}\,\frac{1}{112+\frac{1}{5\cdot 17\cdot 31\cdot(2\cdot 13\cdot 17+1)-\frac{15}{43}}}$$

$$=1/137.035999037435$$

${}^{27}_{13}Al_{14}$ ${}^{31}_{15}P_{16}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39,40}_{19}K_{20,21}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{64,68}_{30}Zn_{34,38}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{77}_{34}Se_{43}$ ${}^{88}_{38}Sr_{50}$ ${}^{93}_{41}Nb_{52}$

${}^{98,99}_{43}Tc^{*}_{55,56}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{289=17^2}_{114=6\cdot19}Fl^{ie}_{175}$ ${}^{328=8\cdot41}_{129=3\cdot43}Ch^{ie}_{199}$ ${}^{392,394}_{155=5\cdot31,156=12\cdot13}Ch^{ie}_{237,14\cdot17}$ ${}^{420}_{170}Ch^{ie}_{250}$

$$\alpha_{1-170} = \frac{873}{170\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{34451}{34450})^{68901}}}\,\frac{1}{112+\frac{4171}{8\times 10^{11}}}$$

$$= \frac{3^2\cdot 97}{170\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{[\frac{47(12\cdot 61+1)}{2\cdot 25\cdot 13\cdot 53}]^{3\cdot 7\cdot 17\cdot 193}}}\,\frac{1}{112+\frac{43\cdot 97}{8\cdot 10^{11}}} = 1/137.035999037435$$

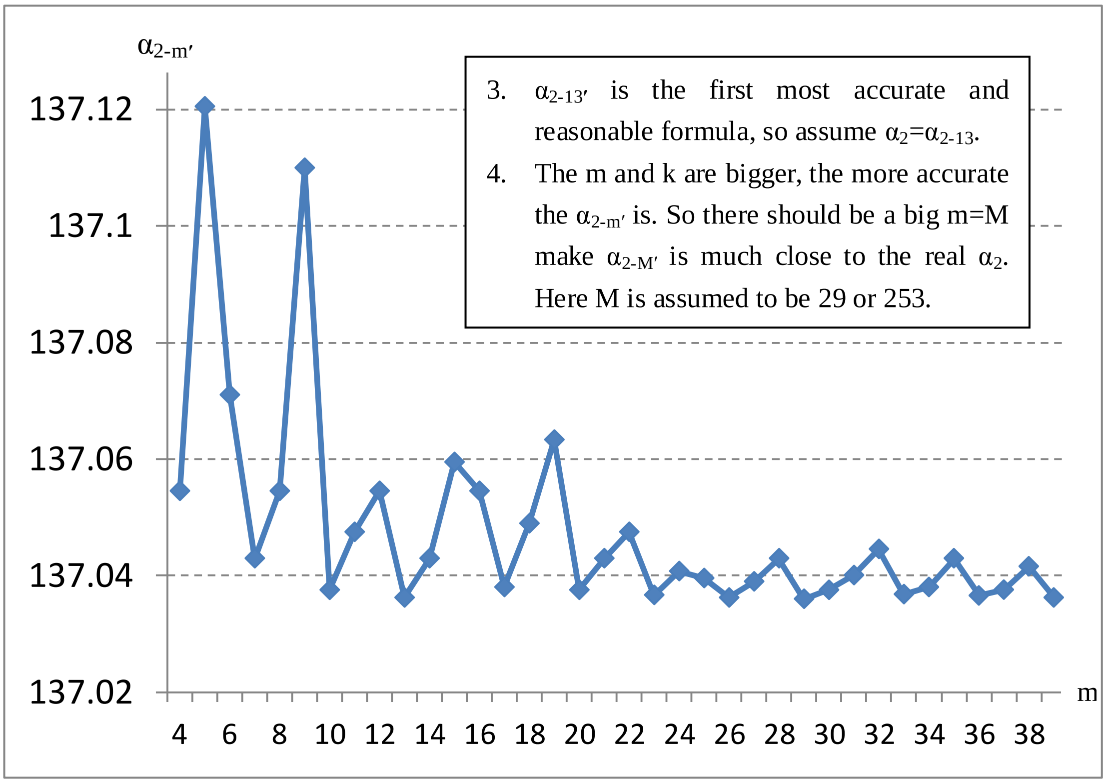
${}^{35,37}_{17}Cl_{18,20}$ ${}^{76,77}_{34}Se_{42,43}$ ${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{95,97}_{42}Mo_{53,55}$ ${}^{97,98}_{43}Tc^{*}_{54,56}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{127}_{53}I_{74}$ ${}^{145}_{61}Pm^{*}_{84}$ ${}^{163}_{66}Dy_{97}$ ${}^{168}_{68}Er_{100}$

${}^{193}_{77}Ir_{116}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{6\cdot53,320}_{127}Ch^{ie}_{191,193}$ ${}^{420}_{170}Ch^{ie}_{250}$

Table 6. Parameters and Results of Approximate Formulas of α_2 (2019/7/3)

m	n	k	α 2-m′	m	n	k	α 2-m′
1	**8**	4	137.933814383	22	170	32	137.047480404
2	16	4	137.933814383	23	177	**161**	137.036664793
3	24	4	137.933814383	**24**	185	62	137.040748949
4	31	20	137.054511358	**25**	193	39	137.039552569
5	39	11	137.120466691	26	200	278	137.036218856
6	47	8	137.070996332	27	**208**	80	137.038980680
7	**54**	48	137.042951195	28	216	48	137.042951195
8	62	20	137.054511358	**29**	**223**	**655**	**137.036002235**
9	70	14	137.109928583	30	231	104	137.037530964
10	77	**104**	137.037530964	31	239	58	137.040063944
11	85	32	137.047480404	**32**	247	41	137.044550585
12	93	20	137.054511358	33	254	**138**	137.036795730
13	**100**	**278**	**137.036218856**	34	262	70	137.038016730
14	108	48	137.042951195	35	270	48	137.042951195
15	**116**	28	137.059466839	**36**	277	**190**	137.036562950
16	124	20	137.054511358	37	285	85	137.037566566
17	131	70	137.038016730	38	293	56	137.041569603
18	139	37	137.048943854	39	300	278	137.036218856
19	147	26	137.063298933	**125**	**961**	**4293**	137.035999678
20	154	104	137.037530964	**253**	**1945**	**28186**	**137.035999128**
21	162	48	137.042951195	269	2068	**41654**	137.035999118

Fig. 8. Results of Approximate Formulas of α_2 (2019/7/3)



$$\alpha_{2-1}=\frac{e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\,\frac{e^2}{(\frac{4}{3})^7}\,\frac{e^2}{(\frac{5}{4})^9}}{2\cdot 2^2}\,\frac{1}{112-1+\frac{1}{3}-\frac{1}{16}+\frac{1}{41\cdot(12\cdot 13+1)+\frac{13}{41}}}=1/137.035999111816$$

${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{73}_{32}Ge_{41}$ ${}^{93}_{41}Nb_{52}$ ${}^{112}_{48}Cd_{64}$ ${}^{128}_{52}Te_{76}$ ${}^{138}_{56}Ba_{82}$ ${}^{157}_{64}Gd_{93}$ ${}^{208}_{82}Pb_{126}$

$$\alpha_{2-4}=\frac{2^2\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{21}{20})^{41}}}{31}\,\frac{1}{112-\frac{1}{66}+\frac{1}{71\cdot(14\cdot 43-1)-\frac{56}{95}}}=1/137.035999111818$$

${}^{28}_{14}Si_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{55}_{25}Mn_{30}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{161}_{66}Dy_{95}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{284}_{113}Nh^{ie}_{171}$

$$\alpha_{2-5}=\frac{5\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{12}{11})^{23}}}{39}\,\frac{1}{112-\frac{1}{14}+\frac{1}{10\cdot 41}-\frac{1}{23\cdot(14\cdot 11\cdot 79+1)+\frac{11}{16}}}=1/137.035999111818$$

${}^{11}_{5}B_{6}$ ${}^{23}_{11}Na_{12}$ ${}^{39,41}_{19}K_{20,22}$ ${}^{51}_{23}V_{28}$ ${}^{78}_{34}Se_{44}$ ${}^{79}_{35}Br_{44}$ ${}^{89}_{39}Y_{50}$ ${}^{134,135,138}_{56}Ba_{78,79,82}$ ${}^{288}_{115=5\cdot 23}Mc^{ie}_{173}$ ${}^{402}_{158}Ch^{ie}_{244}$

$$\alpha_{2-6}=\frac{6\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{9}{8})^{17}}}{47}\,\frac{1}{112-\frac{1}{2\cdot 17}+\frac{1}{2\cdot(36\cdot 17+1)-\frac{4}{47}}}=1/137.035999111818$$

${}^{34}_{16}S_{18}$ ${}^{35}_{17}Cl_{18}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{82,83,84}_{36}Kr_{46,47,48}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{136}_{56}Ba_{80}$ ${}^{166,167,168}_{68}Er_{98,99,100}$ ${}^{223}_{87}Fr^{*}_{136}$

$$\alpha_{2-7}=\frac{7\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{48}{47})^{95}}}{6\cdot 3^2}\,\frac{1}{112+\frac{1}{13\cdot 34}-\frac{1}{2\cdot 29\cdot(24\cdot 89+1)+\frac{11}{34}}}=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{50,52,53,54}_{24}Cr_{26,28,29,30}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{63,65}_{29}Ba_{34,36}$ ${}^{78,82}_{34}Se_{44,48}$ ${}^{94}_{40}Zr_{54}$ ${}^{96}_{42}Mo_{54}$ ${}^{98,100}_{44}Ru_{54,56}$ ${}^{227}_{89}Ac^{*}_{138}$
${}^{384=3\cdot 128,386}_{153=9\cdot 17,154=14\cdot 11}Ch^{ie}_{21\cdot 11,8\cdot 29}$

$$\alpha_{2-9}=\frac{3^2\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{15}{14})^{29}}}{70}\,\frac{1}{112-\frac{1}{16}+\frac{1}{11\cdot 43}-\frac{1}{70\cdot 17\cdot(3\cdot 64-1)-\frac{41}{70}}}=1/137.035999111818$$

${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{112}_{48}Cd_{64}$ ${}^{76}_{33}As^{*}_{43}$ ${}^{79}_{35}Br_{44}$ ${}^{93}_{41}Nb_{52}$ ${}^{99}_{43}Td^{*}_{56}$ ${}^{121}_{51}Sb_{70}$ ${}^{157}_{64}Gd_{93}$ ${}^{172}_{70}Yb_{102}$

$$\alpha_{2-10}=\frac{10\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{5\cdot 21}{8\cdot 13})^{11\cdot 19}}}{77}\,\frac{1}{112-\frac{1}{3\cdot 14\cdot 19}+\frac{1}{14\cdot(4\cdot 27\cdot(2\cdot 15\cdot 19+1)-1)}}=1/137.035999111818$$

