

# Discipline of knowledge and the graphical law, part II

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(Dated: December 13, 2019)

## Abstract

We study Oxford English dictionaries of economics, geography and psychology; look into Concise Oxford English dictionaries of linguistics and medical and consult Dorlands pocket medical dictionary respectively. We draw the natural logarithm of the number of entries, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised. We find that the graphs are closer to the curves of reduced magnetisation vs reduced temperature for the Bethe-Peierls approximation of the Ising model with four nearest neighbours, in absence and presence of little temperature dependent external magnetic fields i.e. magnetisation curves for various constant values of  $\beta H$ . For economics, geography and two medical dictionaries  $\beta H$  is zero. For linguistics and psychology dictionaries  $\beta H$  is 0.02. Moreover, we have redone the analysis for the Oxford Dictionary of Construction, Surveying and Civil Engineering as well as for the Oxford Dictionary of Science and have found that the entries underlie magnetisation curves for the the Bethe-Peierls approximation of the Ising model with four nearest neighbours with  $\beta H = 0.02$  and  $\beta H = 0.01$  respectively.  $\beta$  is  $\frac{1}{k_B T}$  where, T is temperature, H is external magnetic field and  $k_B$  is Boltzmann constant.

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## I. INTRODUCTION

”Knowledge is almighty”.. Quote unknown.

Magnetic field is omnipresent. Wherever we go, we are in the fabric of one or, another kind of magnetic field. This happened in our past as far as we know. Do we see imprint of magnetic field in our understanding of the world? To understand the world we have progressively developed system of knowledge from antiquity, have classified the system of knowledge into different disciplines. Do we find footprints of magnetic field in the patterns in which those disciplines are laid out? The query led us to our investigation, [1]. We continue to pursue along that line into five more disciplines of knowledge in this paper.

Tool for us is counting of the entries of a dictionary of the respective discipline. Dictionaries of a discipline come in various forms. Do those underlie the same pattern as seen from magnetisation viewpoint? This drove us to investigate, which we are going to expound in this paper, two medical dictionaries, one concise and another pocket written by two different set(s) of people.

In our previous work, [1], we have found that the Oxford dictionaries of the disciplines of philosophy, sociology and Dictionary of Law and Administration (2000, National Law Development Foundation, Para Road, CCS Building, Shivpuram, Lucknow-17, India) to underlie the magnetisation curve in Bethe-Peierls approximation with four nearest neighbours.

In the language side, we have studied a set of natural languages, [2] and have found existence of a magnetisation curve under each language. We termed this phenomenon as graphical law. This was followed by finding of graphical law behind bengali, [3], Basque languages,[4] and Romanian language,[5].

We have found, [2], three type of languages. For the first kind, the points associated with a language fall on one curve of magnetisation, of Ising model with non-random coupling. For the second kind, the points associated with a language fall on one curve of magnetisation, once we remove the letter with maximum number of words or, letters with maximum and next-maximum number of words or, letters with maximum, next-maximum and nextnext-maximum number of words, from consideration. There are third kind of languages, for which the points associated with a language fall on one curve of magnetisation with fitting not that well or, with high dispersion. Those third kind of languages seem to underlie magnetisation curves for a Spin-Glass in presence of an external magnetic field.

We describe how a graphical law is hidden within six different dictionaries belonging to five disciplines of knowledge, in this article. The planning of the paper is as follows. We give an introduction to the standard curves of magnetisation of Ising model in the section II. This section is semi-technical. If a reader is not interested to know the relevance of the comparator curves in the subject of magnetisation, she or, he can start from the section III. In the section III, we describe analysis of words of Economics, [6]. In the sections IV, we dwell on words of Geography, [7]. In the following section, section V, we study words of Linguistics, [8]. In the section VI, we deal with words of Psychology, [9]. We describe graphical law behind medical science in the sections VII, subjecting two different kinds of dictionaries, one concise, [10] and another pocket, [11] to find the same graphical law holding good behind both. To err is human, so are we. In the later two sections, VIII and IX, we reanalyse and replace with the correct graphical laws for the subjects Construction etc., [12] and Science, [13]. This supersedes our eariler analysis in the paper,[1]. Sections X, XI, XII, XIII, XIV are Discussion, Summary, appendix, Acknowledgement and bibliography respectively.

## II. MAGNETISATION

### A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like paramagnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by  $L = \frac{1}{N}\sum_i\sigma_i$ , where  $\sigma_i$  is i-th spin, N being total number of spins. L can vary from minus one to one.  $N = N_+ + N_-$ , where  $N_+$  is the number of up spins,  $N_-$  is the number of down spins.  $L = \frac{1}{N}(N_+ - N_-)$ . As a result,  $N_+ = \frac{N}{2}(1 + L)$  and  $N_- = \frac{N}{2}(1 - L)$ . Magnetisation or, net magnetic moment,  $M$  is  $\mu\sum_i\sigma_i$  or,  $\mu(N_+ - N_-)$  or,  $\mu NL$ ,  $M_{max} = \mu N$ .  $\frac{M}{M_{max}} = L$ .  $\frac{M}{M_{max}}$  is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[14], for the lattice of spins, setting  $\mu$  to one, is  $-\epsilon\sum_{n,n}\sigma_i\sigma_j - H\sum_i\sigma_i$ , where n.n refers to nearest neighbour pairs. The difference  $\Delta E$  of energy if we flip an up spin to down spin is, [15],  $2\epsilon\gamma\bar{\sigma} + 2H$ , where  $\gamma$  is the number of nearest neighbours of a spin. According to Boltzmann principle,  $\frac{N_-}{N_+}$  equals  $exp(-\frac{\Delta E}{k_B T})$ , [16]. In the Bragg-Williams approximation,[17],  $\bar{\sigma} = L$ , considered in the thermal average sense. Consequently,

$$\ln\frac{1+L}{1-L} = 2\frac{\gamma\epsilon L + H}{k_B T} = 2\frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2\frac{L + c}{\frac{T}{T_c}} \quad (1)$$

where,  $c = \frac{H}{\gamma\epsilon}$ ,  $T_c = \gamma\epsilon/k_B$ , [18].  $\frac{T}{T_c}$  is referred to as reduced temperature.

Plot of  $L$  vs  $\frac{T}{T_c}$  or, reduced magnetisation vs. reduced temperature is used as reference curve. In the presence of magnetic field,  $c \neq 0$ , the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [15]. W. L. Bragg was a professor of Hans

Bethe. Rudlof Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudlof Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

### B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [14],[15],[16],[17],[18], due to Bethe-Peierls, [19], reduced magnetisation varies with reduced temperature, for  $\gamma$  neighbours, in absence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (2)$$

$\ln \frac{\gamma}{\gamma-2}$  for four nearest neighbours i.e. for  $\gamma = 4$  is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe datas generated from the equation(1) and the equation(2) in the table, I, and curves of magnetisation plotted on the basis of those datas. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.1. Empty spaces in the table, I, mean corresponding point pairs were not used for plotting a line.

### C. Bethe-peierls approximation in presence of four nearest neighbours, in presence of external magnetic field

In the Bethe-Peierls approximation scheme, [19], reduced magnetisation varies with reduced temperature, for  $\gamma$  neighbours, in presence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{factor-1}{e^{\frac{2\beta H}{\gamma}} factor^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (3)$$

BW	BW(c=0.01)	BP(4)	reduced magnetisation
0	0	0	1
0.435	0.439	0.563	0.978
0.439	0.443	0.568	0.977
0.491	0.495	0.624	0.961
0.501	0.507	0.630	0.957
0.514	0.519	0.648	0.952
0.559	0.566	0.654	0.931
0.566	0.573	0.7	0.927
0.584	0.590	0.7	0.917
0.601	0.607	0.722	0.907
0.607	0.613	0.729	0.903
0.653	0.661	0.770	0.869
0.659	0.668	0.773	0.865
0.669	0.676	0.784	0.856
0.679	0.688	0.792	0.847
0.701	0.710	0.807	0.828
0.723	0.731	0.828	0.805
0.732	0.743	0.832	0.796
0.756	0.766	0.845	0.772
0.779	0.788	0.864	0.740
0.838	0.853	0.911	0.651
0.850	0.861	0.911	0.628
0.870	0.885	0.923	0.592
0.883	0.895	0.928	0.564
0.899	0.918		0.527
0.904	0.926	0.941	0.513
0.946	0.968	0.965	0.400
0.967	0.998	0.965	0.300
0.987		1	0.200
0.997		1	0.100
1	1	1	0

TABLE I. Reduced magnetisation vs reduced temperature datas for Bragg-Williams approximation, in absence of and in presence of magnetic field,  $c = \frac{H}{\gamma\epsilon} = 0.1$ , and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours .

Derivation of this formula ala [19] is given in the appendix.

$\ln \frac{\gamma}{\gamma-2}$  for four nearest neighbours i.e. for  $\gamma = 4$  is 0.693. For four neighbours,

$$\frac{0.693}{\ln \frac{factor-1}{e^{-\frac{2\beta H}{\gamma}} factor^{-\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (4)$$

In the following, we describe datas in the table, II, generated from the equation(4) and curves of magnetisation plotted on the basis of those datas. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.06$ . calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that  $\beta H = 0.05$ . calculated from the equation(4). BP(m=0.02) stands for reduced temperature

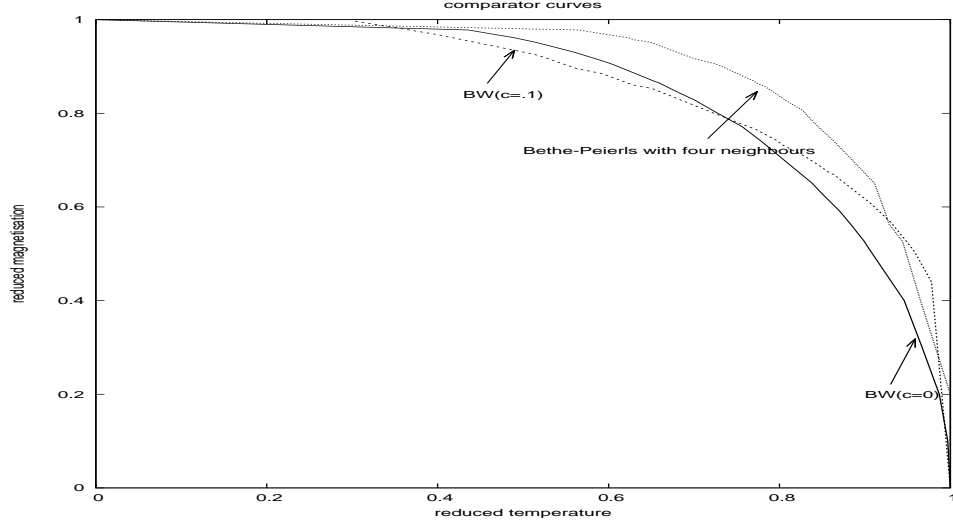


FIG. 1. Reduced magnetisation vs reduced temperature curves for Bragg-Williams approximation, in absence(dark) of and presence(inner in the top) of magnetic field,  $c = \frac{H}{\gamma\epsilon} = 0.1$ , and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours (outer in the top).

in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field,  $H$ , such that  $\beta H = 0.04$ . calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field,  $H$ , such that  $\beta H = 0.02$ . calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field,  $H$ , such that  $\beta H = 0.01$ . calculated from the equation(4). The data set is used to plot fig.2. Empty spaces in the table, II, mean corresponding point pairs were not used for plotting a line.

BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE II. Bethe-Peierls approx. in presence of little external magnetic fields

#### D. Spin-Glass

In the case coupling between (among) the spins, not necessarily n.n, for the Ising model is (are) random, we get Spin-Glass, [20–26]. When a lattice of spins randomly coupled and in an external magnetic field, goes over to the Spin-Glass phase, magnetisation increases steeply like  $\frac{1}{T-T_c}$  upto the the phase transition temperature, followed by very little increase, [20, 25], in magnetisation, as the ambient temperature continues to drop. This happens at least in the replica approach of the Spin-Glass theory, [22, 23].



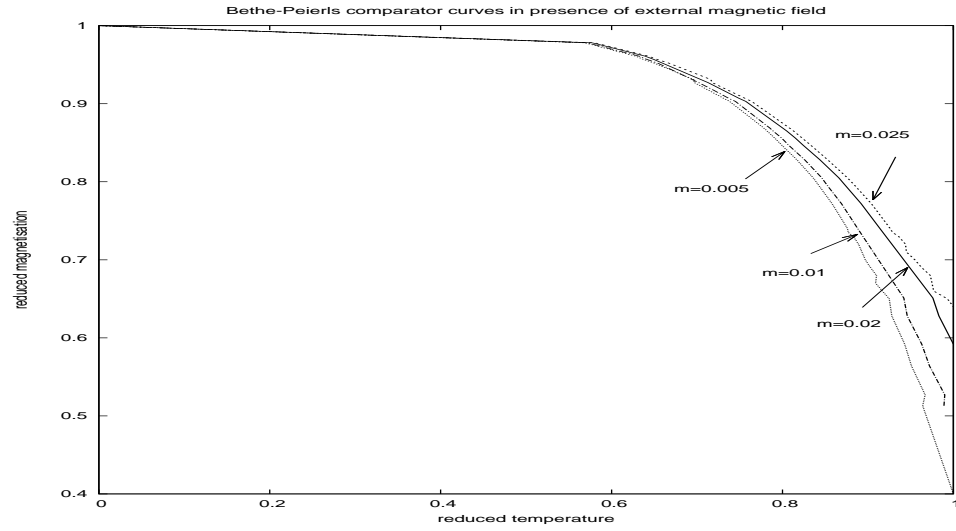


FIG. 2. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with  $\beta H = 2m$ .

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
167	181	434	199	231	174	112	68	218	21	18	130	201	127	90	290	30	203	317	184	75	44	75	2	8	8

TABLE III. Economics words

### III. ANALYSIS OF ECONOMICS

”Wealth or, well being?”

Economics is a subject which is concerned with wealth as well-being of people. GDP, inflation, interest rate are the commonest words of the subject. The first concrete formulation of the discipline is due to Adam Smith who was a professor of moral philosophy, in his ”Wealth of Nations”. There are probably as many subdisciplines, right now, of the discipline as there are disciplines of knowledge. It will be instructive to look for graphical law in each subdiscipline.

To have a feeling, we enter into an economics dictionary, namely the Oxford economics dictionary,[6]. There, we count the entries, loosely speaking words, one by one from the beginning to the end, starting with different letters. The result is the following table, III. Highest number of words, four thirty four, starts with the letter C followed by words numbering three hundred seventeen beginning with S, two hundred ninety with the letter P. To visualise we plot the number of words against respective letters in the dictionary sequence,[6] in the figure fig.3.

For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by  $f$  and the respective rank, [27], denoted by  $k$ .  $k$  is a positive integer starting from one. Moreover, we attach a limiting rank,  $k_{lim}$ , and a limiting number of words. The limiting rank is maximum rank plus one, here it is twenty five and the limiting number of words is one. As a result both  $\frac{\ln f}{\ln f_{max}}$  and  $\frac{\ln k}{\ln k_{lim}}$  varies from zero to one. Then we tabulate in the adjoining table, IV, and plot  $\frac{\ln f}{\ln f_{max}}$  against  $\frac{\ln k}{\ln k_{lim}}$  in the figure fig.4.

We then ignore the letter with the highest of words, tabulate in the adjoining table, IV, and redo the plot, normalising the  $\ln f$ s with next-to-maximum  $\ln f_{nextmax}$ , and starting from  $k = 2$  in the figure fig.5. Normalising the  $\ln f$ s with next-to-next-to-maximum  $\ln f_{nextnextmax}$ , we tabulate in the adjoining table, IV, and starting from  $k = 3$  we draw in the figure fig.6. Normalising the  $\ln f$ s with next-to-next-to-next-to-maximum  $\ln f_{nextnextnextmax}$  we record in

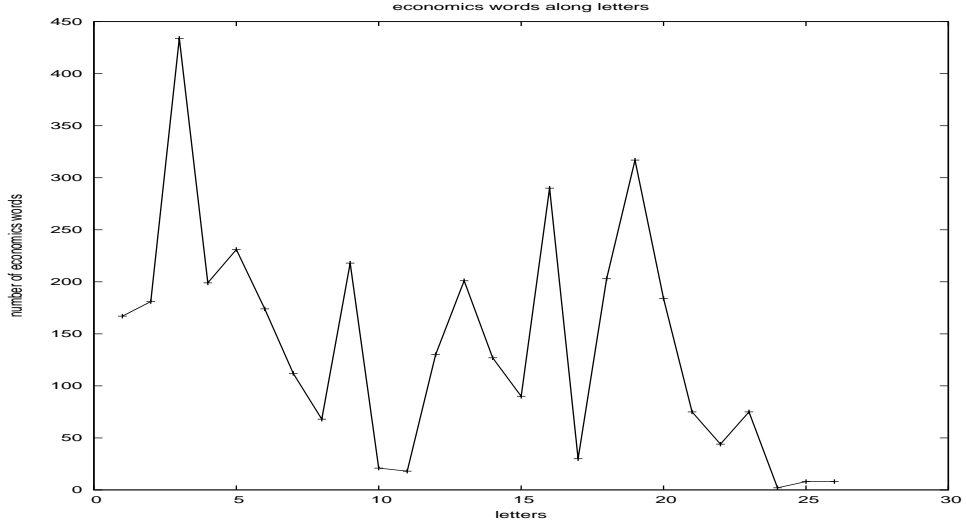


FIG. 3. Vertical axis is number of words in the economics dictionary,[6]. Horizontal axis is the letters of the English alphabet. Letters are represented by the sequence number in the alphabet.

