The Dirac moduli space

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Abstract

We define a moduli space called the Dirac moduli space with help of the Dirac operator.

1 The Dirac operator

Let (M, g) be a spin manifold, then we can define the Dirac operator D with help of the Levi-Civita connection ∇ [F].

$$D(\psi) = \sum_i e_i.\nabla_{e_i}(\psi)$$

(e_i) is an orthonormal basis.

$$D = \mu \circ \nabla$$

with μ the Clifford multiplication.

2 The Dirac moduli space

The Dirac euqations are defined over (X, ψ) a vector field and a spinor:

$$X.\nabla_X\psi = \alpha\nabla_X X.\psi + \nu||X||^2\psi$$

$$\mathcal{D}(X.\psi) = (\alpha + 1)dX.\psi + \mu X.\psi$$

with $\mathcal{D}$ the Dirac operator and α, ν, μ are constants. The gauge group is $\mathcal{G} = C^\infty(M, \mathbf{R}_+^*)$ and acts over the solutions of the Dirac equations $S(X, \psi)$:

$$f.(X, \psi) = (f^b X, f^a \psi)$$

with $\alpha = a/b$. The moduli space is:

$$\mathcal{M} = S(X, \psi)/\mathcal{G}$$

References

[F] T.Friedrich, "Dirac Operators in Riemannian Geometry", vol 25, AMS, 2000.

[GHL] S.Gallot, D.Hulin, J.Lafontaine, "Riemannian geometry", 3ed., Springer, Berlin, 2004.