${}^{14}_{7}N_{7}$ ${}^{27}_{13}Al_{14}$ ${}^{28}_{14}Si_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{56}_{26}Fe_{30}$ ${}^{97,99}_{42}Mo_{54,56}$ ${}^{131,136}_{54}Xe_{77,82}$ ${}^{138,139}_{57}La_{81,82}$ ${}^{191}_{77}Ir_{2\cdot 57}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{285}_{112}Cn^{*}_{173}$
${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{418,420}_{168}Ch^{ie}_{250,252}$

$$\alpha_{2-11}=\frac{11\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{33}{32})^{65}}}{85}\,\frac{1}{112-\frac{1}{106}+\frac{1}{30\cdot(4\cdot 163+1)-\frac{35}{52}}}=1/137.035999111818$$

${}^{79}_{35}Br_{44}$ ${}^{79}_{35}Br_{44}$ ${}^{96,99,100,104}_{44}Ru_{52,55,56,60}$ ${}^{125}_{52}Te_{73}$ ${}^{127}_{53}I_{74}$ ${}^{145}_{60}Nd_{85}$ ${}^{210}_{85}At^{*}_{125}$ ${}^{268}_{105}Db^{*}_{163}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{270}_{107}Bh^{*}_{163}$
${}^{2\cdot 163}_{128,129}Ch^{ie}_{198=18\cdot 11,197}$ ${}^{410}_{163}Ch^{ie}_{247}$ ${}^{420}_{170}Ch^{ie}_{250}$

$$\alpha_2=\alpha_{2\text{-}13}=\frac{13\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{9\cdot 31}{2\cdot 139})^{557}}}{10^2}\,\frac{1}{112-\frac{1}{3\cdot 29\cdot 64}}=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{27}_{13}\mathrm{Al}_{14}$ ${}^{29}_{14}\mathrm{Si}_{15}$ ${}^{54,56,57,58}_{26}\mathrm{Fe}_{28,30,31,32}$ ${}^{58}_{28}Ni_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{93}_{41}Nb_{52}$ ${}^{100}_{44}Ru_{56}$ ${}^{112,116}_{48}Cd_{64,68}$
${}^{116}_{50}Sn_{66}$ ${}^{128}_{52}Te_{76}$ ${}^{136,137,38}_{56}\mathrm{Ba}_{80,81,82}$ ${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$ ${}^{149}_{62}Sm_{87}$ ${}^{156,157,160}_{64}Gd_{92,93,96}$ ${}^{166,167,168}_{68}Er_{98,99,100}$
${}^{169}_{69}Tm_{100}$ ${}^{192}_{76}Os_{116}$ ${}^{208}_{82}Pb_{126}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{237}_{93}Np^{*}_{144}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{278+7}_{112}Cn^{*}_{166+7}$ ${}^{330,332}_{130,131}Ch^{ie}_{200,201}$ ${}^{348=12\cdot 29}_{139}Ch^{ie}_{209}$

$$\alpha_{2-15}=\frac{15\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{29}{28})^{57}}}{2^2\cdot 29}\,\frac{1}{112-\frac{1}{4\cdot 13}+\frac{1}{12\cdot(36\cdot 43+1)-\frac{1}{16}}}=1/137.035999111818$$

${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{31}_{15}P_{16}$ ${}^{56}_{26}Fe_{30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{84,86}_{36}Kr_{48,50}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{222}_{86}Rn^{*}_{136}$

$$\alpha_{2-17}=\frac{17\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{71}{70})^{3\cdot 47}}}{131}\,\frac{1}{112-\frac{1}{6\cdot 101}+\frac{1}{23\cdot(30\cdot 35^2-1)+\frac{6}{23}}}=1/137.035999111818$$

${}^{35}_{17}Cl_{18}$ ${}^{51}_{23}V_{28}$ ${}^{64,66,68}_{30}Zn_{34,36,38}$ ${}^{80,82}_{34}Se_{46,48}$ ${}^{81}_{35}Br_{46}$ ${}^{83}_{36}Kr_{47}$ ${}^{106}_{46}Pd_{60}$ ${}^{107}_{47}Ag_{60}$ ${}^{121,122}_{51}Sb_{70,71}$ ${}^{144}_{60}Nd_{84}$ ${}^{171,172}_{70}Yb_{101,102}$
${}^{175}_{71}Lu_{104}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{330,4\cdot 83}_{130,131}Ch^{ie}_{200,201}$

$$\alpha_{2-18}=\frac{18\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{38}{37})^{75}}}{139}\,\frac{1}{112-\frac{1}{2\cdot 47}+\frac{1}{2\cdot 31\cdot(16\cdot 17-1)+\frac{83}{137}}}=1/137.035999111818$$

${}^{31}_{15}P_{16}$ ${}^{32,33,34,36}_{16}S_{16,17,18,20}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{36,38}_{18}Ar_{18,20}$ ${}^{69}_{31}Ga_{38}$ ${}^{70,72,74,76}_{32}Ge_{38,40,42,44}$ ${}^{74,76}_{34}Se_{40,42}$ ${}^{83,84}_{36}Kr_{47,48}$ ${}^{109}_{47}Ag_{62}$
${}^{148,150,152}_{62}Sm_{86,88,90}$ ${}^{130,132,136,137,138}_{56}Ba_{74,76,80,81,82}$ ${}^{185,187}_{75}\mathrm{Re}_{110,112}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{348}_{139}Ch^{ie}_{209}$ ${}^{418}_{166}Ch^{ie}_{252}$

$$\alpha_{2-19}=\frac{19\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{27}{26})^{53}}}{3\cdot 49}\frac{1}{112-\frac{1}{44}+\frac{1}{16\cdot(4\cdot 37+1)-\frac{23}{6\cdot 47+1}}}=1/137.035999111818$$

${}^{23}_{11}Na_{12}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{46,47,48,49,50}_{22}Ti_{24,25,26,27,28}$ ${}^{51}_{23}V_{28}$ ${}^{54,56,57,58}_{26}Fe_{28,30,31,32}$ ${}^{59}_{27}Co_{32}$ ${}^{83}_{36}Kr_{47}$ ${}^{85,87}_{37}Rb_{48,50}$

${}^{95}_{42}Mo_{53}$ ${}^{98,100}_{44}Ru_{54,56}$ ${}^{107,109}_{47}Ru_{60,62}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{127}_{53}I_{74}$ ${}^{6\cdot 49}_{118}Og^{ie}_{176=16\cdot 11}$ ${}^{370,374=22\cdot 17}_{147,148}Ch^{ie}_{223,226}$

$$\alpha_{2-23}=\frac{23\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{2\cdot 81}{7\cdot 23})^{17\cdot 19}}}{3\cdot 59}\frac{1}{112-\frac{1}{2\cdot(40\cdot 23-1)+\frac{9}{32\cdot 10}}}=1/137.035999111818$$

${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39}_{19}K_{20}$ ${}^{40,43}_{20}Ca_{20,23}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{70,72}_{32}Ge_{38,40}$ ${}^{80,82}_{34}Se_{46,48}$ ${}^{90,91,94}_{40}Zr_{50,51,54}$

${}^{102,105,110}_{46}Pd_{56,59,64}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{294=6\cdot 49}_{117=9\cdot 13}Ts^{ie}_{177=3\cdot 59}$

$$\alpha_{2-24}=\frac{2^2\cdot 6\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{63}{62})^{125}}}{5\cdot 37}\frac{1}{112-\frac{1}{257}+\frac{1}{10\cdot(12\cdot 13\cdot 83+1)+\frac{23}{81}}}=1/137.035999111818$$

${}^{24,24,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{50,52,54}_{24}Cr_{26,28,30}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Rb_{50}$ ${}^{108}_{46}Pd_{62}$

${}^{110,111,112}_{48}Cd_{62,63,64}$ ${}^{124,125,126,130}_{52}Te_{72,73,74,78}$ ${}^{137}_{56}Ba_{81}$ ${}^{184,186}_{74}W_{110,112}$ ${}^{205}_{81}Tl_{124}$ ${}^{208,209}_{83}Bi^{*}_{125,126}$ ${}^{257}_{100}Fm^{*}_{157}$

${}^{312=24\cdot 13}_{125}Ch^{ie}_{187=11\cdot 17}$ ${}^{32\cdot 13,418}_{166}Ch^{ie}_{250,252}$ ${}^{426=6\cdot 71}_{169}Ch^{ie}_{257}$

$$\alpha_{2-25}=\frac{5^2\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{40}{39})^{79}}}{193}\frac{1}{112-\frac{1}{8\cdot 43}+\frac{1}{18\cdot 23\cdot(32\cdot 27-1)-\frac{3}{7}}}=1/137.035999111818$$

${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{40}_{18}Ar_{22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{55}_{25}Mn_{30}$ ${}^{59}_{27}Co_{32}$ ${}^{72,74}_{32}Ge_{40,42}$ ${}^{92,96,98}_{42}Mo_{50,54,56}$ ${}^{97,99}_{43}Tc^{*}_{54,56}$ ${}^{193}_{77}Co_{115}$

${}^{192,194}_{78}Pt_{114,116}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{24\cdot 13,314}_{125,18\cdot 7}Ch^{ie}_{287,188}$ ${}^{318,320}_{127}Ch^{ie}_{191,193}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{3\cdot 128,2\cdot 193}_{153=9\cdot 17,154=7\cdot 22}Ch^{ie}_{7\cdot 33,8\cdot 29}$

$$\alpha_{2-27}=\frac{3\cdot 3^2\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{81}{80})^{7\cdot 23}}}{4^2\cdot 13}\frac{1}{112-\frac{1}{5\cdot 82}+\frac{1}{2\cdot 27\cdot 43\cdot(3\cdot 64+1)-\frac{19}{26}}}=1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{39,40,41}_{19}K_{20,21,22}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{56,58}_{26}Fe_{30,32}$ ${}^{59}_{27}Co_{32}$ ${}^{86,87,88}_{38}Sr_{48,49,50}$ ${}^{93}_{41}Nb_{52}$ ${}^{97,99}_{43}Tc^{*}_{54,56}$ ${}^{112}_{48}Cd_{64}$