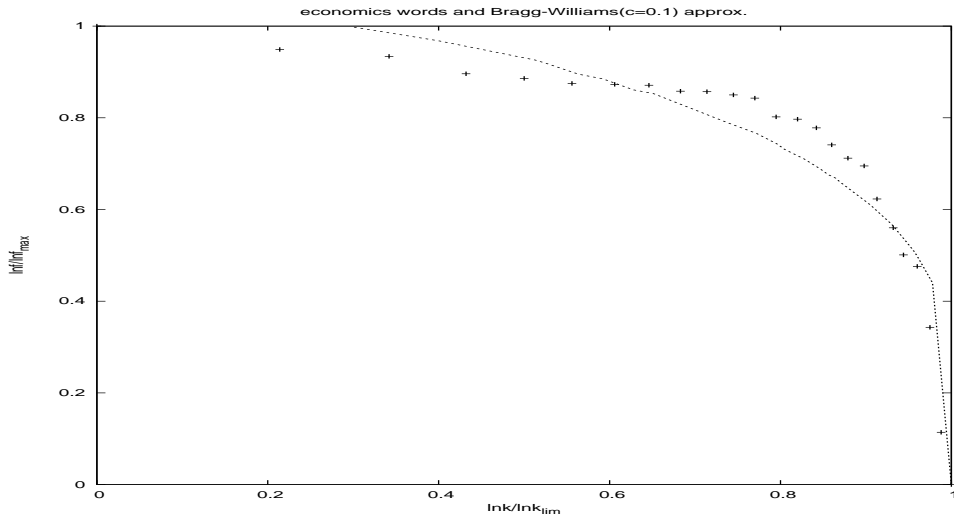


FIG. 4. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the economics dictionary with the fit curve being Bragg-Williams in presence of magnetic field,  $c = \frac{H}{\gamma\epsilon} = 0.1$ .

the adjoining table, IV, and plot starting from  $k = 4$  in the figure fig.7.

k	lnk	lnk/lnk <sub>lim</sub>	f	lnf	lnf/lnf <sub>max</sub>	lnf/lnf <sub>nextmax</sub>	lnf/lnf <sub>nnmax</sub>	lnf/lnf <sub>nnnmax</sub>
1	0	0	434	6.07	1	Blank	Blank	Blank
2	0.69	0.214	317	5.76	0.949	1	Blank	Blank
3	1.10	0.342	290	5.67	0.934	0.984	1	Blank
4	1.39	0.432	231	5.44	0.896	0.944	0.959	1
5	1.61	0.500	218	5.38	0.886	0.934	0.949	0.989
6	1.79	0.556	203	5.31	0.875	0.922	0.937	0.976
7	1.95	0.606	201	5.30	0.873	0.920	0.935	0.974
8	2.08	0.646	199	5.29	0.871	0.918	0.933	0.972
9	2.20	0.683	184	5.21	0.858	0.905	0.919	0.958
10	2.30	0.714	181	5.20	0.857	0.903	0.917	0.956
11	2.40	0.745	174	5.16	0.850	0.896	0.910	0.949
12	2.48	0.770	167	5.12	0.843	0.889	0.903	0.941
13	2.56	0.795	130	4.87	0.802	0.845	0.859	0.895
14	2.64	0.820	127	4.84	0.797	0.840	0.854	0.890
15	2.71	0.842	112	4.72	0.778	0.819	0.832	0.868
16	2.77	0.860	90	4.50	0.741	0.781	0.794	0.827
17	2.83	0.879	75	4.32	0.712	0.750	0.762	0.794
18	2.89	0.898	68	4.22	0.695	0.733	0.744	0.776
19	2.94	0.913	44	3.78	0.623	0.656	0.667	0.695
20	3.00	0.932	30	3.40	0.560	0.590	0.600	0.625
21	3.04	0.944	21	3.04	0.501	0.528	0.536	0.559
22	3.09	0.960	18	2.89	0.476	0.502	0.510	0.531
23	3.14	0.975	8	2.08	0.343	0.361	0.367	0.382
24	3.18	0.988	2	.693	0.114	0.120	0.122	0.127
25	3.22	1	1	0	0	0	0	0

TABLE IV. Economics words:ranking,natural logarithm,normalisations

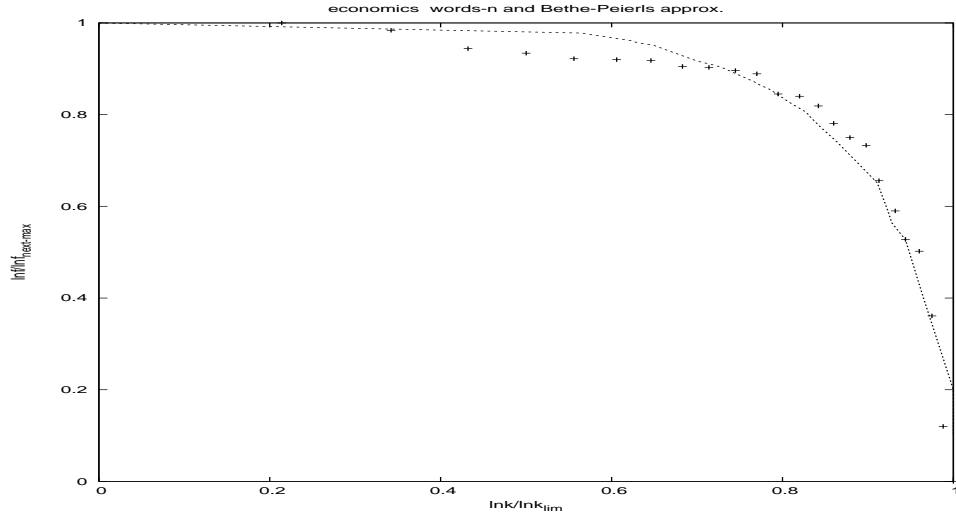


FIG. 5. Vertical axis is  $\frac{\ln f}{\ln f_{next-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the economics dictionary with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours.

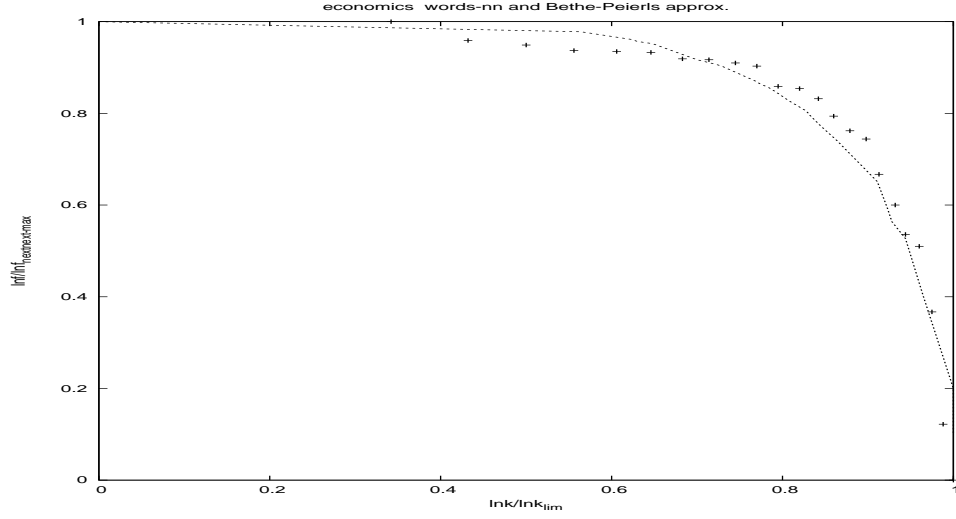


FIG. 6. Vertical axis is  $\frac{\ln f}{\ln f_{\text{nextnext-max}}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{\text{lim}}}$ . The + points represent the words of the economics dictionary with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours.

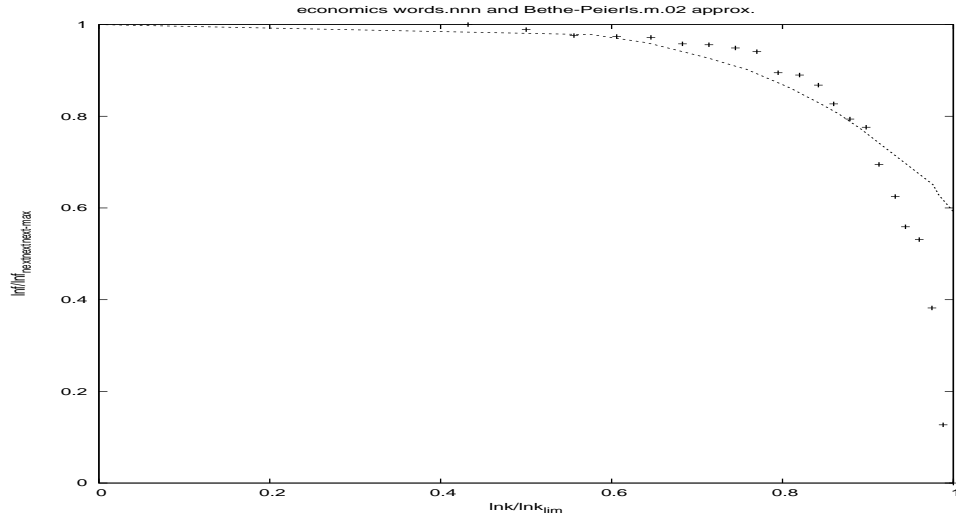


FIG. 7. Vertical axis is  $\frac{\ln f}{\ln f_{\text{nextnextnext-max}}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{\text{lim}}}$ . The + points represent the words of the words of the economics dictionary with the fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field,  $m = 0.02$  or,  $\beta H = 0.04$ .

### A. conclusion

From the figures (fig.4-fig.7), we observe that there is a curve of magnetisation, behind words of economics. This is magnetisation curve in the Bethe-Peierls approximation with

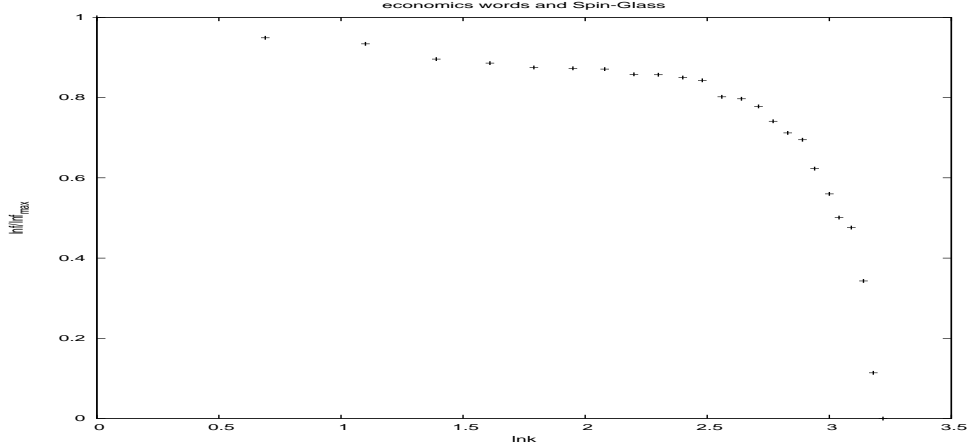


FIG. 8. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the economics dictionary.

four nearest neighbours.

Moreover, the associated correspondance is,

$$\frac{\ln f}{\ln f_{next-to-next-to-maximum}} \longleftrightarrow \frac{M}{M_{max}},$$

$$\ln k \longleftrightarrow T.$$

$k$  corresponds to temperature in an exponential scale, [28]. As temperature decreases, i.e.  $\ln k$  decreases,  $f$  increases. The letters which are recording higher entries compared to those which have lesser entries are at lower temperature. As the subject of economics expands, the letters like ...,P, S, C which get enriched more and more, fall at lower and lower temperatures. This is a manifestation of cooling effect, as was first observed in [29], in another way.

Moreover, for the shake of completeness we draw  $\frac{\ln f}{\ln f_{max}}$  against  $\ln k$  in the figure fig.(8) to explore for the possible existence of a magnetisation curve of a Spin-Glass in presence of an external magnetic field, underlying economics words. In the figure 8, the pointsline does not have a clearcut transition Hence, the words of the economics, is not suited to be described, to underlie a Spin-Glass magnetisation curve, [20], in the presence of magnetic field.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
197	158	338	158	164	139	153	123	138	8	18	140	159	80	66	235	11	149	317	152	45	29	49	3	4	13

TABLE V. Geography words

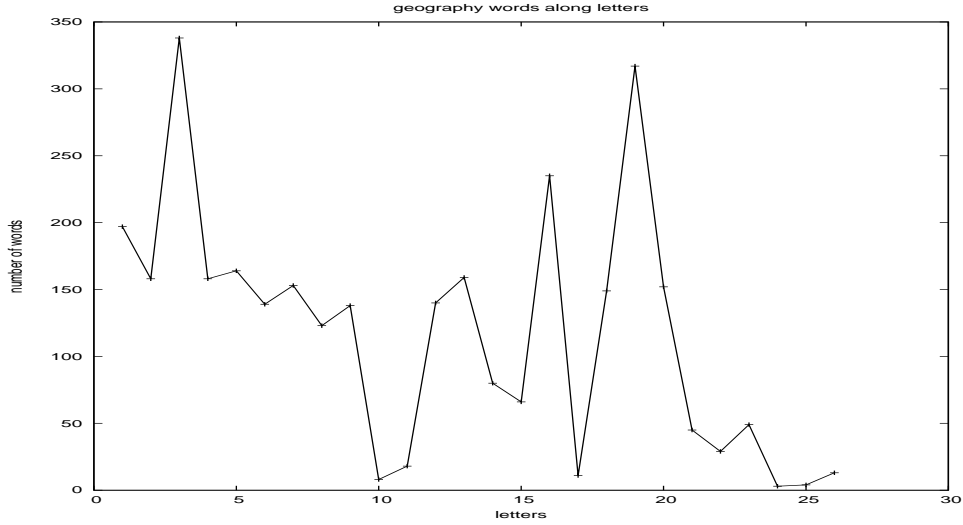


FIG. 9. Vertical axis is number of words in the geography dictionary,[7]. Horizontal axis is the letters of the English alphabet. Letters are represented by the sequence number in the alphabet.

#### IV. ANALYSIS OF GEOGRAPHY

”A traveller who does not tell the truth,  
is not a traveller.”....a traveller.

It is through the experiences of the travellers, explorers, adventurers, navigators etal. from the ancient times developed the discipline of geography. Marco Polo, Hiuen Tsang are the text-book travellers. Colombus, Vasco-da-gamma are the text-book explorers/adventurers/navigators. We go through one geography dictionary,[7]. From the dictionary, the author came to know that oasis is where the water table meets the surface in an arid area. Then we count the words, strictly speaking entries, one by one from the beginning to the end, starting with different letters. The result is the table, V. Highest number of words, three hundred thirty eight, start with the letter C followed by words numbering three hundred seventeen, start with the letter S, two hundred thirty five beginning with P etc. To visualise we plot the number of words against respective letters in the dictionary sequence,[7] in the figure fig.9. For the purpose of exploring graphical law, we assort the letters according to the number of

k	lnk	lnk/lnk <sub>lim</sub>	f	lnf	lnf/lnf <sub>max</sub>	lnf/lnf <sub>nmax</sub>	lnf/lnf <sub>nnmax</sub>	lnf/lnf <sub>nnnmax</sub>
1	0	0	338	5.82	1	Blank	Blank	Blank
2	0.69	0.212	317	5.76	0.990	1	Blank	Blank
3	1.10	0.337	235	5.46	0.938	0.948	1	Blank
4	1.39	0.426	197	5.28	0.907	0.917	0.967	1
5	1.61	0.494	164	5.10	0.876	0.885	0.934	0.966
6	1.79	0.549	159	5.07	0.871	0.880	0.928	0.960
7	1.95	0.598	158	5.06	0.870	0.879	0.927	0.959
8	2.08	0.638	153	5.03	0.864	0.873	0.921	0.953
9	2.20	0.675	152	5.02	0.863	0.872	0.919	0.951
10	2.30	0.706	149	5.00	0.859	0.868	0.916	0.947
11	2.40	0.736	140	4.94	0.849	0.858	0.905	0.936
12	2.48	0.761	139	4.934	0.848	0.857	0.904	0.934
13	2.56	0.785	138	4.927	0.847	0.855	0.902	0.933
14	2.64	0.810	123	4.81	0.826	0.835	0.881	0.911
15	2.71	0.831	80	4.38	0.753	0.760	0.802	0.830
16	2.77	0.850	66	4.19	0.720	0.727	0.767	0.794
17	2.83	0.868	49	3.89	0.668	0.675	0.712	0.737
18	2.89	0.887	45	3.81	0.655	0.661	0.698	0.722
19	2.94	0.902	29	3.37	0.579	0.585	0.617	0.638
20	3.00	0.920	18	2.89	0.497	0.502	0.529	0.547
21	3.04	0.933	13	2.56	0.440	0.444	0.469	0.485
22	3.09	0.948	11	2.40	0.412	0.417	0.440	0.455
23	3.14	0.963	8	2.08	0.357	0.361	0.381	0.394
24	3.18	0.975	4	1.39	0.239	0.241	0.255	0.263
25	3.22	0.988	3	1.10	0.189	0.191	0.201	0.208
26	3.26	1	1	0	0	0	0	0

TABLE VI. Geography words:ranking,natural logarithm,normalisations

words, in the descending order, denoted by  $f$  and the respective rank, denoted by  $k$ .  $k$  is a positive integer starting from one. Moreover, we attach a limiting rank,  $k_{lim}$ , and a limiting number of words. The limiting rank is maximum rank plus one, here it is twenty six and the limiting number of words is one. As a result both  $\frac{lnf}{lnf_{max}}$  and  $\frac{lnk}{lnk_{lim}}$  varies from zero to one. Then we tabulate in the adjoining table, VI, and plot  $\frac{lnf}{lnf_{max}}$  against  $\frac{lnk}{lnk_{lim}}$  in the figure fig.10.

We then ignore the letter with the highest number of words, tabulate in the adjoining table, VI, and redo the plot, normalising the  $lnfs$  with next-to-maximum  $lnf_{nextmax}$ , and starting from  $k = 2$  in the figure fig.11. Normalising the  $lnfs$  with next-to-next-to-maximum  $lnf_{nextnextmax}$ , we tabulate in the adjoining table, VI, and starting from  $k = 3$  we draw in the figure fig.12. Normalising the  $lnfs$  with next-to-next-to-next-to-maximum  $lnf_{nextnextnextmax}$  we record in the adjoining table, VI, and plot starting from  $k = 4$  in the figure fig.13.

From the figures (fig.10-fig.13), we observe that there is a curve of magnetisation, behind



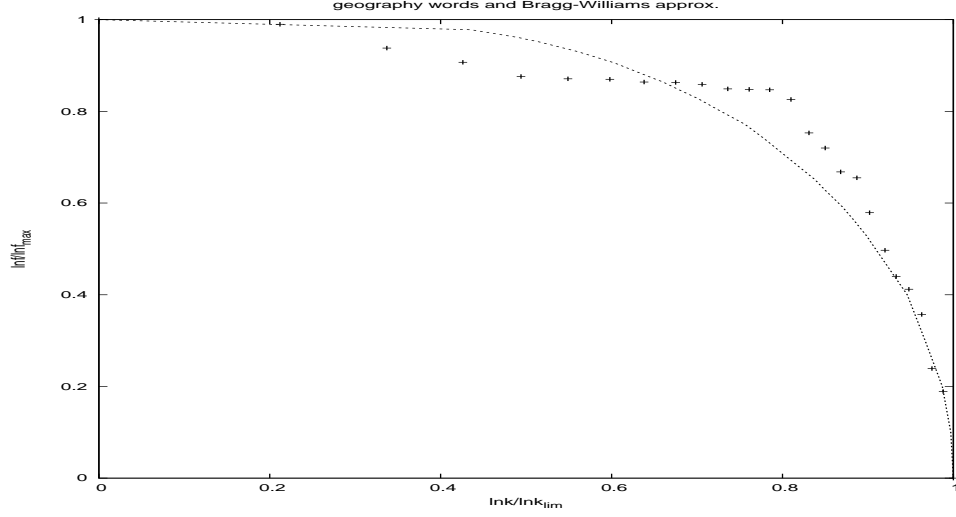


FIG. 10. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the geography dictionary with fit curve being Bragg-Williams curve in absence of magnetic field.

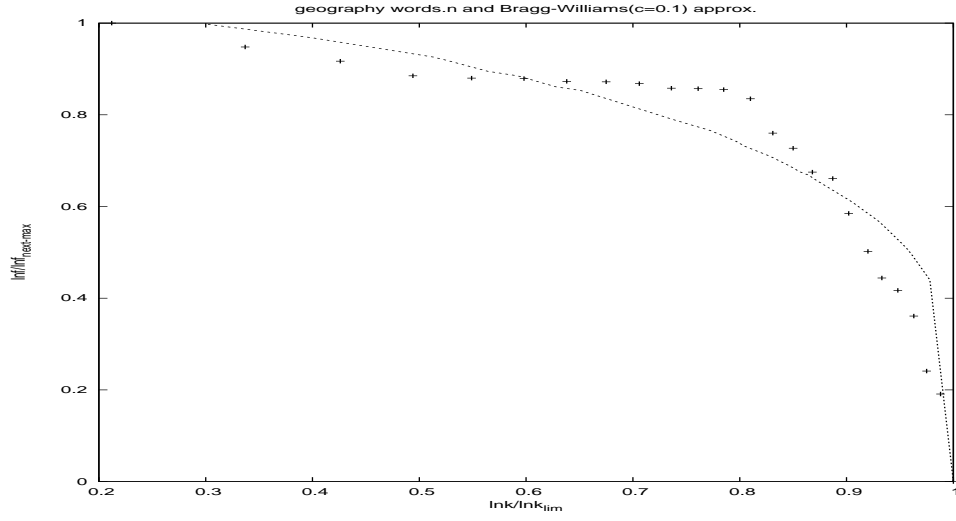


FIG. 11. Vertical axis is  $\frac{\ln f}{\ln f_{next-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the geography dictionary with fit curve being Bragg-Williams curve in presence of magnetic field,  $c = \frac{H}{\gamma\epsilon} = 0.1$ .

words of geography. This is magnetisation curve in the Bethe-Peierls approximation with four nearest neighbours. Moreover, the associated correspondance is,

$$\frac{\ln f}{\ln f_{next-to-next-to-maximum}} \longleftrightarrow \frac{M}{M_{max}},$$

$$\ln k \longleftrightarrow T.$$

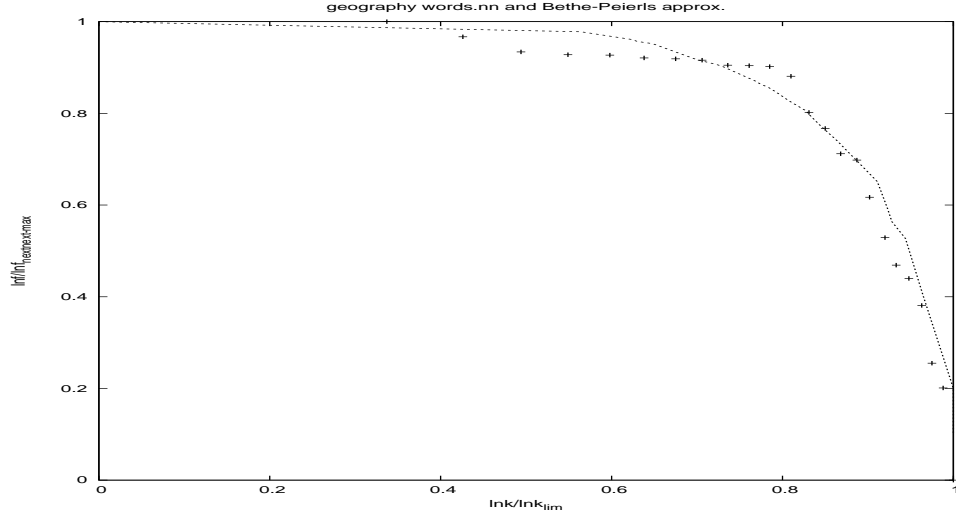


FIG. 12. Vertical axis is  $\frac{\ln f}{\ln f_{nextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the geography dictionary with fit curve being Bethe-Peierls curve with four nearest neighbours.

