Noname manuscript No.
(will be inserted by the editor)

Infinity furthers anomaly in the application of complex numbers

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Received: date / Accepted: date

Abstract In the paper an anomaly in complex number theory is reported. Similar to a previous note, the ingredients of the analysis are Euler's identity and the DeMoivre rule for $n = 2$. If a quadratic and definitely not weak equation has two solutions, then, a contradiction can be derived from ± 1 functions in complex number theory. A constructivist finite approach to cos and sin is briefly discussed to resolve the anomaly.

Keywords Basic complex number theory · Euler's identity and the DeMoivre rule · conflicting solution

Mathematics Subject Classification (2010) 00A05 · 03A05

1 Introduction

Complex number theory is a well established and broadly applied theory of numbers. Through the introduction of complex numbers with a real part based on the unit 1 and an imaginary part based on the unit $i = \sqrt{-1}$, we have that each polynomial function, $f(z)$ of degree n has n solutions $z_1, z_2, \dots z_n$ for $f(z) = 0$. Let us look at a particular $n = 2$ polynomial, $f(z) = z^2 - c$, and $c \not\equiv 0$ and possibly complex. It is therefore easy to establish that there are *two distinct* solutions $z_1 = \sqrt{c}$ and $z_2 = -\sqrt{c}$. By introduction of a number $\eta = \pm 1$ we are then allowed to write $z(\eta) = \eta\sqrt{c}$. E.g. we may write $z_1 = z(1)$ and $z_2 = z(-1)$. This is all very basic and in case of the quadratic refers back to the famous abc formula..

In the present short paper the previous elementary material is, despite its widely accepted use, studied more deeply. It is found that perhaps there is

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a problem with consistency. In the paper a possible anomaly in elementary complex number theory [1] in relation to sign type functions, is reported.

In the paper only one textbook reference is presented because it is unknown to the author if other modern research into this matter exists. In elementary complex number theory [1] there are two basic principles that will be employed here. The first is Euler's identity. This is $\forall_{t\in\mathbb{R}}\ e^{it} = \cos(t) + i\sin(t)$. The second one is the power rule of DeMoivre. This is, $\forall_{n\in\mathbb{N}}\left(\cos(x) + i\sin(x)\right)^n = \cos(nx) + i\sin(nx)$. Here we will look at $n = 2$.

2 Quadratic form

If the theory of complex numbers is consistent then it must be impossible to, with the use of valid derivation steps, arrive at a contradiction. If, on the other hand, a contradiction is validly arrived at, the result is perhaps similar to Gödel's result [4] but then accomplished in concrete mathematics [5].

Basing ourselves on the introductory remarks, let us look at the following quadratic equation.

$$z^2 = \exp\left[2i(\varphi+\psi)^2\right] \tag{1}$$

When $\eta = \pm 1$ we also may write

$$z^2 = \eta^2 \exp\left[2i(\varphi+\psi)^2\right] \tag{2}$$

If we, subsequently, take $\varphi + \psi = \sqrt{\pi}$ then, $z^2 = 1$. Therefore, $z = \pm 1$. This can, obviously, be written like $z(\eta_0) = \eta_0$ and $\eta_0 = \pm 1$ or

$$z = \eta e^{i\pi} = \eta_0 \tag{3}$$

Let us in the next step explicitly compute $(\varphi+\psi)^2$. We have

$$(\varphi+\psi)^2 = \varphi^2 + \psi^2 + 2\varphi\psi \tag{4}$$

This is all quite elementary. Let us, to continue, define α as

$$\alpha =_{def} \varphi\left(\varphi - \sqrt{\pi}\right) \tag{5}$$

Hence, when $\beta =_{def} \frac{1}{2}\left(\varphi^2 + \psi^2\right) = \frac{\pi}{2} + \alpha$, the equation (4) can be rewritten as

$$(\varphi+\psi)^2 = 2\beta - 2\alpha \tag{6}$$

Therefore, noting from (1) that $z(\eta_0) = \eta_0$ and $\eta_0 = \pm 1$, we can arrive, looking at (3) via

$$z = \eta_0 = \eta e^{2i\beta - 2i\alpha} \tag{7}$$

and $\eta_0' = \eta_0\eta$ at

$$\exp(2i\alpha) = \eta_0' \exp(2i\beta) \tag{8}$$

Now from $-\eta = \eta_0$ in (3) it follows, $\eta_0' = \eta_0 \eta = -1$ so,

$$\exp(2i\alpha) = -\exp(2i\beta) \tag{9}$$

With the use of Euler's rule and, subsequently, the DeMoivre rule [1] for $n = 2$, we may rewrite the left- and right-hand of (9) such that