${}^{126,128,130}_{52}Te_{74,76,78}$ ${}^{208=16\cdot 13}_{82}Pt_{126}$ ${}^{344=8\cdot 43,346,348}_{136,137,138=6\cdot 23}Fy^{ie}_{208,209,210}$

$$\alpha_{2-29}=\frac{29\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{16\cdot 41}{5\cdot 131})^{3\cdot 19\cdot 23}}}{223}\frac{1}{112-\frac{1}{29\cdot 59\cdot(12\cdot 19+1)+\frac{19}{29}}}=1/137.035999111818$$

${}^{39,41}_{19}K_{20,22}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{73}_{32}Ge_{41}$ ${}^{131}_{54}Xe_{77}$ ${}^{139}_{57}La_{82}$ ${}^{141}_{59}\mathrm{Pr}_{82}$ ${}^{169}_{69}Tm_{100}$ ${}^{9\cdot 23,208}_{82}Pb_{125,126}$ ${}^{223}_{87}Fr^{*}_{136}$

${}^{330,332}_{130,131}Ch^{ie}_{200,201}$ ${}^{16\cdot 23}_{145=5\cdot 29}Ch^{ie}_{223}$

$$\alpha_{2-31}=\frac{31\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{59}{58})^{9\cdot13}}}{7\cdot34+1}\frac{1}{112-\cfrac{1}{7\cdot43+\cfrac{9}{5\cdot113}}}=1/137.035999111819$$

${}^{63,65}_{29}Cu_{34,36}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{76,77,78}_{34}Se_{42,43,44}$ ${}^{89}_{39}Y_{50}$ ${}^{99}_{43}Tc^{*}_{56}$ ${}^{136,138,140,141}_{58}Ce_{78,80,82,84}$ ${}^{146,147,148,152}_{62}Sm_{84,85,86,90}$

${}^{222}_{86}Rn^{*}_{136}$ ${}^{284}_{113}Nh^{ie}_{171=9\cdot19}$ ${}^{22\cdot17}_{148}Ch^{ie}_{226}$ ${}^{8\cdot49,394}_{155=5\cdot31,156=12\cdot13}Ch^{ie}_{237,7\cdot34}$

$$\alpha_{2-32}=\frac{2\cdot4^2\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{42}{41})^{83}}}{13\cdot19}\frac{1}{112-\cfrac{1}{11\cdot13}+\cfrac{1}{6\cdot37\cdot(5\cdot210-1)+\cfrac{10}{11}}}$$

$=1/137.035999111818$

${}^{39,41}_{19}K_{20,22}$ ${}^{70,72,73,74,76}_{32}Ge_{38,40,41,42,44}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{209}_{83}Bi^{*}_{126}$ ${}^{210}_{84}Po^{*}_{126}$ ${}^{235}_{92}U^{*}_{143}$ ${}^{410}_{163}Ch^{ie}_{247=13\cdot19}$ ${}^{418,420}_{166,168}Ch^{ie}_{252}$

$$\alpha_{2-33}=\frac{33\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{139}{138})^{277}}}{2\cdot(2\cdot8^2-1)}\frac{1}{112-\cfrac{1}{32\cdot48-\cfrac{36}{35\cdot13}}}=1/137.035999111818$$

${}^{75}_{33}As_{42}$ ${}^{84}_{36}Kr_{48}$ ${}^{112}_{48}Cd_{64}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{326,326/328}_{128,129}Ch^{ie}_{198=6\cdot33,197/199}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{12\cdot29}_{139}Ch^{ie}_{209=11\cdot19}$

$$\alpha_{2-36}=\frac{6^2\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{191}{190})^{3\cdot127}}}{2\cdot138+1}\frac{1}{112-\cfrac{1}{10\cdot7\cdot31+\cfrac{13}{25\cdot23}}}=1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{50,51}_{23}V_{27,28}$ ${}^{53}_{24}Cr_{29}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56,57}_{26}Fe_{28,30,31}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{89}_{39}Rb_{50}$ ${}^{95}_{42}Mo_{53}$ ${}^{108}_{46}Pd_{62}$

${}^{112,115,119,122}_{50}Sn_{62,65,69,72}$ ${}^{127}_{53}I_{74}$ ${}^{124,126,128,130}_{52}Te_{72,74,76,78}$ ${}^{185,187}_{75}Re_{110,112}$ ${}^{190}_{78}Pt_{112}$ ${}^{288=8\cdot36}_{115=5\cdot25}Mc^{ie}_{173}$

${}^{318=6\cdot53,320}_{127}Ch^{ie}_{191,193}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{378,380,382}_{150,151,152}Ch^{ie}_{228,229,230}$

$$\alpha_{2-37}=\frac{37\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{2\cdot43}{5\cdot17})^{9\cdot19}}}{3\cdot5\cdot19}\frac{1}{112-\cfrac{1}{4\cdot3\cdot5\cdot13}+\cfrac{1}{5\cdot37^2\cdot149}}=1/137.035999111818$$

${}^{27}_{13}Al_{14}$ ${}^{39}_{19}K_{20}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{89}_{39}Y_{50}$ ${}^{126}_{52}Te_{74}$ ${}^{144,145,146,147,148,150}_{60}Nd_{84,85,86,88,90}$ ${}^{210}_{85}Po^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$

${}^{284}_{113}Nh^{ie}_{171=9\cdot19}$ ${}^{289=17^2}_{114=2\cdot57}Tl^{ie}_{175}$ ${}^{2\cdot149,300}_{119=7\cdot17,120}Ch^{ie}_{179,180}$ ${}^{22\cdot17}_{148=4\cdot37,149}Ch^{ie}_{226,225}$

$$\alpha_{2-38}=\frac{2\cdot19\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{57}{56})^{113}}}{6\cdot7^2-1}\frac{1}{112-\cfrac{1}{3\cdot73}+\cfrac{1}{30(8\cdot27\cdot17+1)-\cfrac{12}{13}}}=1/137.035999111816$$

${}^{19}_{9}F_{10}$ ${}^{39}_{19}K_{20}$ ${}^{56}_{26}Fe_{30}$ ${}^{64,66,68}_{30}Zn_{34,36,38}$ ${}^{87,88}_{38}Sr_{49,50}$ ${}^{125}_{52}Te_{73}$ ${}^{180,181}_{73}Ta_{107,108}$ ${}^{284}_{113}Nh^{ie}_{171=9\cdot19}$ ${}^{289=17^2}_{114=2\cdot57}Tl^{ie}_{175}$

$$\alpha_{2-125}=\frac{5\cdot 5^2\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{4294}{4293})^{8587}}}{31^2}\frac{1}{112-\frac{1}{2159481}}$$

$$=\frac{5\cdot 5^2\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{2\cdot 19\cdot 113}{81\cdot 53})^{31\cdot(12\cdot 23+1)}}}{31^2}\frac{1}{112-\frac{1}{3\cdot 101\cdot(8\cdot 81\cdot 11-1)}}=1/137.035999111818$$

$^{39}_{19}K_{20}$ $^{50,51}_{23}V_{27,28}$ $^{53}_{24}Cr_{29}$ $^{69,71}_{31}Ga_{38,40}$ $^{79,81}_{35}Br_{44,46}$ $^{84,86,88}_{38}Sr_{46,48,50}$ $^{5\cdot 19}_{42}Mo_{53}$ $^{100,101}_{44}Ru_{56,57}$ $^{108}_{46}Pd_{62}$

$^{137}_{56}Ba_{81}$ $^{150,152}_{62}Sm_{88,90}$ $^{9\cdot 19}_{70}Yb_{101}$ $^{205}_{81}Tl_{124}$ $^{11\cdot 19}_{84}Po^*_{125}$ $^{210}_{85}At^*_{125}$ $^{226}_{88}Ac^*_{138}$ $^{258}_{101}Md^*_{157}$ $^{24\cdot 13,314}_{125}Ch^{ie}_{187,188}$

$$\alpha_{2-253}=\frac{253\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{28187}{28186})^{56373}}}{1945}\frac{1}{112-\frac{10411}{8\times 10^{11}}}$$

$$=\frac{11\cdot 23\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{[\frac{71(36\cdot 11+1)}{2\cdot 17(36\cdot 23+1)}]^{3\cdot 19\cdot 23\cdot 43}}}{5(4\cdot 97+1)}\frac{1}{112-\frac{29(360-1)}{8\times 10^{11}}}=1/137.035999111818$$

$^{23}_{11}Na_{12}$ $^{35,37}_{17}Cl_{18,20}$ $^{39,41}_{19}K_{20,22}$ $^{50,51}_{23}V_{27,28}$ $^{63,65}_{29}Cu_{34,36}$ $^{77,78,80}_{34}Se_{43,44,46}$ $^{80,82,84,86}_{36}Kr_{44,46,48,50}$

$^{97,98,99}_{43}Tc^*_{54,55,56}$ $^{163}_{66}Dy_{97}$ $^{175,176}_{71}Lu_{104.105}$ $^{247}_{97}Bk^*_{150}$ $^{248=4\cdot 71}_{113}Nh^{ie}_{171=9\cdot 19}$ $?^{384=3\cdot 128,386}_{153=9\cdot 17,154=14\cdot 11}Ch^{ie}_{21\cdot 11,8\cdot 29}$

$$\alpha_{2-269}=\frac{269\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{41655}{41654})^{83309}}}{2068}\frac{1}{112-5.317\times 10^{-9}}$$

$$=\frac{(4\cdot 67+1)\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{15\cdot(6\cdot(16\cdot 29-1)-1)}{2\cdot 59\cdot(6\cdot 59-1)})^{227\cdot(6\cdot 61+1)}}}{4\cdot 11\cdot 47}\frac{1}{112-\frac{13\cdot(24\cdot 17+1)}{10^{12}}}$$

$$=1/137.035999111818$$

$^{50,52,53}_{24}Cr_{26,28,29}$ $^{76,77,78,80}_{34}Se_{42,43,44,46,48}$ $^{3\cdot 47}_{59}Pr_{82}$ $^{167,168}_{68}Er_{99,100}$ $^{223}_{87}Fr^*_{136}$ $^{294}_{118}Og^{ie}_{176}$ $^{340}_{134,135}Ch^{ie}_{206,205}$ $?^{376=8\cdot 47}_{149}Ch^{ie}_{227}$

In above formulas, there are many amazing coincidences. As 136=8×17 and 138=6×23, 17 and 23 both appear in α_{1-1}, α_{1-17}, α_{1-22}, α_{1-23}, α_{1-25}, α_{1-59}, α_{1-103}, α_{1-133}, α_{2-17} and α_{2-23}, 17 frequently appears in α_1 and 23 frequently appears in α_2. 157 and 257 in α_{1-50} should relate to $_{100}Fm^*_{157}$, 173 in α_{1-16} should relate to $_{112}Cn^*_{173}$, and so on. As the factors in formulas of α are reasonably assumed to relate to nuclides, some ideal extended elements such as $_{136,137,138}Fy_{208,209,210}$ and $_{169}Ch_{257}$ are predicted.

