A NOTE ON MANIFOLDS VS NETWORKS AS MATHEMATICS MODELS IN MODERN PHYSICS

LUCIAN M. IONESCU

Abstract. Some stages of development of Manifold Theory are inspected, and how they evolved into the modern discrete frameworks of lattice and spin networks, with help from Topology and Homological Algebra.

Recalling experimental evidence that reality *is* discrete, notably quantum Hall effect, includes more recent findings of quantum knots and spin-net condensates.

Thus Pythagoras, Zeno and Plato were right after all: "Number rules the Universe", perhaps explaining the "unreasonable effectiveness of mathematics", but not quite, why Quantum Physics' scattering amplitudes are often Number Theory's multiple zeta values.

Contents

1. Introduction

Riemann's foundations on geometry, culminating with their use by Einstein in his General Relativity[1], have clear limitations in view of Quantum Theories[2], evolving in parallel with Topology and Computer Science to a new version of Feynman Calculus: Topological Quantum Computing [1].

Not only more adequate then its predecessors, discrete mathematical objects with topological properties have been "hardware" implemented by experimentalists, e.g.

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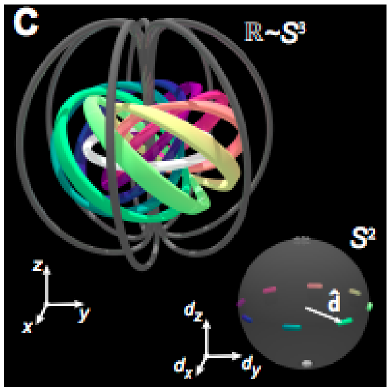
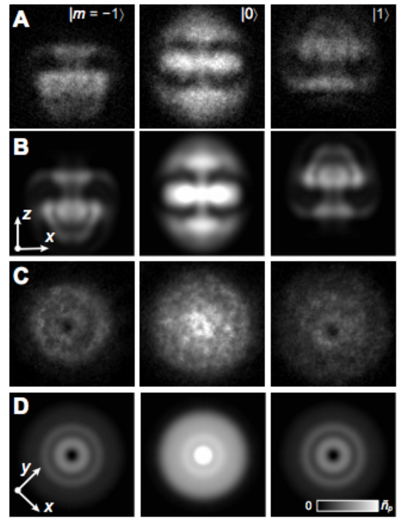
[1]With help from his classmate and friend, *mathematician* Marcel Grossman.

[2]Where we emphasize: "quantum" refers to, and has to do mainly with "discrete", not "uncertain" or "unpredictible", and interactions as links enabling multiple-conectedness, and the feedback essential in cybernetics and control: see weak measurements in Quantum Optics.



(A) Theory



(B) Experiment

FIGURE 1. Experimental creation of a theoretical soliton quantum knot [2], Fig.1 and Fig.3

"real-material" quantum knots [2] [3]. We reproduce Fig.1, loc. cit. p.1, explaining the "Structure of the soliton knot and the method of its creation", and the comparison with the experiment (Fig.3, loc. cit., p.2), as incentive for the reader's further explorations of the topic:

The visual similarity to atomic orbitals, hints to unity of interface, versus diversity of mathematical implementations, weather using the traditional Schrodinger differential equation, spherical harmonics and harmonic analysis, or the more "modern" (Wigner 1930's; Langlands Program 1960's; Quantum Groups 1980's) approach via representation theory.

This amazing "mathematical control" on reality is perhaps due to the reason that reality "is pixalated" after all [5], providing another clue regarding why Mathematics is so "unreasonably effective":

[3]Reminiscent of Abrikosov vortices and quantized space associated to quantum Hall effect [4].

