

In general relativity the synchronization of the clocks occurs using photons, but if gravitons exist, and have the same speed as photons, then the synchronization can be performed with gravitational wave detectors.

From a quantum point of view, the curvature of the gravitational field is equivalent to the interaction of photons with gravitons, therefore there is a delay, and a deviation, in the path of photons due to the interaction with gravitons.

On the other hand if gravitons interact with photons, then gravitational waves interact with photons, therefore interact with electromagnetic fields: there should be an electromagnetic curvature (so that Charged Black Holes exist for gravitational waves).

The Coulomb's law and Newton's law of universal gravitation are the same laws with different constant.

The magnetic field can be seen as a deformation of the electric field induced by the relativity of the motion of a particle near a wire (Berkeley Physics Course), therefore a neutral wire of opposite masses (particle antiparticles wire) induces a gravitomagnetic field in a moving mass due to the deformation of the gravitational field of the individual masses.

The Einstein field equation are the high energy definition of the gravitational field, because of the special relativity induce the same effect on the mass in movement near a mass wire, then I suppose that the Gravitoelectromagnetism field equation must be equal:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi(-G)}{c^4}T_{\mu\nu} \implies \mathbf{F} = (-G)\frac{m_1m_2}{r^2}\hat{\mathbf{r}}_{21} = \frac{1}{4\pi\epsilon_g}\frac{m_1m_2}{r^2}\hat{\mathbf{r}}_{21}$$

the same law is written for the electromagnetic field:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi(-K)}{c^4}T_{\mu\nu} \implies \mathbf{F} = (K)\frac{q_1q_2}{r^2}\hat{\mathbf{r}}_{21} = \frac{1}{4\pi\epsilon}\frac{q_1q_2}{r^2}\hat{\mathbf{r}}_{21}$$

so that the low energy laws are the classical laws.

It is possible to simplify the electromagnetic expression using the equalities:

$$\begin{aligned}\epsilon_g &= \frac{1}{4\pi(-G)} \\ \mu_g\epsilon_g &= \frac{1}{c^2} \\ \mu_g &= \frac{1}{c^2\epsilon_g} = \frac{4\pi(-G)}{c^2}\end{aligned}$$

each quantum electrodynamics equation become a quantum gravitoelectromagnetic equation using the variables $\mu_g(-G)$ and $\epsilon_g(-G)$.

The laws of gravitoelectromagnetism and electromagnetism are:

Gravitoelectromagnetism	Electromagnetism
$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$	$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi(-K)}{c^4}T_{\mu\nu}$
$\nabla \cdot \mathbf{E}_g = 4\pi(-G) \rho_g$	$\nabla \cdot \mathbf{E} = 4\pi K \rho$
$\nabla \cdot \mathbf{B}_g = 0$	$\nabla \cdot \mathbf{B} = 0$
$\nabla \times \mathbf{E}_g = -\frac{\partial \mathbf{B}_g}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
$\nabla \times \mathbf{B}_g = \frac{4\pi(-G)}{c^2} \mathbf{J}_g + \frac{1}{c^2} \frac{\partial \mathbf{E}_g}{\partial t}$	$\nabla \times \mathbf{B} = \frac{4\pi K}{c^2} \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}$
$\partial_\mu (F_g)^{\mu\nu} = \frac{4\pi(-G)}{c^2} (J_g)^\nu$	$\partial_\mu F^{\mu\nu} = \frac{4\pi K}{c^2} J^\nu$

the electromagnetic field is quantizable, so that the Gravitoelectromagnetic equation should be quantizable; these are the same relativistic laws, the same low energy laws, so that it is possible a Quantum Gravitodynamics with the same theoretical steps, using the same variable and the same methods (like Feynmann diagrams, lagrangian and Proca equations using 4-vector potentials).

If there is a 4-vector gravitational potential $(A_g)^\mu = (\phi_g, \mathbf{A}_g)$ where:

$$\begin{aligned} \mathbf{B}_g &= \nabla \times \mathbf{A}_g \\ \mathbf{E}_g &= -\frac{\partial \mathbf{A}_g}{\partial t} - \nabla \phi_g \\ (F_g)^{\mu\nu} &= \partial^\mu (A_g)^\nu - \partial^\nu (A_g)^\mu \end{aligned}$$

using a Lorentz gauge:

$$\nabla \cdot \mathbf{A}_g + \frac{1}{c^2} \frac{\partial \phi_g}{\partial t} = 0$$

the equations for electromagnetic and gravitoelectromagnetic waves are:

$$\begin{aligned} \square \mathbf{A}_g &= \frac{1}{c^2} \frac{\partial^2 \mathbf{A}_g}{\partial t^2} - \Delta \mathbf{A}_g = \frac{4\pi(-G)}{c^2} \mathbf{J}_g \\ \square \mathbf{A} &= \frac{4\pi K}{c^2} \mathbf{J} \end{aligned}$$

the lagrangian for the gravitoelectromagnetism and electromagnetism are:

$$\begin{aligned} \mathcal{L}_g &= -\frac{c^2}{16\pi(-G)} (F_g)^{\mu\nu} (F_g)_{\mu\nu} - (J_g)^\mu (A_g)_\mu \\ \mathcal{L} &= -\frac{c^2}{16\pi K} F^{\mu\nu} F_{\mu\nu} - J^\mu A_\mu \end{aligned}$$

in fact, the Maxwell's equation can be obtained from the Eulero-Lagrange equation:

$$\begin{aligned} \partial_\mu \left\{ \frac{\partial \mathcal{L}}{\partial [\partial_\mu (A_g)_\nu]} \right\} - \frac{\partial \mathcal{L}}{\partial (A_g)_\nu} &= 0 \\ \partial_\mu \left\{ \frac{\partial \mathcal{L}}{\partial [\partial_\mu A_\nu]} \right\} - \frac{\partial \mathcal{L}}{\partial A_\nu} &= 0 \end{aligned}$$

the gravitoelectromagnetism equation for particles with mass and half-integer spin is:

$$\mathcal{L}_g = \bar{\psi}_g \left\{ i\gamma^\mu [\partial_\mu + iM(A_g)_\mu + iM(B_g)_\mu + ieA_\mu + ieB_\mu - m] \psi \right\} + \frac{1}{4}(F_g)^{\mu\nu}(F_g)_{\mu\nu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

where $(A_g)_\mu$ is the gravitomagnetic potential and $(B_g)_\mu$ is the gravitomagnetic field (the sign of the gravitoelectric and gravitomagnetic field energy is changed, to change the sign in the wave equation for gravitomagnetism), the quantity in the Dirac equation are natural units, and e,M are the charge and the mass of the external field.