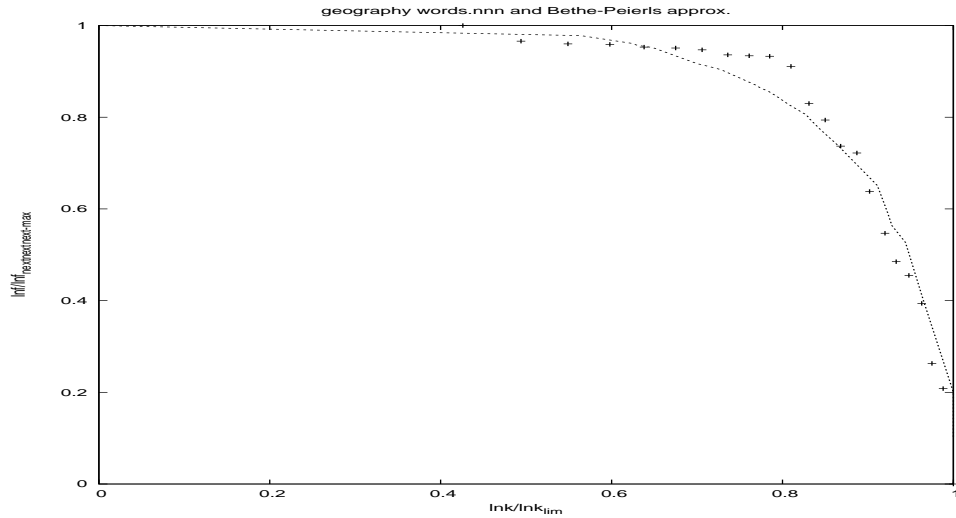


FIG. 13. Vertical axis is  $\frac{\ln f}{\ln f_{nextnextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the geography dictionary with fit curve being Bethe-Peierls curve with four nearest neighbours.

$k$  corresponds to temperature in an exponential scale, [28]. As temperature decreases, i.e.  $\ln k$  decreases,  $f$  increases. The letters which are recording higher entries compared to those which have lesser entries are at lower temperature. As the subject of geography expands, the letters like...,P, S, C which get enriched more and more, fall at lower and lower tem-

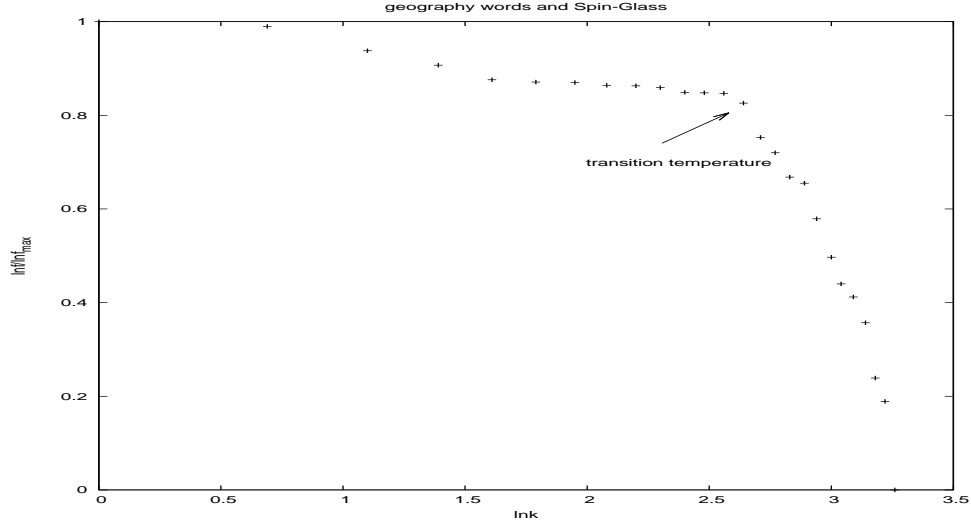


FIG. 14. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the geography dictionary.

peratures. This is a manifestation of cooling effect, as was first observed in [29], in another way. Moreover, for the shake of completeness we draw  $\frac{\ln f}{\ln f_{max}}$  against  $\ln k$  in the figure fig.(8) to explore for the possible existence of a magnetisation curve of a Spin-Glass in presence of an external magnetic field, underlying geography words. We note that the pointslines in the fig.14, has a more or, less clear-cut transition point. Hence, words of geography is suited to be described by a Spin-Glass magnetisation curve, [20], also, in the presence of an external magnetic field.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
303	119	355	214	170	130	120	103	195	17	40	171	198	133	111	249	20	168	397	162	62	68	48	4	12	12

TABLE VII. Linguistics words

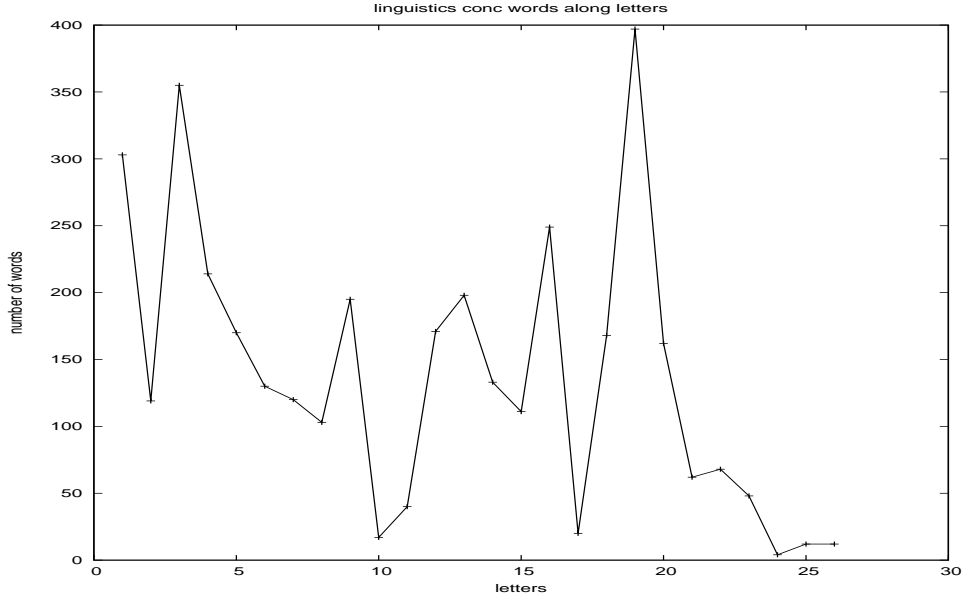


FIG. 15. Vertical axis is number of words in the linguistics dictionary,[8]. Horizontal axis is the letters of the English alphabet. Letters are represented by the sequence number in the alphabet.

## V. ANALYSIS OF LINGUISTICS

”tin korosh tok badalta hai pani,  
sat korosh tok badalta hai bani.” ....Magadhi saying.

It is with with grammatical and structural aspects of bani or, languages, that the discipline of linguistics is primarily concerned with. Syllable, diphothong, phonem, morpheme, grapheme, phone are among the daily lores for a linguist. We read through one linguistics dictionary,[8]. Then we count the words, strictly speaking entries, one by one from the beginning to the end, starting with different letters. The result is the table, VII. Highest number of words, three hundred ninety seven, start with the letter S followed by words numbering three hundred fifty five beginning with C, words numbering three hundred three beginning with A etc. Moreover, we represent the number of words pictorially, against respective letters in the dictionary sequence,[8] in the figure fig.15. For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, de-

k	lnk	lnk/ $\ln k_{lim}$	f	lnf	lnf/ $\ln f_{max}$	lnf/ $\ln f_{nmax}$	lnf/ $\ln f_{nnmax}$	lnf/ $\ln f_{nnnmax}$
1	0	0	397	5.98	1	Blank	Blank	Blank
2	0.69	0.212	355	5.87	0.982	1	Blank	Blank
3	1.10	0.337	303	5.71	0.955	0.973	1	Blank
4	1.39	0.426	249	5.52	0.923	0.940	0.967	1
5	1.61	0.494	214	5.37	0.898	0.915	0.940	0.973
6	1.79	0.549	198	5.29	0.885	0.901	0.926	0.958
7	1.95	0.598	195	5.27	0.881	0.898	0.923	0.955
8	2.08	0.638	171	5.141	0.860	0.876	0.900	0.931
9	2.20	0.675	170	5.136	0.859	0.875	0.899	0.930
10	2.30	0.706	168	5.12	0.856	0.872	0.897	0.928
11	2.40	0.736	162	5.09	0.851	0.867	0.891	0.922
12	2.48	0.761	133	4.89	0.818	0.833	0.856	0.886
13	2.56	0.785	130	4.87	0.814	0.830	0.853	0.882
14	2.64	0.810	120	4.79	0.801	0.816	0.839	0.868
15	2.71	0.831	119	4.78	0.799	0.814	0.837	0.866
16	2.77	0.850	111	4.71	0.788	0.802	0.825	0.853
17	2.83	0.868	103	4.63	0.774	0.789	0.811	0.839
18	2.89	0.887	68	4.22	0.706	0.719	0.739	0.764
19	2.94	0.902	62	4.13	0.691	0.704	0.723	0.748
20	3.00	0.920	48	3.87	0.647	0.659	0.678	0.701
21	3.04	0.933	40	3.69	0.617	0.629	0.646	0.668
22	3.09	0.948	20	3.00	0.502	0.511	0.525	0.543
23	3.14	0.963	17	2.83	0.473	0.482	0.496	0.513
24	3.18	0.975	12	2.48	0.415	0.422	0.434	0.449
25	3.22	0.988	4	1.39	0.232	0.237	0.243	0.252
26	3.26	1	1	0	0	0	0	0

TABLE VIII. Linguistics words: ranking,natural logarithm,normalisations

noted by  $f$  and the respective rank, denoted by  $k$ .  $k$  is a positive integer starting from one. Moreover, we attach a limiting rank,  $k_{lim}$ , and a limiting number of words. The limiting rank is maximum rank plus one, here it is twenty six and the limiting number of words is one. As a result both  $\frac{\ln f}{\ln f_{max}}$  and  $\frac{\ln k}{\ln k_{lim}}$  varies from zero to one. Then we tabulate in the adjoining table, VIII, and plot  $\frac{\ln f}{\ln f_{max}}$  against  $\frac{\ln k}{\ln k_{lim}}$  in the figure fig.16. We then ignore the letter with the highest of words, tabulate in the adjoining table, VIII, and redo the plot, normalising the  $\ln f$ s with next-to-maximum  $\ln f_{nextmax}$ , and starting from  $k = 2$  in the figure fig.17. Normalising the  $\ln f$ s with next-to-next-to-maximum  $\ln f_{nextnextmax}$ , we tabulate in the adjoining table, VIII, and starting from  $k = 3$  we draw in the figure fig.18. Normalising the  $\ln f$ s with next-to-next-to-next-to-maximum  $\ln f_{nextnextnextmax}$  we record in the adjoining table, VIII, and plot starting from  $k = 4$  in the figure fig.19.

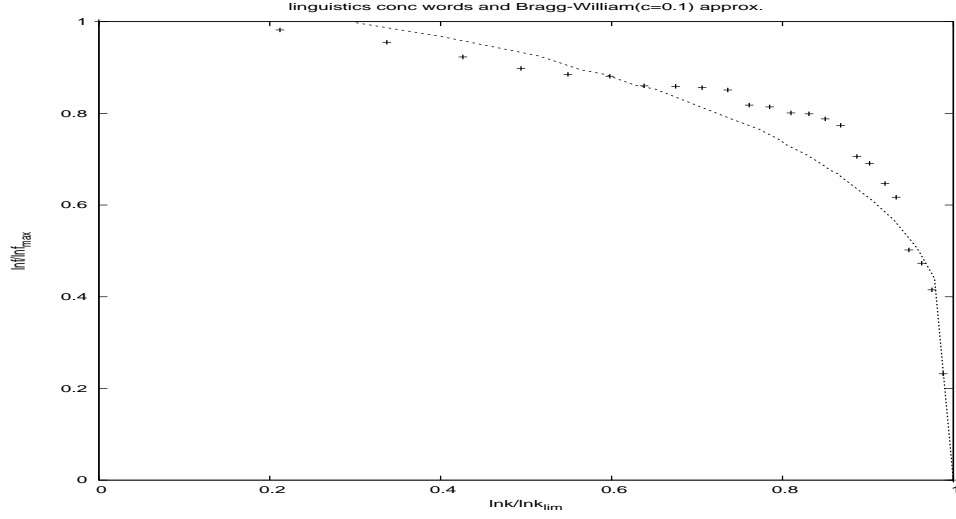


FIG. 16. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the linguistics dictionary with fit curve being Bragg-Williams curve with magnetic field,  $c = \frac{H}{\gamma\epsilon} = 0.1$ .

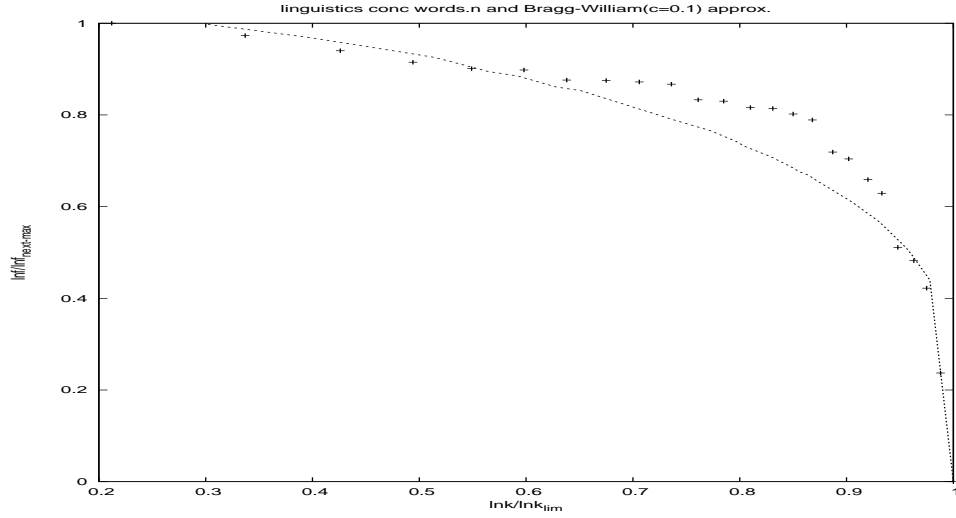


FIG. 17. Vertical axis is  $\frac{\ln f}{\ln f_{next-max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the linguistics dictionary with fit curve being Bragg-Williams curve with magnetic field,  $c = \frac{H}{\gamma\epsilon} = 0.1$ .

### A. conclusion

In the plot fig.19, the points match nicely with the magnetisation curve in the Bethe-Peierls approximation in presence of little magnetic field. Hence, words of linguistics can be characterised by the magnetisation curve in the Bethe-Peierls approximation in presence of

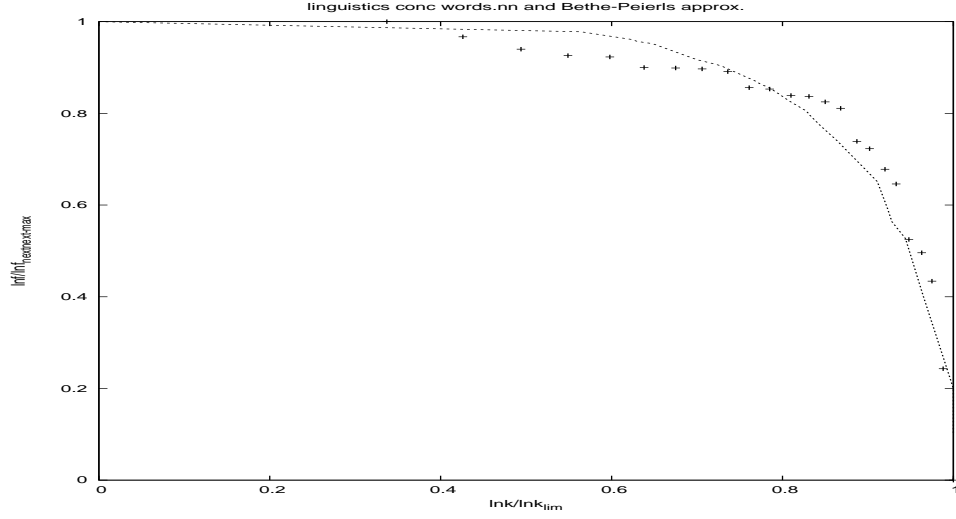


FIG. 18. Vertical axis is  $\frac{\ln f}{\ln f_{nextnext-max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the linguistics dictionary with fit curve being Bethe-Peierls curve with four nearest neighbours.

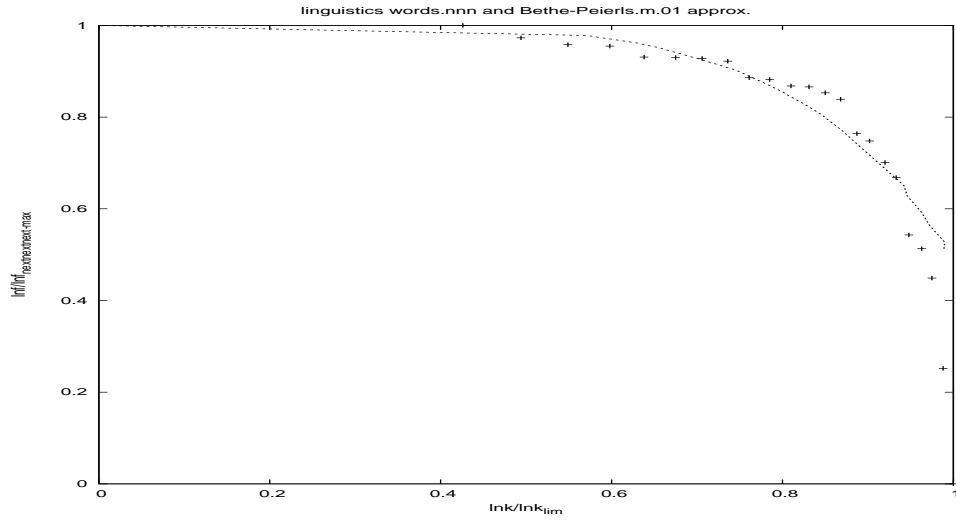


FIG. 19. Vertical axis is  $\frac{\ln f}{\ln f_{nextnextnext-max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the linguistics dictionary with fit curve being Bethe-Peierls curve with four nearest neighbours, in presence of little magnetic field,  $m=0.01$  or,  $\beta H = 0.02$ .

little magnetic field,  $m=0.01$  i.e.  $\beta H = 0.02$ . Moreover, there is an associated correspondance is,

$$\frac{\ln f}{\ln f_{nextnextnext-max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$\ln k \longleftrightarrow T.$$

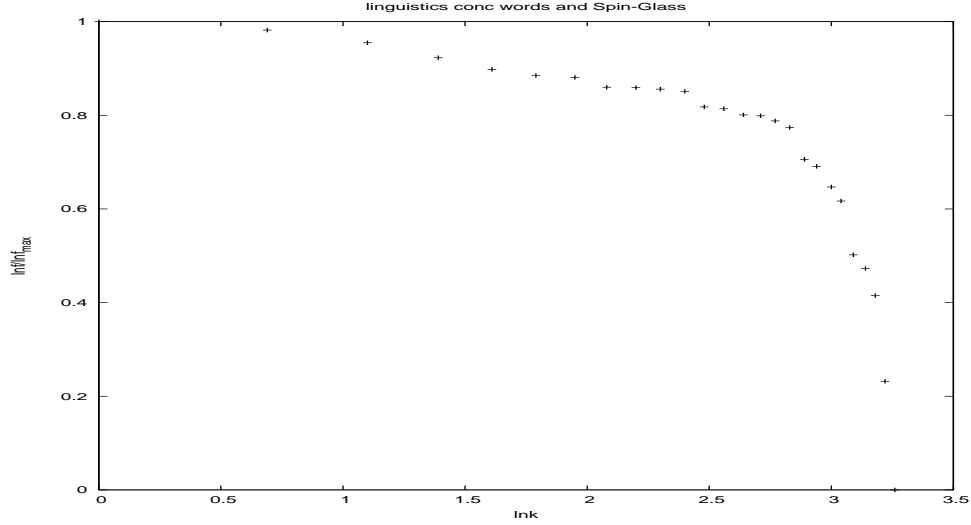


FIG. 20. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the linguistics dictionary.