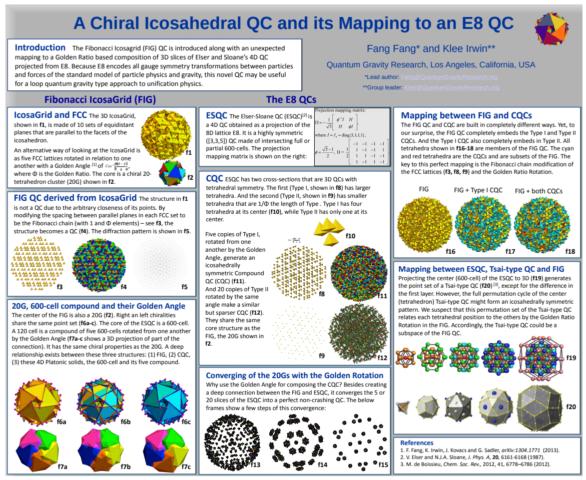


Figure 2. Quasi-crystlas as models for space-time, towards Quantum Gravity [6].

The trend marked by lattice models and spin networks (discrete / topological frameworks), evolved towards a more daring claim: even the fermion-boson quantum reality emerges from a *spin-net bosonic condensate*; is then reality, virtual? Perhaps, e.g.Elon Mask [7], S. Lloyd [12, 11, 9] etc.; why not a "dream", or even a "dream in a dream" ... but let's focus!

The new paradigm in science *is Quantum Information Processing* [10], which includes the *Master-Slave Duality*, an interface between Classical Computing and Quantum Computing/Physics. It is based on the theory of Flows on Networks, and may benefit from some general intuitive overview and projected prospects.

The article browses some foundational aspects of physics, and points towards important changes regarding the mathematical foundations of Quantum Physics, required by recent discoveries in *low energy physics*.

2. Historical Overview

An overview of the development of manifold theory progresses into the two modern directions for overcoming its limitations and request for a "quantum theory: discrete models and the elimination of "space" for geometry and "time" for dynamics.

Berhard Riemann's foundations on geometry, the birth of *Riemannian Geometry* made possible applications like General Relativity and later on, Gauge Theory on principal bundles and beyond [13, 14].

Discrete models on lattices and graphs, naturally import the classical foundation concepts, which are independent on the number system and local model used; on the other hand the symplectic approach balancing position and momenta, evolve

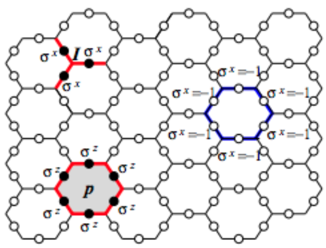


FIG. 2: The constraint term $\prod_{\text{legs of } I} \sigma_i^x$ and magnetic term $\prod_{\text{edges of } p} \sigma_j^z$ in Z_2 lattice gauge theory. In the dual picture, we regard the links with $\sigma^x = -1$ as being occupied by a string, and the links with $\sigma^x = +1$ as being unoccupied. The constraint term then requires the strings to be closed - as shown on the right.

A Z_2 lattice gauge theory dual too Ising model is the appropriate quantum computing array for an emergent "pixalated reality", like the colored image on a PC display (or TV).

FIGURE 3. String-net condensates: see [18] for details.

towards purely algebraic methods belonging to representation theory, as in the work of Weinberg, and beyond to the works of Reshetikhin-Turaev on invariants of knots and 3D-manifolds [15].

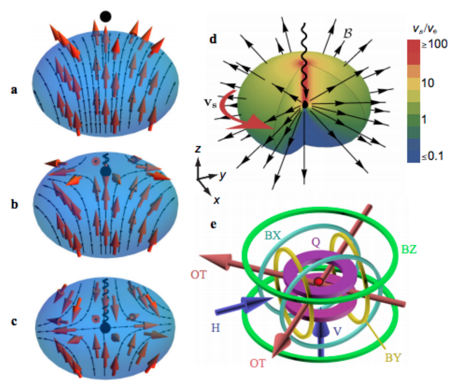
In hindsight, the drawbacks of the manifold approach to modeling dynamics are of two types: A) preferring "position" as a primary concept, and deriving "change", external as "motion", and internal as "quantum state" (e.g. particle decays etc.); and B) Imagining a continuum, based on (essentially) Cauchy sequences of approximations called "Real" numbers (see [16, 17]).

Regarding (A), the obvious alternative is the Network: places as "position markers" and arrows as "vectors". The obvious alternative to (B) is discrete models, i.e. *quantum*, e.g. lattice models, spin networks, graphical calculus in ribbon categories etc.