15. Radius of Electron and Proton

The classical electron radius r_e has been calculated very accurately. However, the proton charge radius r_p hasn't yet been determined precisely. Recent two experiments measured r_p and had given the best results up to now which was r_p=0.833(19) fm[9] and r_p=0.831(19) fm[10], and hence CODADA revised its recommended data of r_p to 0.8414(19) fm. Here we give our calculation results of r_e and r_p. And it seems there is α_p similar to α. α_p could be called "the second fine-structure constant".

Ratio of Bohr radius of hydrogen atom to classical electron radius:

$$\frac{a_0}{r_e}=\frac{1}{\alpha_c^2}=\frac{1}{\alpha_1\alpha_2}=112\times(168-\frac{1}{3}+\frac{1}{2^2\cdot 3\cdot 47}-\frac{1}{2\cdot 3\cdot 29\cdot 53\cdot 59-79/47})=18788.865042381$$

$$r_e=\alpha_c^2 a_0=\alpha_1\alpha_2 a_0=\frac{5.29177210903(80)\times 10^{-11}m}{18788.865042381}=2.81794032658(43)\ fm$$

Comparable to CODATA recomended value $r_e=2.8179403262(13)\ fm$ but more precise.

Ratio of Bohr radius of hydrogen atom to the proton charge radius should have the similiar form, and is assumed to have the following hypothetical formulas:

$$\frac{a_0}{r_p}=\frac{1}{\alpha_{p/c}^2}=\frac{1}{\alpha_{p/1}\alpha_{p/2}}=225\cdot(282+\frac{1}{3}-\frac{1}{12\cdot 47}+\frac{1}{6\cdot 29\cdot 53\cdot 59-79/47})=63524.60147736$$

$$=247\cdot(257+\frac{1}{5}-\frac{1}{5\cdot 13}+\frac{1}{30\cdot(28\cdot(2\cdot 100-1)+1)+\frac{8}{45}})$$

$$=(252+\frac{1}{24}-\frac{1}{2\cdot 17\cdot 37}+\frac{1}{11\cdot 13\cdot 19\cdot(2\cdot 11\cdot 19+1)+\frac{11}{20}})^2=252.040872632515^2$$

$$r_p=\alpha_{p/c}^2 a_0=\alpha_{p/1}\alpha_{p/2}a_0=\frac{5.29177210903(80)\times 10^{-11}m}{63524.60147736}=0.833027202999(13)\ fm$$

$\alpha_{p/c}\approx\alpha_{p/1}\approx\alpha_{p/2}\approx 252.04$, α_p could be called the second fine-structure constant.

${}^{7}_{3}Li_{4}$ ${}^{12,13}_{6}C_{6,7}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{63,66}_{26}Cu_{34,36}$ ${}^{83,84,86}_{36}Kr_{47,48,50}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{103}_{45}Rh_{58}$ ${}^{107,109}_{47}Ag_{60,62}$ ${}^{107}_{53}I_{2\cdot 37}$

${}^{136,138,140,142}_{58}Ce_{78,80,82,84}$ ${}^{3\cdot 47}_{59}Pr_{82}$ ${}^{3\cdot 53}_{65}Tb_{2\cdot 47}$ ${}^{3\cdot 59,178}_{72}Hf_{105,106}$ ${}^{197}_{79}Au_{2\cdot 59}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Fr^{*}_{138}$ ${}^{293}_{116}Lv^{ie}_{177}$ ${}^{402}_{158}Ch^{ie}_{244}$

${}^{27}_{13}Al_{14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{58,60}_{28}Ni_{30,32}$ ${}^{64,66,67,68}_{30}Zn_{34,36,37,38}$ ${}^{100}_{44}Ru_{56}$ ${}^{103}_{45}Rh_{58}$

${}^{112,114-120,122,124}_{50}Sn_{62,64-70,72,74}$ ${}^{3\cdot 53}_{65}Tb_{2\cdot 47}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{13\cdot 19=247}_{97}Es^{*}_{150}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{410}_{163}Ch^{ie}_{247}$ ${}^{426}_{169}Ch^{ie}_{257}$

${}^{23}_{11}Na_{12}$ ${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{27}_{13}Al_{14}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{39-41}_{19}K_{20-22}$ ${}^{42,44,46,48}_{20}Ca_{22,24,26,28}$ ${}^{46-50}_{22}Ti_{24-28}$

${}^{50,52,53,54}_{24}Cr_{26,28,29,30}$ ${}^{74,76-78,80,82}_{34}Se_{40,42-44,46,48}$ ${}^{85,87}_{37}Rb_{48,50}$ ${}^{86-88}_{38}Sr_{48-50}$ ${}^{90-92,94,96}_{40}Zr_{50-52,54,56}$ ${}^{184,186}_{74}W_{110,112}$

${}^{16\cdot 13}_{82}Pb_{126}$ ${}^{11\cdot 19}_{83}Bi^{*}_{126}$ ${}^{11\cdot 19}_{84}Po^{*}_{125}$ ${}^{235}_{92}U^{*}_{143}$ ${}^{13\cdot 19}_{97}Es^{*}_{150}$ ${}^{252}_{99}Es^{*}_{9\cdot 17}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{410}_{163}Ch^{ie}_{13\cdot 19}$ ${}^{418}_{166,168}Ch^{ie}_{252,250}$ 2019/12/19-23

$$\alpha_{p/1}=\frac{5\cdot 3^2}{8\cdot(2\pi)_{58}}\frac{1}{225+\frac{1}{4\cdot 112}-\frac{1}{5\cdot 19\cdot 83\cdot 103+\frac{19}{20}}}=1/252.040872632515$$

${}^{19}_{9}F_{10}$ ${}^{39}_{19}K_{20}$ ${}^{55}_{25}Mn_{30}$ ${}^{87}_{38}Sr_{49}$ ${}^{95}_{42}Mo_{53}$ ${}^{112}_{48}Cd_{64}$ ${}^{103}_{45}Rh_{58}$ ${}^{140,142}_{58}Ce_{82,84}$ ${}^{158,160}_{64}Gd_{94,96}$ ${}^{7\cdot 23}_{66}Dy_{95}$

${}^{173}_{70}Yb_{103}$ ${}^{185,187}_{75}Rw_{110,112}$ ${}^{11\cdot 19}_{83}Bi^{*}_{126}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{3\cdot 81}_{95}Am^{*}_{148}$ ${}^{262}_{103}Lr^{*}_{159}$ ${}^{15\cdot 19}_{112}Cn^{*}_{173}$ ${}^{418}_{166}Ch^{ie}_{252}$

$$\alpha_{p/2}=\frac{23\cdot(2\pi)_{227}}{2\cdot 9^2}\frac{1}{225-\frac{1}{3\cdot 16\cdot 29-\frac{71}{2\cdot 67}}}=1/252.040872632514$$

${}^{50,51}_{23}V_{27,28}$ ${}^{50,51,52,53,54}_{24}Cr_{26,27,28,29,30}$ ${}^{59}_{27}Co_{32}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{79,81}_{35}Br_{44.46}$ ${}^{112}_{48}Cd_{64}$ ${}^{134,136,137,138}_{56}Ba_{78,80,81,82}$ ${}^{138}_{57}La_{81}$

${}^{136,138,140}_{58}Ce_{78,80,82}$ ${}^{175,176}_{71}Lu_{104,105}$ ${}^{7\cdot 29,205}_{81}Tl_{122,124}$ ${}^{223}_{87}Fr^{*}_{136}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{340}_{134,135}Ch^{ie}_{206,205}$ ${}^{8\cdot 47}_{149}Ch^{ie}_{227}$ 2020/1/2

$$\alpha_{p/2}=\frac{22\cdot(2\pi)_{164}}{5\cdot 31}\frac{1}{225-\frac{1}{7\cdot 137-\frac{7\cdot 13}{197}}}=1/252.040872632512 \qquad 2020/1/3$$

${}^{47,48,49,50}_{22}Ti_{25.26,27,28}$ ${}^{56,57}_{26}Fe_{30.31}$ ${}^{69,71}_{31}Ga_{38.40}$ ${}^{89}_{39}Y_{50}$ ${}^{99,100}_{44}Ru_{55.56}$ ${}^{134,136,137,138}_{56}Ba_{78,80,81,82}$ ${}^{5\cdot 31}_{64}Gd_{7\cdot 13}$ ${}^{164}_{66}Dy_{98}$ ${}^{196,198}_{78}Pt_{118,120}$

${}^{197}_{79}Au_{118}$ ${}^{206,207,208}_{82}Pb_{124,125,126}$ ${}^{223}_{87}Ra^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{326,326/328}_{128,129}Ch^{ie}_{198,197/199}$ ${}^{372=12\cdot 31}_{147}Ch^{ie}_{225}$ ${}^{8\cdot 49,2\cdot 197}_{155=5\cdot 31,156=12\cdot 13}Ch^{ie}_{237,238}$ ${}^{416=32\cdot 13}_{164}Ch^{ie}_{252}$

$$\alpha_{p/2}=\frac{21\cdot(2\pi)_{126}}{2^2\cdot 37}\frac{1}{225-\frac{1}{16\cdot 29}+\frac{1}{20\cdot 13^2\cdot 179+\frac{8}{17}}}=1/252.040872632515 \qquad 2020/1/3$$

${}^{45}_{21}Sc_{24}$ ${}^{63,65}_{29}Cu_{34.36}$ ${}^{85,87}_{37}Rb_{48.50}$ ${}^{126}_{52}Te_{74}$ ${}^{148}_{60}Nd_{88}$ ${}^{169}_{69}Tm_{100}$ ${}^{179}_{72}Hf_{107}$ ${}^{16\cdot 13}_{82}Pb_{126}$ ${}^{298,300}_{119,120}Ch^{ie}_{179,180}$ ${}^{312,314}_{125,126}Ch^{ie}_{187,188}$ ${}^{426}_{169}Ch^{ie}_{257}$

16. Direct Relationships of 2π with Nuclides

In Chen's formulas of the fine-structure constant, there are 2π-e formulas, in which k gets certain numbers and relate to nucleon numbers of some nuclides. So in the end of this paper we feel curious about whether 2π directly relate to nuclides.