$k$  corresponds to temperature in an exponential scale, [28]. As temperature decreases, i.e.  $\ln k$  decreases,  $f$  increases. The letters which are recording higher entries compared to those which have lesser entries are at lower temperature. As the subject of linguistics expands, the letters like .....,A, C, S which get enriched more and more, fall at lower and lower temperatures. This is a manifestation of cooling effect, as was first observed in [29], in another way. Again, to be sure we draw  $\frac{\ln f}{\ln f_{max}}$  against  $\ln k$  in the figure fig.20 to explore for the possible existence of a magnetisation curve of a Spin-Glass in presence of an external magnetic field, underlying linguistics. We note that the points in the fig.20 does not have a clear-cut transition point for the words of linguistics dictionary, [8].



A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
684	375	892	463	450	363	303	379	311	36	89	282	601	336	295	1035	26	377	996	445	68	158	103	12	23	30

TABLE IX. Psychology words

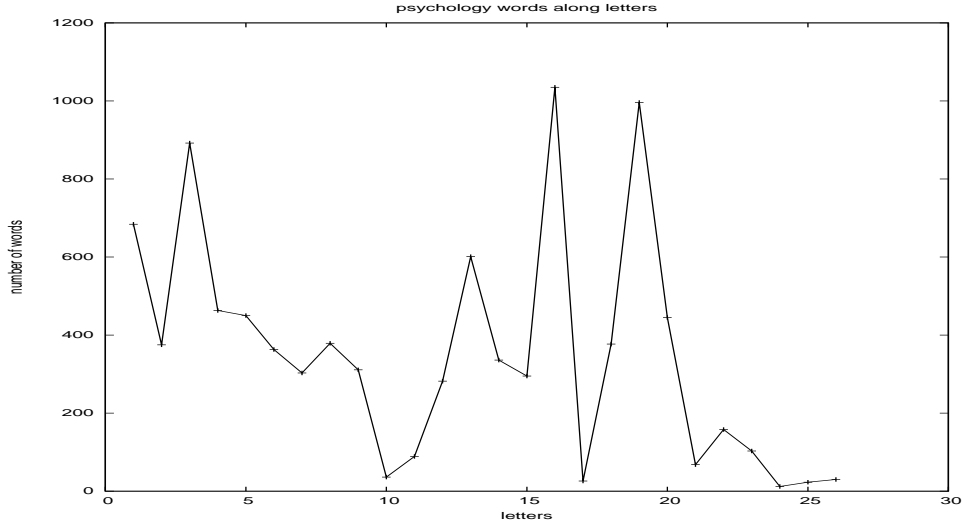


FIG. 21. Vertical axis is number of words in the psychology dictionary,[9]. Horizontal axis is the letters of the English alphabet. Letters are represented by the sequence number in the alphabet.

## VI. ANALYSIS OF PSYCHOLOGY

Psychology is a subject dealing with mental state of a human being in isolation or, in presence of different orders of societal structures. The subject got consolidated through the effort of legendary Sigmund Freud. We delve into the psychology dictionary,[9]. Then we count the words, strictly speaking entries, one by one from the beginning to the end, starting with different letters. The result is the table, IX. Highest number of words, one thousand thirty five, start with the letter P followed by words numbering nine hundred ninety six with the letter S, eight hundred ninety two beginning with C, etc. To visualise we plot the number of words again respective letters in the dictionary sequence,[9] in the adjoining figure, fig.21. For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by  $f$  and the respective rank, denoted by  $k$ .  $k$  is a positive integer starting from one. Moreover, we attach a limiting rank,  $k_{lim}$ , and a limiting number of words. The limiting rank is maximum rank plus one, here it is twenty seven and the limiting number of words is one. As a result both  $\frac{\ln f}{\ln f_{max}}$

k	lnk	lnk/lnk <sub>lim</sub>	f	lnf	lnf/lnf <sub>max</sub>	lnf/lnf <sub>next-max</sub>	lnf/lnf <sub>nnmax</sub>	lnf/lnf <sub>nnnmax</sub>
1	0	0	1035	6.94	1	Blank	Blank	Blank
2	0.69	0.209	996	6.90	0.994	1	Blank	Blank
3	1.10	0.333	892	6.79	0.978	0.984	1	Blank
4	1.39	0.421	684	6.53	0.941	0.946	0.962	1
5	1.61	0.488	601	6.40	0.922	0.928	0.943	0.980
6	1.79	0.542	463	6.14	0.884	0.890	0.904	0.940
7	1.95	0.591	450	6.11	0.880	0.886	0.900	0.936
8	2.08	0.630	445	6.10	0.879	0.884	0.898	0.934
9	2.20	0.667	379	5.94	0.856	0.861	0.875	0.910
10	2.30	0.697	377	5.932	0.855	0.860	0.874	0.908
11	2.40	0.727	375	5.927	0.854	0.859	0.873	0.908
12	2.48	0.752	363	5.89	0.848	0.854	0.867	0.902
13	2.56	0.776	336	5.82	0.838	0.843	0.857	0.891
14	2.64	0.800	311	5.74	0.827	0.832	0.845	0.879
15	2.71	0.821	303	5.71	0.823	0.828	0.841	0.874
16	2.77	0.839	295	5.69	0.820	0.825	0.838	0.871
17	2.83	0.858	282	5.64	0.812	0.817	0.831	0.864
18	2.89	0.876	158	5.06	0.729	0.733	0.745	0.775
19	2.94	0.891	103	4.63	0.667	0.671	0.682	0.709
20	3.00	0.909	89	4.49	0.647	0.651	0.661	0.688
21	3.04	0.921	68	4.22	0.608	0.612	0.622	0.646
22	3.09	0.936	36	3.58	0.516	0.519	0.527	0.548
23	3.14	0.952	30	3.40	0.490	0.493	0.501	0.521
24	3.18	0.964	26	3.26	0.470	0.472	0.480	0.499
25	3.22	0.976	23	3.14	0.452	0.455	0.462	0.481
26	3.26	0.988	12	2.48	0.357	0.359	0.365	0.380
27	3.30	1	1	0	0	0	0	0

TABLE X. Psychology words: ranking,natural logarithm,normalisations

and  $\frac{\ln k}{\ln k_{lim}}$  varies from zero to one. Then we tabulate in the adjoining table, X, and plot  $\frac{\ln f}{\ln f_{max}}$  against  $\frac{\ln k}{\ln k_{lim}}$  in the figure fig.22. We then ignore the letter with the highest of words, tabulate in the adjoining table, X, and redo the plot, normalising the  $\ln f$ s with next-to-maximum  $\ln f_{next-max}$ , and starting from  $k = 2$  in the figure fig.23. Normalising the  $\ln f$ s with next-to-next-to-maximum  $\ln f_{nextnext-max}$ , we tabulate in the adjoining table, X, and starting from  $k = 3$  we draw in the figure fig.24. Normalising the  $\ln f$ s with next-to-next-to-next-to-maximum  $\ln f_{nextnextnext-max}$  we record in the adjoining table, X, and plot starting from  $k = 4$  in the figure fig.25.

### A. conclusion

In the plot fig.25, the points match with the magnetisation curve in the Bethe-Peierls approximation with four nearest neighbours, in presence of little magnetic field,  $m = 0.01$  or,

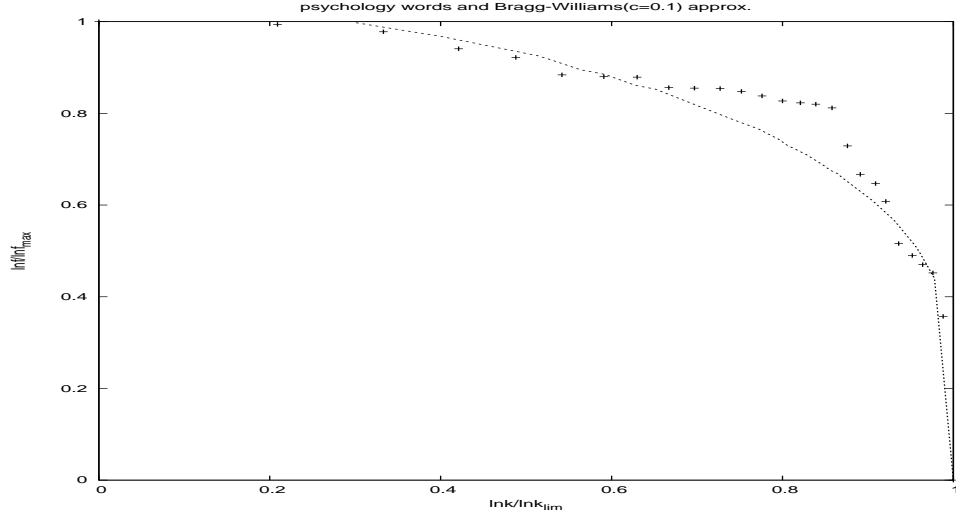


FIG. 22. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent words of the psychology dictionary with fit curve being Bragg-Williams curve in presence of magnetic field,  $c = \frac{\mu H}{\gamma J} = 0.1$ .

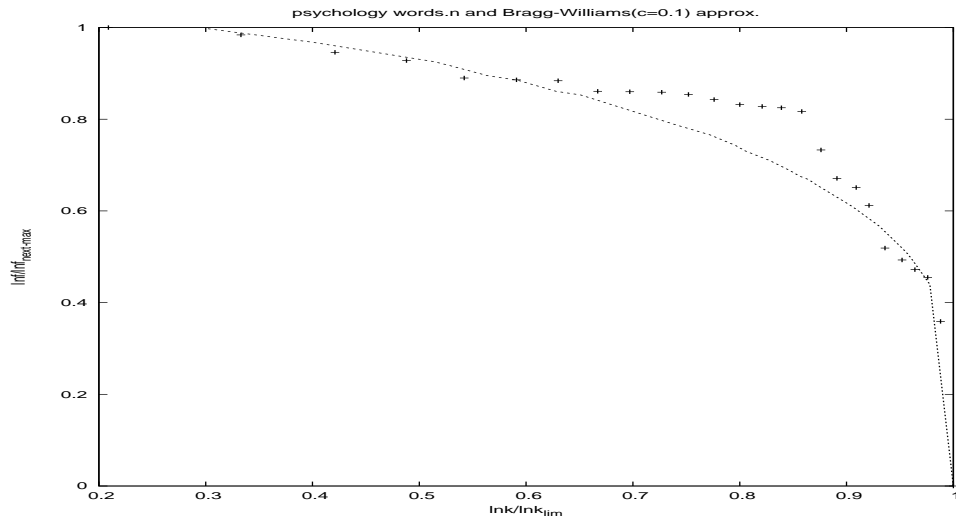


FIG. 23. Vertical axis is  $\frac{\ln f}{\ln f_{next-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent words of the psychology dictionary with fit curve being Bragg-Williams curve in presence of magnetic field,  $c = \frac{H}{\gamma \epsilon} = 0.1$ .

$\beta H = 0.02$ . Hence, words of psychology can be characterised by the magnetisation curve in the Bethe-Peierls approximation with four nearest neighbours, in presence of little magnetic

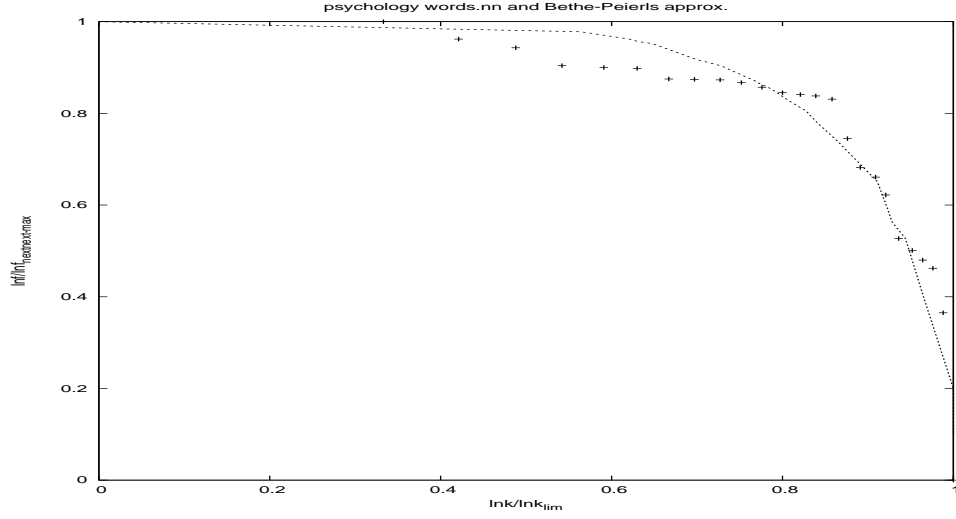


FIG. 24. Vertical axis is  $\frac{\ln f}{\ln f_{nextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent words of the psychology dictionary with fit curve being Bethe-Peierls curve with four nearest neighbours.

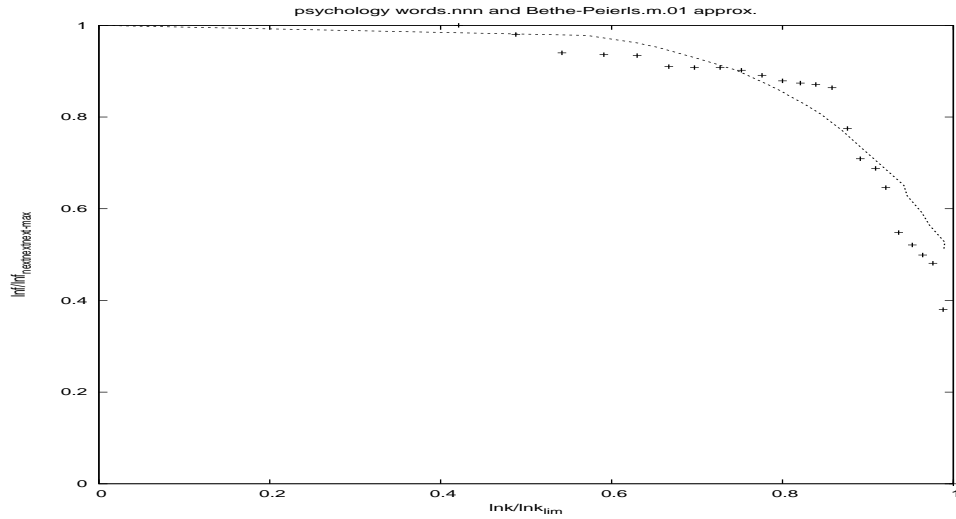


FIG. 25. Vertical axis is  $\frac{\ln f}{\ln f_{nextnextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent words of the psychology dictionary with fit curve being Bethe-Peierls curve in presence of four nearest neighbours and little magnetic field,  $m = 0.01$  or,  $\beta H = 0.02$ .

field,  $\beta H = 0.02$ . Moreover, there is an associated correspondance is,

$$\frac{\ln f}{\ln f_{nextnextnext-max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$\ln k \longleftrightarrow T.$$

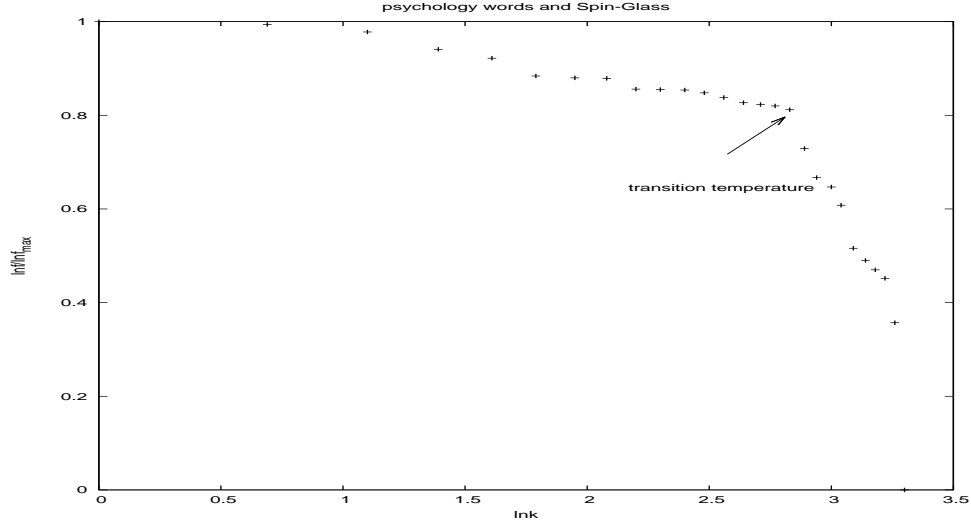


FIG. 26. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the psychology dictionary.

$k$  corresponds to temperature in an exponential scale, [28]. As temperature decreases, i.e.  $\ln k$  decreases,  $f$  increases. The letters which are recording higher entries compared to those which have lesser entries are at lower temperature. As the subject of psychology expands, the letters like .....,C,S,P which get enriched more and more, fall at lower and lower temperatures. This is a manifestation of cooling effect, as was first observed in [29], in another way. Moreover, we draw  $\frac{\ln f}{\ln f_{max}}$  against  $\ln k$  in the figure fig.26 to explore for the possible existence of a magnetisation curve of a Spin-Glass in presence of an external magnetic field, underlying psychology words. We note that the pointslines in the fig.26, has a clear-cut transition point. Hence, words of psychology is suited to be described by a Spin-Glass magnetisation curve, [20], also, in the presence of an external magnetic field.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1018	502	1200	579	480	416	428	690	465	43	143	465	833	375	405	1396	33	424	1029	703	142	311	93	38	12	38

TABLE XI. Concise medical dictionary words

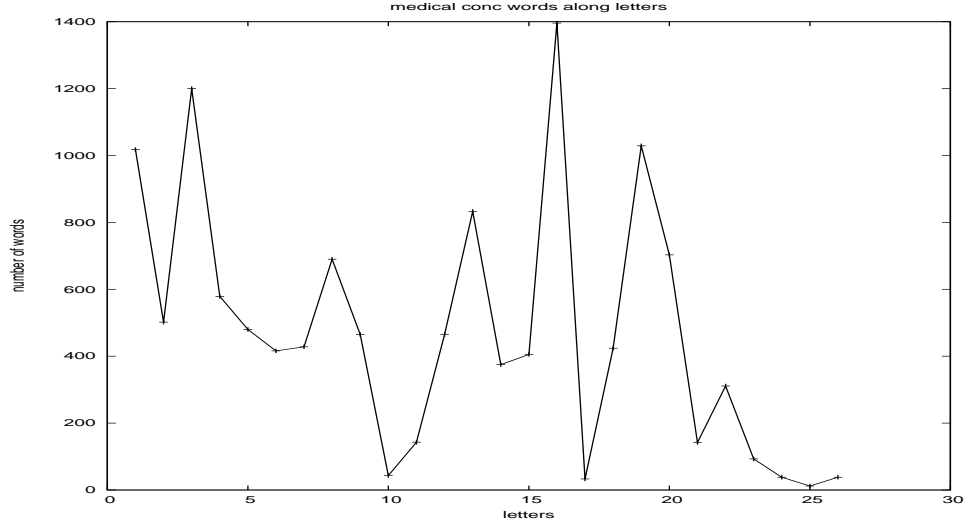


FIG. 27. Vertical axis is number of words in the concise medical dictionary,[10]. Horizontal axis is the letters of the English alphabet. Letters are represented by the sequence number in the alphabet.