Now, regarding (A), are there "places" as points of space? Heisenberg's uncertainty relations really means a "No!"; it's not about how precise we can measure a supposedly "real space position".

And regarding (B), the world is "pixalated" [5], but nodes linked into a network, not necessarily a matrix, but rather with the structure of a cortex. This enables an obvious conceptual unification of fermions and bosons, as no longer aspects to be considered separately, as how they evolved historically until now.

This brings us to the notable progress away from a traditional "space-Time Geometry-Dynamics approach, namely the approach to an emergent dynamics from a "no motion/ properties flows" quantum computing perspective, which very recently is being proposed: from the present author Quantum Information Dynamics 2005 [9], to string-net condensation 2017 Xiao-Gang Wen [18], Fig. 2, p.3, reproduced here in Fig.3 (see loc. cit. for details).



The similarities with the creation of quantum knots are apparent. We are programming reality conform with a mathematics that does not work in the classical framework of a continuum space-time, even quantum, perhaps because reality is mathematical: quantum logic and quantum computing ("pixelated").

Figure 4. Monopoles creation: see [21] for details.

Modern experiments tend to confirm that Nature is using our theoretical solitons and knots at deeper levels, as in the quantum knots experimentally constructed, making Topological Quantum Computing a reality, as it will be described briefly next.

3. Monopoles, Anyons and Networks

What monopoles are and whether they may exist in principle, depends on the type of framework used. If allowing multiple connectedness in the underlying "Space", they are modeled based on topological aspects (Vector Bundles [13], Aharonov-Bohm Effect [19], Abrikosov vortices, a.k.a. fluxons [20] etc.).

Recently they have been identified in synthetic magnetic fields [21] (see Fig.4, and loc. cit. for details).

Other evidence of their need in modeling quantum phenomena is provided by Abrykosov vortices and quantum Hall effect; briefly, "Space is Quantized" [22] (or [23]: "pieces of Space-Time"), and there is no need for pointwise charges [24].

The more recent fractional quantum Hall effect confirms the natural development of Quantum Computing towards Topological Quantum Computing, based on the concept of anyon [1, 25].

The concepts of qubits and quantum registers have been complemented by *topological quantum memory* [26] which enables stability of quantum states and entanglement.

4. Emergence of Quantum Concepts of Classical Origin

Not only classical reality emerges from quantum world, but then later, modeled classically (still) as interacting particles, fermions and bosons, subject to "uncertainty" and "unpredictibility" (under *strong measurements*), emerge from an underlying substrate (core model), for example implemented as a lattice, called in [27] a *string-net* *condensate* [28]. Topological order [29] and topological entropy become foundational aspects [30].

This underlying "medium", also called *quantum ether* [27], from which photons and electrons emerge, is the modern quantum version of the old idea of an "ether", supporting the propagation of interactions (Electromagnetic waves).

Of course, one need not stop there: a "pixelated 3D-TV model" with qubits as baryons and ribbons as mesons, subject to a Turaev Graphical Calculus in Ribbon Categories, was proposed by the present author [42], to be developed [4].

5. Quantum Knots and String-Nets

By now quantum knots have been created in the lab [2], and the lattice models have evolved towards focusing on topological aspects (algebraic topology / homology theory methods), towards a background-free *Finite String Theory* based on *string-net condensates* [18], as anticipated in [47].

As explained in [2], *quantum knots* are topological stable objects within field theories, a useful technique, similar to embedding networks in space, graphs on Riemann surfaces, or even manifolds in a sufficiently high dimensional R^n. Experimentally obtained by controlling a superfluid demonstrates how theoretical objects like the Hopf fibration, essential in modeling monopoles, solitons and qubits, can be modeled as actual physical subsystems (see loc. cit. Fig. 2 and Fig. 3).

One essential feature is holistic character of a Bose-Einstein condensate, "a manifestation of macroscopic occupation of a single quantumstate" [3]. The condensation of part of the material into one entangled structure yields homological/homotopical preperties characteristic of an infotronic circuit within an underlying background lattice.

