The fuzzy probabilities

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Abstract

We introduce the notion of fuzzy probabilities bound to the fuzzy sets.

1   The fuzzy sets

A fuzzy set is a function with values in $[0, 1]$. The fuzzy set is open if the function is lower semi continuous.

2   The measure of a fuzzy set

If we take a fuzzy set f, then its measure is:

$$m(f) = \int f d\mu$$

3   The fuzzy probabilities

We consider the fuzzy sets of a space X with a measure of probability. The fuzzy probability of X is the measure over the fuzzy sets. We have the theorem:

Theorem The fuzzy Lebesgue measurable sets are the Lebesgue measurable functions with values in $[0, 1]$.

Proof:

We show first that the step functions are fuzzy Lebesgue measurable sets.

$$\sum_i a_i 1_{E_i}$$

Then the superior limit of a countable set of step functions is a fuzzy Lebesgue measurable set so that we can approximate a Lebesgue measurable function by the limit and the difference is positiv with zero integral and so is a fuzzy Lebesgue measurable set.

4   Bibliography

A.Kaufmann, "Introduction à la théorie des sous-ensembles flous", Masson et cie, 1973.