## VII. ANALYSIS OF MEDICAL DICTIONARIES

### A. Analysis of concise medical dictionary

”Health is wealth”...English Proverb

We count the words, strictly speaking entries, of the concise medical dictionary,[10], one by one from the beginning to the end, starting with different letters. The result is the table, XI. Highest number of words, one thousand three hundred ninety six, start with the letter P followed by words numbering one thousand two hundred beginning with C, one thousand twenty nine with the letter S etc. To visualise we plot the number of words against respective letters in the dictionary sequence,[10] in the adjoining figure, fig.27. For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by  $f$  and the respective rank, denoted by  $k$ .  $k$  is a positive integer starting from one. Moreover, we attach a limiting rank,  $k_{lim}$ , and a limiting number

k	lnk	lnk/ $\ln k_{lim}$	f	lnf	lnf/ $\ln f_{max}$	lnf/ $\ln f_{next-max}$	lnf/ $\ln f_{nnmax}$	lnf/ $\ln f_{nnnmax}$
1	0	0	1396	7.241	1	Blank	Blank	Blank
2	0.69	0.214	1200	7.090	0.9791	1	Blank	Blank
3	1.10	0.342	1029	6.936	0.9579	0.978	1	Blank
4	1.39	0.432	1018	6.926	0.9565	0.977	0.999	1
5	1.61	0.500	833	6.73	0.929	0.949	0.970	0.972
6	1.79	0.556	703	6.56	0.906	0.925	0.946	0.947
7	1.95	0.606	690	6.54	0.903	0.922	0.943	0.944
8	2.08	0.646	579	6.36	0.878	0.897	0.917	0.918
9	2.20	0.683	502	6.22	0.859	0.877	0.897	0.898
10	2.30	0.714	480	6.17	0.852	0.870	0.890	0.891
11	2.40	0.745	465	6.14	0.848	0.866	0.885	0.887
12	2.48	0.770	428	6.06	0.837	0.855	0.874	0.875
13	2.56	0.795	424	6.05	0.836	0.853	0.872	0.874
14	2.64	0.820	416	6.03	0.833	0.850	0.869	0.871
15	2.71	0.842	405	6.00	0.829	0.846	0.865	0.866
16	2.77	0.860	375	5.93	0.819	0.836	0.855	0.856
17	2.83	0.879	311	5.74	0.793	0.810	0.828	0.829
18	2.89	0.898	143	4.962	0.685	0.700	0.715	0.716
19	2.94	0.913	142	4.956	0.684	0.699	0.715	0.716
20	3.00	0.932	93	4.53	0.626	0.639	0.653	0.654
21	3.04	0.944	43	3.76	0.519	0.530	0.542	0.543
22	3.09	0.960	38	3.64	0.503	0.513	0.525	0.526
23	3.14	0.975	33	3.50	0.483	0.494	0.505	0.505
24	3.18	0.988	12	2.48	0.342	0.350	0.358	0.358
25	3.22	1	1	0	0	0	0	0

TABLE XII. Concise medical dictionary words: ranking,natural logarithm,normalisations

of words. The limiting rank is maximum rank plus one, here it is twenty five and the limiting number of words is one. As a result both  $\frac{\ln f}{\ln f_{max}}$  and  $\frac{\ln k}{\ln k_{lim}}$  varies from zero to one. Then we tabulate in the adjoining table, XII, and plot  $\frac{\ln f}{\ln f_{max}}$  against  $\frac{\ln k}{\ln k_{lim}}$  in the figure fig.28. We then ignore the letter with the highest of words, tabulate in the adjoining table, XII, and redo the plot, normalising the  $\ln f$ s with next-to-maximum  $\ln f_{next-max}$ , and starting from  $k = 2$  in the figure fig.29. Normalising the  $\ln f$ s with next-to-next-to-maximum  $\ln f_{nextnext-max}$ , we tabulate in the adjoining table, XII, and starting from  $k = 3$  we draw in the figure fig.30. Normalising the  $\ln f$ s with next-to-next-to-next-to-maximum  $\ln f_{nextnextnext-max}$  we record in the adjoining table, XII, and plot starting from  $k = 4$  in the figure fig.31.

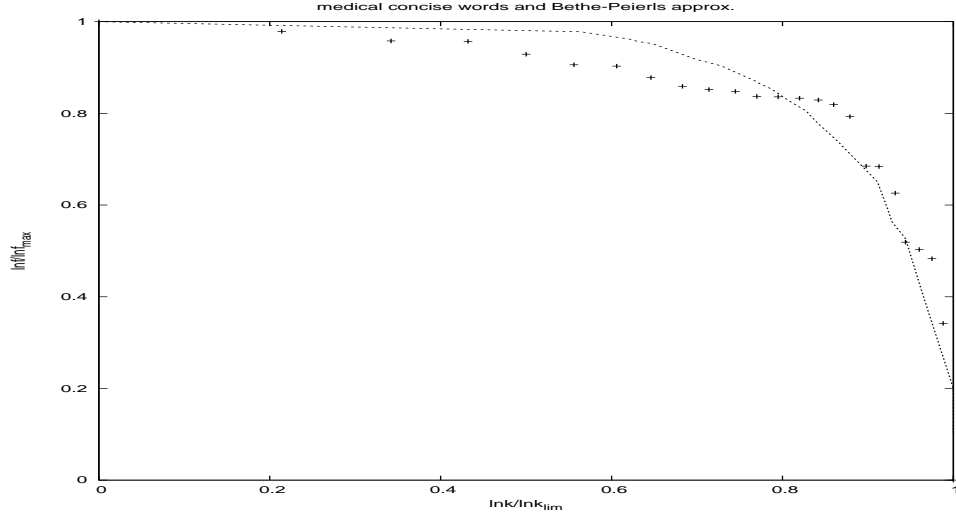


FIG. 28. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the concise medical dictionary with fit curve being Bethe-Peierls curve in presence of four nearest neighbours.

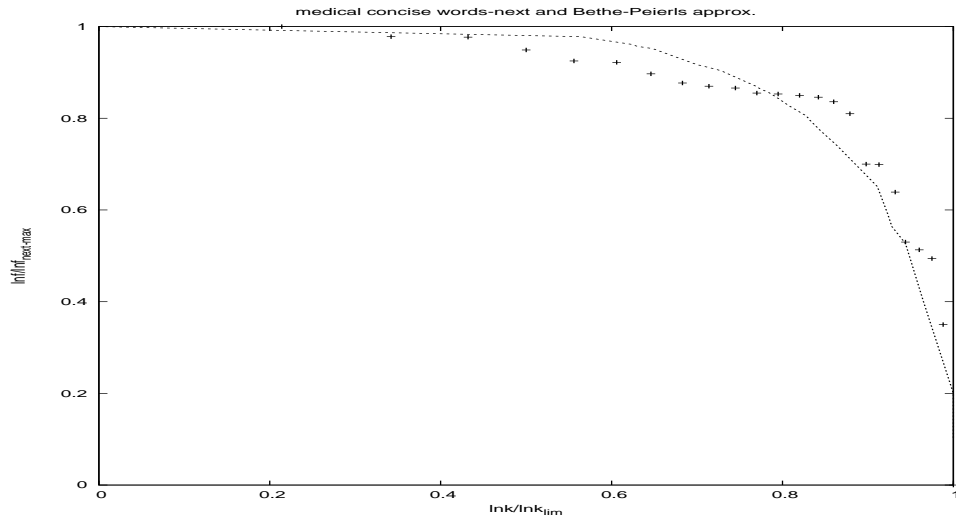


FIG. 29. Vertical axis is  $\frac{\ln f}{\ln f_{next-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the concise medical dictionary with fit curve being Bethe-Peierls curve in presence of four nearest neighbours.

1. conclusion

From the figures (fig.28-fig.31), we observe that there is a curve of magnetisation, behind words of concise medical dictionary. This is magnetisation curve in the Bethe-Peierls ap-



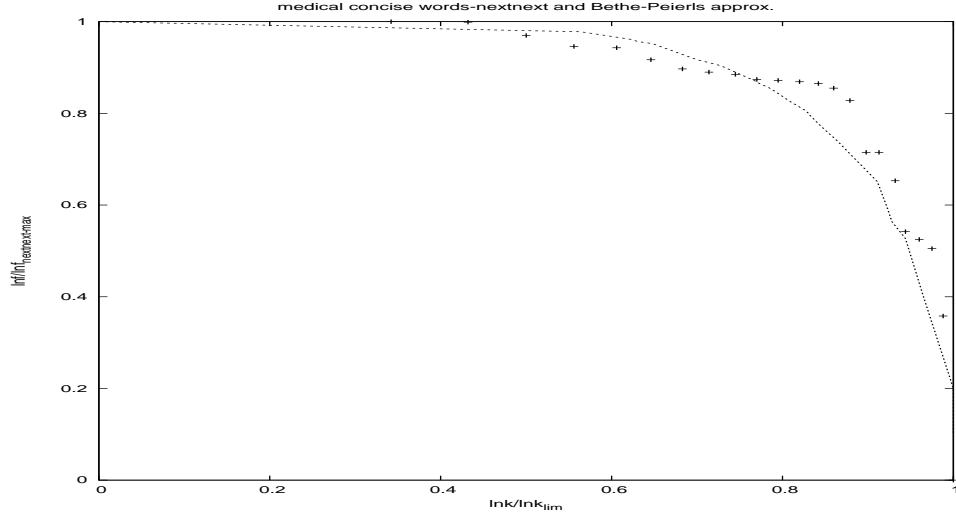


FIG. 30. Vertical axis is  $\frac{\ln f}{\ln f_{nextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the concise medical dictionary with fit curve being Bethe-Peierls curve in presence of four nearest neighbours.

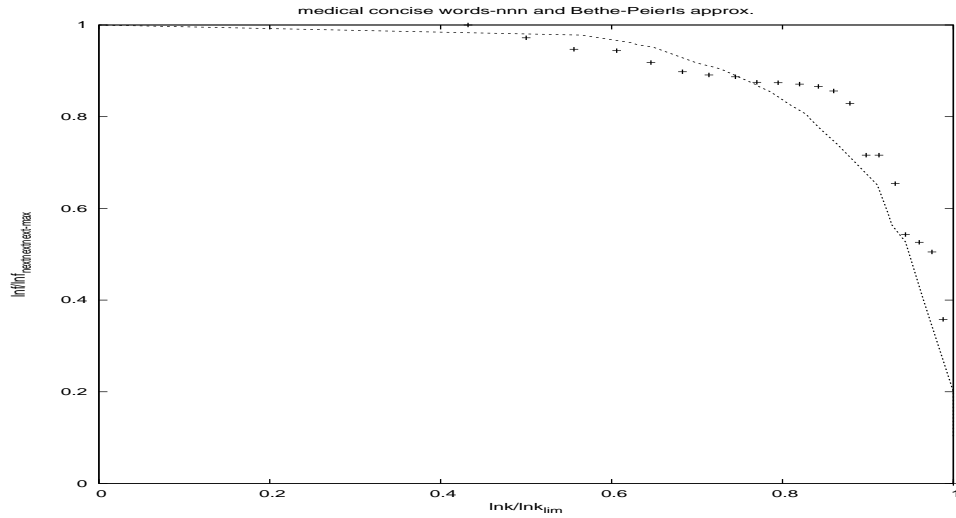


FIG. 31. Vertical axis is  $\frac{\ln f}{\ln f_{nextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the concise medical dictionary with fit curve being Bethe-Peierls curve in presence of four nearest neighbours.

proximation with four nearest neighbours. Moreover, the associated correspondance is,

$$\frac{\ln f}{\ln f_{next-max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$\ln k \longleftrightarrow T.$$

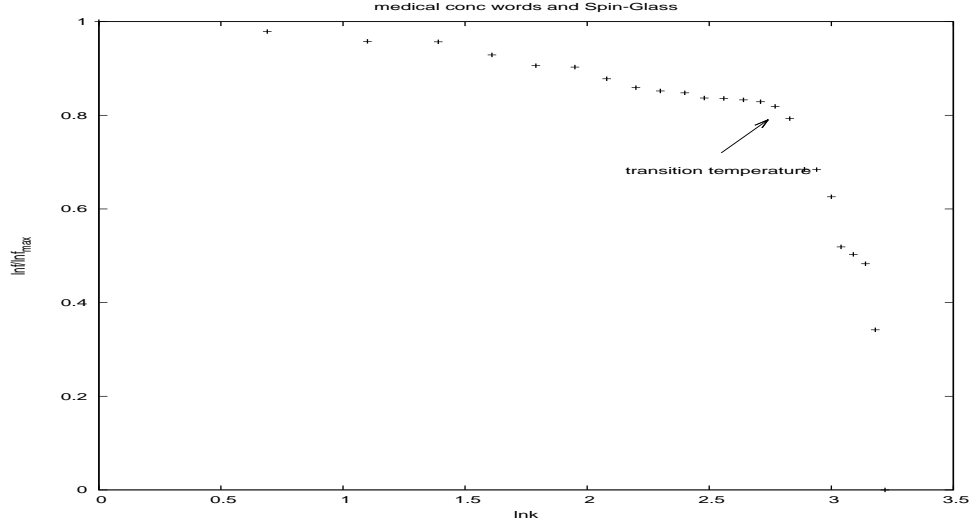


FIG. 32. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the concise medical dictionary.

$k$  corresponds to temperature in an exponential scale, [28]. As temperature decreases, i.e.  $\ln k$  decreases,  $f$  increases. The letters which are recording higher entries compared to those which have lesser entries are at lower temperature. As concise medical dictionary expands, the letters like ....S, C, P which get enriched more and more, fall at lower and lower temperatures. This is a manifestation of cooling effect, as was first observed in [29], in another way. Moreover, for the shake of completeness we draw  $\frac{\ln f}{\ln f_{max}}$  against  $\ln k$  in the figure fig.(32) to explore for the possible existence of a magnetisation curve of a Spin-Glass in presence of an external magnetic field, underlying words of concise medical dictionary. In the figure 32, the pointsline does not have a clearcut transition Hence, the words of the concise medical dictionary, is not suited to be described, to underlie a Spin-Glass magnetisation curve, [20], in the presence of magnetic field.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
2602	845	2572	1213	1397	616	756	1518	836	38	217	836	1640	690	827	2833	52	703	1945	1320	372	531	61	68	14	94

TABLE XIII. Pocket medical dictionary words

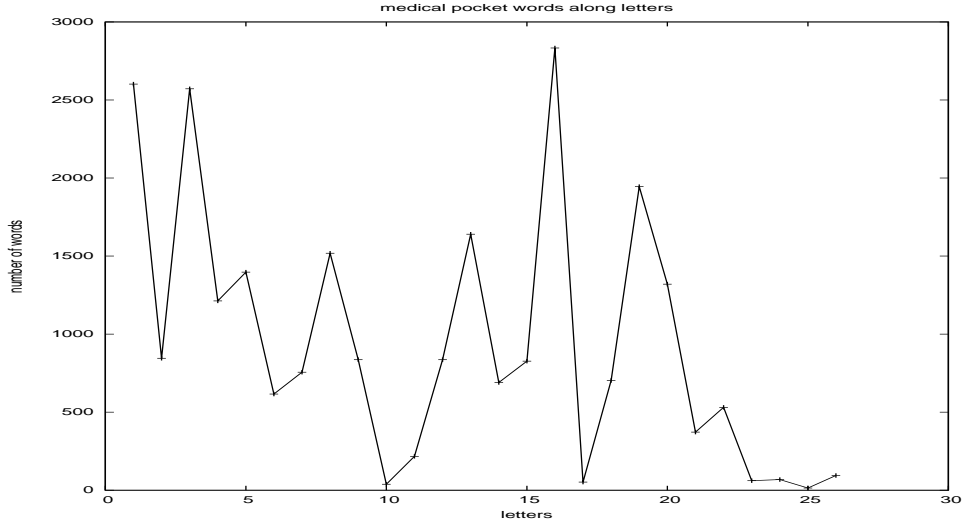


FIG. 33. Vertical axis is number of words in the pocket medical dictionary,[11]. Horizontal axis is the letters of the English alphabet. Letters are represented by the sequence number in the alphabet.

### B. Analysis of pocket medical dictionary

”Size matters”: a commoner’s perception.

We count the words, strictly speaking entries, of the pocket medical dictionary,[11], one by one from the beginning to the end, starting with different letters. The result is the table, XIII. Highest number of words, two thousand eight hundred thirty three, start with the letter P followed by words numbering two thousand six hundred two beginning with A, two thousand five hundred seventy two with the letter C etc. To visualise we plot the number of words against respective letters in the dictionary sequence,[11] in the adjoining figure, fig.33. For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by  $f$  and the respective rank, denoted by  $k$ .  $k$  is a positive integer starting from one. Moreover, we attach a limiting rank,  $k_{lim}$ , and a limiting number of words. The limiting rank is maximum rank plus one, here it is twenty six and the limiting number of words is one. As a result both  $\frac{\ln f}{\ln f_{max}}$  and  $\frac{\ln k}{\ln k_{lim}}$  varies

k	lnk	lnk/ $lnk_{lim}$	f	lnf	lnf/ $lnf_{max}$	lnf/ $lnf_{nmax}$	lnf/ $lnf_{nnmax}$	lnf/ $lnf_{nnnmax}$
1	0	0	2833	7.95	1	Blank	Blank	Blank
2	0.69	0.212	2602	7.86	0.989	1	Blank	Blank
3	1.10	0.337	2572	7.85	0.987	0.999	1	Blank
4	1.39	0.426	1945	7.57	0.952	0.963	0.964	1
5	1.61	0.494	1640	7.40	0.931	0.941	0.943	0.978
6	1.79	0.549	1518	7.33	0.922	0.933	0.934	0.968
7	1.95	0.598	1397	7.24	0.911	0.921	0.922	0.956
8	2.08	0.638	1320	7.19	0.904	0.915	0.916	0.950
9	2.20	0.675	1213	7.10	0.893	0.903	0.904	0.938
10	2.30	0.706	845	6.74	0.848	0.858	0.859	0.890
11	2.40	0.736	836	6.73	0.847	0.856	0.857	0.889
12	2.48	0.761	827	6.72	0.845	0.855	0.856	0.888
13	2.56	0.785	756	6.63	0.834	0.844	0.845	0.876
14	2.64	0.810	703	6.56	0.825	0.835	0.836	0.867
15	2.71	0.831	690	6.54	0.823	0.832	0.833	0.864
16	2.77	0.850	616	6.42	0.808	0.817	0.818	0.848
17	2.83	0.868	531	6.27	0.789	0.798	0.799	0.828
18	2.89	0.887	372	5.92	0.745	0.753	0.754	0.782
19	2.94	0.902	217	5.38	0.677	0.684	0.685	0.711
20	3.00	0.920	94	4.54	0.571	0.578	0.578	0.600
21	3.04	0.933	68	4.22	0.531	0.537	0.538	0.557
22	3.09	0.948	61	4.11	0.517	0.523	0.524	0.543
23	3.14	0.963	52	3.95	0.497	0.503	0.503	0.522
24	3.18	0.975	38	3.64	0.458	0.463	0.464	0.481
25	3.22	0.988	14	2.64	0.332	0.336	0.336	0.349
26	3.26	1	1	0	0	0	0	0

TABLE XIV. Pocket medical dictionary words: ranking,natural logarithm,normalisations

from zero to one. Then we tabulate in the adjoining table, XIV, and plot  $\frac{lnf}{lnf_{max}}$  against  $\frac{lnk}{lnk_{lim}}$  in the figure fig.34. We then ignore the letter with the highest of words, tabulate in the adjoining table, XIV, and redo the plot, normalising the  $lnfs$  with next-to-maximum  $lnf_{nextmax}$ , and starting from  $k = 2$  in the figure fig.35. Normalising the  $lnfs$  with next-to-next-to-maximum  $lnf_{nextnextmax}$ , we tabulate in the adjoining table, XIV, and starting from  $k = 3$  we draw in the figure fig.36. Normalising the  $lnfs$  with next-to-next-to-next-to-maximum  $lnf_{nextnextnextmax}$  we record in the adjoining table, XIV, and plot starting from  $k = 4$  in the figure fig.37. Matching of the plots in the figures fig.(34-37) with comparator curves i.e. Bethe-Peierls curve in presence of four nearest neighbours, dispersion reduces over higher orders of normalisations and the points in the figure fig.35 go the best along

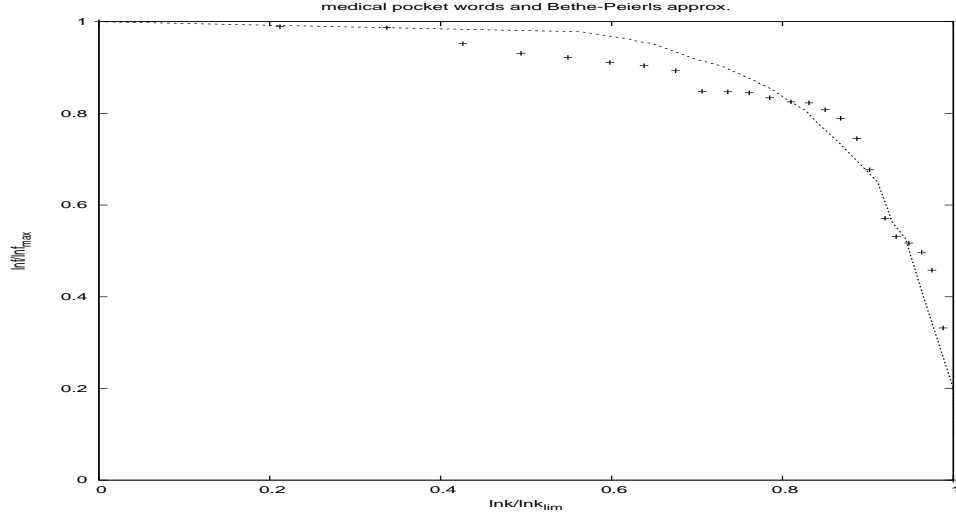


FIG. 34. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the pocket medical dictionary with fit curve being Bethe-Peierls curve in presence of four nearest neighbours.