$$(2\pi)_k=\left(\frac{e}{e^{\gamma_{c/k}}}\right)^2=e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{k+1}{k})^{2k+1}}, \qquad \alpha_1=\frac{n}{m\cdot(2\pi)_k}\frac{1}{112+\delta_1} \qquad \alpha_2=\frac{m\cdot(2\pi)_k}{n}\frac{1}{112-\delta_2}$$

$$2\pi=6.2831853\cdots\approx\frac{4\cdot 157}{100}=6.28\approx\frac{3\cdot 7\cdot 44\cdot 68}{100^2}=6.2832$$
${}^{7}_{3}Li_{4}$ ${}^{100}_{44}Ru_{56}$ ${}^{157}_{64}Gd_{93}$ ${}^{168}_{68}Er_{100}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{400}_{157}Ch^{ie}_{243}$

$$2\pi\approx\frac{4\cdot 157}{100}=\frac{157}{25}=6.28$$
${}^{55}_{25}Mn_{30}$ ${}^{100}_{44}Ru_{56}$ ${}^{157}_{64}Gd_{93}$ ${}^{118,119,120}_{50}Sn_{68,69,70}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{185,187}_{75}Re_{110,112}$

${}^{200}_{80}Hg_{120}$ ${}^{257}_{100}Fm^{*}_{157}$ ${}^{258}_{101}Md^{*}_{157}$ ${}^{259}_{102}No^{*}_{157}$ ${}^{312,2\cdot 157}_{125,126}Ch^{ie}_{11\cdot 17,188}$ ${}^{400}_{157}Ch^{ie}_{243}$

$$2\pi\approx\frac{16\cdot 3\cdot 7\cdot 11\cdot 17}{100^2}=\frac{48\cdot 7\cdot 11\cdot 17}{100^2}=\frac{3\cdot 112\cdot 11\cdot 17}{100^2}=\frac{3\cdot 7\cdot 44\cdot 68}{100^2}=\frac{3\cdot 7\cdot 22\cdot 136}{100^2}=\frac{3\cdot 28\cdot 22\cdot 34}{100^2}=\cdots=6.2832$$
${}^{7}_{3}Li_{4}$ ${}^{20,21,22}_{10}Ne_{10,11,12}$ ${}^{23}_{11}Na_{12}$ ${}^{45}_{21}Sc_{24}$ ${}^{46,47,49,50}_{22}Ti_{24,25,27,28}$ ${}^{61}_{28}Ni_{33}$ ${}^{55}_{25}Mn_{30}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{78,80}_{34}Se_{44,46}$ ${}^{98}_{42}Mo_{56}$ ${}^{100}_{44}Ru_{56}$ ${}^{112}_{48}Cd_{64}$

${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{168}_{68}Er_{100}$ ${}^{185,11\cdot 17}_{75}Re_{110,112}$ ${}^{209}_{84}Po^{*}_{125}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{223}_{87}Fa^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{278+7}_{112}Cn^{*}_{173}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$

$$2\pi\approx\frac{44}{7}=\frac{2\cdot 22}{7}=6.2857\cdots$$
${}^{50}_{22}Ti_{28}$ ${}^{61}_{28}Ni_{33}$ ${}^{100}_{44}Ru_{56}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{294}_{118}Og^{ie}_{176}$ ${}^{22\cdot 17}_{148=4\cdot 37}Ch^{ie}_{226}$

$$2\pi\approx\frac{201}{32}=\frac{3\cdot 67}{32}=6.2812\cdots$$
${}^{32}_{16}S_{16}$ ${}^{59}_{27}Co_{32}$ ${}^{67}_{30}Zn_{37}$ ${}^{112}_{48}Cd_{64}$ ${}^{117}_{50}Sn_{67}$ ${}^{128,134}_{54}Xe_{74,80}$ ${}^{134}_{56}Ba_{78}$ ${}^{165}_{67}Ho_{98}$ ${}^{201}_{80}Hg_{121}$ ${}^{332}_{131}Ch^{ie}_{201}$ ${}^{402}_{158}Ch^{ie}_{244}$

$$2\pi\approx\frac{245}{39}=\frac{5\cdot 7^2}{3\cdot 13}=6.2820\cdots$$
${}^{7}_{3}Li_{4}$ ${}^{27}_{13}Al_{14}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{89}_{39}Y_{50}$ ${}^{79,81}_{35}Br_{44,46}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{24\cdot 13,314}_{125,126}Ch^{ie}_{187,188}$

$$2\pi\approx\frac{289}{46}=\frac{17^2}{2\cdot 23}=6.2826\cdots$$
${}^{3\cdot 17}_{23}V_{28}$ ${}^{78,80}_{34}Se_{44,46}$ ${}^{6\cdot 17}_{46}Pd_{56}$ ${}^{168}_{68}Er_{100}$ ${}^{169}_{69}Tm_{100}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{11\cdot 17}_{75}Re_{112}$ ${}^{222}_{86}Rn^{*}_{136}$

${}^{223}_{87}Fa^{*}_{136}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{227}_{89}Ac^{*}_{138}$ ${}^{238}_{92}U^{*}_{146}$ ${}^{17\cdot 17}_{114}Fl^{ie}_{175}$ ${}^{344,346,348}_{136,137,138}Fy^{ie}_{208,209,210}$ ${}^{22\cdot 17}_{148=4\cdot 37}Ch^{ie}_{226}$

$$2\pi\approx\frac{333}{53}=\frac{9\cdot 37}{53}=6.2830\cdots$$
${}^{85,87}_{37}Rb_{48,50}$ ${}^{3\cdot 37=111}_{48}Cd_{7\cdot 9}$ ${}^{127}_{53}I_{74}$ ${}^{180,184,189}_{74}W_{106,110,112}$ ${}^{222}_{86}Rn^{*}_{136}$ ${}^{269}_{106}Sg^{*}_{163}$ ${}^{280}_{111}Rg^{*}_{169}$ ${}^{22\cdot 17}_{148}Ch^{ie}_{226}$

$$2\pi\approx\frac{377}{60}=\frac{13\cdot 29}{4\cdot 3\cdot 5}=6.2833\cdots$$
${}^{24,25,26}_{12}Mg_{12,13,14}$ ${}^{28,29,30}_{14}Si_{14,15,16}$ ${}^{31}_{15}P_{16}$ ${}^{54,56}_{26}Fe_{28,30}$ ${}^{63,65}_{29}Cu_{34,36}$ ${}^{116,120}_{50}Sn_{66,70}$ ${}^{24\cdot 13,314}_{125,126}Ch^{ie}_{187,188}$

${}^{140,142}_{58}Ce_{82,84}$ ${}^{144,145,146,148,150}_{60}Nd_{84,85,86,88,90}$ ${}^{200}_{80}Hg_{120}$ ${}^{223}_{87}Fa^{*}_{136}$

$$2\pi\approx\frac{465}{74}=\frac{30\cdot 31}{4\cdot 37}=6.2837\cdots$$
${}^{31}_{15}P_{16}$ ${}^{67}_{30}Zn_{37}$ ${}^{69,71}_{31}Ga_{38,40}$ ${}^{6\cdot 31}_{74}W_{112}$ ${}^{85,67}_{37}Rb_{48,50}$ ${}^{4\cdot 37}_{60}Nd_{88}$ ${}^{157}_{64}Gd_{93}$ ${}^{243}_{95}Am^{*}_{4\cdot 37}$ ${}^{22\cdot 17}_{148=4\cdot 37}Ch^{ie}_{226}$

$$2\pi\approx\frac{509}{81}=\frac{2\cdot 3\cdot 5\cdot 17-1}{9^2}=6.2839\cdots$$
${}^{19}_{9}F_{10}$ ${}^{35,37}_{17}Cl_{18,20}$ ${}^{64,70}_{30}Zn_{34,40}$ ${}^{80,82}_{34}Se_{46,48}$ ${}^{136,137,138}_{56}Ba_{80,81,82}$ ${}^{203,205}_{81}Tl_{61,62}$ ${}^{210}_{85}At^{*}_{125}$

${}^{3\cdot 81}_{95}Am^{*}_{148}$ ${}^{345-348}_{136-138}Fy^{*}_{209,210}$ ${}^{22\cdot 17}_{148=4\cdot 37}Ch^{ie}_{226}$ ${}^{400}_{157}Ch^{ie}_{3\cdot 81}$

$$2\pi\approx\frac{622}{99}=\frac{4\cdot(8\cdot 3\cdot 13-1)}{9\cdot 22}=6.2828\cdots$$
${}^{23}_{11}Na_{12}$ ${}^{27}_{13}Al_{14}$ ${}^{46,48,29}_{22}Ti_{24,26,27}$ ${}^{50,52,54}_{24}Cr_{26,28,30}$ ${}^{54,56,58}_{26}Fe_{28,30,32}$ ${}^{99}_{44}Ru_{55}$ ${}^{167}_{68}Er_{99}$ ${}^{252}_{99}Es^{*}_{153}$

$$2\pi\approx\frac{2\cdot 355}{113}=\frac{4\cdot 5\cdot 71}{2\cdot 113}=6.2831858\cdots$$
${}^{71}_{31}Ga_{40}$ ${}^{112,113}_{48}Cd_{64,65}$ ${}^{113,115}_{49}In_{64,66}$ ${}^{120,122}_{50}Sn_{70,72}$ ${}^{2\cdot 71}_{60}Nd_{82}$ ${}^{171}_{70}Yb_{101}$ ${}^{175}_{71}Lu_{104}$ ${}^{186}_{74}W_{112}$

${}^{187}_{75}Re_{112}$ ${}^{188,189}_{76}Os_{112,113}$ ${}^{226}_{88}Ra^{*}_{138}$ ${}^{232}_{90}Th^{*}_{2\cdot 71}$ ${}^{4\cdot 71}_{113}Nh^{ie}_{171}$ ${}^{22\cdot 17}_{148=4\cdot 37}Ch^{ie}_{226}$ ${}^{426=6\cdot 71}_{169}Ch^{ie}_{257}$ 2020/1/8-10

The approximate rational numbers of 2π (could be called 2π formulas) relate to nuclides marvelously. This means 2π (along with 2π-e formula) plays important roles in atomic nuclei, and acts as a rational number rather than an irrational number in the world of atomic nuclei.