$$(\cos(\alpha) + i\sin(\alpha))^2 = -(\cos(\beta) + i\sin(\beta))^2 \tag{10}$$

This quadratic equation then leads us to

$$\cos(\alpha) + i\sin(\alpha) = \eta_1 \sqrt{-1}\,(\cos(\beta) + i\sin(\beta)) \tag{11}$$

with

$$\eta_1 = \pm 1 \tag{12}$$

and $i = \sqrt{-1}$. Note for completeness that $\left(\eta_1\sqrt{-1}\right)^2 = -1$. Note also that $\cos(\alpha) + i\sin(\alpha) \not\equiv 0$ and $\cos(\beta) + i\sin(\beta) \not\equiv 0$. Looking at the $\beta = (\pi/2) + \alpha$ on the right hand of (11) we can, subsequently, observe that

$$\begin{aligned} \cos(\beta) &= \cos\left(\frac{\pi}{2} + \alpha\right) = -\sin(\alpha) \\ \sin(\beta) &= \sin\left(\frac{\pi}{2} + \alpha\right) = \cos(\alpha) \end{aligned} \tag{13}$$

Hence,

$$\cos(\alpha) + i\sin(\alpha) = \eta_1 \sqrt{-1}\,(-\sin(\alpha) + i\cos(\alpha)) \tag{14}$$

In the next step we compare real and imaginary parts on left and right hand side of (14) and observe, $\eta_1 = \pm 1$ from(12) in (14). This leads us to the following two equations.

$$\begin{aligned} \sin(\alpha) &= -\eta_1 \sin(\alpha) \\ \cos(\alpha) &= -\eta_1 \cos(\alpha) \end{aligned} \tag{15}$$

These two equations in (15) can, for $\eta_1 = 1$, in (12), only be fulfilled when $\cos(\alpha) = \sin(\alpha) = 0$. The value $\eta_1 = 1$ is obviously valid for $\eta_1^2 = 1$. However, it is well known that $\nexists_{x \in \mathbb{R}} \cos(x) = \sin(x) = 0$. Hence, a contradiction can be concluded in a valid way. If, however, cos and sin are redefined on a finite interval, then there is the possibility to have "boundary values" where $\cos(x) = \sin(x) = 0$.

3 Conclusion

3.1 Recap

In the paper a contradiction is derived from $e^{2i\alpha} = -e^{2i\beta}$ which originates from $e^{i\pi} = -1$. Using Euler's identity, the DeMoivre rule and $\beta = (\pi/2) + \alpha$, there are two solutions

$$\cos(\alpha) + i\sin(\alpha) = i\left(-\sin(\alpha) + i\cos(\alpha)\right)$$
$$\cos(\alpha) + i\sin(\alpha) = -i\left(-\sin(\alpha) + i\cos(\alpha)\right)$$

with $i = \sqrt{-1}$. The first equation of the above two is contradictory. There is no reason whatsoever to *exclusively* have the second equation with the factor $-i$ in (10)- (13). The reason is *both* $i^2 = -1$ and $(-i)^2 = -1$ are valid.

3.2 Discussion

The presented anomaly is complex number theoretic but there are consequences for applied mathematics. For instance, it can be conjectured that there are consequences for the application of Fourier analysis and transforms in e.g. spectroscopy [3, p11-17] and the anomaly found in the present paper.

Further, we may ask if nature is following anomalies when the analysis with complex numbers is so very effective. Such a question looks quite philosophical. It can, nevertheless, be practical when e.g. "weirdness" pops up in the analysis. The author conjectures that weirdness of results of computations where complex numbers are involved, may have an explanation in the discovered anomaly.

Finally it is noted that the presented result, here interpreted as the necessity for a boundary where $\cos(x) = \sin(x) = 0$, shows some resemblance with Bishop's constructivistic approach to the foundation of mathematics [6] and to its earlier version, intuitionism of the famous Dutch mathematician Brouwer [7, p 273] and [8].

Ultimately, the paper is a plea for a finitistic applied mathematics that can, when e.g. weirdness arises, also be translated into finite computer programs. If that cannot be done then anomaly from infinity might be at work in one's concepts.

Conflict of interest

The author declares that he has no conflict of interest.
The author was not funded for this research.
The author trusts that the reviewers also do not have a conflict of interest.

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