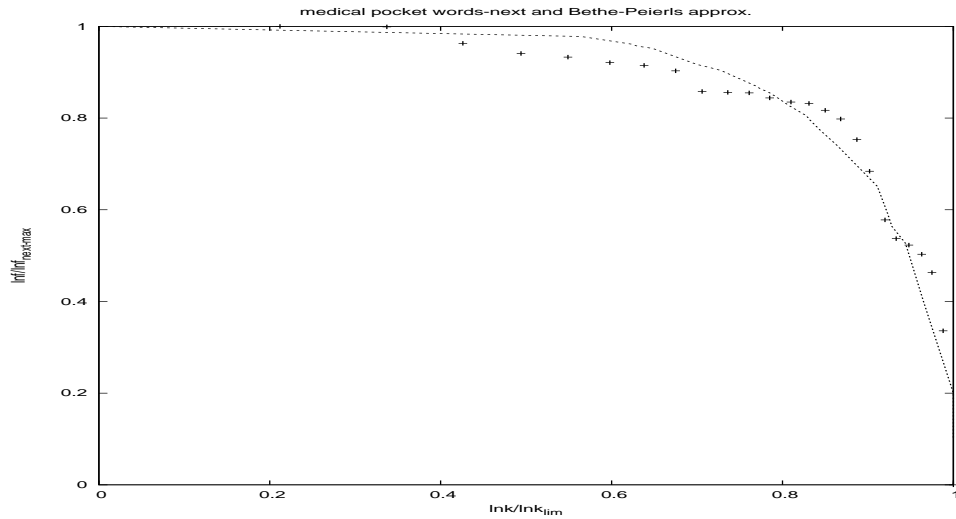


FIG. 35. Vertical axis is  $\frac{\ln f}{\ln f_{next-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the pocket medical dictionary with fit curve being Bethe-Peierls curve in presence of four nearest neighbours.

the Bethe-Peierls curve in presence of four nearest neighbours. Hence the words of pocket medical dictionary can be characterised by Bethe-Peierls curve in presence of four nearest neighbours.

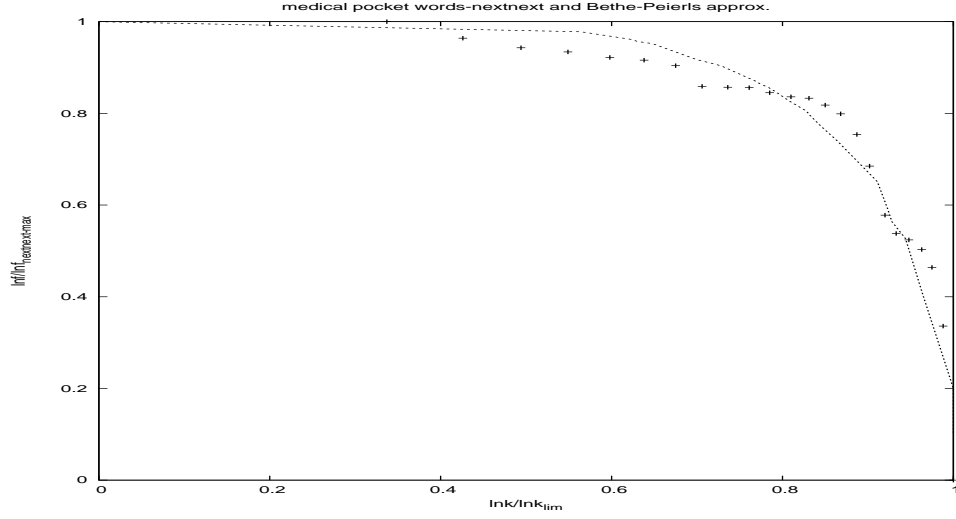


FIG. 36. Vertical axis is  $\frac{\ln f}{\ln f_{nextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the pocket medical dictionary with fit curve being Bethe-Peierls curve in presence of four nearest neighbours.

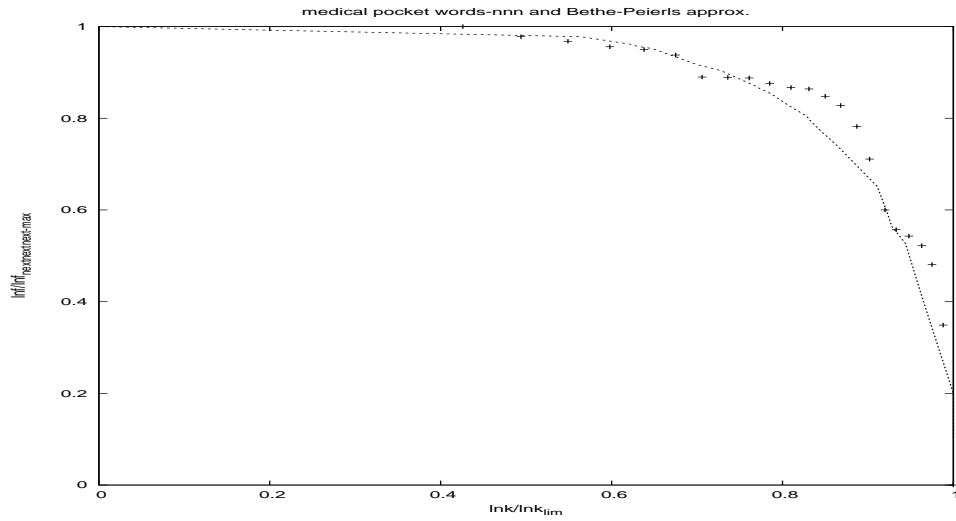


FIG. 37. Vertical axis is  $\frac{\ln f}{\ln f_{nextnextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the pocket medical dictionary with fit curve being Bethe-Peierls curve in presence of four nearest neighbours.

1. conclusion

From the figures (fig.34-fig.37), we observe that there is a curve of magnetisation, behind the words of pocket medical dictionary. This is Bethe-Peierls curve in presence of four nearest

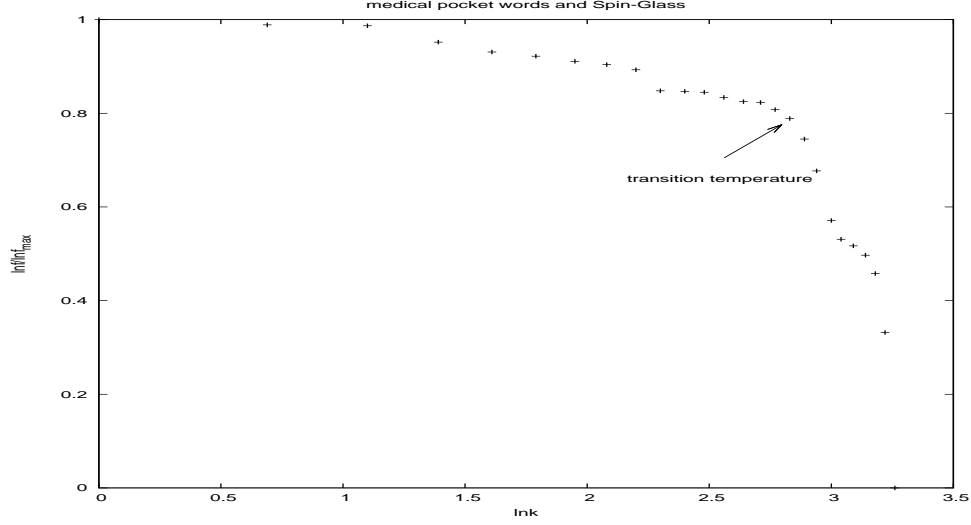


FIG. 38. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the pocket medical dictionary.

neighbours. Moreover, the associated correspondance is,

$$\frac{\ln f}{\ln f_{next-max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$\ln k \longleftrightarrow T.$$

$k$  corresponds to temperature in an exponential scale, [28]. As temperature decreases, i.e.  $\ln k$  decreases,  $f$  increases. The letters which are recording higher entries compared to those which have lesser entries are at lower temperature. As the words of pocket medical dictionary expands, the letters like ...C, A, P which get enriched more and more, fall at lower and lower temperatures. This is a manifestation of cooling effect, as was first observed in [29], in another way. But to be certain, we draw  $\frac{\ln f}{\ln f_{max}}$  against  $\ln k$  in the figure fig.38 to explore for the possible existence of a magnetisation curve of a Spin-Glass in presence of an external magnetic field, underlying words of pocket medical dictionary. We note that the points in the fig.38, does not have a clear-cut transition point. Hence, the words of pocket medical dictionary is not suited to be described by a Spin-Glass magnetisation curve, [20], in the presence of an external magnetic field.

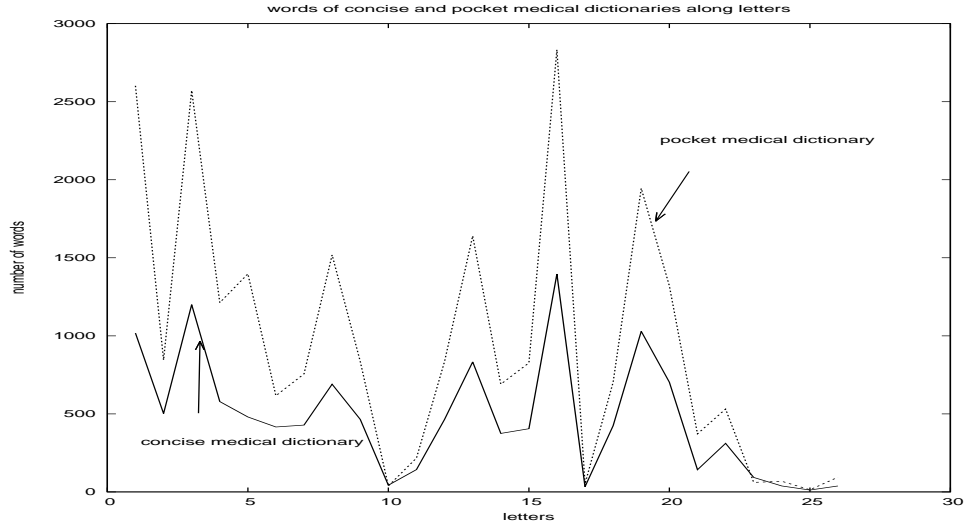


FIG. 39. Vertical axis is number of words and horizontal axis is respective letters. Letters are represented by the number in the alphabet or, dictionary sequence,[10, 11].

### C. comparison between two medical dictionaries

We notice that the maxima fall on the same letters for both the dictionaries. Moreover, as we have observed in the previous two subsections, that the sets of graphs are similar. Both the dictionaries underlie the same magnetisation curve. It will be interesting to find that the same pattern continues if we take a third medical dictionary.



A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
508	539	593	501	385	634	335	421	265	78	102	340	347	154	283	727	67	495	1289	519	140	185	214	7	19	27

TABLE XV. Words of dictionary of Construction etc.

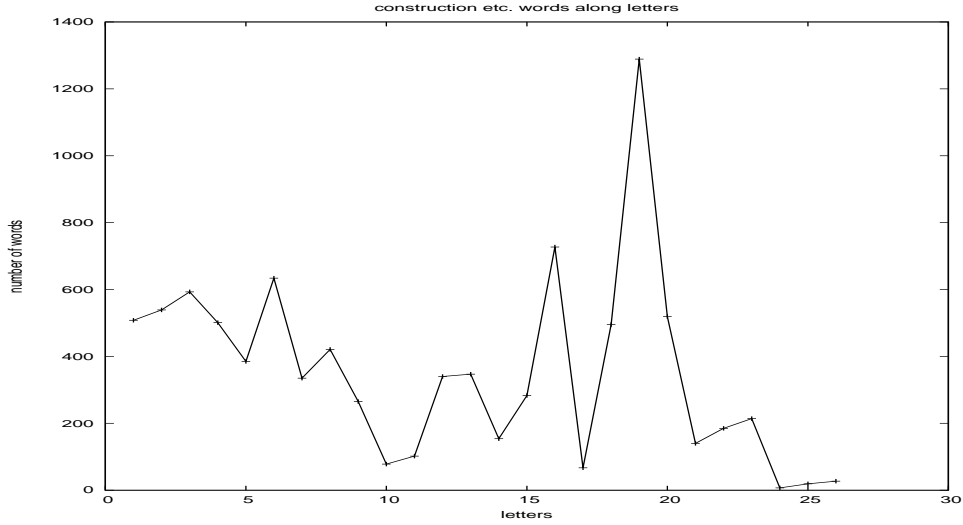


FIG. 40. Vertical axis is number of words in the dictionary of construction etc.,[12]. Horizontal axis is the letters of the English alphabet. Letters are represented by the sequence number in the alphabet.

## VIII. REANALYSIS OF CONSTRUCTION

”To err is human”: quote unknown

We take a relook in the dictionary of construction etc.,[12]. There we have counted, [2], the words, strictly speaking entries, one by one from the beginning to the end, starting with different letters. The result is the table, XV. Highest number of words, one thousand two hundred eighty nine, start with the letter S followed by words numbering seven hundred twenty seven beginning with P, six hundred thirty four with the letter F etc. To visualise we plot the number of words again respective letters in the dictionary sequence,[12] in the adjoining figure, fig.40. For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by  $f$  and the respective rank, denoted by  $k$ .  $k$  is a positive integer starting from one. Moreover, we attach a limiting rank,  $k_{lim}$ , and a limiting number of words. The limiting rank is maximum rank plus one, here it is twenty seven and the limiting number of words is one. As a result both  $\frac{\ln f}{\ln f_{max}}$

k	lnk	lnk/lnk <sub>lim</sub>	f	lnf	lnf/lnf <sub>max</sub>	lnf/lnf <sub>next-max</sub>	lnf/lnf <sub>nnmax</sub>	lnf/lnf <sub>nnnmax</sub>
1	0	0	1289	7.16	1	Blank	Blank	Blank
2	0.69	0.209	727	6.59	0.920	1	Blank	Blank
3	1.10	0.333	634	6.45	0.901	0.979	1	Blank
4	1.39	0.421	593	6.39	0.892	0.970	0.991	1
5	1.61	0.488	539	6.29	0.878	0.954	0.975	0.984
6	1.79	0.542	519	6.25	0.873	0.948	0.969	0.978
7	1.95	0.591	508	6.23	0.870	0.945	0.966	0.975
8	2.08	0.630	501	6.22	0.869	0.944	0.964	0.973
9	2.20	0.667	495	6.20	0.866	0.941	0.961	0.970
10	2.30	0.697	421	6.04	0.844	0.917	0.936	0.945
11	2.40	0.727	385	5.95	0.831	0.903	0.922	0.931
12	2.48	0.752	347	5.85	0.817	0.888	0.907	0.915
13	2.56	0.776	340	5.83	0.814	0.885	0.904	0.912
14	2.64	0.800	335	5.81	0.811	0.882	0.901	0.909
15	2.71	0.821	283	5.65	0.789	0.857	0.876	0.884
16	2.77	0.839	265	5.58	0.779	0.847	0.865	0.873
17	2.83	0.858	214	5.37	0.750	0.815	0.833	0.840
18	2.89	0.876	185	5.22	0.729	0.792	0.809	0.817
19	2.94	0.891	154	5.04	0.704	0.765	0.781	0.789
20	3.00	0.909	140	4.94	0.690	0.750	0.766	0.773
21	3.04	0.921	102	4.62	0.645	0.701	0.716	0.723
22	3.09	0.936	78	4.36	0.609	0.662	0.676	0.682
23	3.14	0.952	67	4.20	0.587	0.637	0.651	0.657
24	3.18	0.964	27	3.30	0.461	0.501	0.512	0.516
25	3.22	0.976	19	2.94	0.411	0.446	0.456	0.460
26	3.26	0.988	7	1.95	0.272	0.296	0.302	0.305
27	3.30	1	1	0	0	0	0	0

TABLE XVI. Words of dictionary of Construction etc.: ranking,natural logarithm,normalisations

and  $\frac{\ln k}{\ln k_{lim}}$  varies from zero to one. Then we tabulate in the adjoining table, XVI, and plot  $\frac{\ln f}{\ln f_{max}}$  against  $\frac{\ln k}{\ln k_{lim}}$  in the figure fig.41. We then ignore the letter with the highest of words, tabulate in the adjoining table, XVI, and redo the plot, normalising the  $\ln f$ s with next-to-maximum  $\ln f_{next-max}$ , and starting from  $k = 2$  in the figure fig.42. Normalising the  $\ln f$ s with next-to-next-to-maximum  $\ln f_{nextnext-max}$ , we tabulate in the adjoining table, XVI, and starting from  $k = 3$  we draw in the figure fig.43. Normalising the  $\ln f$ s with next-to-next-to-next-to-maximum  $\ln f_{nextnextnext-tmax}$  we record in the adjoining table, XVI, and plot starting from  $k = 4$  in the figure fig.44.

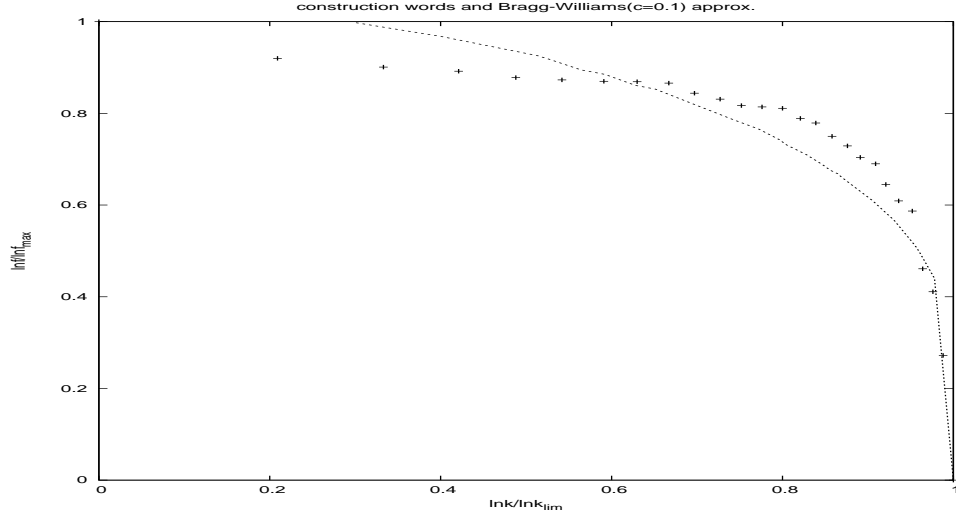


FIG. 41. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the dictionary of construction etc. with fit curve being Bragg-Williams curve in presence of magnetic field,  $c = \frac{H}{\gamma\epsilon} = 0.1$ .