This exemplifies how quantum phenomena (fermion-bosonic physics) may emerge from a simple, programable (controlled) quantum computing array: an "artificial vacuum in condensed matter" [31] (See Summary).

6. Conclusions and Further Developments

The takeoff of Science, with Galileo and Newton, and understandably forgetting the Ancient Greeks heritage due to its scope - modeling large scale motion, was "served" well by Riemannian Differential Geometry and its extensions: from Einstein's General Relativity and Kaluza-Klein's inclusion of Electromagnetism, to Weyl's Gauge Theory, which still goes strong with the current Standard Model.

Yet, there is a time for change, and that is *now*, with the boom of Information Theory, classical and quantum, on one hand; the other: Nature can be rightfully and profitably thought of as Classical/Quantum Information Prosessing, when modeling systems, from Commercial Airlines to Elementary Particle "Decays".

[4]What the Buddhists call "The Wall"; not related to Pink Floyd's ... I guess?!?

As Heraclit said it long time ago: "Panta Rei"; together with the atomic postulate (including space-time according to Zeon), everything is *Flow on Networks*.

A confirmation of this is the going-on process of translating the "old" concepts and theories, from the continuum framework to a discrete, graph/network based framework: [32, 33, 34] etc. (just a too small sample!).

A hint that the modeling process has only to gain both conceptually, and computationally (continuum is usually not "tractable"), is the tight connection between low-dimensional topology and manifold theory [35, 15, 36]. For example, in 2D "fat" Feynman graphs (ribbon graphs) "are" Riemann Surfaces, or in 3D, manifolds can be constructed via Dehn surgery, Heegart splitting, rather playing the role of graphs in Topological Quantum "Field" Theory, CFT etc.

Just "upgrading" Elementary Particle Physics from quark line diagrams in the context of gauge theory of fields to Turaev Ribbon calculus / Topological Quantum Computing is a must, in order to address what mass is and why there is really only *one* interaction.

But will this account for / justify an emergent geometry and topology of 3-manifolds [35], plus of course enabling quantum tunneling and the "wierd" aspects of quantum phenomena, in a similar way quantization of classical systems yields quantum systems having as a classical limit their familiar "parents", behaving in a "normal", as expected way?

Bottom line, Heisenberg understood it before even classical computing (computer science) was well established: "there is no composite ..." The need for a holistic approach to Quantum Physics is evident from many perspectives [37]; for a more technical reason, atomic orbitals, as linear combinations of Slater determinants, are "quantum circuits", and can only be approximated using a particle oriented approach, in a classical framework of Hamiltonian or Lagrangian framework, even quantized once, twice, or more!

Indeed fermions and bosons are indistinguishable, i.e. form a network / circuit, with its "weird" properties of "which-way-did-it-go" because it has a homology; "elementary particles" really split at "crossings" (according to a parameter called "time"), the same way Computer Scientists found that it is better to split info on our classical WWW into packets sent on various rout. Mathematically, the coproduct is as essential in modern mathematical (algebraic) structures as the product; and physicists "new" that long time ago: creation and annihilation operators, Feynman diagrams etc. Invoking "wave-particle duality" is for the "public" ...

The time has come to incorporate the Particle Data on which the Standard Model is built in a new theory, beyond lattice models, with Quantum Computing at its foundations, as a language [38]. The history of theories in nuclear physics is instructive, together with the successes of Face-Centered-Cubical model, conforming that the nucleus behaves like a crystal [39].

Still, in such a lattice/network framework for quantum computing / physics, what remains to be quantized ("3rd quantization"), is the quantum phase: each subsystem has a "clock" defined by a certain frequency. Revisiting the foundations, the unit of quantum information, the qubit $SU(2)$, needs to be a discrete structure [42]; the "coincidence" between scattering amplitudes and arithmetic (Number Theory) / algebraic periods (Algebraic Geometry) [40] has such a much deeper reason, to be discovered next.

But then, "Why Is M-Theory the Leading Candidate for Theory of Everything?" [46] Well, because it's well funded; otherwise, by far, it's not *The Way*! [5]

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Department of Mathematics, Illinois State University, IL 61790-4520
E-mail address: `lmiones@ilstu.edu`