Some Chen's formulas of the fine-structure constant and 2π formulas correlate with each others with the same factors and all together relate to the same nuclides. For example, $\alpha_{1\text{-}50}$ and $2\pi\approx 4\times 157/100$ have the same 157 and 100 factors, $\alpha_{1\text{-}50}$ and $2\pi\approx 3\times 7\times 44\times 68/100^2$ have the same 100, 7, 11 and 16 factors, and they relate to the same corresponding nuclides. They also have common factors with $\alpha_{1\text{-}7}$ and $\alpha_{2\text{-}13}$ which should relate to $2\pi\approx 5\times 7^2/3/13$ and $2\pi\approx 13\times 29/4/3/5$.

$$\alpha_{1-9}=\frac{47}{3^2\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{10}{9})^{19}}}\,\frac{1}{112+\frac{1}{4\cdot 17}-\frac{1}{2(8\cdot 9\cdot 37-1)+\frac{3\cdot 17}{157}}}$$

$$\alpha_{1-50}=\frac{2\cdot 257}{100\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{14\cdot 13}{181})^{3\cdot 11^2}}}\,\frac{1}{112+\frac{1}{29\cdot 61+\frac{157}{16\cdot 11}}}$$

$$2\pi\approx\frac{4\cdot 157}{100}=\frac{157}{25}=6.28\qquad 2\pi\approx\frac{3\cdot 7\cdot 44\cdot 68}{100^2}=\frac{3\cdot 7\cdot 11\cdot 17}{25^2}=6.2832$$

$$\alpha_{1-7}=\frac{6^2}{7\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{113}{112})^{225}}}\,\frac{1}{112+\frac{1}{75^2}}\qquad \alpha_{2\text{-}13}=\frac{13\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{279}{278})^{557}}}{10^2}\,\frac{1}{112-\frac{1}{3\cdot 29\cdot 64}}$$

$$2\pi\approx\frac{245}{39}=\frac{5\cdot 7^2}{3\cdot 13}=6.2820\cdots\qquad 2\pi\approx\frac{377}{60}=\frac{13\cdot 29}{4\cdot 3\cdot 5}=6.2833\cdots$$

$\alpha_{1\text{-}22}$ relates to $2\pi\approx 2\times 22/7$, $2\pi\approx 17^2/7/23$ and $2\pi\approx 2\times 355/113$ as follows. And $2\pi\approx 17^2/7/23$ also relates to $\alpha_{1\text{-}1}$, $\alpha_{1\text{-}17}$, $\alpha_{1\text{-}22}$, $\alpha_{1\text{-}23}$, $\alpha_{1\text{-}25}$, $\alpha_{1\text{-}59}$, $\alpha_{1\text{-}103}$, $\alpha_{1\text{-}133}$, $\alpha_{2\text{-}17}$ and $\alpha_{2\text{-}23}$, in which both 17 and 23 factors appear.

$$\alpha_{1-22}=\frac{113}{22\cdot e^2\,\frac{e^2}{(\frac{2}{1})^3}\,\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{27\cdot 29}{2\cdot 17\cdot 23})^{5\cdot(2\cdot 157-1)}}}\,\frac{1}{112+\frac{1}{2\cdot[2\cdot 3\cdot 17\cdot(10\cdot 19+1)+1]+\frac{29}{49}}}$$

$$2\pi\approx\frac{2\cdot 22}{7}=6.2857\cdots,\ 2\pi\approx\frac{17^2}{2\cdot 23}=6.2826\cdots,\ 2\pi\approx\frac{2\cdot 355}{113}=\frac{4\cdot 5\cdot 71}{2\cdot 113}=6.2831858\cdots$$

$\alpha_{1\text{-}13}$ and $\alpha_{1\text{-}43}$ relate to $2\pi\approx 3\times 67/32$, $2\pi\approx 5\times 7^2/39$, $2\pi\approx 17^2/46$ and others as follows.

$$\alpha_{1-13}=\frac{67}{13\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{47}{2\cdot 23})^{3\cdot 31}}}\frac{1}{112+\frac{1}{137}-\frac{1}{6(2\cdot 27\cdot 59+1)+\frac{9}{50}}}$$

$$\alpha_{1-43}=\frac{13\cdot 17}{43\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{3\cdot 67}{200})^{401}}}\frac{1}{112+\frac{1}{8\cdot(12\cdot 83+1)+\frac{4}{3\cdot 13}}}$$

$$2\pi\approx\frac{3\cdot 67}{32},\ 2\pi\approx\frac{5\cdot 7^2}{3\cdot 13},\ 2\pi\approx\frac{289}{46}=\frac{17^2}{2\cdot 23},\ 2\pi\approx\frac{13\cdot 29}{60},\ 2\pi\approx\frac{30\cdot 31}{4\cdot 37}$$

$\alpha_{1\text{-}11}$, $\alpha_{1\text{-}36}$, $\alpha_{2\text{-}24}$, $\alpha_{2\text{-}23}$, $\alpha_{2\text{-}37}$ and $\alpha_{2\text{-}125}$ relate to $2\pi\approx 9\times 37/53$, $2\pi\approx 15\times 31/2/37$ and $2\pi\approx(30\times 17\text{-}1)/81$ as follows.

$$\alpha_{1-11}=\frac{57}{11\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{19}{18})^{37}}}\frac{1}{112+\frac{1}{35}-\frac{1}{88\cdot 41-\frac{5\cdot 53}{22\cdot 13}}}$$

$$\alpha_{1-36}=\frac{5\cdot 37}{6^2\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{3\cdot 79}{4\cdot 59})^{11\cdot 43}}}\frac{1}{112+\frac{1}{5\cdot(31\cdot 42-1)+\frac{3\cdot 31}{14\cdot 13}}}$$

$$\alpha_{2-24}=\frac{2^2\cdot 6\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{63}{2\cdot 31})^{125}}}{5\cdot 37}\frac{1}{112-\frac{1}{257}+\frac{1}{10\cdot(12\cdot 13\cdot 83+1)+\frac{23}{81}}}$$

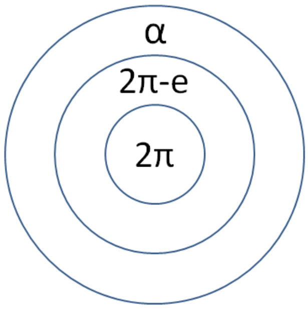
$$\alpha_{2-23}=\frac{23\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{2\cdot 81}{7\cdot 23})^{17\cdot 19}}}{3\cdot 59}\frac{1}{112-\frac{1}{2\cdot(40\cdot 23-1)+\frac{9}{32\cdot 10}}}$$

$$\alpha_{2-37}=\frac{37\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{2\cdot 43}{5\cdot 17})^{9\cdot 19}}}{3\cdot 5\cdot 19}\frac{1}{112-\frac{1}{4\cdot 3\cdot 5\cdot 13}+\frac{1}{5\cdot 37^2\cdot 149}}$$

$$\alpha_{2-125}=\frac{5\cdot 5^2\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{2\cdot 19\cdot 113}{81\cdot 53})^{31\cdot(12\cdot 23+1)}}}{31^2}\frac{1}{112-\frac{1}{101\cdot(20\cdot(12\cdot 89+1)+1)}}$$

$$2\pi\approx\frac{9\cdot 37}{53}=6.2830\cdots\quad 2\pi\approx\frac{15\cdot 31}{2\cdot 37}=6.2837\cdots\quad 2\pi\approx\frac{30\cdot 17-1}{81}=6.2839\cdots$$

In overall, there are multi-correlations among these formulas and nuclides. It seems there should be a mathematical shell model of nuclides, in which the core is 2π formulas and the middle layer is 2π-e formulas and the outer layer is Chen's formulas of the fine-structure constant (**Fig. 9**). The nucleon numbers, stability and abundance of nuclides are regulated by these formulas, especially by their integer factors.



Chen's Mathematic Shell Model of Nuclides

Dr. Gang Chen (2020/1/12-13)

Fig. 9

17. Ideal Extended Elements

In the deduction of Chen's formulas of the fine-structure constant, it was reasonably assumed the factors in them related to nucleon numbers of nuclides, and it seems this assumption is quite correct. So by somewhat correlation and decoding methodology, some ideal extended elements were predicted (**Table 7**). In addition, nuclides can even relate to naked 2π's approximate rational numbers (2π formulas). Some typical example of correlations of ideal extended elements with formulas of α and 2π are listed as follows.

Example 1: Correlations of 100, 157, 257, 169, *et al.*

$$\alpha_{1-50}=\frac{2\cdot 257}{100\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{14\cdot 13}{181})^{3\cdot 11^2}}}\cdot\frac{1}{112+\dfrac{1}{29\cdot 61+\dfrac{157}{16\cdot 11}}}$$

$$\alpha_{2-24}=\frac{2^2\cdot 6\cdot e^2\dfrac{e^2}{(\frac{2}{1})^3}\dfrac{e^2}{(\frac{3}{2})^5}\cdots\dfrac{e^2}{(\frac{63}{62})^{125}}}{5\cdot 37}\cdot\frac{1}{112-\dfrac{1}{257}+\dfrac{1}{10\cdot(12\cdot 13\cdot 83+1)+\dfrac{23}{81}}}$$

also refer to α_{1-5}, α_{1-9} and α_{1-22}

$$2\pi\approx\frac{4\cdot 157}{100}\quad 2\pi\approx\frac{16\cdot 3\cdot 7\cdot 11\cdot 17}{100}\quad 2\pi\approx\frac{4\cdot 11}{7}\quad 2\pi\approx\frac{17^2}{2\cdot 23}\quad 2\pi\approx\frac{30\cdot 31}{4\cdot 37}\quad 2\pi\approx\frac{4\cdot 5\cdot 71}{2\cdot 113}$$

$${}^{99,100}_{44}Ru_{55,66}\quad {}^{168}_{68}Er_{100}\quad {}^{169}_{69}Tm_{100}\quad {}^{208,209}_{83}Bi^{*}_{125,126}\quad {}^{226}_{88}Ra^{*}_{138}\quad {}^{257}_{100}Fm^{*}_{157}$$

$${}^{302}_{121=11^2}Ch^{ie}_{181}\quad {}^{312=24\cdot 13}_{125}Ch^{ie}_{187=11\cdot 17}\quad {}^{374=2\cdot 11\cdot 17}_{148=4\cdot 37}Ch^{ie}_{226}\quad {}^{400}_{157}Ch^{ie}_{243=3^5}\quad {}^{402}_{158}Ch^{ie}_{244=4\cdot 61}\quad {}^{426=6\cdot 71}_{169=13^2}Ch^{ie}_{257}$$