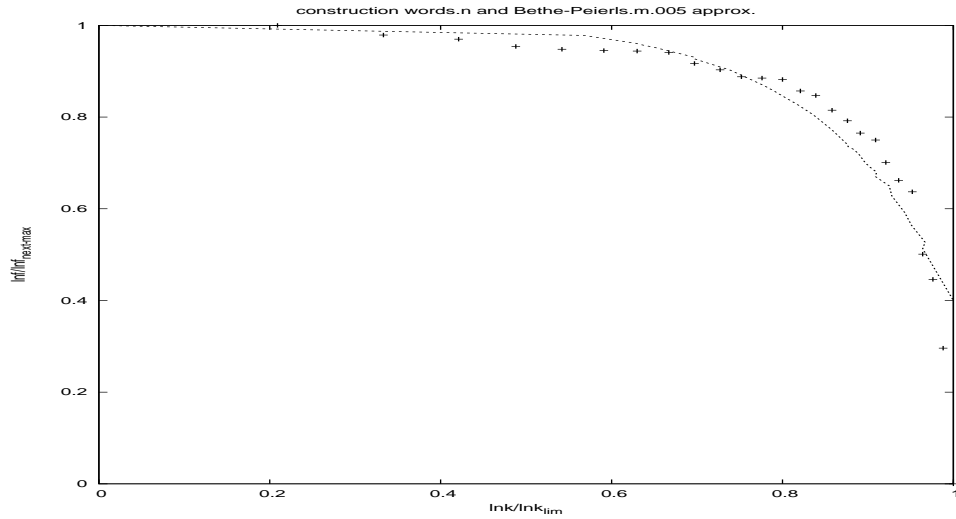


FIG. 42. Vertical axis is  $\frac{\ln f}{\ln f_{next-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the dictionary of construction etc. with fit curve being Bethe-Peierls curve with four nearest neighbours in presence of little magnetic field  $m = 0.005$  or,  $\beta H = 0.01$ .

### A. conclusion

From the figures (fig.41-fig.44), we observe that there is a curve of magnetisation, behind words of construction etc. This is magnetisation curve in the Bethe-Peierls approximation

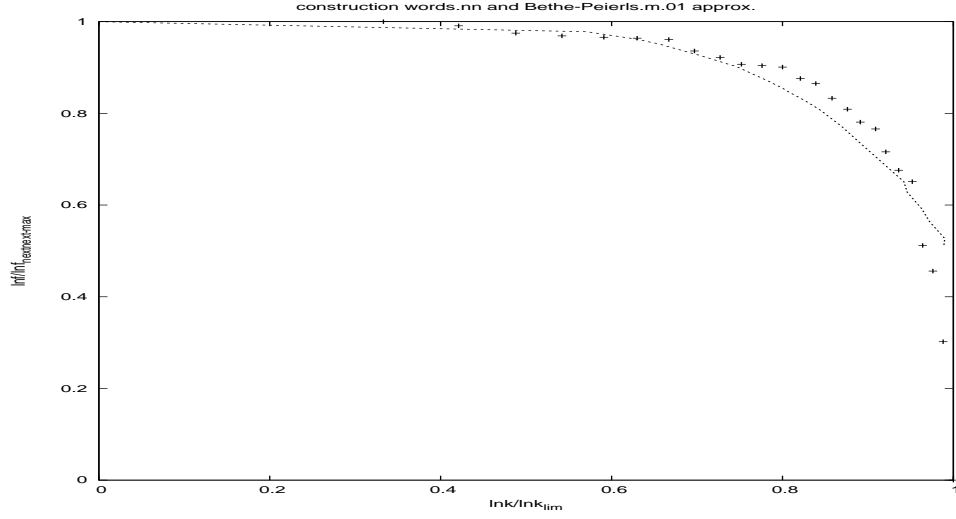


FIG. 43. Vertical axis is  $\frac{\ln f}{\ln f_{nextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the dictionary of construction etc. with fit curve being Bethe-Peierls curve with four nearest neighbours in presence of little magnetic field  $m = 0.01$  or,  $\beta H = 0.02$ .

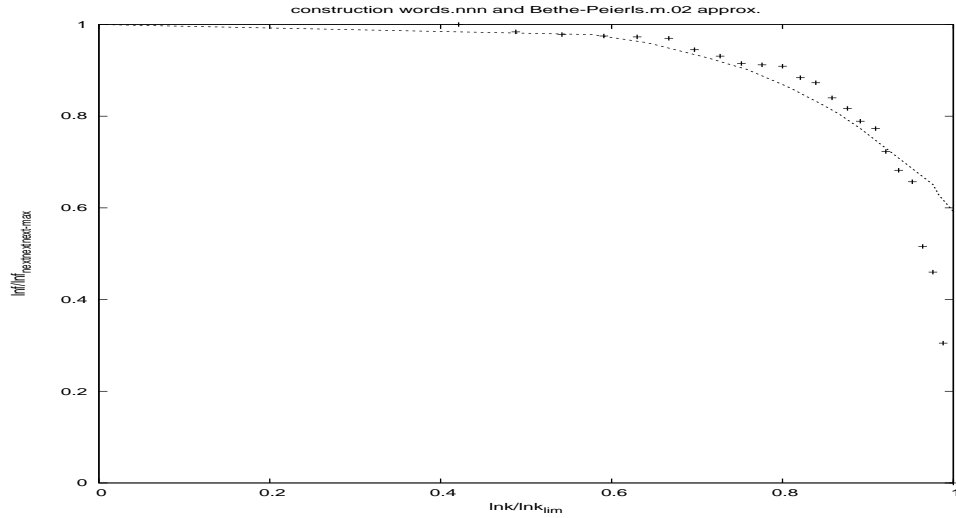


FIG. 44. Vertical axis is  $\frac{\ln f}{\ln f_{nextnextnext-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the dictionary of construction etc. with fit curve being Bethe-Peierls curve with four nearest neighbours in presence of little magnetic field  $m = 0.02$  or,  $\beta H = 0.04$ .

with four nearest neighbours, in presence of little magnetic field,  $m = 0.01$  or,  $\beta H = 0.02$ . Moreover, the associated correspondance is,

$$\frac{\ln f}{\ln f_{next-to-next-to-maximum}} \longleftrightarrow \frac{M}{M_{max}},$$

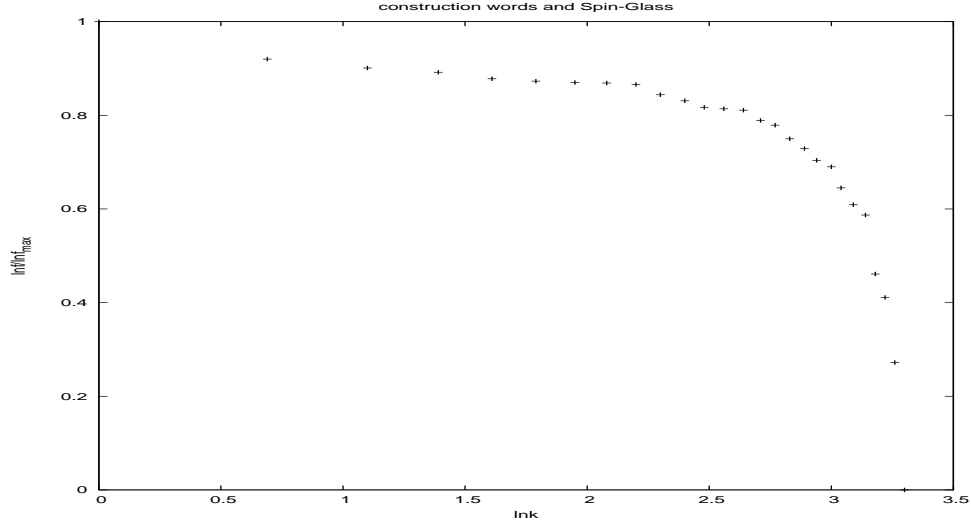


FIG. 45. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the dictionary of construction etc.

$$\ln k \longleftrightarrow T.$$

$k$  corresponds to temperature in an exponential scale, [28]. As temperature decreases, i.e.  $\ln k$  decreases,  $f$  increases. The letters which are recording higher entries compared to those which have lesser entries are at lower temperature. As the subject of construction etc. expands, the letters like.... F, P, S which get enriched more and more, fall at lower and lower temperatures. This is a manifestation of cooling effect, as was first observed in [29], in another way. Moreover, for the shake of completeness we draw  $\frac{\ln f}{\ln f_{max}}$  against  $\ln k$  in the figure fig.(45) to explore for the possible existence of a magnetisation curve of a Spin-Glass in presence of an external magnetic field, underlying words of construction etc. In the figure 45, the pointsline does not have a clearcut transition Hence, the words of the construction etc., is not suited to be described, to underlie a Spin-Glass magnetisation curve, [20], in the presence of magnetic field.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
877	573	1157	497	551	367	400	464	367	45	108	391	655	292	244	955	74	394	943	478	81	185	122	26	19	47

TABLE XVII. Words of dictionary of Science

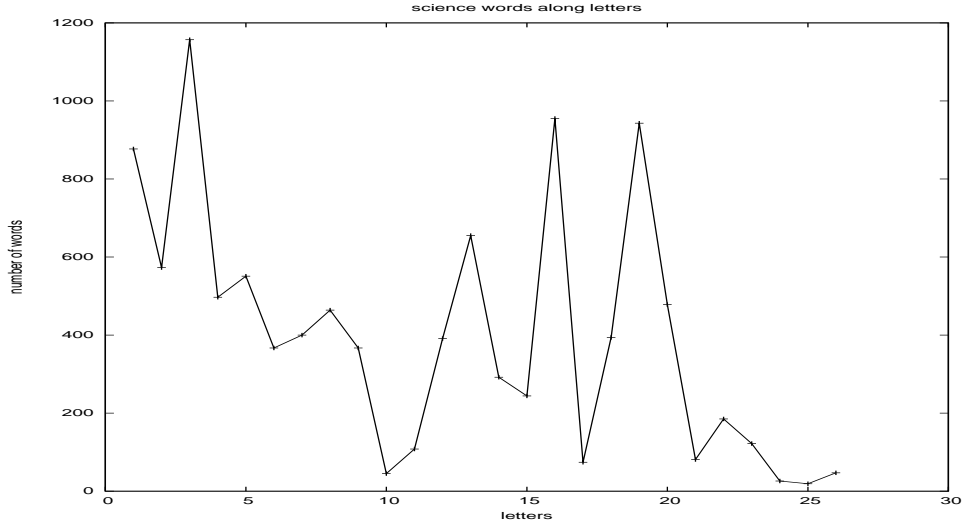


FIG. 46. Vertical axis is number of words in the dictionary of science,[13]. Horizontal axis is the letters of the English alphabet. Letters are represented by the sequence number in the alphabet.

## IX. REANALYSIS OF SCIENCE

We are in an era of science. To understand the discipline as a layman we have picked up a science dictionary, namely Oxford dictionary of Science, [13]. There we have counted, [2], the words, strictly speaking entries, one by one from the beginning to the end, starting with different letters. The result is the table, XVII. Highest number of words, one thousand one hundred fifty seven, start with the letter C followed by words numbering nine hundred fiftyfive beginning with P, nine hundred forty three with the letter S etc. To visualise we plot the number of words again respective letters in the dictionary sequence,[13] in the adjoining figure, fig.46. For the purpose of exploring graphical law, we assort the letters according to the number of words, in the descending order, denoted by  $f$  and the respective rank, denoted by  $k$ .  $k$  is a positive integer starting from one. Moreover, we attach a limiting rank,  $k_{lim}$ , and a limiting number of words. The limiting rank is maximum rank plus one, here it is twenty six and the limiting number of words is one. As a result both  $\frac{\ln f}{\ln f_{max}}$  and  $\frac{\ln k}{\ln k_{lim}}$  varies from zero to one. Then we tabulate in the adjoining table,XVIII, and plot  $\frac{\ln f}{\ln f_{max}}$  against

k	lnk	lnk/ $lnk_{lim}$	f	lnf	lnf/ $lnf_{max}$	lnf/ $lnf_{nmax}$	lnf/ $lnf_{nnmax}$	lnf/ $lnf_{nnnmax}$
1	0	0	1157	7.05	1	Blank	Blank	Blank
2	0.69	0.212	955	6.86	0.973	1	Blank	Blank
3	1.10	0.337	943	6.85	0.972	0.999	1	Blank
4	1.39	0.426	877	6.78	0.962	0.988	0.990	1
5	1.61	0.494	655	6.48	0.919	0.945	0.946	0.956
6	1.79	0.549	573	6.35	0.901	0.926	0.927	0.937
7	1.95	0.598	551	6.31	0.895	0.920	0.921	0.931
8	2.08	0.638	497	6.21	0.881	0.905	0.907	0.916
9	2.20	0.675	478	6.17	0.875	0.899	0.901	0.910
10	2.30	0.706	464	6.14	0.871	0.895	0.896	0.906
11	2.40	0.736	400	5.99	0.850	0.873	0.874	0.883
12	2.48	0.761	394	5.98	0.848	0.872	0.873	0.882
13	2.56	0.785	391	5.97	0.847	0.870	0.872	0.881
14	2.64	0.810	367	5.91	0.838	0.862	0.863	0.872
15	2.71	0.831	292	5.68	0.806	0.828	0.829	0.838
16	2.77	0.850	244	5.50	0.780	0.802	0.803	0.811
17	2.83	0.868	185	5.22	0.740	0.761	0.762	0.770
18	2.89	0.887	122	4.80	0.681	0.700	0.701	0.708
19	2.94	0.902	108	4.68	0.664	0.682	0.683	0.690
20	3.00	0.920	81	4.39	0.623	0.640	0.641	0.647
21	3.04	0.933	74	4.30	0.610	0.627	0.628	0.634
22	3.09	0.948	47	3.85	0.546	0.561	0.562	0.568
23	3.14	0.963	45	3.81	0.540	0.555	0.556	0.562
24	3.18	0.975	26	3.26	0.462	0.475	0.476	0.481
25	3.22	0.988	19	2.94	0.417	0.429	0.429	0.434
26	3.26	1	1	0	0	0	0	0

TABLE XVIII. Words of dictionary of Science: ranking,natural logarithm,normalisations

$\frac{lnk}{lnk_{lim}}$  in the figure fig.47. We then ignore the letter with the highest of words, tabulate in the adjoining table,XVIII,and redo the plot, normalising the  $lnfs$  with next-to-maximum  $lnf_{next-max}$ , and starting from  $k = 2$  in the figure fig.48. Normalising the  $lnfs$  with next-to-next-to-maximum  $lnf_{nextnext-max}$ , we tabulate in the adjoining table,XVIII, and starting from  $k = 3$  we draw in the figure fig.49. Normalising the  $lnfs$  with next-to-next-to-next-to-maximum  $lnf_{nextnextnext-max}$  we record in the adjoining table,XVIII, and plot starting from  $k = 4$  in the figure fig.50.

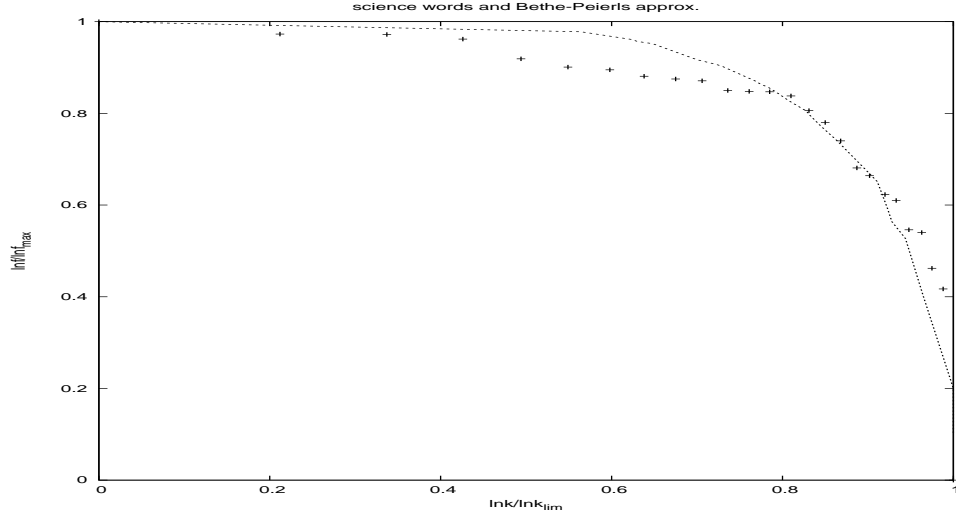


FIG. 47. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the dictionary of science with fit curve being Bethe-Peierls curve in presence of four neighbours.

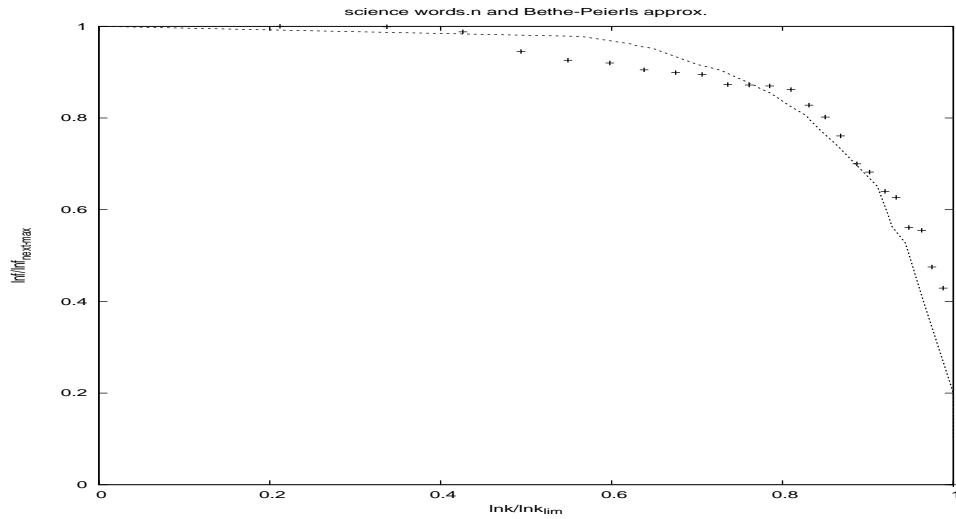


FIG. 48. Vertical axis is  $\frac{\ln f}{\ln f_{next-max}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{lim}}$ . The + points represent the words of the dictionary of science with fit curve being Bethe-Peierls curve in presence of four nearest neighbours.

### A. conclusion

From the figures (fig.47-fig.50), we observe that there is a curve of magnetisation, behind words of science. This is magnetisation curve in the Bethe-Peierls approximation with four nearest neighbours, in presence of magnetic field  $m = 0.005$  or,  $\beta H = 0.01$ . Moreover, the



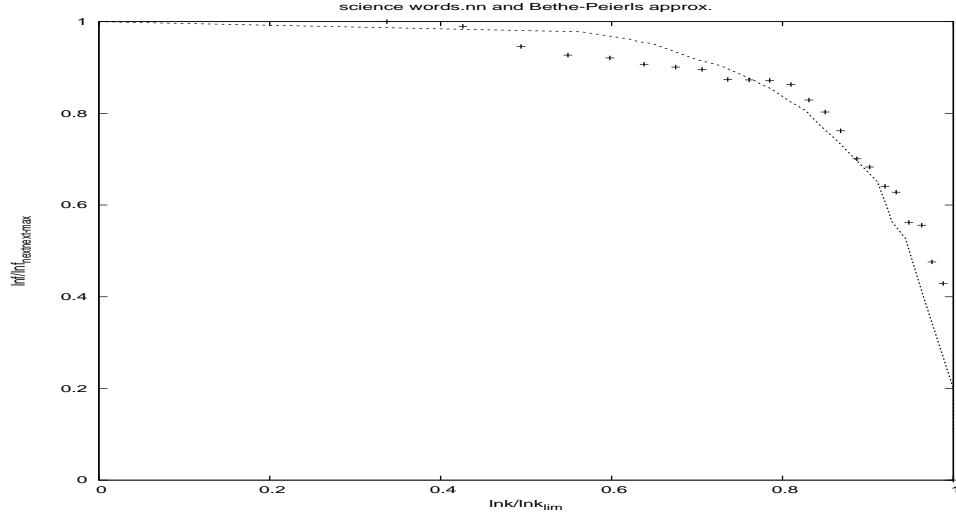


FIG. 49. Vertical axis is  $\frac{\ln f}{\ln f_{\text{nextnext-max}}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{\text{lim}}}$ . The + points represent the words of the dictionary of science with fit curve being Bethe-Peierls curve in presence of four nearest neighbours.

