The Dirac-Ricci operator

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Abstract

We define a Dirac type operators called the Dirac-Ricci operator with help of the Ricci curvature.

1 The Dirac operator

Let (M,g) be a spin manifold, then we can define the Dirac operator D with help of the Levi-Civita connection ∇ [F].

$$D(\psi)=\sum_i e_i.\nabla_{e_i}(\psi)$$

(e_i) is an orthonormal basis.

$$D=\mu\circ\nabla$$

with μ the Clifford multiplication.

2 The Dirac-Ricci operator

The Dirac-Ricci operator can be defined by the following formula:

$$DR(\psi)=\sum_i Ric(e_i).\nabla_{e_i}(\psi)$$

Ric is the Ricci curvature [GHL] viewed as an endomorphism of the tangent bundle.

$$DR=\mu\circ(Ric\otimes 1)\circ\nabla$$

If M is an Einstein manifold [Be] then the Dirac-Ricci operator is reduced to the Dirac operator.

3 The Dirac-Lichnerowicz-Ricci formula

The Dirac-Ricci operator is symmetric with respect of the spinor product and we have the Dirac-Lichnerowicz-Ricci formula:

$$DR^2=-\Delta_R+\alpha$$

with:

$$\Delta_R = \sum_{i,j} g(Ric(e_i), Ric(e_j))\nabla_{e_i}\nabla_{e_j}$$

and α is a lower term; it is a scalar if $\nabla Ric = 0$.

References

[Be] A.Besse, "Einstein Manifolds", Springer Verlag, Berlin, 1987.

[F] T.Friedrich, "Dirac Operators in Riemannian Geometry", vol 25, AMS, 2000.

[GHL] S.Gallot, D.Hulin, J.Lafontaine, "Riemannian geometry", 3ed., Springer, Berlin, 2004.