Example 2: Correlations of 83, 126, 84, 125, 209, 112, 173, 285, 115 and 137

$$\alpha_{1-16}=\frac{83}{4^2\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{17}{16})^{33}}}\frac{1}{112+\frac{1}{28}-\frac{1}{6\cdot(18\cdot41+1)+\frac{173}{2\cdot(2\cdot75-1)}}}$$

$$\alpha_{1-25}=\frac{3\cdot43}{5^2\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{35}{34})^{3\cdot23}}}\frac{1}{112+\frac{1}{11\cdot19}-\frac{1}{13^2(2\cdot136-1)+\frac{11}{25}}}$$

$$\alpha_{1-32}=\frac{15\cdot11}{2\cdot2^2\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{41}{40})^{81}}}\frac{1}{112+\frac{1}{25\cdot29-\frac{5\cdot83}{19\cdot23}}}$$

$$\alpha_{2-10}=\frac{10\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{5\cdot21}{8\cdot13})^{11\cdot19}}}{77}\frac{1}{112-\frac{1}{3\cdot14\cdot19}+\frac{1}{14\cdot(4\cdot27\cdot(2\cdot15\cdot19+1)-1)}}$$

$$\alpha_{2-32}=\frac{2\cdot4^2\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{42}{41})^{83}}}{13\cdot19}\frac{1}{112-\frac{1}{11\cdot13}+\frac{1}{6\cdot37\cdot(5\cdot210-1)+\frac{10}{11}}}$$

$$\alpha_{1-13}=\frac{67}{13\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{47}{2\cdot23})^{3\cdot31}}}\frac{1}{112+\frac{1}{137}-\frac{1}{6(2\cdot27\cdot59+1)+\frac{9}{50}}}$$

$$\alpha_{1-17}=\frac{2^2\cdot22}{17\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{21}{20})^{41}}}\frac{1}{112+\frac{1}{137}-\frac{1}{2\cdot19\cdot23\cdot59-\frac{30}{100}}}$$

$$\alpha_{1-27}=\frac{139}{27\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{67}{66})^{7\cdot19}}}\frac{1}{112+\frac{1}{11\cdot47+\frac{18}{23}+\frac{1}{6\cdot23\cdot137}}}$$

$$\alpha_{2-18}=\frac{18\cdot e^2\frac{e^2}{(\frac{2}{1})^3}\frac{e^2}{(\frac{3}{2})^5}\cdots\frac{e^2}{(\frac{38}{37})^{75}}}{139}\frac{1}{112-\frac{1}{2\cdot47}+\frac{1}{2\cdot31\cdot(16\cdot17-1)+\frac{83}{137}}}$$

$$2\pi\approx\frac{16\cdot3\cdot7\cdot11\cdot17}{100}\quad 2\pi\approx\frac{4\cdot11}{7}\quad 2\pi\approx\frac{17^2}{2\cdot23}$$

$${}^{136=8\cdot17,137,138=6\cdot23}_{56}Ba_{80,81,82}\quad {}^{209=11\cdot19}_{83}Bi^{*}_{126}\quad {}^{209}_{84}Po^{*}_{125}\quad {}^{210}_{85}At^{*}_{125}\quad {}^{285=15\cdot19}_{112}Cn^{*}_{173}\quad {}^{346=2\cdot173}_{137}Fy^{ie}_{209}$$

Table 7. Correlations of Ideal Extended Elements with Formulas of α and 2π

Ideal Extended Elements	**Page**	**α**	**2π**
$_{113}Nh_{171}$	10 19 21 28 29 31	α_c^2 $\alpha_{1-5,7}$ $\alpha_{2-22,23,31,37,38,253}$	2π≈4×355/226
$_{114}Fl_{175}$	19 23 28 31	$\alpha_{1-11,133,155}$ $\alpha_{2-37,38}$	2π≈17^2/46
$_{115}Mc_{173}$	20 21 25 31	$\alpha_{1-1,16,23}$ α_{2-5}	2π≈17^2/46
$_{116}Lv_{177}$ $_{117}Ts_{177}$	10 20 22 27 31	$\alpha_{1-13,59}$ α_{2-23} $1/\alpha_c^2$	2π≈622/99

${}_{118}Og_{176}$	20 22 23 27	$\alpha_{1-17,20,50,59,133}$ $\alpha_{2-19,269}$	$2\pi\approx 44/7$
${}_{119,120,121}Ch_{179,180,181}$	21 22 28 31	$\alpha_{1-23,29,50}$ α_{2-37} $\alpha_{p/2}$	$2\pi\approx 44/7$
${}_{125,126}Ch_{187,188}$	11 19 20 21 27 29 31	α_{1-7} $\alpha_{1-9,25}$ $\alpha_{2-25,125}$ $\alpha_{p/2}$	$2\pi\approx 628/100$ *et al.*
${}_{127}Ch_{191,193}$	23 27 28	α_{1-170} $\alpha_{2-25,36}$	
${}_{128,129}Ch_{198,197/199}$	19 23 26 28 31	$\alpha_{1-4,96,155}$ $\alpha_{2-11,33}$ $\alpha_{p/2}$	
${}_{130,131}Ch_{200,201}$	20 22 26 27	$\alpha_{1-13,43}$, $\alpha_{2-13,17,27}$	
${}_{132,133}Ch_{202,203}$	21 23	$\alpha_{1-27,133}$	
${}_{134,135}Ch_{206,205}$	19 20 21 22 29 30 31	$\alpha_{1-5,13,17,27,43,81}$ α_{2-269} $\alpha_{p/2}$	$2\pi\approx 3\times 134/64$
${}_{136,137,138}Fy_{208,209,210}$	11 20 21 26 27 28	$\alpha_{1-13,16,17,25,27}$ $\alpha_{2-10,18,25,27,33,36}$	
${}_{139}Ch_{209}$	10 21 26 28 30	α_c^2 α_{1-27} $\alpha_{2-13,33}$ α_c^2	
${}_{144-149}Ch_{222-227}$	11 19 21 23 27 28 29 30 31	$\alpha_{1-7,22,29,31,96}$ $\alpha_{2-19,29,31,37,269}$ $\alpha_{p/2}$	$2\pi\approx 930/148$ $2\pi\approx 1420/226$ *et al.*
${}_{150,151,152}Ch_{228,229,230}$	19 20 28	$\alpha_{1-4,19}$ α_{2-36}	
${}_{153,154}Ch_{231,232}$	11 19 20 21 22 23 25 27 29	$\alpha_{1-6,20,22,23,33,96}$ $\alpha_{2-7,25,253}$	
${}_{155,156}Ch_{237,238}$	11 19 20 21 22 23 28	$\alpha_{1-6,20,31,36,59,155}$ α_{2-31} $\alpha_{p/2}$	
${}_{157}Ch_{243}$	11 20 21 22 31	$\alpha_{1-9,22,50}$	$2\pi\approx 628/100\approx 509/81$
${}_{158}Ch_{244}$	10 11 22 25 10 30	$1/\alpha_c^2$ α_{1-50} α_{2-5} $1/\alpha_c^2$	$2\pi\approx 201/32$
${}_{163}Ch_{247}$	11 23 26 28 30	α_{1-96} $\alpha_{2-11,111}$ $1/\alpha_{p/c}^2$	
${}_{166,168}Ch_{250/252}$	11 20 21 22 26 27 28 30 31	$\alpha_{1-16,25,32,33,43}$ $\alpha_{2-10,18,24,32}$ $1/\alpha_{p/c}^2$ $\alpha_{p/1}$	
${}_{169}Ch_{257}$	19 22 27 31	$\alpha_{1-5,50}$ α_{2-24}	$2\pi\approx 710/113$ *et al.*
${}_{170}Ch_{250}$	23 26	$\alpha_{1-155,170}$ α_{2-11}	

18. Chen's Picture of Elements and Ideal Extended Elements

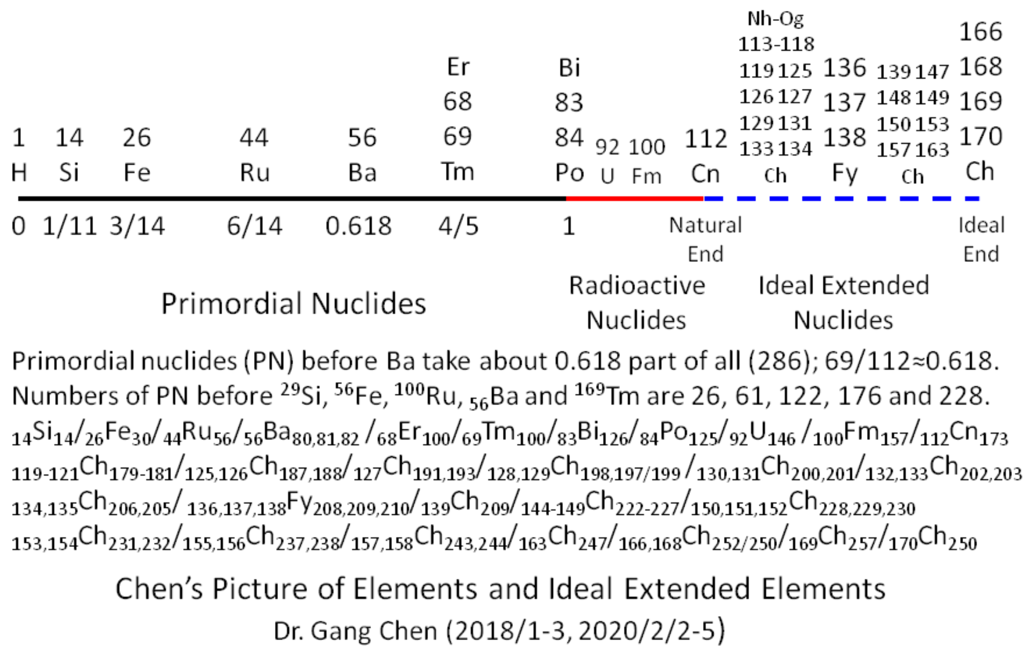


Fig. 10

The relationships between elements and ideal extended elements (the frontier of elements) and an overall picture of them were depicted as above.

20. Discussion and Conclusion

Regarding the fine-structure constant, Richard Feynman said: "is it related to π or perhaps to the base of natural logarithms? "[4] Our answer is that it relate to 2π-e formula. He also deduced that the maximum element should be the 137th element Fynmanium (Fy) based on the analyses of the electron line velosity of his ideal hydrogen-like atoms. Our answer is that the natural end of the elements is the 112th element Copernicium (Cn^*) , but the elements could have some ideal extensions, and above all, the fine-structure constant does relate to elements.

So, based on the analyses of ideal and real natural maximum element, Chen's Chirality and Poetry Model of Atomic Nuclei[7] and 2π-e formula[6,7,8], we deduced two series of Chen's formulas of the fine-structure constant which gave two values α_1=1/137.035999037435 and α_2=1/137.035999111818. The factors in the formulas are much coincident to nucleon numbers of some nuclides, this means the formulas should be correct (too many coincidences mean too few possibilities to be wrong). And we indicated the reason of $\alpha\approx1/137.036$ is that it's almost the equal ratio factor between 112 and 168 (more precisely 168-1/3) which are the key stable numbers (magic numbers) in Chen's Chirality and Poetry Model of Atomic Nuclei[7].