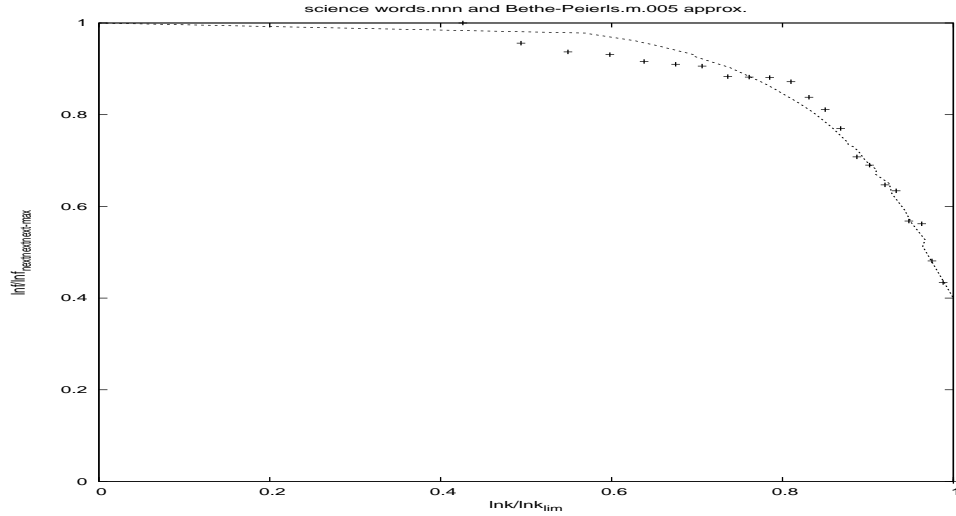


FIG. 50. Vertical axis is  $\frac{\ln f}{\ln f_{\text{nextnext-max}}}$  and horizontal axis is  $\frac{\ln k}{\ln k_{\text{lim}}}$ . The + points represent the words of the dictionary of science with fit curve being Bethe-Peierls curve in presence of four nearest neighbours in presence of magnetic field  $m = 0.005$  or,  $\beta H = 0.01$ .

associated correspondance is,

$$\frac{\ln f}{\ln f_{\text{next-to-next-to-next-to-maximum}}} \longleftrightarrow \frac{M}{M_{\text{max}}},$$

$$\ln k \longleftrightarrow T.$$

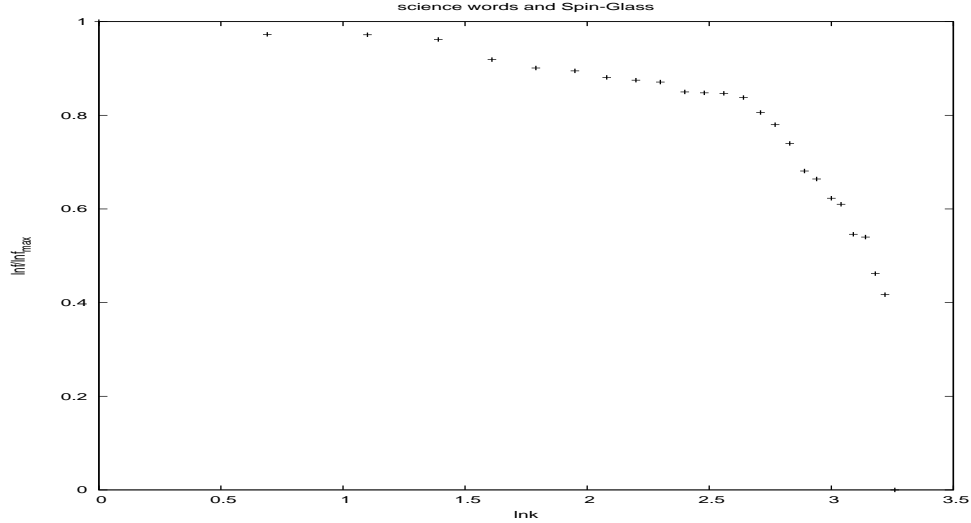


FIG. 51. Vertical axis is  $\frac{\ln f}{\ln f_{max}}$  and horizontal axis is  $\ln k$ . The + points represent the words of the dictionary of science.

$k$  corresponds to temperature in an exponential scale, [28]. As temperature decreases, i.e.  $\ln k$  decreases,  $f$  increases. The letters which are recording higher entries compared to those which have lesser entries are at lower temperature. As science expands, the letters like ...,S, P, C which get enriched more and more, fall at lower and lower temperatures. This is a manifestation of cooling effect, as was first observed in [29], in another way. Moreover, for the shake of completeness we draw  $\frac{\ln f}{\ln f_{max}}$  against  $\ln k$  in the figure fig.(51) to explore for the possible existence of a magnetisation curve of a Spin-Glass in presence of an external magnetic field, underlying science words. In the figure 51, the pointsline does not have a clearcut transition Hence, the words of the science dictionary, is not suited to be described, to underlie a Spin-Glass magnetisation curve, [20], in the presence of magnetic field.

## X. DISCUSSION

We have observed that there is a curve of magnetisation, behind entries of dictionary of economics,[6]. This is magnetisation curve in the Bethe-Peierls approximation with four nearest neighbours. The magnetisation curve in the Bethe-Peierls approximation with four nearest neighbours also underlie entries of dictionary of geography,[7]. Entries of dictionary of linguistics,[8], can be characterised by the magnetisation curve in the Bethe-Peierls approximation with four nearest neighbours in presence of little magnetic field,  $m = 0.01$

or,  $\beta H = 0.02$  like entries of dictionary of psychology,[9]. For entries of both the medical dictionaries, [10],[11], underlying magnetisation curve is the Bethe-Peierls approximation with four nearest neighbours. It opens up another line of investigation. Whether the same magnetisation curve underlies any medical dictionary? Whether a particular subject is characterised by one magnetisation curve irrespective of whatever dictionaries of that subject we analyse?

Moreover, behind entries of construction etc.,[12], magnetisation curve is the Bethe-Peierls approximation with four nearest neighbours, in presence of little magnetic field,  $m = 0.01$  or,  $\beta H = 0.02$  whereas, behind entries of science,[13], magnetisation curve is the Bethe-Peierls approximation with four nearest neighbours, in presence of magnetic field  $m = 0.005$  or,  $\beta H = 0.01$ .

We note that in the approximation scheme due to Bethe-Peierls, [19], reduced magnetisation varies with reduced temperature, for  $\gamma$  neighbours, in absence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}$$

$\ln \frac{\gamma}{\gamma-2}$  for four nearest neighbours i.e. for  $\gamma = 4$  is 0.693. In the two beginning papers, [1] and [2], an error crept in advertantly in the form of 0.693 appearing in place of  $\ln \frac{\gamma}{\gamma-2}$  for all  $\gamma$ , in the numerator, though this is correct only for  $\gamma = 4$ , invalidating all the magnetisation curves being termed as Bethe-Peierls curves excepting for  $\gamma = 4$ . This was detected by the author few months back. Whether the above relation with 0.693 appearing in place of  $\ln \frac{\gamma}{\gamma-2}$  for all  $\gamma$  in the numerator, is a valid another approximation of Ising model or, not is not known at least to the author. This necessiated the reinvestigation of the dictionaries of construction etc and science which appeared earlier in the paper, [1]. We will take up the reinvestigation of the languages which were labelled by Bethe-Peierls curves for  $\gamma \neq 4$  in the paper, [2], soon.

It will not be surprising if graphical law emerges in other kind of dictionaries like dictionary of place names, dictionary of street names, dictionary of names of people etc.

## XI. SUMMARY

Graphical law: Oxford Dictionaries and Dorland's Pocket Medical Dictionary

Economics	Geography	Concise Linguistics	Psychology	Concise Medical	Pocket Medical	Construction etc	Science
BP(4; $\beta H = 0$ )	BP(4; $\beta H = 0$ )	BP(4; $\beta H = 0.02$ )	BP(4; $\beta H = 0.02$ )	BP(4; $\beta H = 0$ )	BP(4; $\beta H = 0$ )	BP(4; $\beta H = 0.02$ )	BP(4; $\beta H = 0.01$ )

where,  $BP(4;\beta H = 0)$  represents magnetisation curve under Bethe-Peierls approximation with four nearest neighbours in absence of external magnetic field i.e.  $H$  is equal to zero and  $BP(4;\beta H = 0.02)$  stands for magnetisation curve under Bethe-Peierls approximation with four nearest neighbours in presence of external magnetic field i.e.  $\beta H$  is equal to 0.02. Moreover, we recall from [1],

Graphical law: Oxford Dictionaries and Dictionary of Law etc.

Philosophy	Sociology	Dictionary of Law and Administration
$BP(4;\beta H = 0)$	$BP(4;\beta H = 0)$	$BP(4;\beta H = 0)$

Moreover, the steps in finding a graphical law are as follows:

- (i) Count a dictionary from beginning to end word( entry) by word along the letters.
- (ii) Arrange the numbers of words in descending order. Denote by  $f$ .
- (iii) Assign an increasing rank i.e. a sequence starting from one to a limiting number with the sequence of decreasing number of words. The limiting number corresponds to last word number, put by hand if not there, being as one. Denote the sequence as  $k$ .  $k=k_d$  for  $f=1$ .
- (iv) Take natural logarithm of  $f$  and  $k$ . Normalise  $\ln k$  i.e. consider  $\frac{\ln k}{\ln k_d}$ .
- (v) Normalise  $\ln f$  i.e. consider  $\frac{\ln f}{\ln f_{max}}$  and plot  $\frac{\ln f}{\ln f_{max}}$  against  $\frac{\ln k}{\ln k_d}$ .
- (vi) Superpose comparator curves of section II one by one onto the plot and find which one is the closest to the plot.
- (vii) Leave  $f_{max}$ . Normalise  $\ln f$  i.e. consider  $\frac{\ln f}{\ln f_{next-max}}$  and plot  $\frac{\ln f}{\ln f_{next-max}}$  against  $\frac{\ln k}{\ln k_d}$  and superpose comparator curves of section II one by one onto the plot and find which one is the closest to the plot.
- (viii) Leave  $f_{max}$  and  $f_{next-max}$ . Normalise  $\ln f$  i.e. consider  $\frac{\ln f}{\ln f_{nextnext-max}}$  and plot  $\frac{\ln f}{\ln f_{nextnext-max}}$  against  $\frac{\ln k}{\ln k_d}$  and superpose comparator curves of section II one by one onto the plot and find which one is the closest to the plot.
- (ix) continue and adjudge which one is the best fit between a plot of a normalised  $\ln f$  vs normalised  $\ln k$  with a comparator curve.
- (ix) Refer the best fit as the magnetisation curve behind the dictionary.

## XII. APPENDIX

### A. Bethe-Peierls approximation in presence of magnetic field

Let us consider an Ising model of spins with  $\gamma$  nearest neighbours for each spin and subjected to a constant external magnetic field  $H$ . Let us pick up one spin and its nearest neighbourhood in the lattice. Let  $P(+1, n)$  be the probability of  $n$  spins in the up state and  $\gamma-n$  spins in the down spin state when the central spin is in the up state. Let  $P(-1, n)$  be the probability of  $n$  spins in the up state and  $\gamma-n$  spins in the down spin state when the central spin is in the down state.

$$P(+1, n) = \frac{1}{q_H} C_n^\gamma e^{\beta\epsilon(2n-\gamma)} z^n e^{\beta H} e^{\beta(2n-\gamma)H},$$

$$P(-1, n) = \frac{1}{q_H} C_n^\gamma e^{\beta\epsilon(\gamma-2n)} z^n e^{-\beta H} e^{\beta(2n-\gamma)H}$$

where  $q_H$  is a normalisation factor fixed by the condition that the total probability to get one particular spin among the neighbours either up or, down is one i.e.

$$\sum_{n=0}^{\gamma} [P(+1, n) + P(-1, n)] = 1,$$

where,

$$\sum_{n=0}^{\gamma} P(+1, n) = \frac{e^{\beta H}}{q_H} (e^{-\beta(\epsilon+H)} + e^{\beta(\epsilon+H)} z)^\gamma,$$

$$\sum_{n=0}^{\gamma} P(-1, n) = \frac{e^{-\beta H}}{q_H} (e^{-\beta(-\epsilon+H)} + e^{\beta(-\epsilon+H)} z)^\gamma.$$

This determines

$$q_H = e^{\beta H} (e^{-\beta(\epsilon+H)} + e^{\beta(\epsilon+H)} z)^\gamma + e^{-\beta H} (e^{\beta(\epsilon-H)} + e^{-\beta(\epsilon-H)} z)^\gamma$$

where,  $z$  introduces coupling of a spin and the nearest neighbourhood with the rest spins of the lattice. Moreover,

$$\frac{1}{\gamma} \sum_{n=0}^{\gamma} n P(+1, n) = \frac{z e^{\beta H}}{q_H} e^{\beta(\epsilon+H)} (e^{-\beta(\epsilon+H)} + e^{\beta(\epsilon+H)} z)^{\gamma-1},$$

$$\frac{1}{\gamma} \sum_{n=0}^{\gamma} n P(-1, n) = \frac{z e^{-\beta H}}{q_H} e^{\beta(-\epsilon+H)} (e^{-\beta(-\epsilon+H)} + e^{\beta(-\epsilon+H)} z)^{\gamma-1}.$$

Again,  $\sum_{n=0}^{\gamma} n P(+1, n)$  is the average number of pairs with the central spin up and another spin up in the nearest neighbourhood forming a pair. Total number of pairs with the central

spin in one end and another spin from the nearest neighbourhood is  $\gamma$ . Hence average probability to find an upspin in the nearest neighbourhood pairing with the central spin being in the up state is  $\frac{1}{\gamma}\sum_{n=0}^{\gamma}nP(+1, n)$ . Similarly,  $\sum_{n=0}^{\gamma}nP(-1, n)$  is the average number of pairs with the central spin down and another spin up in the nearest neighbourhood forming a pair. Total number of pairs with the central spin in one end and another spin from the nearest neighbourhood is  $\gamma$ . Hence average probability to find an upspin in the nearest neighbourhood pairing with the central spin being in the down state is  $\frac{1}{\gamma}\sum_{n=0}^{\gamma}nP(+1, n)$ . Therefore, average probability to find an up spin in the nearest neighbourhood of the central spin is  $\frac{1}{\gamma}\sum_{n=0}^{\gamma}n[P(+1, n) + P(-1, n)]$ . Moreover, distinction made in describing one spin as central and another spin as one in the neighbourhood is artificial with respect to the lattice. This implies probability of finding an up spin at the center is the same as the average probability of finding an up spin in the nearest neighbourhood. Consequently,

$$\sum_{n=0}^{\gamma}P(+1, n) = \frac{1}{\gamma}\sum_{n=0}^{\gamma}n[P(+1, n) + P(-1, n)],$$

resulting in,

$$\begin{aligned} z &= \frac{(e^{-\beta(\epsilon+H)} + e^{\beta(\epsilon+H)}z)^{\gamma-1}}{(e^{-\beta(-\epsilon+H)} + e^{\beta(-\epsilon+H)}z)^{\gamma-1}} \\ &= \left(\frac{1 + e^{2\beta(\epsilon+H)}z}{e^{2\beta\epsilon} + e^{2\beta H}z}\right)^{\gamma-1}. \end{aligned}$$

Moreover, this ensues

$$z^{\frac{1}{\gamma-1}} = \frac{1 + e^{2\beta(\epsilon+H)}z}{e^{2\beta\epsilon} + e^{2\beta H}z},$$

which in turn implies

$$e^{2\beta\epsilon} = \frac{1 - z^{\frac{\gamma}{\gamma-1}}e^{2\beta H}}{z^{\frac{1}{\gamma-1}} - ze^{2\beta H}}.$$

From which follows on taking natural logarithm on both sides,

$$2\beta\epsilon = \ln \frac{1 - z^{\frac{\gamma}{\gamma-1}}e^{2\beta H}}{z^{\frac{1}{\gamma-1}} - ze^{2\beta H}}$$

Again, reduced magnetisation,  $L$  or,  $\frac{M}{M_{max}}$  is given by

$$\begin{aligned} \frac{1+L}{2} &= \sum_{n=0}^{\gamma}P(+1, n) \\ &= \frac{e^{\beta H}(e^{-\beta(\epsilon+H)} + e^{\beta(\epsilon+H)}z)^{\gamma}}{e^{\beta H}(e^{-\beta(\epsilon+H)} + e^{\beta(\epsilon+H)}z)^{\gamma} + e^{-\beta H}(e^{\beta(\epsilon-H)} + e^{-\beta(\epsilon-H)}z)^{\gamma}}, \end{aligned}$$

which leads to

$$L = \frac{z^{\frac{\gamma}{\gamma-1}} - e^{-2\beta H}}{z^{\frac{\gamma}{\gamma-1}} + e^{-2\beta H}},$$

or,

$$z = e^{-2\beta H \left(\frac{\gamma-1}{\gamma}\right)} \left(\frac{1+L}{1-L}\right)^{\frac{\gamma-1}{\gamma}}$$

or,

$$z^{\frac{\gamma}{\gamma-1}} e^{2\beta H} = \frac{1+L}{1-L}$$

This results in

$$2\beta\epsilon = \ln \frac{1 - \frac{1+L}{1-L}}{e^{-\frac{2\beta H}{\gamma} \left(\frac{1+L}{1-L}\right)^{\frac{1}{\gamma}}} - e^{\frac{2\beta H}{\gamma} \left(\frac{1+L}{1-L}\right)^{\frac{\gamma-1}{\gamma}}}}$$

## B. critical temperature, $T_c$

$$z = \left(\frac{1 + e^{2\beta(\epsilon+H)}z}{e^{2\beta\epsilon} + e^{2\beta H}z}\right)^{\gamma-1}$$

Setting  $H = 0$ , as critical temperature,  $T_c$ , for  $H \neq 0$  is taken to be that of  $H = 0$  case, one gets

$$z = \left(\frac{1 + e^{2\beta\epsilon}z}{e^{2\beta\epsilon} + z}\right)^{\gamma-1}$$

One writes this as

$$z = f(z)$$

where,

$$f(z) = \left(\frac{1 + e^{2\beta\epsilon}z}{e^{2\beta\epsilon} + z}\right)^{\gamma-1}$$

Obviously,  $z = 1$  is a solution of this equation.  $z \neq 1$  is also a solution for  $T < T_c$ . Moreover,  $\beta = \frac{1}{k_B T}$  where,  $k_B$  is Boltzmann constant. Again,

$$\frac{df(z)}{dz} = (\gamma - 1) \frac{(1 + e^{2\beta\epsilon}z)^{\gamma-2}}{(e^{2\beta\epsilon} + z)^\gamma} (e^{4\beta\epsilon} - 1)$$

Consequently,

$$\frac{df(z)}{dz} \Big|_{z=1} = (\gamma - 1) \frac{1}{(e^{2\beta\epsilon} + 1)^2} (e^{4\beta\epsilon} - 1)$$

Moreover, when

$$\frac{df(z)}{dz} \Big|_{z=1} > 1$$

$f(z)$  intersects the  $z = z$  line at  $z = 1$  and other two points. This is a different phase, occuring for  $T < T_c$ . The onset of phase transition, hence, is at

$$\frac{df(z)}{dz} \Big|_{z=1} = 1.$$

which implies after some algebra,

$$e^{2\beta_c\epsilon} = \frac{\gamma}{\gamma - 2},$$

which on taking natural logarithm of both sides, reduces to

$$2\beta_c\epsilon = \ln \frac{\gamma}{\gamma - 2},$$

which finally yields with the result of previous subsection,

$$\frac{2\beta_c\epsilon}{2\beta\epsilon} = \frac{\ln \frac{\gamma}{\gamma - 2}}{\ln \frac{1 - \frac{1+L}{1-L}}{e^{-\frac{2\beta H}{\gamma}} \left(\frac{1+L}{1-L}\right)^{\frac{1}{\gamma}} - e^{\frac{2\beta H}{\gamma}} \left(\frac{1+L}{1-L}\right)^{\frac{\gamma-1}{\gamma}}}}$$

i.e

$$\frac{T}{T_c} = \frac{\ln \frac{\gamma}{\gamma - 2}}{\ln \frac{\frac{1+L}{1-L} - 1}{e^{\frac{2\beta H}{\gamma}} \left(\frac{1+L}{1-L}\right)^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} \left(\frac{1+L}{1-L}\right)^{\frac{1}{\gamma}}}}.$$

. where,  $\beta = \frac{1}{k_B T}$  and  $k_B = 1.38 \times 10^{-23}$  Joule/Kelvin.

### XIII. ACKNOWLEDGEMENT

The author would like to thank nehu library for allowing the author to use Dorland's pocket Medical Dictionary,[11]. We have used gnuplot for plotting the figures in this paper.

### XIV. BIBLIOGRAPHY

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