With Chen's formulas of the fine-structure constant, we predicted the nucleon numbers of some ideal extended elements from 119th to 170th; we theoretically or mathematically calculated the speed of light in atomic units, i.e., $c_{au}=1/\alpha_c= 1/(\alpha_1\alpha_2)^{1/2}$ =137.035999074627; we deduced a concise Schrödinger-Chen equation of hydrogen atom which included α_1/α_2 factor; in analogy to α and its formulas, α_p (the second fine-structure constant) and its formulas were hypothesized, and the proton charge radius r_p was supposed to be 0.833027203 fm; in the end we discovered that the approximate rational values of 2π marvelously and directly related to nuclides. Based on these, a mathematic shell model of elements was established and a picture of elements and ideal extended elements was depicted.

In their relations to nuclides, 2π formulas can only be certain approximate rational numbers and 2π-e formulas in Chen's formulas of the fine-structure constant can only take certain k values. So we also believe the two values of the fine-structure constant should be rational numbers with definite digits rather than irrational numbers with infinite digits, and actually the fine-structure constant is transformed to nucleon numbers of 136,137 and 138 in the world of nuclides.

In a recent paper[11], physicist Nicolas Gisin commented that in 1920s there once was a debate between mathematicians David Hilbert and Luitzen Egbertus Jan Brouwer. Hilbert was promoting formalized mathematics, in which every real number with its infinite series of digits is a completed individual object. On the other side the Luitzen Egbertus Jan Brouwer was defending the view that each point on the line should be represented as a never-ending process that develops in time, a view known as intuitionistic mathematics. Hilbert and his supporters clearly won the debate. In consequence, formalized mathematics has been adopted as the language of physics. In the end of his paper, Nicolas Gisin said: “Physics can be as successful if built on intuitionistic mathematics, even if this breaks its marriage to determinism. Contrary to usual expectations, I bet that the next physical theory will not be even more abstract than quantum field theory, but might well be closer to human experience.”

In this paper we adopted mathematical language like intuitionistic mathematics, but we go ahead even more. The formulas of 2π, 2π-e and the fine-structure constant consist of integer factors and relate to nucleon numbers of nuclides, and hence correlate with each others. So in this paper we may use super-intuitionistic mathematics or decoding methodology with features of multi-correlations of integer factors or rational numbers with nucleon numbers of nuclides, and it seems it is the real language in the world of nuclides. As we know an atomic nucleus is a N-body system and chaos should be its real state, so it seems N-body chaos returns to integers. In overall, Leopold Kronecker’s famous saying “God made the integers, all else is the work of man” should be correct in the world of nuclides or even atoms. It seems an irrational number can only be a rational number to play roles in the real world.

“God is a pure mathematician!” declared British astronomer Sir James Jeans(1877-1946). The physical Universe does seem to be organised around elegant mathematical relationships[3]. The fine-structure constant may be the most important number in physics. As it is dimensionless, it could be called the proportional ruler of the nature or the bridge of mathematics and physics. And we have successfully given reasonable and precise formulas of it. In some sense, we explain the bridge between mathematics and physics, or we may realize the unification of mathematics and physics. It seems we prove the saying “God is a pure mathematician”. At least, it seems that good mathematics means good physics, and some pure mathematical numbers do have scientific meanings.

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Acknowledgements

The main author Dr. Gang Chen studied in the Department of Chemistry at Sichuan University from 1983 to 1987 (Bachelor's degree), in Institute of Chemistry of the Academy of Sciences of China from 1987-1990 (Master degree), in the Hong Kong Polytechnic University from 1999 to 2004 (Ph. D and research assistant) and in Kyoto University from 2004 to 2005 (postdoctoral research).

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Appendix I: Research History

Items	Page	Discover/Create	Revise/Supplement
2π-e formula	3	2013/4-12	
Formulas related to 2π-e formula	4	2013/4-12	
Preliminary applications of 2π-e formula and its related formulas	6	2013/4-12	
Chen's Periodic Table of Elements and Natural Group Theory	6	2014-2017/12	
Chirality and Poetry Model of Atomic Nuclei	6	2017/12-2018/3	
Chen's theory of the fine-structure constant	6	2018/4-6	2018/7-2020/1
Original Inspiration for Formulas of the Fine-structure Constant	6	2018/4/12	
Logical deduction of Chen's formulas of the fine-structure constant	6 7	2018/4/12-24	
α_1 ($\alpha_{1\text{-}7}$)	6 7	2018/4/12	2018/4/20 (+1/75^2)
α_2 ($\alpha_{2\text{-}13}$)	7	2018/4/24	2018/9/18-20 (280→278 *et al.*)
Calculation tables and diagrams of $\alpha_{1/2}$	8	2018/4/12-24	2018/9/18-20
$\alpha_{1\text{-}(3/2)}$	9	2019/4/25	
$\alpha_{2\text{-}(3/2)}$	9	2019/4/25	
α_c^2	10	2018/6/8-9, 9/18-19; 2019/4/17-19	
$1/\alpha_c^2$	10	2019/12/14	
112/137≈137/168 *et al.*	11	2018/4-6	
$_{136,137,138}Fy_{208,209,210}$	11	2019/12-2020/1	
$_{125,126}Ch$, $_{144\text{-}149}Ch$, $_{153,154}Ch$, $_{155,156}Ch$, $_{157}Ch$, $_{163}Ch$, $_{164\text{-}168}Ch$, $_{169}Ch$	11	2019/12-2020/1	
c_{au} formulas	12	2019/12/16	2020/1/5-8
The special 29 and 75 factors	12-15	2019/4/22-24	
α_1/α_2 in Schrödinger Equation of Hydrogen Atom	16	2018/4-6	2019/12/13
α_1/α_2	16	2019/8/28-29	
The two kinds of general formulas of the fine-structure constant	17	2019/6/27	2019/7/2-3
Parameters and results of approximate formulas of α_1	18	2019/7/2	
$\alpha_{1\text{-}1}$	19	2019/7/2	
$\alpha_{1\text{-}2}$	19	2019/6/26	
$\alpha_{1\text{-}3}$	19	2019/5/26	2019/6/26
$\alpha_{1\text{-}4}$	19	2019/6/27	

α_{1-5}	19	2019/6/27	
α_{1-6}	19	2019/6/27	
α_{1-7} (α_1)	19	2018/4/12	2018/4/20 (+1/75^2)
α_{1-9}	20	2019/6/28	
α_{1-11}	20	2019/6/29	
α_{1-13}	20	2019/6/29	
α_{1-16}	20	2019/6/29-30	
α_{1-17}	20	2019/6/30	2019/10/29
α_{1-19}	20	2019/7/1	
α_{1-20}	20	2018/4-6	2019/6/26
α_{1-22}	21	2019/5/25	2019/12/12
α_{1-23}	21	2019/7/4	
α_{1-25}	21	2019/7/4	
α_{1-27}	21	2019/5/25-26	
α_{1-29}	21	2019/5/25	
α_{1-31}	21	2019/7/4	
α_{1-32}	21	2019/7/5	
α_{1-33}	22	2019/7/5	
α_{1-34}	22	2019/5/24	
α_{1-36}	22	2019/5/24	
α_{1-43}	22	2019/12/15	
α_{1-50}	22	2018/4-6	2019/6/26
α_{1-59}	22	2019/5/25	
α_{1-81}	22	2019/5/25-26	
α_{1-96}	23	2019/5/25-26	2019/12/29
α_{1-103}	23	2019/12/15	
α_{1-133}	23	2019/5/26	2019/12/29
α_{1-140}	23	2019/5/26	
α_{1-155}	23	2019/12/15	
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Parameters and results of approximate formulas of α_2	24	2019/7/3	
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α_{2-4}	25	2019/5/28	
α_{2-5}	25	2019/6/22	
α_{2-6}	25	2019/6/23	
α_{2-7}	25	2019/5/24	
α_{2-9}	25	2019/6/21	
α_{2-10}	26	2019/5/28	
α_{2-11}	26	2019/6/22	
α_{2-13} (α_2)	26	2018/4/24	2018/9/18-20 (280→278 *et al.*)

α_{2-15}	26	2019/6/19	
α_{2-17}	26	2019/6/24	
α_{2-18}	26	2019/6/24	
α_{2-19}	27	2019/6/24	
α_{2-23}	27	2019/6/23	
α_{2-24}	27	2019/6/23	
α_{2-25}	27	2019/6/24	
α_{2-27}	27	2019/6/21	
α_{2-29}	27	2019/6/20	
α_{2-31}	28	2019/7/6	
α_{2-32}	28	2019/7/6	
α_{2-33}	28	2019/7/6	
α_{2-36}	28	2019/6/25	
α_{2-37}	28	2019/7/7	
α_{2-38}	28	2019/7/7	
α_{2-125}	29	2019/5/25	2019/12/20
α_{2-253}	29	2019/7/3	2019/7/9
α_{2-269}	29	2019/7/7	2019/12/19
a_0/r_e, r_e; a_0/r_p, r_p	30	2019/12/19-23	
$\alpha_{p/1}$	30	2020/1/2	
$\alpha_{p/2}$	30-31	2020/1/2-3	
Direct relationships of 2π with nuclides	31	2020/1/8-10	
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$_{130,131}Ch_{200,201}$	20 22 26 27	2020/1/26-28	
$_{119,120,121}Ch_{179,180,181}$	21 28 31	2020/1/28-29	2020/2/5 (add 121)
$_{128,129}Ch_{198,197/199}$	23 26 31	2020/1/28-29	2020/1/31
$_{139}Ch_{209}$	21 26 28	2020/1/29	
$_{132,133}Ch_{202,203}$	21 23	2020/1/31	
$_{169}Ch_{257}$	11 22 27	2020/1/29-30	
$_{157}Ch_{243}$	11 20 21 22 31	2019/1/8	
$_{158}Ch_{243}$	11 20 22 25 30 31	2019/1/31	
$_{169}Ch_{257}$	11 22 27	2020/1/29-30	
$_{134,135}Ch_{206,205}$	20	2020/1/31	
$_{127}Ch_{191,193}$	27	2020/1/31	2020/2/1 (add 191)
$_{150,151,152}Fy_{228,229,230}$	19 28	2020/2/2	
$_{170}Ch_{250}$	23 26	2020/2/3	
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